LTH 423
hep-ph/9803405

The soft scalar mass β-function

I. Jack, D.R.T. Jones and A. Pickering

Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

We present an exact formula for the β-function for the soft-breaking scalar mass in an $N = 1$ supersymmetric gauge theory, in the form of an operator acting on the anomalous dimension. In particular we give the exact form for the correction due to the ϵ-scalar mass, and show that it has a particularly simple form in the renormalisation scheme corresponding to the exact NSVZ gauge β-function.

March 1998

It has been known for some time that the gauge β-function β_g for a supersymmetric theory can be expressed (in a suitable renormalisation scheme) in terms of the anomalous dimension matrix γ of the chiral superfields, according to the so-called NSVZ formula[1]. Moreover, for the case of a softly-broken theory, there has been considerable progress [2]–[7] in writing exact expressions for the β-functions for the soft-breaking parameters in terms of γ. (The methods of Ref. [3], for incorporating soft-breaking parameters by replacing couplings by superfields, have been extended and used in applications to Beyond the Standard Model physics, in Refs. [7] and [8].) In this paper we discuss the form for the soft-breaking $\phi^*\phi$ mass β-function; in general this contains a contribution which, at least when using DRED (supersymmetric dimensional regularisation with minimal subtraction), is related to the ϵ-scalar mass renormalisation. At present this contribution has been calculated explicitly only at lowest order in perturbation theory. Here we propose an exact form for it and present compelling evidence (we believe) in favour of our result.

For a $N = 1$ supersymmetric gauge theory with superpotential

$$W(\Phi) = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j, \tag{1}$$

we take the soft breaking Lagrangian L_{SB} as follows:

$$\begin{aligned} L_{SB}(\Phi, W_A) = &-\left\{\int d^2\theta\eta\left(\frac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}b^{ij}\Phi_i\Phi_j + \frac{1}{2}MW_A{}^{\alpha}W_{A\alpha}\right) + \text{h.c.}\right\} \\ &- \int d^4\theta\bar{\eta}\eta\bar{\Phi}^j(m^2)^i{}_j(e^{2gV})_i{}^k\Phi_k. \end{aligned} \tag{2}$$

Here $\eta = \theta^2$ is the spurion external field and M is the gaugino mass. Use of the spurion formalism has a long history; in this context see in particular Ref. [9]. In [4], [5] it was shown that β_h, β_b and β_M are given by the following simple expressions:

$$\beta_h^{ijk} = \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} - 2\gamma_1^i{}_l Y^{ljk} - 2\gamma_1^j{}_l Y^{ilk} - 2\gamma_1^k{}_l Y^{ijl} \tag{3a}$$

$$\beta_b^{ij} = \gamma^i{}_l b^{lj} + \gamma^j{}_l b^{il} - 2\gamma_1^i{}_l \mu^{lj} - 2\gamma_1^j{}_l \mu^{il} \tag{3b}$$

$$\beta_M = \mathcal{O}\left(\frac{\beta_\alpha}{\alpha}\right) \tag{3c}$$

where we have written $\alpha = g^2$,

$$\mathcal{O} = \left(M\alpha\frac{\partial}{\partial\alpha} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\right), \tag{4}$$

and

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j. \tag{5}$$

The result for β_{m^2} is[4]

$$(\beta_{m^2})^i{}_j = \left[\Delta + \tilde{X}(\alpha, Y, Y^*, h, h^*, m, M)\frac{\partial}{\partial\alpha}\right]\gamma^i{}_j. \tag{6}$$

Here

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2MM^*\alpha\frac{\partial}{\partial\alpha} + \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}, \tag{7}$$

$Y_{lmn} = (Y^{lmn})^*$, and

$$\tilde{Y}^{ijk} = (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} + (m^2)^k{}_l Y^{ijl}. \tag{8}$$

