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Four-quark systems*

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In order to understand the binding of four static quarks, flux distributions corresponding to these binding energies are studied in quenched SU(2) and also related to a model for the energies. The potential relevant to string breaking between two heavy-light mesons is measured in quenched SU(3) using stochastic estimates of light quark propagators with the Sheikholeslami-Wohlert action.

Our goal is to understand multi-quark interactions from first principles. Since "multi-quark" means that the system can be decomposed into more than one color singlet, the simplest such system consists of four quarks. In this work we try to understand this simple case, relevant for meson-meson interactions, hoping that generalization to more complicated cases will be relatively straightforward.

We first give a brief introduction to a model that reproduces one hundred ground- and excited state energies of four static quarks [1]. These energies have been calculated earlier in SU(2) gauge theory on $\approx 20^3 \times 32$ lattices with $a \approx 0.1$ fm – a continuum extrapolation has been performed [2]. The flux distribution producing four-quark binding has been measured [3] and we attempt to relate it to our model.

Preliminary results are reported from a more realistic calculation concerning the potential between two heavy-light mesons in quenched SU(3). The heavy quarks are taken to be static and the light quark propagators are calculated using stochastic estimators with an $O(a)$ improved fermion action. This work also concerns string breaking.

1. The model

We like to think of the simplest multi-quark system, consisting of four quarks, as a bridge between a single hadron and interacting multi-hadron systems. Earlier, simulations of static four-quark systems have been performed for a general set of geometries [2,4, references therein]. These quenched SU(2) calculations have produced binding energies up to 120 MeV. In order to understand these results a phenomenological model based on two-body potentials and multi-quark interaction terms has been developed.

The basis states of the model come from the three possible ways to pair four quarks into two mesons in SU(2). These pairing are taken to be in the ground state (A, B, C) or first excited state (A^*, B^*, C^*) of the two-body potential, giving six states $(A, \ldots, C^*)$ in total. The basic equation of the model is

$$[\mathbf{V} - E(4)\mathbf{N}] = 0, \tag{1}$$

where $\mathbf{N}$ contains overlaps of any two states $(N_{AB} = \langle A|B\rangle, \ldots)$, $\mathbf{V}$ has the interactions $(V_{AB} = \langle A|V|B\rangle, \ldots)$ and $E(4)$ is the four-quark energy. The binding energy is defined as $B(4) = E(4) - \min(V_{AA}, V_{BB}, V_{CC})$, i.e. as the difference of the four-quark energy and the energy of the lowest-lying two-body pairing.

A central element in the model is a multi-quark interaction term f, defined as $\langle A|B\rangle = \langle B|C\rangle = \langle A|C\rangle = -f/2$. For small distances $f = 1$ and for large $f = 0$. The potentials V have a perturbative one-gluon exchange prefactor and lattice two-body potentials v_{ij}, giving the form $V_{ij} = -\frac{1}{3}\sum_{i \leq j} \vec{\tau}_i \cdot \vec{\tau}_j v_{ij}$. This expression alone would lead to unphysical van der Waals forces, which are removed by the f. The interaction between basis states A and B is thus $V_{AB} = -\frac{f}{2}(V_{AA} + V_{BB} - V_{CC})$.

The factor f can be parameterized as $f = \exp(-b_s k_f S)$, where b_s is the string tension and S

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the minimum area bounded by the four quarks. A simple version of the model with just states A, B and $f = 1$ is reproduced by perturbation theory to $O(g^4)$[5].

For quarks at the corners of a regular tetrahedron binding increases with size, which suggests two-body excitations are relevant, as they are closer to the ground state for larger distances. Therefore, we have included excited basis states in our model. This gives new f-factors $\langle A^*|B^*\rangle = \ldots = -f^c/2$, $\langle A^*|B\rangle = \ldots = -f^a/2$, the former of which is parametrized like f and the latter as $f^a = b_S f^a_S S \exp(-b_s k_a S)$. The interaction matrix elements involving two-body excitations are parameterized in a manner similar to the ground state matrix elements, e.g. $\langle A^*|V|B^*\rangle = -\frac{f^c}{2}[V_{AA} + V_{BB} - V_{CC} + c_0(V^*_{AA} + V^*_{BB} - V^*_{CC})]$.

When this model is fitted to 100 ground and excited state four-quark energies it turns out that f_c is always consistent with one, suggesting that two-body excitations interact in a perturbative manner. We are left with four independent parameters $k_f = 1.5(1)$, $k_a = 0.55(3)$, $f^a_S = 0.51(2)$, $c_0 = 3.9(3)$ that fit the energies with $\chi^2/\text{d.o.f.} \approx 1$. The values of these parameters are stable when energies from smaller lattice spacings are used. If the excited basis states are left out we get $\chi^2 \approx 3.2$, which mostly comes from the maximally degenerate regular tetrahedra.

2. Four-quark flux distribution

The flux distribution is measured in quenched SU(2) using

$$f^{\mu\nu}_R(\mathbf{r}) = \left[\frac{\langle W(R,T)\Box^{\mu\nu}_{\mathbf{r}}\rangle - \langle W(R,T)\rangle\langle\Box^{\mu\nu}\rangle}{\langle W(R,T)\rangle}\right],$$

where $W(R,T)$ is the Wilson loop, or more generally, a combination of Wilson loops corresponding to the ground or excited state of the multi-quark system. Since extracting a signal is not easy we use link-integration on the Wilson loops. As in the calculation of energies, we utilize a variational approach in order to extract the ground or an excited state of the system. The variational basis is formed of Wilson loops with various levels of fuzzing.

