A Fast Method to Measure the Volume of a Large Cavity

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ABSTRACT How to quickly and accurately measure the volume of a large cavity is challenging. This paper presents an efficient method to measure the volume of a large conducting cavity. The proposed method is based on statistical wave theory. By measuring the $Q$ factor in the time and frequency domains, the volume of the cavity can be extracted. In the time domain, the $Q$ factor can be extracted directly from the time domain response, while in the frequency domain, the $Q$ factor depends on the volume of the cavity and the transferred power; the transferred power can be measured directly. By correcting the frequency domain $Q$ with the radiation efficiency of antennas, the $Q$ factors obtained from both the time and frequency domains are equal in a well-stirred chamber; this provides an opportunity to measure the volume of the cavity. Measurements are conducted to verify the proposed method. Although the measurement is conducted using electromagnetic waves, acoustic waves can also be used; in this case, the approach can be applied to any cavity, not limited to a conducting cavity. The advantages and the limitations of the proposed method are also discussed.

INDEX TERMS Cavity volume measurement, statistical electromagnetics, statistical acoustics.

I. INTRODUCTION

In the shipbuilding industry, to measure the volume/capacity of a ship hold is important and useful. Once the available volume and weight are known, the cargos to be shipped can be optimised to maximise the efficiency/benefit of the shipment. And another application is, after a ship was built or modified, the available volume need to be measured to make sure that it is the same as expected. It is easy to obtain the weight information from the draft marks on the bow of a ship. However, the volume information below the deck is not easy to measure because of the complex environment. Conventionally, a laser is used to build a 3D model and the volume can be calculated from the 3D model which is time consuming and expensive [1]. Moreover, inside structures like tubes and stairs need to be removed to make sure that the laser can be applied. Similarly, the same problem exists in measuring the available volume of a large granary or storehouse (Fig. 1). Since the inner structure can be complex and irregular, conventional measurement methods could be hard to implement.

In this paper, we present a new and efficient method to measure the volume of an electrically large conducting cavity using electromagnetic waves; the method can be applied not only to the volume measurement of a ship, but also to arbitrary shape metallic cavities (high $Q$ cavities). The method is based on measuring the $Q$ factor of a cavity. By measuring the
\( Q \) factor in both the time and frequency domains, the volume of the cavity can be obtained. However, we need to know the total efficiency of the transmitting (Tx) and receiving (Rx) antennas, which can be measured in a cavity with a known volume (as a calibration process). Although the measurement is done using electromagnetic waves in this paper, acoustic waves can also be used, and the cavity does not need to be metallic.

The paper is organised as follows: the theory of the proposed method is first introduced in Section II; Section III presents the measurement conducted in a reverberation chamber (RC) to verify the proposed method and finally discussion and conclusions are given in Section IV.

II. THEORY

Recently, Holloway et al proposed three methods to measure the radiation efficiency of antennas in an RC without using a reference antenna [2]. We have found that this theory can also be applied to the volume measurement when the radiation efficiencies of antennas are known. A typical two-antenna measurement setup is shown in Fig. 2. The stirrers in the RC are driven by step motors, for each stirrer position, the \( S \)-parameters are collected by using a vector network analyser (VNA) and saved in a computer. In practice we may not have stirrers, but the source stirred approach can be applied [3]; we can use one antenna at various positions/orientations or a MIMO (multiple-input and multiple-output) antenna array.