The function $\tilde{X}$ introduced in Eq. (6) is related to X as defined in Ref. [4] by the equation $\tilde{X} = 2gX$. The term in $\tilde{X}$ does not appear in a naive application of the spurion formalism, because (when using DRED) it fails to allow for the fact that the ϵ-scalars associated with DRED acquire a mass through radiative corrections [10]. Indeed, in DRED, β_{m^2} will actually depend on the ϵ-scalar mass. It is, however, possible to define a scheme, DRED$'$, related to DRED, such that β_{m^2} is independent of the ϵ-scalar mass [11]. In this scheme β_{m^2} is given by Eq. (6) with the leading contribution to $\tilde{X}$ given by[6]

$$\tilde{X} = -4S\alpha^2(16\pi^2)^{-1} \tag{9}$$

where

$$S = r^{-1}\mathrm{tr}[m^2C(R)] - MM^*C(G). \tag{10}$$

(Here r is the number of generators of the gauge group and $C(R)$ and $C(G)$ are the quadratic matter and adjoint Casimirs respectively; see [10] for more on our notation). In Ref. [6] we showed that if there is a RG-invariant trajectory $Y = Y(\alpha)$, then we also have RG-invariant trajectories given by

$$h^{ijk} = -2M\alpha\frac{dY^{ijk}}{d\alpha} \tag{11a}$$

$$b^{ij} = -2M\alpha\frac{d\mu^{ij}}{d\alpha} \tag{11b}$$

$$(m^2)^i{}_j = 2\frac{\alpha^2}{\beta_\alpha}MM^*\frac{d\gamma^i{}_j}{d\alpha}, \tag{11c}$$

on which $\tilde{X}$ takes the form

$$\tilde{X} = 2\alpha M M^* \left[\frac{\alpha}{\beta_\alpha}\frac{d\beta_\alpha}{d\alpha} - 2\right]. \tag{12}$$

Kobayashi et al[12] discussed a possible generalisation of our results to a RG-invariant mass sum rule. They also gave a result for $\tilde{X}$ on the RG trajectory in the scheme corresponding to the NSVZ results (we shall specify this scheme in more detail later). In our conventions, this result is given by

$$\tilde{X}^{\rm NSVZ} = -4\frac{\alpha^2}{16\pi^2}\frac{S}{[1-2\alpha C(G)(16\pi^2)^{-1}]}. \tag{13}$$

(In fact, this equation may be obtained by substituting into Eq. (12) the NSVZ result for β_α given by

$$\beta_\alpha^{\rm NSVZ} = 2\frac{\alpha^2}{16\pi^2}\left[\frac{Q - 2r^{-1}{\rm tr}[\gamma C(R)]}{1-2\alpha C(G)(16\pi^2)^{-1}}\right], \tag{14}$$

where $Q = T(R) - 3C(G)$, and using Eq. ($11c$).) Our principal claim in this paper is that Eq. (13) is true *in general*, and not just on the RG trajectory.

To substantiate this claim, and to clarify the nature of $\tilde{X}$ in general, it is necessary to discuss the transformation properties of $\tilde{X}$ under a change of scheme. We consider schemes related by redefinitions $\alpha \to \alpha'(\alpha, Y, Y^*)$ and $M \to M'(\alpha, h, M, Y, Y^*)$. It is important not to redefine Y or h if Eqs. (3) are to remain true, and it is easy to convince oneself that m^2 must not be redefined if Eq. (6) is to remain true, which implies

$$\beta'_{m^2}(\alpha', Y, Y^*, h, h^*, m, M') = \beta_{m^2}(\alpha, Y, Y^*, h, h^*, m, M). \tag{15}$$

The transformation properties of $\tilde{X}$ then follow from Eq. (15). The first ingredient is the fact that γ and γ_1 transform in general according to:

$$\gamma'(\alpha', Y, Y^*) = \gamma(\alpha, Y, Y^*), \tag{16a}$$

$$\gamma'_1(\alpha', Y, Y^*, M', h) = \gamma_1(\alpha, Y, Y^*, M, h). \tag{16b}$$

