LTH-424

Anomalous dimension of gluonic operator in polarized deep inelastic scattering at $O(1/N_f)$

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Abstract. We compute the $O(1/N_f)$ correction to the predominantly gluonic flavour singlet twist-2 anomalous dimension used in polarized deep inelastic scattering. It is consistent with known two loop perturbation theory and we determine the three loop contribution at $O(1/N_f)$. The treatment of the ϵ-tensor in the large N_f d-dimensional critical point formalism is also discussed.

There has been intense experimental activity in recent years into understanding the origin of the spin of the nucleons. This is primarily because of the EMC effect, [1], which appeared to suggest that the proton spin was due to a sea quark effect with no significant contribution from the valence quarks. Consequently there has been theoretical interest in the problem. For example, one avenue of research has been to improve theoretical predictions from perturbative QCD by going to higher loop orders, [2]. A problem related to this concerns the construction of the perturbative sector of the operator product expansion used in polarized deep inelastic scattering which is important, for example, for current and future HERA experiments. Unlike the unpolarized sector, polarized calculations of twist-2 operator dimensions have only recently become available at two loops, [3, 4], as a function of the operator moment, n, or equivalently as a function of x, the conjugate variable of the Mellin transform in the DGLAP formalism of Dokshitzer, Gribov, Lipatov, Altarelli and Parisi, [5, 6]. Although the two loop results of [3, 4] represent progress for polarized scattering, building on the one loop results of [7], there is clearly a need to push calculations as far as is possible. Indeed for unpolarized scattering the relevant twist-2 operator dimensions are available exactly at three loops for the even moments, $n \leq 8$, [8] (and additionally $n = 10$ for the flavour non-singlet case, [9]). Although the explicit n-dependence of the three loop result is the main goal, since it is needed to carry out the full two loop renormalization group evolution, some progress has been made in that direction already. Recently, the finite parts of the two loop graphs have been determined in [10] as a function of x. These represent the first stage of a full three loop calculation since they will contribute to the final value when multiplied by one loop counterterms. Further, some information is available from the large N_f expansion where the $1/N_f$ leading order non-singlet anomalous dimensions have been deduced to all orders in the coupling, explicitly as a function of n, [11], with N_f the number of quark flavours. (The method used is based on a critical point argument where one considers the theory in the neighbourhood of a d-dimensional non-trivial zero of the β-function and computes critical exponents. The initial development was for the $O(N)$ σ model in [12]. Through the critical renormalization group equation these exponents are related to the perturbative renormalization group functions.) The restriction of [11] to compare with the exact three loop non-singlet results of [9] gave agreement where there is overlap. In anticipation of a full polarized calculation at three loops the large N_f unpolarized calculations have been extended in [13] to the polarized case. However, in the calculation of [13] only the dimension of the twist-2 flavour singlet operator which was predominantly fermionic in nature, was considered. The aim of this article is to complete the gap in the twist-2 singlet polarized sector by computing the first non-trivial large N_f contribution to the flavour singlet operator which is gluonic in character. In providing this result we will not only be able to check the recent two loop results but also be able to provide the *first* insight into the n-dependence of the three loop dimension, albeit at $O(1/N_f)$. As the calculation of the unpolarized singlet gluonic operators was treated at length in [14], we will be brief concerning the technical details of the integration and focus on the salient differences which arise in the polarized case.

We recall that the basic twist-2 flavour singlet polarized operators are, [7],

$$\begin{aligned} \mathcal{O}_q^{\mu_1\ldots\mu_n} &= i^{n-1}\mathcal{S}\bar{\psi}\gamma^5\gamma^{\mu_1}D^{\mu_2}\ldots D^{\mu_n}\psi ~-~ \text{trace terms} \\ \mathcal{O}_g^{\mu_1\ldots\mu_n} &= \tfrac{1}{2}i^{n-2}\mathcal{S}\epsilon^{\mu_1\alpha\beta\gamma}\,\mathrm{tr}\,G^a_{\beta\gamma}D^{\mu_2}\ldots D^{\mu_{n-1}}G^{a\,\mu_n}{}_\alpha ~-~ \text{trace terms} \end{aligned} \quad (1)$$

