LTH-418

Non-zeta knots in the renormalization of the Wess-Zumino model?

P.M. Ferreira & J.A. Gracey,
Theoretical Physics Division,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool,
L69 7ZF,
United Kingdom.

Abstract. We solve the Schwinger Dyson equations of the $O(N)$ symmetric Wess-Zumino model at $O(1/N^3)$ at the non-trivial fixed point of the d-dimensional β-function and deduce a critical exponent for the wave function renormalization at this order. By developing the ϵ-expansion of the result, which agrees with known perturbation theory, we examine the distribution of transcendental coefficients and show that only the Riemann ζ series arises at this order in $1/N$. Unlike the analogous calculation at the same order in the bosonic $O(N)$ ϕ^4-theory non-zeta transcendentals, associated with for example the $(3,4)$-torus knot, cancel.

One of the more exciting developments in renormalization theory in recent years has been the realization that there is a relation between the Riemann zeta and non-zeta transcendental numbers which appear in the renormalization group functions of a variety of theories and the positive knots of mathematics, [1, 2, 3]. Briefly, in [1] it was demonstrated that a multiloop Feynman diagram could be easily associated with a link diagram which, after applying basic knot manipulations and a skeining relation, could be reduced to an underlying positive knot listed in the classification tables, plus trivial unknots. It turned out that one could build up a one-to-one correspondence between different positive knots which would appear in a Feynman diagram and the irrational numbers, like the Riemann zeta series, $\zeta(n)$, which appeared in the renormalization constant of the *same* graph. For example, if a diagram contained the number $\zeta(3)$ in its simple pole with respect to the regularization then it always occurred in a Feynman diagram whose associated link diagram reduced to the trefoil or $(2,3)$-torus knot. More generally, the presence of $\zeta(2n-3)$, for $n \geq 3$, in a renormalization constant corresponds to a $(2,2n-3)$-torus knot in the link diagram of the original Feynman diagram, [1, 2]. Indeed intense investigation revealed that other *non*-zeta transcendentals which are represented by independent double and triple infinite series were 1-1 associated with other sets of torus knots, [3]. Moreover it was found that there was an intimate relation between the braid word representation of the knot and the structure of its knot number when it is expressed as an infinite sum. This empirical evidence has been examined with great intensity more recently and has so far remained robust, [4]. An overview of the status of the area can be found in [5]. One advantage for renormalization theory of having such an association is that one immediately knows from the simple link diagram of a Feynman diagram which independent transcendentals to expect in its calculation. This can therefore be adapted as a calculational aid for the very high order loop diagrams in which they arise. For example, performing an algebraic-numerical calculation one can project onto the appropriate number basis for the diagram. Indeed this method was successfully used in [3] to compute all the primitively divergent graphs at six and seven loop which contribute to the $\overline{\mathrm{MS}}$ β-function of ϕ^4 theory. This was the first order in this theory to involve these new non-zeta transcendentals.