As shown in Ref. [4], Eq. ($16b$) implies that

$$\alpha M = \alpha' M' \frac{\partial\alpha(\alpha', Y, Y^*)}{\partial\alpha'} - h^{ijk}\frac{\partial\alpha(\alpha', Y, Y^*)}{\partial Y^{ijk}}. \tag{17}$$

We showed in Ref. [4] that Eqs. (3) preserve their form if we make scheme redefinitions $\alpha \to \alpha'(\alpha, Y, Y^*)$ and $M \to M'(\alpha, h, M, Y, Y^*)$ as defined above. It is a straightforward exercise to show in a similar fashion that Eq. (15) requires

$$\begin{aligned}\tilde{X} =& \tilde{X}'\frac{\partial\alpha}{\partial\alpha'} + 2M'M'^*\left[\alpha'^2\frac{\partial^2\alpha}{\partial\alpha'^2} + 2\alpha'\frac{\partial\alpha}{\partial\alpha'} - 2\frac{\alpha'^2}{\alpha}\left(\frac{\partial\alpha}{\partial\alpha'}\right)^2\right] \\ &- \left[2M'\alpha' h_{ijk}\left(\frac{\partial^2\alpha}{\partial Y_{ijk}\partial\alpha'} - \frac{2}{\alpha}\frac{\partial\alpha}{\partial Y_{ijk}}\frac{\partial\alpha}{\partial\alpha'}\right) - \tilde{Y}^{ijk}\frac{\partial\alpha}{\partial Y^{ijk}} + \text{c.c.}\right] \\ &+ 2h^{ijk}h_{lmn}\left[\frac{\partial^2\alpha}{\partial Y^{ijk}\partial Y_{lmn}} - \frac{2}{\alpha}\frac{\partial\alpha}{\partial Y^{ijk}}\frac{\partial\alpha}{\partial Y_{lmn}}\right],\end{aligned} \tag{18}$$

which may be rewritten more succinctly as

$$\tilde{X} = \tilde{X}'\frac{\partial\alpha}{\partial\alpha'} + (\Delta' - \Delta)\alpha, \tag{19}$$

where Δ is given in Eq. (7). Reassuringly, one can then check that on an RG-invariant trajectory, Eq. (12) is true in any scheme related by $\alpha \to \alpha'(\alpha, Y, Y^*)$ and $M \to M'(\alpha, h, M, Y, Y^*)$ (as in Eq. (17)). Indeed, on an RG-invariant trajectory, we find

$$\begin{aligned}\beta'_{\alpha'} &= \frac{d\alpha'}{d\alpha}\beta_\alpha \\ \alpha' M' &= \alpha M\frac{d\alpha'}{d\alpha}\end{aligned} \tag{20}$$

(using Eqs. (11a) and (17)). Then if $\tilde{X}$ is given in the unprimed scheme by Eq. (12), it follows from Eq. (18) that $\tilde{X}'$ satisfies the same equation in the primed scheme.

The DRED$'$ scheme is well-defined; but what we mean by the NSVZ scheme requires further explanation. We have shown[13] how to construct perturbatively a redefinition $\alpha^{\text{DRED}} \to \alpha^{\text{NSVZ}}$ which takes us from DRED to a scheme in which β_α takes the NSVZ form of Eq. (14). We now define the NSVZ scheme as the result of making this redefinition of α, accompanied by the corresponding redefinition of M as given in Eq. (17). We believe that this scheme corresponds to that used by Hisano and Shifman[3] to derive their exact results for the soft-breaking couplings. This is because their scheme certainly corresponds to β_α of the form Eq. (14), and moreover their result (consisting of an RG-invariant combination of M, b and α) could be shown to lead to our exact formula for β_M in Eq. (3c). The scheme used by Hisano and Shifman is defined by changing from the holomorphic normalisation for the gauge coupling in the Wilsonian action to the canonical normalisation. In this context, it is of interest to consider the reverse transition, from the NSVZ scheme to the holomorphic scheme. (For a discussion of the relationship between the holomorphic and NSVZ couplings see Ref. [14].) Consider a theory with only a single Yukawa coupling y and with both $C(R)$ and γ proportional to the unit matrix. For an example of such a theory, consider $SU(N)$ with three adjoint chiral superfields and the superpotential