where ψ is the quark field, $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$, A^a_μ is the gluon field, f^{abc} are the colour group structure functions, $\mathcal{S}$ denotes symmetrization over the Lorentz indices and $\epsilon_{\mu\nu\sigma\rho}$ is a totally antisymmetric four dimensional pseudotensor. As has been observed in [7], since $\mathcal{O}_q$ and $\mathcal{O}_g$ have the same dimension, they will mix under renormalization in perturbation theory. This leads to a mixing matrix of renormalization constants and hence anomalous dimensions which we denote by

$$\gamma_{ij}(a) \;=\; \left(\begin{array}{cc} \gamma_{qq}(a) & \gamma_{gq}(a) \\ \gamma_{qg}(a) & \gamma_{gg}(a) \end{array} \right) \tag{2}$$

where we use the same definition for the coupling constant, a, as [8]. To three loops the explicit N_f dependence of each of the entries will be

$$\begin{aligned} \gamma_{qq}(a) &= a_1 a + (a_{21}\tilde{N}_f + a_{22})a^2 + (a_{31}\tilde{N}_f^2 + a_{32}\tilde{N}_f + a_{33})a^3 + O(a^4) \\ \gamma_{gq}(a) &= b_1 a + (b_{21}\tilde{N}_f + b_{22})a^2 + (b_{31}\tilde{N}_f^2 + b_{32}\tilde{N}_f + b_{33})a^3 + O(a^4) \\ \gamma_{qg}(a) &= c_1 \tilde{N}_f a + c_2\tilde{N}_f a^2 + (c_{31}\tilde{N}_f^2 + c_{32}\tilde{N}_f + c_{33})a^3 + O(a^4) \\ \gamma_{gg}(a) &= (d_{11}\tilde{N}_f + d_{12})a + (d_{21}\tilde{N}_f + d_{22})a^2 + (d_{31}\tilde{N}_f^2 + d_{32}\tilde{N}_f + d_{33})a^3 + O(a^4) \end{aligned} \tag{3}$$

where $\tilde{N}_f = T(R)N_f$ and the coefficients a_{ij}, b_{ij}, c_{ij} and d_{ij} will depend on n and the colour group Casimirs. This form will be important when we apply the critical point formalism to deduce the $O(1/N_f)$ anomalous dimensions. Moreover, for concreteness and to clarify the conventions we use, we recall the one loop polarized results of Ahmed and Ross are, [7],

$$\begin{aligned} a_1 &= C_2(R)\left[4S_1(n) \;-\; 3 \;-\; \frac{2}{n(n+1)}\right] \quad , \quad b_1 \;=\; -\;\frac{2(n+2)C_2(R)}{n(n+1)} \\ c_1 &= -\;\frac{4(n-1)T(R)}{n(n+1)} \quad , \quad d_{11} \;=\; \frac{4}{3}T(R) \end{aligned} \tag{4}$$

where the finite sums $S_l(n)$ are defined by $S_l(n) = \sum_{r=1}^{n} 1/r^l$. Although we have indicated the form of the mixing matrix in (2), one could in principle consider a basis where it is diagonal. Indeed it turns out that in the $1/N_f$ formalism of [13] that that method produces the anomalous dimensions of the twist-2 eigenoperators of (2). This is primarily due to the fact that the canonical dimensions of the operators of QCD in perturbation theory are the same, whereas computing at the fixed point of QCD in d-dimensional spacetime, where the large N_f expansion is constructed, the canonical dimensions are different. Indeed it is elementary to deduce this difference is $O(\epsilon)$ where $d = 4 - 2\epsilon$. Further, it was shown in [13] that the critical exponent of the operator $\mathcal{O}_f$ corresponded to the eigen-anomalous dimension of (1) with a predominantly fermionic content evaluated at the non-trivial zero of the β-function. Likewise in [14], it was demonstrated that the remaining operator corresponded to the predominantly gluonic operator. More concretely, diagonalizing (2) produces the eigen-anomalous dimensions

$$\lambda_\pm(a) \;=\; \frac{1}{2}(\gamma_{qq} + \gamma_{gg}) \;\pm\; \frac{1}{2}\left[(\gamma_{qq} - \gamma_{gg})^2 + 4\gamma_{qg}\gamma_{gq}\right]^{\frac{1}{2}} \tag{5}$$