Clearly to gain some insight into the underlying knot theory requires very high order multiloop calculations which is a technical and difficult exercise, [1, 3, 4]. However, in [6] non-zeta transcendentals which occur in graphs with subgraph divergences were accessed in the wave function renormalization of $O(N)$ ϕ^4 theory through the large N expansion. (The knot relation in graphs with subgraph divergences has also been investigated in perturbation theory in [7].) A result had been available in [8] in d-dimensions at $O(1/N^3)$ for the critical exponent η which is related to the wave function anomalous dimension through the critical renormalization group. It contained the value of a two loop integral which when expanded in powers of ϵ, where $d = 4 - 2\epsilon$, contained non-zeta transcendentals. The first of these had been known for a long time and had been denoted in [9] by $U_{62} = \sum_{n>m>0}^{\infty} (-1)^{n-m} n^{-6} m^{-2}$. However, it is now associated in the more modern approach with the $(3,4)$-torus knot, [3], and hence through the braid word representation of this knot U_{62} has been replaced by the independent, but more systematically defined, non-alternating double sum $F_{53} = \sum_{n>m>0}^{\infty} n^{-5} m^{-3}$. This discovery yielded the expansion of the two loop integral to much higher order, [6], and established the knot association to a new number of loops. It also provided a basis for making a stronger connection with algebraic number theory, [4]. Although most of the results we have mentioned have been established in simple scalar theories the knot approach has proved useful in gaining insight into long term multiloop problems in theories with symmetries. Indeed the cancellation of $\zeta(3)$ in the quenched β-function of QED at three loops has been explained and extended to higher order cancellations by knot theoretic arguments in [10]. Therefore it would seem that the presence of a symmetry in a theory, such as gauge symmetry, can in some way suppress the appearance of these transcendental numbers. Moreover it would be interesting to understand what effect other symmetries, such as *four* dimensional supersymmetry, will have on the structure of the renormalization group functions. This is the subject of this paper where we will study the Wess-Zumino model, [11], with an $O(N)$ symmetry to $O(1/N^3)$ in a large N expansion. The aim is to first calculate the critical exponent, η, corresponding to the wave function renormalization and then to examine the expression obtained for it to ascertain how supersymmetry modifies the appearance of zeta and non-zeta transcendentals compared with the underlying bosonic ϕ^4-theory. This would therefore extend some of the analysis of [6]. We note that in the underlying bosonic model, $O(N)$ ϕ^4 theory, it was possible to establish not only the first location of U_{62} in the wave function renormalization at $O(1/N^3)$ but also to provide its explicit coefficient, [6, 12]. Knowledge of this is clearly important if one is ever to have a systematic knot based method to perform the renormalization since the coefficients of the transcendentals that appear in some renormalization scheme will also need to be determined. Therefore the large N results will provide important, non-trivial and independent checks. We recall that large N calculations in models such as the Wess-Zumino model are possible at $O(1/N^2)$ through critical renormalization group methods and solution of the Schwinger Dyson equations order by order in $1/N$ at a non-trivial fixed point of the d-dimensional β-function. The method had been originally applied to the $O(N)$ bosonic σ model in an impressive series of articles, [13, 8]. More recently, similar calculations in supersymmetric theories have been made possible in [14] by extending the methods of [13] to superspace for the $O(N)$ Wess-Zumino model which, it is hoped, can be extended to study supersymmetric gauge theories.

We now turn to the specifics of calculating η in the Wess-Zumino model with an $O(N)$ symmetry and note that its action in superspace is given by

$$S ~=~ \int d^dx \left[\int d^4\theta \left(\bar{\Phi}^i \Phi^i + \frac{\bar{\sigma}\sigma}{g^2} \right) ~-~ \frac{1}{2} \int d^2\theta \, \sigma \Phi^2 ~-~ \frac{1}{2} \int d^2\bar{\theta} \, \bar{\sigma} \bar{\Phi}^2 \right] \tag{1}$$

where Φ^i and σ are chiral superfields, $1 \leq i \leq N$ and g is the coupling constant which has been rescaled into the σ kinetic term to ensure that the interaction in d-dimensions is in a form for which the conformal integration technique or uniqueness, [13], can be readily applied to ease the calculation of high order integrals. The four loop d-dimensional β-function of the model is deduced from the result for a general symmetry group given in [15] which used the non-renormalization theorem, [11, 16], and generalized earlier results, [17, 18]. Specifying to an $O(N)$ group it is given by, [15],

$$\begin{aligned} \beta(g) ~=~& \frac{1}{2}(d-4)g ~+~ \frac{1}{2}(N+4)g^2 ~-~ 2(N+1)g^3 ~+~ \left(\frac{1}{2}(N^2+11N+4) + 3(N+4)\zeta(3) \right) g^4 \\ &+~ \left(\frac{1}{6}(N^3 - 36N^2 - 84N - 20) - 3(N^2+16N+24)\zeta(3) \right. \\ &\left. \quad +~ \frac{3}{2}(N+4)^2\zeta(4) - 10(N+2)(N+4)\zeta(5) \right) g^5 ~+~ O(g^6) \end{aligned} \tag{2}$$

where $\zeta(n)$ is the Riemann zeta function. As we will be calculating information relevant to the Φ-superfield renormalization, we note also that, [15],

$$\begin{aligned} \gamma_\Phi(g) ~=~& g ~-~ \frac{1}{2}(N+2)g^2 ~-~ \frac{1}{4}\left(N^2 - 10N - 4 - 24\zeta(3) \right) g^3 \\ &-~ \left(\frac{1}{24}(3N^3 + 16N^2 + 152N + 40) - \frac{\zeta(3)}{4}(N^2 - 4N - 36)(N+4) \right. \\ &\left. \quad -~ 3\zeta(4)(N+4) + 20\zeta(5)(N+2) \right) g^4 ~+~ O(g^5) \end{aligned} \tag{3}$$