$$W = yd^{abc}\phi_1^a\phi_2^b\phi_3^c. \tag{21}$$

Let us define the transformation

$$\frac{1}{\alpha^{\rm H}} = \frac{1}{\alpha^{\rm NSVZ}} + \frac{2}{16\pi^2}C(G)\ln\alpha^{\rm NSVZ} - \frac{2}{3}\frac{1}{16\pi^2}T(R)\ln(yy^*), \tag{22}$$

with an associated transformation of M dictated by Eq. (17). In the "holomorphic" scheme corresponding to $\alpha^{\rm H}$, $\beta_{\alpha^{\rm H}}$ is one-loop exact, i.e. $16\pi^2\beta_{\alpha^{\rm H}} = 2Q(\alpha^{\rm H})^2$. (In our example Eq. (21), we have, in fact, $Q = 0$ and so in this case $\beta_{\alpha^{\rm H}}$ vanishes to all orders). Remarkably, we find using Eq. (18) that $\tilde{X}^{\rm H} = 0$, assuming that $\tilde{X}^{\rm NSVZ}$ is given by Eq. (13). This means that in the holomorphic scheme, all contributions to the soft β-functions beyond one loop can be traced to γ. The fact that the form of $\tilde{X}^{\rm NSVZ}$ leads to the vanishing of $\tilde{X}^{\rm H}$ is intriguing. In a more general case with $C(R), \gamma$ not proportional to the unit matrix it is straightforward to write down a generalisation of Eq. (22) such that β_{α^H} is one-loop exact; we have simply to replace $\frac{2}{3}T(R)\ln(yy^*)$ by $4r^{-1}{\rm tr}[ZC(R)]$ where $\mu\frac{dZ}{d\mu} = \gamma$. However, we do not then find that $\tilde{X}^{\rm H}$ is zero, except of course on an RG-invariant trajectory, Eq. (12).

Our proposed exact result for $\tilde{X}$ is in the NSVZ scheme; however, the existence of $\tilde{X}$ was first identified in the DRED$'$ scheme, and ascribed to the ϵ-scalar mass. In fact, we shall also derive an exact formula for $\tilde{X}$ in DRED$'$, related to the β-function $\beta_{\tilde{m}^2}$ for the ϵ-scalar mass $\tilde{m}^2$. We shall then show that our proposed result for $\tilde{X}$ in the NSVZ scheme is related to our result for $\tilde{X}$ in DRED$'$ by Eq. (18), up to the limits of our perturbative calculations.

We now give our result for $\tilde{X}$ in DRED$'$. Suppose that $\beta_{\tilde{m}^2}$ is given by

$$\beta_{\tilde{m}^2} = N_1 + N_2\tilde{m}^2, \tag{23}$$

where $N_1(\alpha, Y, Y^*, h, h^*, m, M)$ does not depend on $\tilde{m}$, and may be written

$$N_1 = \sum_{L=1}^{\infty} N_1^{(L)}, \tag{24}$$

where $N_1^{(L)}$ is the L-loop contribution to N_1. Our claim now is that

$$\tilde{X}^{\mathrm{DRED}'} = -\sum_{L=1}^{\infty} \frac{\alpha}{L} N_1^{(L)}. \tag{25}$$