The critical value of the QCD coupling has been given before in [14] and is

$$\begin{aligned} a_c &= \frac{3\epsilon}{4T(R)N_f} \;+\; \left[\frac{33}{16}C_2(G)\epsilon \;-\; \left(\frac{27}{16}C_2(R) + \frac{45}{16}C_2(G)\right)\epsilon^2 \right. \\ &\quad \left. +\; \left(\frac{99}{64}C_2(R) + \frac{237}{128}C_2(G)\right)\epsilon^3 \;+\; \left(\frac{77}{64}C_2(R) + \frac{53}{128}C_2(G)\right)\epsilon^4\right]\frac{1}{T^2(R)N_f^2} \end{aligned} \tag{6}$$

where we have included the contribution from the recently computed four loop β-function, [15]. Therefore substituting (6) in (5) and expanding in powers of $1/N_f$, we find that the gluonic dimension to third order in ϵ is given by

$$\begin{aligned} \lambda_+(a_c) &= \frac{3d_{11}\epsilon}{4} \;+\; \frac{1}{T(R)N_f}\left[\frac{3}{4}\left(d_{12} + \frac{b_1c_1}{d_{11}} + \frac{11}{4}C_2(G)d_{11}\right)\epsilon \right. \\ &\quad +\; \frac{9}{16}\left(d_{21} + \frac{b_{21}c_1}{d_{11}} - (3C_2(R) + 5C_2(G))\,d_{11}\right)\epsilon^2 \\ &\quad \left. +\; \frac{27}{64}\left(d_{31} + \frac{b_{31}c_1}{d_{11}} + \left(\frac{11}{3}C_2(R) + \frac{79}{18}C_2(G)\right)d_{11}\right)\epsilon^3\right] \;+\; O\left(\frac{\epsilon^4}{N_f^2}\right) \end{aligned} \tag{7}$$

To reproduce this at $O(\epsilon^2)$ together with the subsequent terms at $O(1/N_f)$ one applies the critical point formalism to the operator $\mathcal{O}_g$ which exploits the fixed point equivalence of QCD with the non-abelian Thirring model, [16], at leading order in $1/N_f$. As the method has been well documented, [14], we will comment only on those technical details of the computation which are peculiar to $\mathcal{O}_g$. Indeed as the calculation virtually parallels that of the unpolarized case we refer the interested reader to [14]. There details of the integration procedure for the Feynman diagrams have been recorded, including the construction and solution of the recurrence relations for deducing the contributions from subintegrals in the graphs. These have been implemented in the symbolic manipulation language FORM, [17], which was used to perform the tedious algebra in the integration. The four contributing Feynman graphs for the $O(1/N_f)$ part of (7) are illustrated in fig. 1, where the operator insertion is indicated on an internal gluon line. We have used the same Feynman rule for the insertion and conventions as [3].