The d-dimensional fixed point where we will analyse the Schwinger Dyson equation is given by the non-trivial zero of the β-function, (2). Specifically expanding in powers of $1/N$ it is given by, [14],

$$
\begin{aligned}
g_c \;=\; & \frac{2\epsilon}{N} + \left(-8\epsilon + 16\epsilon^2 - 8\epsilon^3 - \frac{16}{3}\epsilon^4 + O(\epsilon^5) \right) \frac{1}{N^2} \\
& + \left(32\epsilon - 176\epsilon^2 - 8(6\zeta(3) - 37)\epsilon^3 \right. \\
& \left. \quad + \frac{16}{3}(60\zeta(5) - 9\zeta(4) + 18\zeta(3) - 4)\epsilon^4 + O(\epsilon^5) \right) \frac{1}{N^3} + O\left(\frac{\epsilon}{N^4} \right)
\end{aligned} \tag{4}
$$

Evaluating the renormalization group functions at g_c defines the critical exponents. So, for example, $\eta = \gamma_\Phi(g_c)/2$. In [14] an expression for η was deduced at $O(1/N^2)$ by replacing the lines of the Schwinger Dyson equation, truncated at the same order, by the asymptotic scaling forms near g_c of the propagators of the respective fields. For an Euclidean space-time these are,

$$
\begin{aligned}
\langle \bar{\Phi}(-p,\theta)\Phi(p,\theta') \rangle & \sim \frac{A\delta^4(\theta - \theta')}{(p^2)^{\mu-\alpha}} \\
\langle \bar{\sigma}(-p,\theta)\sigma(p,\theta') \rangle & \sim \frac{B\delta^4(\theta - \theta')}{(p^2)^{\mu-\beta}}
\end{aligned} \tag{5}
$$

as $p \to \infty$, where we have set $d = 2\mu$, A and B are unknown momentum independent amplitudes and

$$
\alpha = \mu - 1 + \tfrac{1}{2}\eta \quad , \quad \beta = 1 - \eta \tag{6}
$$

are the exponents of the Φ^i and σ superfields. Their canonical dimensions are fixed by ensuring that each term in the d-dimensional superspace action is consistent with S being dimensionless. Due to the non-renormalization theorem, [11, 16], there is no vertex renormalization and its associated exponent, ordinarily denoted by χ, is zero to all orders in $1/N$. In particular in the critical region the Φ equation of fig. 1 becomes

$$
0 = a(\alpha - \mu + 1) + a(\alpha)z \tag{7}
$$

where $z = a^2(\mu - \alpha)a(\mu - \beta)A^2B$ and $a(x) = \Gamma(\mu - x)/\Gamma(x)$. The σ equation is

$$
0 = a(\beta - \mu + 1) + \tfrac{1}{2}Nza(\beta) \tag{8}
$$

where we have chosen to express each representation of fig. 1 in coordinate space through a Fourier transform. Thus eliminating z gives

$$
\eta = \frac{4a(\alpha)a(\beta - \mu + 1)}{N(2\mu - 2 - \alpha)a(\alpha - \mu + 2)a(\beta)} \tag{9}
$$

Due to the chirality property of the superfields no new two and three loop graphs occur at $O(1/N^2)$ unlike the bosonic model and therefore one merely expands out (9) to $O(1/N^2)$ to deduce η_2, where $\eta = \sum_{i=1}^{\infty} \eta_i/N^i$. Thus, [14],

$$
\begin{aligned}
\eta_1 &= \frac{4\Gamma(2\mu - 2)}{\Gamma(\mu)\Gamma^2(\mu - 1)\Gamma(2 - \mu)} \\
\eta_2 &= \left[\psi(2 - \mu) + \psi(2\mu - 2) - \psi(\mu - 1) - \psi(1) + \frac{1}{2(\mu - 1)} \right] \eta_1^2
\end{aligned} \tag{10}
$$

where $\psi(x)$ is the logarithmic derivative of the Euler Γ-function.

The absence of these higher order graphs and the finiteness of the vertex means that one can use the Schwinger Dyson approach to compute η_3 which we believe is the first instance this has been attempted. Ordinarily one has to use the conformal bootstrap method, [19, 8], to do this.