We denote it as the frequency domain \( Q_{FD} \) [2], [4]

\[
Q_{FD} = \frac{C_{RC} \langle |S_{21}|^2 \rangle}{\eta_{tot} \eta_{2tot}}
\]

where \( S_{21} \) is the transmission coefficient, \( \langle \rangle \) means the average value using any stirring method (e.g. mechanical stir, frequency stir, polarisation stir, source stir, etc.), \( C_{RC} \) is the chamber constant

\[
C_{RC} = \frac{16\pi^2 V}{\lambda^3}
\]

\( V \) is the volume of the cavity, and \( \lambda \) is the free-space wavelength. It should be noted that (2) is accurate if both antennas are ideal antennas (well-matched, no line-of-sight, 100% total efficiency). If we consider the antenna efficiency, and the line-of-sight component, the corrected \( Q_{FD} \) becomes [2]

\[
Q_{FDCor} = \frac{C_{RC} \langle |S_{21,s}|^2 \rangle}{\eta_{tot} \eta_{2tot}}
\]

where \( S_{21,s} \) is the stirred part of the \( S \)-parameter [2]

\[
S_{a,s} = S_a - \langle S_a \rangle
\]

\( \eta_{tot} \) and \( \eta_{2tot} \) are the total efficiency (having taken the loss and impedance matching into account) of antenna 1 and antenna 2 respectively. This measurement method has been widely used to extract the absorbing cross section of an object under test [5], [6].

Meanwhile, the \( Q \) factor can be obtained from the time domain [2]

\[
Q_{TD} = \frac{\lambda^2 c_0 (\tau) \eta_{tot} \eta_{2tot}}{8\pi \langle |S_{21,s}|^2 \rangle} = \frac{\lambda^2 c_0 (\tau) \eta_{tot} \eta_{2tot}}{8\pi \langle |S_{21} - \langle S_{21} \rangle|^2 \rangle}
\]

where \( \tau \) is the decay time constant of the cavity, \( \omega \) is the angular frequency. To reduce the measurement error, \( \tau \) can be measured with different stirrer positions (source positions) and then averaged, thus \( \tau \) can be replaced as \( \langle \tau \rangle \). It should be noted that it is not necessary to measure it directly in the time domain since the time domain response can be obtained from the inverse Fourier transform from the frequency domain response.

If the cavity is well-stirred (the field in the cavity is statistically uniform), the two \( Q \) factors (4) and (6) are equal [2], \( Q_{FDCor} = Q_{TD} \), thus the volume of the cavity can be obtained as

\[
V = \frac{\lambda^2 c_0 (\tau) \eta_{tot} \eta_{2tot}}{8\pi \langle |S_{21,s}|^2 \rangle} = \frac{\lambda^2 c_0 (\tau) \eta_{tot} \eta_{2tot}}{8\pi \langle |S_{21} - \langle S_{21} \rangle|^2 \rangle}
\]

Like Friis equation in free space, Hill’s equation reveals the relationship between the transmission coefficient and the \( Q \) factor in a rich isotropic multipath (RIMP) environment.

\[
e_b = \sqrt{\langle |S_{11,s}|^2 \rangle \langle |S_{22,s}|^2 \rangle / \langle |S_{21,s}|^2 \rangle} = 2
\]

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If only one antenna is used, \( \langle |S_{11}|^2 \rangle = 2\langle |S_{21}|^2 \rangle \) [2], [7]. (7) can be rewritten as

\[
V = \frac{\lambda^2 c_0 (\tau) \eta_{tot}^2}{4\pi \langle |S_{ii}|^2 \rangle} = \frac{\lambda^2 c_0 (\tau) \eta_{tot}^2}{4\pi \langle |S_{ii} - (S_{ii})|^2 \rangle}, \quad i = 1 \text{ or } 2
\] (9)

This can further simplify the measurement system, but more stirrer positions may need to realize \( e_b = 2 \). In practice, it may not be easy to realize a well-stirred cavity and could increase the measurement error.

## III. MEASUREMENT IN AN RC

In this section, we calibrate the total efficiency of antennas using an RC with known volume, and then we change the environment: load the RC using radio absorbing materials (RAMs), open the door of the RC. After the environment is changed we repeat the measurement to validate the proposed method.

### A. CALIBRATION PROCESS

The measurement scenario is shown in Fig. 3, two horn antennas are used as antenna 1 (Rohde & Schwarz® HF 906) and antenna 2 (SATIMO® SH2000). 100 stirrer positions with 3.5 degrees/step are used. At each stirrer position, 10001 frequency points are collected in the frequency range of 2.8 GHz to 4.2 GHz. The volume of the RC is 3.6 m × 4 m × 5.8 m = 83.52 m³.