One technical issue deserves detailed comment. As has been noted elsewhere, [18], one has to deal with the problem of treating an object, such as $\epsilon_{\mu\nu\sigma\rho}$, which is defined only in four dimensions, in a formalism which is manifestly d-dimensional. Such a situation arises not only in large N_f calculations but also in explicit perturbative multiloop computations. Its resolution in the latter case is to treat products of two ϵ-tensors, where they arise, as the determinant of a 4×4 matrix of $\eta_{\mu\nu}$ tensors, [19]. As the latter can have a d-dimensional interpretation one can perform a calculation without reference to the original ϵ-tensors. To compensate for the obvious discrepancy and to restore the Ward identity, one introduces an additional (finite) renormalization constant, [20]. This approach was developed for the treatment of the renormalization of the singlet axial vector current which has an anomaly, [20, 21]. A similar approach is adopted for $1/N_f$ calculations, [22], which has already been applied to the calculation of the exponent corresponding to the anomalous dimension of the axial vector current, [13]. For each graph of the present calculation, we project out the contribution to the exponent by multiplying by $\epsilon_{\mu\nu\sigma\rho}\Delta^\sigma p^\rho$, where Δ^μ is a null 4-vector used to define the Feynman rules on the light cone and p^μ is the external momentum flowing through each diagram. Therefore each integral contains the product of two ϵ-tensors and like [20] we replace these by a determinant of $\eta_{\mu\nu}$ tensors and compute in d-dimensions. However, we also need to introduce a compensating factor since not only is $\epsilon_{\mu\nu\sigma\rho}$ four dimensional but the vectors Δ^μ and p^μ are strictly physical objects and cannot be extended to d-dimensions, [18]. In the split γ-algebra notation of [18], $\Delta^\mu \equiv \Delta^{\bar{\mu}}$ and $p^\mu \equiv p^{\bar{\mu}}$ where the bar denotes the four dimensional subspace of the d-dimensional extended spacetime, which corresponds to the physical spacetime in the limit as $d \to 4$. To summarize we have replaced the correct ϵ-tensor combination given on the right side of

$$\epsilon_{\mu\nu\sigma\rho}\epsilon_{\alpha\beta\gamma\delta}\Delta^\sigma p^\rho \Delta^\gamma p^\delta \;=\; f(\mu)\epsilon_{\mu\nu\bar{\sigma}\bar{\rho}}\epsilon_{\alpha\beta\bar{\gamma}\bar{\delta}}\Delta^{\bar{\sigma}} p^{\bar{\rho}} \Delta^{\bar{\gamma}} p^{\bar{\delta}} \tag{8}$$

by the object on the left side which is formally d-dimensional and involves the η-determinant. The discrepancy in this procedure is represented by a compensating function of the spacetime dimension. We have computed this proportionality function, $f(\mu)$, by considering contractions of the d-dimensional object and comparing it with similar contractions of the object restricted to 4-dimensional spacetime. We found

$$f(\mu) \;=\; (2\mu - 3)(\mu - 1) \tag{9}$$

To check that this is a correct procedure we computed the first graph of fig. 1 by using the determinant substitution approach and then compared the result with that obtained by direct calculation of the integral. Such a computation is possible in this case since the graph is in effect a series of chain integrals in the language of [12]. It turns out that by inspecting each of the other graphs a combination which is effectively the same as (8) arises and therefore (9) is used for each of the graphs. Moreover, this argument was also used to justify the inclusion of a similar factor in the large N_f calculation of the anomalous dimension of the singlet axial vector current, [13]. The final expression for that case was consistent with three loop perturbation theory, [21, 19].

In light of these remarks, we can now quote the final value we obtained for $\lambda_{+,1}(a_c)$ where $\lambda_+(a_c) = \sum_{i=0}^\infty \lambda_{+,i}(a_c)/N_f^i$. After evaluating the contribution from each graph we find