For the Wess-Zumino model, however, it does not seem possible to apply it since that method relies on the vertex dimension being non-zero. Moreover the chirality of the fields ensures that there are only two new topologies at $O(1/N^3)$ and these are given for the respective Schwinger Dyson equations in figs 2 and 3. If we denote the values of the graphs for the Φ equation by Σ_i and those for the σ equation by Π_i then the respective Dyson equations to $O(1/N^3)$ are now

$$\begin{aligned} 0 &= \frac{a(\alpha-\mu+1)}{a(\alpha)} + z + z^3 a^5(\alpha) a^3(\beta) a(\mu-\alpha-\beta)\Sigma_1 + N z^4 a^7(\alpha) a^4(\beta) a(\mu-\alpha-\beta)\Sigma_2 \\ 0 &= \frac{2a(\beta-\mu+1)}{Na(\beta)} + z + z^3 a^6(\alpha) a^2(\beta) a(\mu-2\alpha)\Pi_1 + N z^4 a^8(\alpha) a^3(\beta) a(\mu-2\alpha)\Pi_2 \end{aligned} \quad (11)$$

We note that although the factor $a(\mu-\alpha-\beta)$ that appears with each of the higher order corrections in the Φ Dyson equation is singular when expanded in powers of $1/N$ it will turn out that in the computation of the diagrams in momentum space there will be a compensating factor of $a(\alpha+\beta)$ in its final value. Therefore the final overall contribution will be free of infinities.

The finiteness of the vertex means that one does not need to perform the usual renormalization of the Schwinger Dyson equations. Therefore it only remains to compute the four graphs. As is usual in superfield calculations the first step in such an evaluation is to carry out the manipulation of the supercovariant derivatives present in the super-Feynman rules which is known as the D-algebra. (For an introduction see, for example, [20].) For the topologies illustrated in figs 2 and 3 this will result in the distribution of $\Box$-operators in momentum space on various lines. For the non-planar topology the two upper lines joining to the external vertex are each modified by a $\Box$-operator. For the four loop topology the corresponding lines also gain the same operator as well as the completely internal horizontal propagator. The effect of a $\Box$-operator is to reduce the power of the exponent of that line by unity. For a ϕ line this means the power of its propagator will vanish whereas that of a σ line will be non-zero. One might expect that for the graphs of the Φ-equation the values of the graphs where the $\Box$-operators are distributed on the lower propagators are different. Explicit calculation shows that, at least to the order in $1/N$ which we are interested in, this is not the case. Indeed it turns out that these four graphs are straightforward to compute by the rules of conformal integration. The two non-planar graphs involve the same basic two loop integral which we denote by $P(\mu)$. It is defined graphically in fig. 4 in the coordinate space representation where one integrates over the location of the vertices. Thus,

$$\Pi_1 = \frac{a^2(1)}{a(2-\mu)} P(\mu) \quad (12)$$

and

$$a(\mu-\alpha-\beta)\Sigma_1 = P(\mu) \quad (13)$$

where we have included the compensating factor $a(\alpha+\beta)$ on the left side of the value of the graph. This integral, $P(\mu)$, has not appeared in previous $O(1/N^3)$ calculations in other models and if we define

$$P(\mu) = \frac{a^2(\mu-1)}{(2\mu-3)} \Pi(\mu) \quad (14)$$

then its ϵ-expansion, where $d = 4 - 2\epsilon$, is known to very high orders, [9, 21]. In particular

$$\begin{aligned} \Pi(\mu) &= 6\zeta(3) + 9\zeta(4)\epsilon + 7\zeta(5)\epsilon^2 + \frac{5}{2}[\zeta(6) - 2\zeta^2(3)]\epsilon^3 - \frac{1}{8}[91\zeta(7) + 120\zeta(4)\zeta(3)]\epsilon^4 \\ &- \frac{1}{64}[5517\zeta(8) - 512\zeta(5)\zeta(3) - 768U_{62}]\epsilon^5 + O(\epsilon^6) \end{aligned} \quad (15)$$