The factor of $1/L$ arises here because $\tilde{X}$ is related to the *renormalisation constant* that determines $\beta_{\tilde{m}^2}$, which differs at L loops by a factor of L from $\beta_{\tilde{m}^2}$ itself. The proof of Eq. (25) is as follows: DRED and DRED$'$ are related by a redefinition of m^2 given by

$$(m^2)^{\mathrm{DRED}'} = (m^2)^{\mathrm{DRED}} + \delta m^2, \tag{26}$$

where[7]

$$\delta m^2 = \tilde{m}^2 \sum_{L=1}^{\infty} \frac{1}{L} \alpha \frac{\partial}{\partial \alpha} \gamma^{(L)}. \tag{27}$$

δm^2 was computed at lowest order in Refs. [10], [15], and [11], and corresponds[10][16] to finite contributions to $(m^2)^B$ which contain a simple pole in ϵ multiplied by a factor of ϵ deriving from a loop of ϵ-scalars, and containing an insertion of the ϵ-scalar mass. The rules originally derived by Yamada[9] from the spurion formalism lead to Eq. (6) without the term in $\tilde{X}$. From a diagrammatic point of view, these rules precisely omit (compared to DRED) diagrams with an ϵ-scalar mass counterterm replacing $\tilde{m}^2$ in a diagram contributing to δm^2. In particular, the missing simple pole terms (which account for the difference between $\beta_{m^2}^{\mathrm{DRED}}$ and $\Delta\gamma$, where Δ is defined in Eq. (7)) correspond to replacing $\tilde{m}^2$ in δm^2 by the simple pole in $(\tilde{m}^2)^B$. So we find

$$\beta_{m^2}^{\mathrm{DRED}(L)} = \Delta\gamma^{(L)} - \sum_{r=1}^{L-1} L \frac{N_1^{(L-r)}}{L-r} \frac{1}{r} \alpha \frac{\partial}{\partial \alpha} \gamma^{(r)} + \mathrm{O}(\tilde{m}^2). \tag{28}$$

The factor of L is required to convert the simple pole residue to the β-function. Moreover, we also have

$$\beta_{m^2}^{\mathrm{DRED}'}(m^2 + \delta m^2) = \beta_{m^2}^{\mathrm{DRED}}(m^2) + \mu \frac{\partial}{\partial \mu} \delta m^2, \tag{29}$$

which implies

$$\beta_{m^2}^{\mathrm{DRED}'(L)} = \beta_{m^2}^{\mathrm{DRED}(L)} + \sum_{r=1}^{L-1} N_1^{(L-r)} \frac{1}{r} \alpha \frac{\partial}{\partial \alpha} \gamma^{(r)} + \mathrm{O}(\tilde{m}^2). \tag{30}$$

Combining Eqs. (28) and (29), we have

$$\beta_{m^2}^{\mathrm{DRED}'(L)} = \Delta\gamma^{(L)} - \sum_{r=1}^{L-1} \frac{N_1^{(L-r)}}{L-r} \alpha \frac{\partial}{\partial \alpha} \gamma^{(r)}, \tag{31}$$

which, with Eq. (6), implies Eq. (25). (In Eq. (31) we have assumed the cancellation of the terms in $\tilde{m}^2$ in $\beta_{m^2}^{\mathrm{DRED}'}$; this was shown up to two loops in Ref. [11], and has recently been proved to all orders in Ref. [7].)

We can corroborate this claim for $\tilde{X}^{\mathrm{DRED}}$ by explicit perturbative calculations. The ϵ-scalar mass β-function was calculated in Ref. [10] up to two loops; from which result it is easy to obtain $N_1^{(1)}$ and $N_1^{(2)}$. From Eq. (25) we then obtain

$$\begin{aligned} 16\pi^2\tilde{X}^{(1)} &= -4\alpha^2 S, \\ (16\pi^2)^2\tilde{X}^{\mathrm{DRED}'(2)} &= r^{-1}\alpha^2\mathrm{tr}[WC(R)] - 8\alpha^3C(G)S - 4\alpha^3C(G)QMM^*, \end{aligned} \tag{32}$$

where

$$\begin{aligned} W^j{}_i = \frac{1}{2}Y_{ipq}Y^{pqn}(m^2)^j{}_n + \frac{1}{2}Y^{jpq}Y_{pqn}(m^2)^n{}_i + 2Y_{ipq}Y^{jpr}(m^2)^q{}_r \\ + h_{ipq}h^{jpq} - 8\alpha MM^*C(R)^j{}_i. \end{aligned} \tag{33}$$