$$
\begin{aligned}
\lambda_{+,1}(a_c) \;=\; & - \frac{(n+2)(n-1)\Gamma(n+2-\mu)\Gamma(\mu+1)C_2(R)\eta_1^{\mathrm{O}}}{(\mu-2)^2(n+1)\Gamma(2-\mu)\Gamma(\mu+n)nT(R)} \\
& + \frac{2\mu(\mu-1)S_1(n)C_2(G)\eta_1^{\mathrm{O}}}{(2\mu-1)(\mu-2)T(R)} \\
& - [4\mu^3n^2 + 4\mu^3n - 8\mu^3 - 8\mu^2n^2 - 8\mu^2n + 16\mu^2 + 5\mu n^2 \\
& \quad + 5\mu n - 9\mu - n^2 - n + 2]\Gamma(n+2-\mu)\Gamma(\mu)\mu C_2(G)\eta_1^{\mathrm{O}} \\
& \quad /[8(2\mu-1)(\mu-1)^3(n+1)\Gamma(3-\mu)\Gamma(\mu+n)nT(R)] \\
& - [32\mu^4 - 4\mu^3n^2 - 4\mu^3n - 120\mu^3 + 16\mu^2n^2 + 16\mu^2n \\
& \quad + 160\mu^2 - 20\mu n^2 - 20\mu n - 89\mu + 8n^2 + 8n + 18]\mu C_2(G)\eta_1^{\mathrm{O}} \\
& \quad /[8(2\mu-1)(\mu-1)^3(\mu-2)(n+1)nT(R)]
\end{aligned}
\tag{10}
$$

where we have used REDUCE, [23], to handle tedious amounts of algebra. The quantity η_1^{O} is defined by

$$
\eta_1^{\mathrm{O}} = \frac{(2\mu-1)(\mu-2)\Gamma(2\mu)}{4\Gamma^2(\mu)\Gamma(\mu+1)\Gamma(2-\mu)}
\tag{11}
$$

There are various checks on the validity of (10). First, we find that expanding (10) in powers of ϵ it agrees with two loop perturbation theory, [7, 3, 4]. From one point of view this justifies the use of (8), since (9) is ϵ-dependent and therefore will affect the form of the $O(\epsilon^2)$ terms of (10) when it is expanded. Second, if one substitutes $n = 1$ in (10), we find that it vanishes identically. This is consistent with the observations of [24, 25], where it was shown that the axial anomaly forces the $n = 1$ gluonic operator to have vanishing anomalous dimension. Therefore we are confident that (10) is correct and can make an observation on the form of the 3-loop contribution to the gluonic eigen-anomalous dimension. By expanding (10) to $O(\epsilon^3)$ and using (7), we find that

$$
\begin{aligned}
d_{31} + \frac{b_{31}c_1}{d_{11}} \;=\; & - \frac{64(7n^2+7n+3)(n+2)(n-1)S_1(n)C_2(R)}{9(n+1)^3n^3} \\
& + \frac{64(n+2)(n-1)S_1^2(n)C_2(R)}{3(n+1)^2n^2} \\
& - 4[33n^8 + 132n^7 + 142n^6 - 36n^5 - 263n^4 - 312n^3 \\
& \quad + 280n^2 + 408n + 144]C_2(R)/[27(n+1)^4n^4] \\
& - \frac{8(8n^4+16n^3-19n^2-27n+48)S_1(n)C_2(G)}{27(n+1)^2n^2} \\
& - \frac{2(87n^6+261n^5+249n^4+63n^3-76n^2-64n-96)C_2(G)}{27(n+1)^3n^3}
\end{aligned}
\tag{12}
$$

Evaluating this at $n = 1$, one obtains a value which, from (7), is correctly proportional to the three loop coefficient of a_c. As an aid to future explicit perturbative calculations similar to [8], we have evaluated (12) for odd n, $n \leq 23$, and listed the results in table 1. Unlike in the unpolarized calculations, here it is the odd moments which are important since the original operator does not exist when n is even as can be seen, for example, from the Feynman rules, [3]. A further remark on (12) concerns the status of its $C_2(G)$ sector. In the unpolarized case, [14], it was noted that at $O(1/N_f)$ b_{31} will only involve contributions proportional to $C_2(R)$ and not $C_2(G)$. Assuming the situation is similar here, this implies that the terms involving $C_2(G)$ in (12) are in fact the correct n-dependent part of the coefficient set d_{31} of the original perturbative operator. Finally, several general remarks on the overall structure are in order. First, the full expression is much simpler in form than that of the unpolarized case, [14]. Indeed this was also observed in the fermionic singlet sector as well. Second, in the evaluation of each integral an extra n-dependent structure was present which was proportional to $\Gamma(\mu-1)\Gamma(n)/\Gamma(\mu+n)$. This is similar to the unpolarized situation, [14], and like in that calculation, it also cancels in the final summation.