The presence of the non-zeta transcendental at $O(\epsilon^5)$ encourages us to believe there will be a $(3,4)$-torus knot in the wave function renormalization at $O(1/N^3)$ at some very high order in perturbation theory when one disentangles the information encoded in the relation $\eta = \gamma_\Phi(g_c)/2$ using the $O(1/N^3)$ corrections to g_c available in the β-function critical exponent ω_2 computed in [14]. For Π_2 the integral in fact occurs in the β-function calculation of the Wess-Zumino model, [14], as well as in the original work of [13]. Therefore we merely quote the final result

$$\Pi_2 = \frac{a^2(1)a^2(2\mu-2)}{(\mu-2)^2} \left[\frac{\hat{\Psi}^2(\mu)}{2} + \frac{\hat{\Phi}(\mu)}{2} + \frac{\hat{\Psi}(\mu)}{(\mu-2)} - 3\hat{\Theta}(\mu) \right] \eta_1^2 \tag{16}$$

where $\hat{\Theta}(\mu) = \psi'(\mu-1) - \psi'(1)$, $\hat{\Phi}(\mu) = \psi'(2\mu-3) - \psi'(3-\mu) - \psi'(\mu-1) + \psi'(1)$ and $\hat{\Psi}(\mu) = \psi(2\mu-3) + \psi(3-\mu) - \psi(\mu-1) - \psi(1)$. Therefore, like Σ_1 and Π_1, Π_2 only has zeta transcendentals in its ϵ-expansion. For Σ_2 elementary manipulations yield the three loop integral of fig. 5 which is illustrated in the coordinate space representation in the case when $\delta = 0$. It is evaluated by integration by parts using a temporary analytic regularization introduced by shifting the exponents of some of the lines by an infinitesimal quantity, δ. This is necessary because the set of integrals which result are divergent though their sum is finite. The symmetric choice illustrated in fig. 5 is to minimize the number of subsequent graphs that need to be calculated as well as to ensure that these can still be computed by conformal techniques. We find that

$$a(\mu-\alpha-\beta)\Sigma_2 = a(2\mu-2) \left[\frac{5}{6}\hat{\Omega}(\mu) + 2\hat{\Theta}^2(\mu) \right] \tag{17}$$

where $\hat{\Omega}(\mu) = \psi'''(\mu-1) - \psi'''(1)$. Hence we can now substitute for the values of the integrals in (11) and expand out the leading term of (9) to $O(1/N^3)$ to deduce that in d-dimensions

$$\begin{aligned} \eta_3 = \eta_1^3 \Bigg[& \frac{(7\mu^2-26\mu+25)}{4(\mu-2)^2}\hat{\Psi}^2(\mu) + \frac{(3\mu^2-10\mu+9)}{4(\mu-2)^2}\hat{\Phi}(\mu) - (\mu-1)^2\hat{\Theta}^2(\mu) \\ & - \frac{5(\mu-1)^2}{12}\hat{\Omega}(\mu) + \frac{(26\mu^4-174\mu^3+435\mu^2-479\mu+195)}{2(2\mu-3)(\mu-1)(\mu-2)^3}\hat{\Psi}(\mu) \\ & - \frac{(7\mu^2-16\mu+10)}{4(\mu-2)^2}\hat{\Theta}(\mu) + \frac{(44\mu^4-257\mu^3+557\mu^2-531\mu+188)}{2(2\mu-3)^2(\mu-1)^2(\mu-2)^2} \Bigg] \end{aligned} \tag{18}$$