(We note that the expression for $\beta_{\tilde{m}^2}^{(2)}$ in Ref. [10] contained a factor of 2 misprint in the coefficient of $\alpha^3C(G)S$ which we have corrected here.) Clearly $\tilde{X}^{(1)}$ agrees with Eq. (9), and indeed with the lowest order contribution to $\tilde{X}^{\mathrm{NSVZ}}$ in Eq. (13). From Eq. (13), we also have

$$(16\pi^2)^2\tilde{X}^{\mathrm{NSVZ}(2)} = -8\alpha^3C(G)S. \tag{34}$$

To compare $\tilde{X}^{\mathrm{DRED}'(2)}$ and $\tilde{X}^{\mathrm{NSVZ}(2)}$, we need to know the coupling constant redefinition linking NSVZ and DRED. Writing

$$\alpha^{\mathrm{DRED}} = \alpha^{\mathrm{NSVZ}} + \sum_{L=1}^{\infty}\delta^{(L)}(\alpha^{\mathrm{NSVZ}}, Y, Y^*), \tag{35}$$

where $\delta^{(L)}(\alpha^{\mathrm{NSVZ}}, Y, Y^*)$ is the L-loop redefinition, it was shown in Ref. [13] that $\delta^{(1)} = 0$ and

$$(16\pi^2)^2\delta^{(2)} = \alpha^2\left[r^{-1}\mathrm{tr}\left\{PC(R)\right] - \alpha QC(G)\right\}, \tag{36}$$

where

$$P^i{}_j = \tfrac{1}{2}Y^{ikl}Y_{jkl} - 2\alpha C(R)^i{}_j. \tag{37}$$

It is easy to verify using Eq. (18) that $\tilde{X}^{\mathrm{NSVZ}(2)}$ as given in Eq. (34) transforms into $\tilde{X}^{\mathrm{DRED}'(2)}$ as given in Eq. (32) when α transforms according to Eq. (36). We have also performed a partial three-loop calculation of $\beta_{\tilde{m}^2}$ in order to provide an even more stringent check. We have computed the contributions with a maximal number of Yukawa couplings, of the form $\alpha Y^2Y^{*2}m^2$. We performed the calculation in components, using the ϵ-scalar lagrangian which may be found in Ref. [17]. The computation presents no special difficulties, since the momentum integrals may be reduced to the basic set given in Ref. [13], and we suppress the details. We find a prediction for the three-loop contribution to $\tilde{X}^{\mathrm{DRED}'}$ which we can write in the form

$$\begin{aligned}(16\pi^2)^3\tilde{X}^{\mathrm{DRED}'(3)} =&\alpha^2\left(\tilde{Y}^{ijk}\frac{\partial}{\partial Y^{ijk}}+\tilde{Y}_{ijk}\frac{\partial}{\partial Y_{ijk}}\right)\Big(\frac{1}{12r}\mathrm{tr}(Y^2Y^2C(R))\\ &-\frac{2}{3r}Y^{ijk}Y_{ilm}(Y^2)^l{}_jC(R)^m{}_k\Big)+\ldots.\end{aligned}\tag{38}$$