To conclude with we note that all the $1/N_f$ leading order terms are now available for the anomalous dimensions of the twist-2 operators used in polarized deep inelastic scattering. The next stage in this $1/N_f$ programme would be to consider the unpolarized flavour non-singlet sector and compute the $O(1/N_f^2)$ correction to the dimension of the twist-2 operator as a function of n.

Acknowledgements. This work was carried out with the support of PPARC through a Postgraduate Studentship (JFB) and an Advanced Fellowship (JAG).

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n	$C_2(R)$ coefficient	$C_2(G)$ coefficient
1	$-\frac{44}{9}$	$-\frac{158}{27}$
3	$-\frac{5527}{972}$	$-\frac{2342}{243}$
5	$-\frac{716824}{151875}$	$-\frac{341683}{30375}$
7	$-\frac{155752763}{34574400}$	$-\frac{4524703}{370440}$
9	$-\frac{2698556278}{602791875}$	$-\frac{29607251}{2296350}$
11	$-\frac{282328566167}{62758939320}$	$-\frac{1822332833}{135841860}$
13	$-\frac{5077270251071}{1120171987935}$	$-\frac{278760143099}{20142952830}$
15	$-\frac{915045986064491}{200354394240000}$	$-\frac{16575124393}{1167566400}$
17	$-\frac{189383384467690249}{41180834365746075}$	$-\frac{9360556077617}{645328543860}$
19	$-\frac{4489933300070894563}{970409113734060000}$	$-\frac{10746754033373}{727252911000}$
21	$-\frac{934060610086114519}{200816623282959684}$	$-\frac{3109748568599}{207034063236}$
23	$-\frac{128838538840521478537}{27573179435993423232}$	$-\frac{77700727191509}{5098296739536}$

Table 1. Coefficients of $[d_{31} \ + \ b_{31} c_1 / d_{11}]$ as a function of moment.

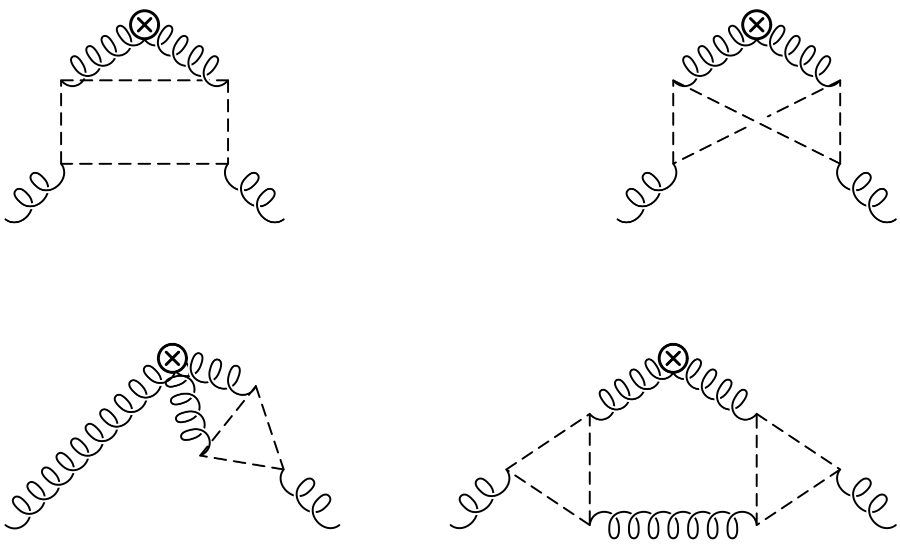

Fig. 1. Leading order diagrams for $\lambda_+(a_c)$.