In the final expression it turns out that the integral $P(\mu)$ has cancelled. This is a direct result of the D-algebra, and therefore supersymmetry, which modifies the exponents of each integral to be similar. A cancellation of some sort between Σ_1 and Π_1 was to have been expected, however, from knowledge of the ϵ-expansion of the expression obtained for η_3 by ignoring the higher order contributions Σ_i and Π_i. In other words that obtained by iterating (9) to $O(1/N^3)$. This approximation, known as Hartree Fock, in fact agrees with the known four loop perturbative result, (3), which implies that although Σ_1 and Π_1 are non-zero, their $\zeta(3)$ contribution at three loops must at least cancel as well as $\zeta(3)$ and $\zeta(4)$ at four loops. (We note the Hartree Fock contribution to η_4 also agrees with the four loop perturbative result.) It could have been the case that Σ_1 and Π_1 took different values but had ϵ-expansion agreement at low orders. Indeed this is the situation at four loops for Σ_2 and Π_2 whose low order ϵ-expansions are the same, correctly involving $\zeta(5)$ as the first term but differing first at level $\zeta(7)$. They give consistency of the Hartree Fock η_3 to four loops but new non-trivial contributions will enter at five and higher loops. Therefore the explicit cancellation of the $P(\mu)$ function is novel and indicates that unlike its underlying bosonic $O(N)$ ϕ^4 theory, there will be no non-zeta knots at all orders in the wave function renormalization at $O(1/N^3)$. Thus the presence of four dimensional supersymmetry would appear to reduce the number of complicated knots which will arise. This is in contrast to the case of the $O(N)$ supersymmetric σ model in two dimensions where it is expected that its η_3 retains the integral yielding a $(3,4)$-torus knot, [22, 6]. One can understand this better by considering the simple properties of supersymmetric theories in differing dimensions. For instance, a four dimensional $\mathcal{N} = 1$ supersymmetric scalar theory involves chiral superfields but in two dimensions it is theories with $\mathcal{N} = 2$ supersymmetry which are chiral and not $\mathcal{N} = 1$ theories such as the $O(N)$ supersymmetric σ model. Therefore this would suggest that theories with $\mathcal{N} = 2$ in two dimensions should not have non-zeta knots at $O(1/N^3)$.

Although (18) clearly will agree with the 4-loop perturbative result for $\gamma_\Phi(g)$ since the higher order graphs give non-zero contributions first at five loops, one question arises as to whether the correct signs and weighting factors have been included. However, the topologies of figs 2 and 3 arise in the computation of the critical exponent which determines the β-function at $O(1/N^2)$, [14]. Therefore we have been careful to include the corrections to the Schwinger Dyson equations with the same relevant factors as in [14], since these were essential to gain agreement with the 4-loop perturbative β-function, (2). In light of these remarks we can now deduce several of the higher order coefficients of $\gamma_\Phi(g)$. If we denote the $O(1/N^3)$ part of $\gamma_\Phi(g)$ as

$$\gamma_\Phi(g) ~=~ g ~+~ \sum_{r=2}^\infty (c_r N^2 + d_r N + e_r) N^{r-3} g^r \tag{19}$$

with $e_2 \equiv 0$ then it is easy to deduce that

$$\begin{aligned} c_5 &= \frac{1}{16}[3\zeta(4) - 2\zeta(3) - 1] \\ d_5 &= \frac{1}{24}[18\zeta(4) - 30\zeta(3) - 29] \\ e_5 &= \frac{1}{24}[-450\zeta(6) + 1416\zeta(5) - 171\zeta(4) + 102\zeta(3) - 108\zeta^2(3) + 304] \end{aligned} \tag{20}$$

and

$$\begin{aligned} c_6 &= \frac{1}{32}[6\zeta(5) - 3\zeta(4) - 2\zeta(3) - 1] \\ d_6 &= \frac{1}{80}[80\zeta(5) - 162\zeta(4) + 248\zeta(3) - 81] \\ e_6 &= \frac{1}{20}[-230\zeta(7) + 1140\zeta(6) - 906\zeta(5) - 36\zeta(4) - 5\zeta(3) - 108\zeta(3)\zeta(4) + 414\zeta^2(3) + 14] \end{aligned} \tag{21}$$

Therefore there are only two unknown coefficients to be determined in the polynomial of N to complete the five loop result. Interestingly the first contribution to the perturbative coefficients from the higher order four loop diagrams of figs 2 and 3 is the $\zeta(7)$ term at six loops. Although we have expressed our result in d-dimensions we cannot quote a value for η_3 in three dimensions since the expression diverges there. This is on a par with the critical exponent $\omega = -\beta'(g_c)$ which is also singular in three dimensions but at $O(1/N^2)$, [14].

In conclusion although we have ruled out non-zeta knots at $O(1/N^3)$ in the wave function renormalization of the $O(N)$ Wess-Zumino model it is not clear if this would be the case at all orders in perturbation theory or for the other renormalization group functions. For instance, one way of ascertaining whether the $(3,4)$-torus knot survives at six loops in, say, the β-function would be to analyse those Feynman diagrams with no subgraph divergences which contributed in ϕ^4 theory to its β-function, [3], and determine their value in the context of the Wess-Zumino model in superspace. Indeed it may be the case that all of these topologies are excluded at the first stage by the chirality of the fields. Alternatively if that is not the case then it would be an intriguing exercise to see how the D-algebra modifies the sum of the contributions from each diagram and if there is an overall cancellation.