Since we have from Eq. (13) that $(16\pi^2)^3\tilde{X}^{\mathrm{NSVZ}(3)} = -16\alpha^4[C(G)]^2S$, all the terms given explicitly in Eq. (38) must be generated by the transformation from NSVZ to DRED$'$. It was shown in Ref. [13] that

$$\delta^{(3)} = \rho\Delta_1 - \frac{4}{3}\Delta_2 + \frac{1}{3}\Delta_3\tag{39}$$

where

$$(16\pi^2)^3\Delta_1 = \alpha^3C(G)\left[r^{-1}\mathrm{tr}[PC(R)] - \alpha QC(G)\right]\tag{40a}$$

$$(16\pi^2)^3\Delta_2 = r^{-1}\mathrm{tr}\left[\alpha^2S_4C(R) - 2\alpha^4QC(R)^2 + 2\alpha^3PC(R)^2\right]\tag{40b}$$

$$(16\pi^2)^3\Delta_3 = \alpha^2r^{-1}\mathrm{tr}[P^2C(R)] - \alpha^4Q^2C(G).\tag{40c}$$

Here $S_{4j}^i = Y^{imn}P^p{}_mY_{jpn}$. (The coefficient ρ has not yet been calculated explicitly[1], but it is immaterial for our present purposes.) It is straightforward to show that this transformation generates the terms in Eq. (38) according to Eq. (18). This completes our claim that $\tilde{X}$ is given by Eq. (13) in NSVZ and by Eq. (25) in DRED$'$.

The authors of Ref. [7] present an elegant prescription for accommodating DRED$'$ within the formalism of Ref. [3]. This leads to a method for extracting β_{m^2} which should lead to results comparable to ours, though they do not give an explicit all-orders formula.

Given our result for $\tilde{X}$, we now have a complete set of exact β-function results in the NSVZ scheme for an arbitrary gauge theory. All the β-functions are given in terms of the anomalous dimension matrix γ of the chiral superfields. Since γ^{NSVZ} has been given through three loops in Ref. [13] there is no obstacle to, for example, performing the usual MSSM running analysis through three loops. Results valid in DRED$'$ could then be obtained at M_Z (say) by using the redefinition of α as given in Eqs. (35), (36) and (40) and the redefinition of M given in Eq. (17).

Acknowledgements

AP was supported by a PPARC Research Grant.

[1] In Ref. [18] a method based on Padé approximants was used to suggest that $\rho \approx 4.8$

References

[1] V. Novikov et al, Nucl. Phys. **B**229 (1983) 381;
V. Novikov et al, Phys. Lett. **B**166 (1986) 329;
M. Shifman and A. Vainstein, Nucl. Phys. **B**277 (1986) 456

[2] Y. Yamada, Phys. Rev. **D**50 (1994) 3537

[3] J. Hisano and M. Shifman, Phys. Rev. **D**56 (1997) 5475

[4] I. Jack and D.R.T. Jones, Phys. Lett. **B**415 (1997) 383

[5] L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, Nucl. Phys. **B**510 (1998) 289

[6] I. Jack, D.R.T. Jones and A. Pickering, hep-ph/9712542

[7] N. Arkani-Hamed, G.F. Giudice, M.A. Luty and R. Rattazzi, hep-ph/9803290

[8] G.F. Giudice and R. Rattazzi, Nucl. Phys. **B**511 (1998) 25

[9] Y. Yamada, Phys. Rev. **D**50 (1994) 3537

[10] I. Jack and D.R.T. Jones, Phys. Lett. **B**333 (1994) 372

[11] I. Jack, D.R.T. Jones, S.P. Martin, M.T. Vaughn and Y. Yamada, Phys. Rev. **D**50 (1994) R5481

[12] T. Kobayashi, J. Kubo and G. Zoupanos, hep-ph/9802267

[13] I. Jack, D.R.T. Jones and C.G. North, Nucl. Phys. **B**486 (1997) 479

[14] N. Arkani-Hamed and H. Murayama, hep-th/9707133

[15] S.P. Martin and M.T. Vaughn, Phys. Rev. **D**50 (1994) 2282

[16] I. Jack, D.R.T. Jones and K.L. Roberts, Z. Phys. **C**63 (1994) 151

[17] D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, Nucl. Phys. **B**167 (1980) 479

[18] I. Jack, D.R.T. Jones and M.A. Samuel, Phys. Lett. **B**407 (1997) 143.