We can check the accuracy of our flux measurement with lattice sum rules. These sum rules relate spatial sums over fields to energies via β-functions such as $b \equiv \partial g/\partial \ln a = 2(S+U)$, e.g.

$$E + E_0 = \sum[S(\mathcal{E}_x + \mathcal{E}_y + \mathcal{E}_z) + U(\mathcal{B}_z + \mathcal{B}_y + \mathcal{B}_x)].$$

Using our earlier two-quark results [6] for S, U this relation allows us to compare the sum over the distribution to a much more accurately measured energy. This is especially convenient when the binding is considered; E_0 is a lattice artefact from quark self-energies, and is removed for the binding energy.

Three-dimensional pictures of our results (Fig. 1) can be viewed at `www.physics.helsinki.fi/~ppennane/pics/`. The binding distribution is found to have approximately constant height in between the quarks, whereas the first excited state has a "cloverleaf" symmetry with sign changes. Replacing energies with flux distributions in our model without excitated basis states gives $k_f < 1$, which agrees with fits to energies [3]. When excited basis states are included the ground-state interaction has shorter range; $k_f > 1$, suggesting that ground state potentials are important at small distances and excited ones at larger distances.

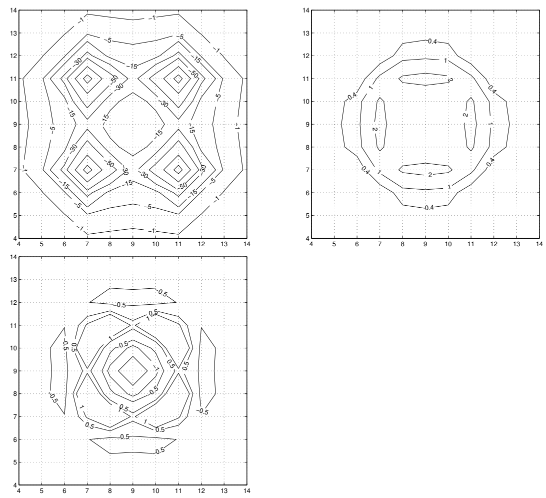

Figure 1. The distribution of energy around four static quarks in a square with side $R = 4$ corresponding to a) total energy b) binding energy c) first excited state binding energy.

3. Two heavy-light mesons

We calculate the potential between two heavy-light mesons as a function of the heavy quark separation R in quenched SU(3) at $\beta = 5.7$. The heavy quarks are taken to be static and the light ones have approximately the s mass ($M_{\rm PS}/M_{\rm V} = 0.65$) with the propagators obtained as stochastic pseudofermionic estimates with maximal variance reduction [7]. The principal advantage of the stochastic estimates is the ability to get propagators to and from most points on the lattice; hence the nickname "all-to-all".

At $R = 0$ the meson-antimeson ($B\bar{B}$) system should have a mass similar to a pion, while the BB system should resemble a heavy-light baryon; the former is indeed observed. The preliminary results in Fig. 2 do not show the one-π exchange effect of deuson models which leads to differences in the BB and $B\bar{B}$ potentials at large distances. As expected, at small distances the $B\bar{B}$ potential is more attractive.

As discussed in other papers in these proceedings [8], the heavy-light meson-antimeson system is relevant to string breaking. Unfortunately calculations of this system at large enough distances are very hard, but the all-to-all propagator scheme offers a way to overcome the difficulties. In our case the $B\bar{B}$ and static $Q\bar{Q}$ energies become equal at $R \approx 1.3$ fm [9]; this is reached by our ongoing calculations on a $16^3 \times 24$ lattice. A variational operator mixing analysis [10] similar to that presented by Philipsen in a Higgs model is then performed. By running our code on UKQCD unquenched configurations we hope to shed light on the differences due to sea quarks in the mixing region.

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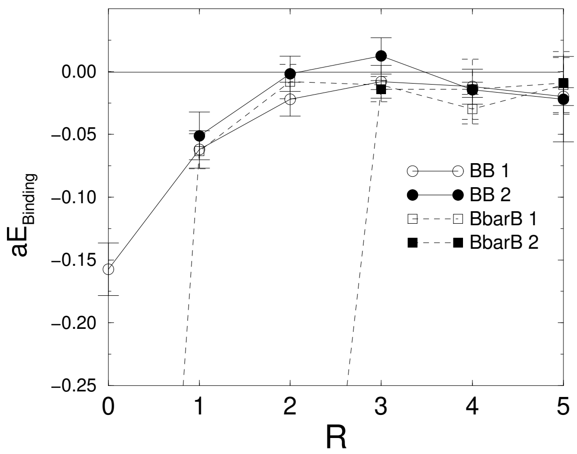

Figure 2. Preliminary results for potential between two B mesons or a B and $\bar{B}$ – the data marked with a "2" includes light-quark exchange. Here $aM_B = 0.863(1)$.

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