Acknowledgements. This work was carried out with support from PPARC through an Advanced Fellowship, (JAG), and JNICT by a scholarship, (PF). We also thank Prof D.R.T. Jones and Dr I. Jack for several useful discussions.

References.

[1] D. Kreimer, Phys. Lett. **B354** (1995), 117.

[2] D. Kreimer, J. Knot Theor. Ram. **6** (1997), 479.

[3] D.J. Broadhurst & D. Kreimer, Int. J. Mod. Phys. **C6** (1995), 519.

[4] D.J. Broadhurst, hep-th/9604128; D.J. Broadhurst & D. Kreimer, Phys. Lett. **B393** (1997), 403; J.M. Borwein, D.M. Bradley & D.J. Broadhurst, hep-th/9611004, El. J. Comb. **4**(2) (1997), R5; D. Kreimer, hep-th/9612010; D.J. Broadhurst & D. Kreimer, hep-th/9612011; D.J. Broadhurst, hep-th/9612012.

[5] D. Kreimer, "Knots and Feynman Diagrams", to be published by Cambridge University Press.

[6] D.J. Broadhurst, J.A. Gracey & D. Kreimer, Z. Phys. **C75** (1997), 559.

[7] D. Kreimer, hep-th/9610128.

[8] A.N. Vasil'ev, Yu.M. Pis'mak & J.R. Honkonen, Theor. Math. Phys. **50** (1982), 127.

[9] D.J. Broadhurst, Z. Phys. **C32** (1988), 249.

[10] D.J. Broadhurst, R. Delbourgo & D. Kreimer, Phys. Lett. **B366** (1996), 421.

[11] J. Wess & B. Zumino, Phys. Lett. **49B** (1974), 52.

[12] J.A. Gracey, Nucl. Inst. Meth. **A389** (1997), 361.

[13] A.N. Vasil'ev, Yu.M. Pis'mak & J.R. Honkonen, Theor. Math. Phys. **46** (1981), 157; A.N. Vasil'ev, Yu.M. Pis'mak & J.R. Honkonen, Theor. Math. Phys. **47** (1981) 291.

[14] P.M. Ferreira & J.A. Gracey, 'The β-function of the Wess-Zumino model at $O(1/N^2)$' preprint LTH-416.

[15] P.M. Ferreira, I. Jack & D.R.T. Jones, Phys. Lett. **B392** (1997), 376.

[16] J. Iliopoulos & B. Zumino, Nucl. Phys. **B76** (1974), 310; S. Ferrara, J. Iliopoulos & B. Zumino, Nucl. Phys. **B77** (1974), 413.

[17] P.K. Townsend & P. van Nieuwenhuizen, Phys. Rev. **D20** (1979), 1832; L.F. Abbott & M.T. Grisaru, Nucl. Phys. **B169** (1980), 415; A. Sen & M.K. Sundaresan, Phys. Lett. **B101** (1981), 61.

[18] L.V. Avdeev, S.G. Gorishny, A.Yu. Kamenshchick & S.A. Larin, Phys. Lett. **B117** (1982), 321.

[19] A.A. Migdal, Sov. Phys. JETP **28** (1969), 1036; A.M. Polyakov, Sov. Phys. JETP **28** (1969), 533; A.A. Migdal, Phys. Lett. **37B** (1969), 198; 386; A.M. Polyakov, ZhETF Pis. Red. **12** (1970), 538; G. Parisi & L. Peliti, Lett. Nuovo Cim. **2** (1971), 627; G. Parisi, Lett. Nuovo Cim. **4** (1971), 777; G. Parisi & L. Peliti, Phys. Lett. **41A** (1972), 331.

[20] M.T. Grisaru, W. Siegel & M. Roček, Nucl. Phys. **B159** (1979), 429; S.J. Gates Jr., M.T. Grisaru, M. Roček & W. Siegel, "Superspace or one thousand and one lessons in supersymmetry" Frontiers in Physics (Benjamin Cummings, Reading, 1983).

[21] D.T. Barfoot & D.J. Broadhurst, Z. Phys. **C41** (1988), 81.

[22] J.A. Gracey, Phys. Lett. **B262** (1991), 49.

