Monotone local linear estimation of transducer functions

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Local polynomial regression has received a great deal of attention in the past. It is a highly adaptable regression method when the true response model is not known. However, estimates obtained in this way are not guaranteed to be monotone. In some situations the response is known to depend monotonically upon some variables. Various methods have been suggested for constraining nonparametric local polynomial regression to be monotone. The earliest of these is known as the Pool Adjacent Violators algorithm (PAVA) and was first suggested by Brunk (1958). Kappenman (1987) suggested that a non-parametric estimate could be made monotone by simply increasing the bandwidth used until the estimate was monotone.

More recently Hall and Huang (2001) have suggested a tilting method based on changing the weights used in the estimating procedure. The weights are modified in such a way that they are as close as possible to the original weights. Finally Dette et al. (2006) have suggested a monotonicity constraint which they call the DNP method. Their method involves calculating a density estimate of the unconstrained regression estimate, and using this to calculate an estimate of the inverse of the regression function.

Fan, Heckman and Wand (1995) generalized local polynomial regression to quasi-likelihood based settings. Obviously such estimates are not guaranteed to be monotone, whilst in many practical situations monotonicity of response is required. In this talk we discuss how the above mentioned monotonicity constraint methods can be adapted to the quasi-likelihood setting. We are particularly interested in the estimation of monotone psychometric function and, more generally, biological transducer functions, for which the response is often known to follow a distribution which belongs to the exponential family.

We consider some of the key theoretical properties of the monotonized local linear estimators in the quasi-likelihood setting. We compare these four methods by means of a simulation study. We investigate a variety of response models, including binary, Poisson and Gaussian. In each study we calculate monotone estimates of the response curve using each method and compare their bias, variance, MSE and MISE. We also apply these methods to analysis of data from various hearing and vision studies. We show some of the deficiencies of using local polynomial estimates, as opposed to local likelihood estimates.

References:

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