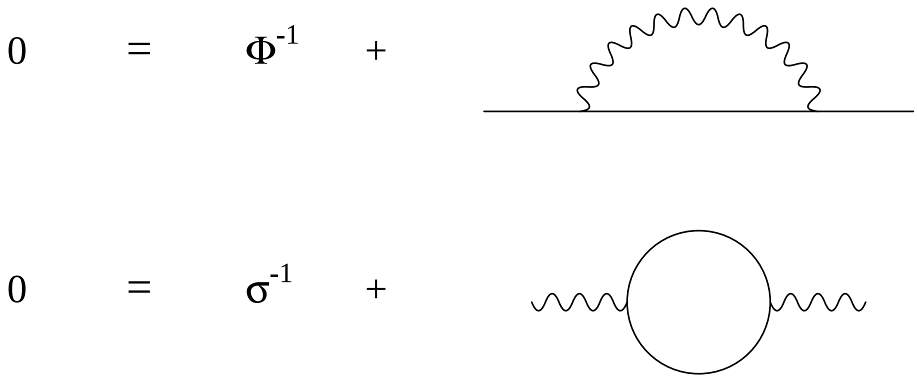

Fig. 1. Leading order Schwinger Dyson equations.

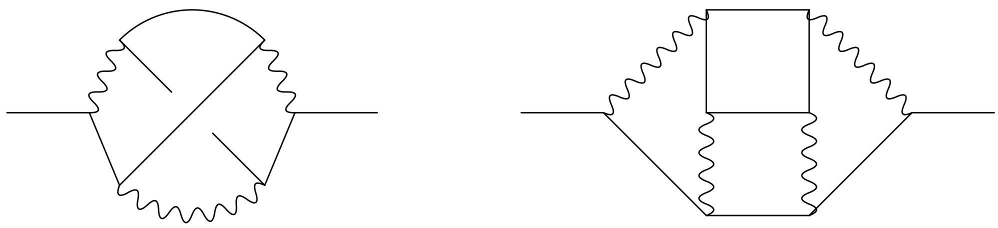

Fig. 2. Additional graphs contributing to the Φ Dyson equation at $O(1/N^3)$.

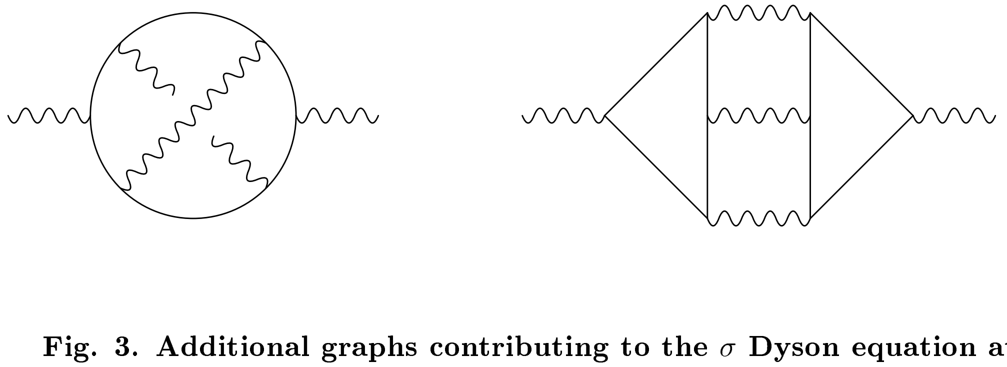

Fig. 3. Additional graphs contributing to the σ Dyson equation at $O(1/N^3)$.

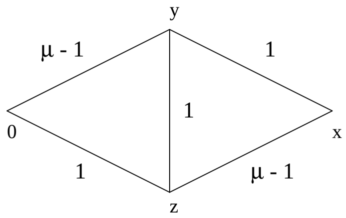

Fig. 4. Definition of the integral $P(\mu)$.

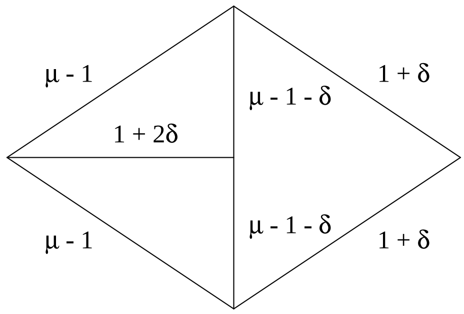

Fig. 5. Intermediate integral in the calculation of Σ_2.