![Calibration measurement in an RC.](image)

If we use the two-antenna method in (7), the measured \( Q_{FD} \) in (2) is given in Fig. 4, at each frequency a frequency stir/window with 100 nearest frequencies are used.

Note \( Q_{FD} \) is not accurate when the total efficiency of antennas are not excluded (\( Q_{FD} \) and \( Q_{TD} \) are not the same when the total efficiencies of antennas are not 100%), but \( Q_{TD} \) is not affected by the total efficiency of antennas. We need to know \( Q_{TD} \) to obtain \( \eta_{1tot} \eta_{2tot} \) [2]

\[
\eta_{1tot} \eta_{2tot} = \frac{Q_{FD}}{Q_{FDCor}} = \frac{Q_{FD}}{Q_{TD}}
\] (10)

In the time domain, the power in the cavity decays exponentially and follows \( P_0 e^{-t/\tau} \) where \( \tau \) is the decay time constant. To obtain the \( \tau \) value, we can measure it in the time domain directly [8] or measure the \( S \)-parameters in the frequency domain and then apply the inverse fast Fourier transform (IFFT) to the \( S \)-parameters (this method has a larger dynamic range). Because \( \tau \) is frequency dependent, a 10-order elliptic band-pass filter with 200 MHz bandwidth is used to filter the \( S \)-parameters, as shown in Fig. 5(a), then the IFFT is applied to the filtered \( S \)-parameters. The least-square fit is then applied to \( \ln(\text{IFFT}(S_{21}^2)) \) to extract the slope \( k \), and \( \tau = -1/k \) can be obtained. To avoid the curve fit error caused by the noise level, only the strong part of the signal is used for the least-square fit as shown in Fig. 5(b). By sweeping

![\( \tau \) extraction procedure.](image)
the centre frequency of the filter, $\tau$ with different center frequencies are obtained (Fig. 6) [9], the averaged $\tau$ is used to calculate $Q_{TD}$ (Fig. 7). Finally, $\eta_{1tot} \eta_{2tot}$ can be obtained using (10) and shown in Fig. 8.

**B. MEASUREMENT PROCESS**

After the calibration process is finished, $\eta_{1tot} \eta_{2tot}$ is obtained, and can be used to measure other cavities. Only $S_{21}$ and $\tau$ in (7) need to be measured. To validate the proposed method, we change the environment: load the RC with RAMs, open the door with 45 degrees and 90 degrees (shown in Fig. 9 respectively). In practice, it may have an entrance without a conducting door, so we open the door to emulate an imperfect cavity.

The same technique as in the calibration process is used to obtain the $\tau$ value in different scenarios, as shown in Fig. 10. The measured $\langle |S_{21,s}|^2 \rangle$ is given in Fig. 11.
Finally we apply (7) to obtain the volume of the cavity, shown in Fig. 12. The whole calibration and measurement procedure are also repeated by using one antenna format (9), results are shown in Fig. 13 and Fig. 14 respectively.

It should be noted that, in practice we may not have stirrers to change the field distribution in the cavity, but source stir [3] and frequency stir are also applicable. To verify it, we keep the stirrers steady; mount antenna 2 on a rotation platform (shown in Fig. 15) and 100 platform rotation angles with 3.5 degrees/step are used. The whole calibration and measurement procedure are repeated, results are given in Fig. 16 and Fig. 17. Note antenna 1 is not rotated, results from antenna 1 only is not available.

The results are summarised in Table 1 and discussed in the next section.

IV. DISCUSSION AND CONCLUSIONS

A fast measurement method to measure the volume of a large cavity has been proposed in this paper. Large means the cavity...
is large compared with the wavelength of the used electromagnetic/acoustic wave. The method is non-destructive and the system can be assembled as a portable volume probe to measure large cavities like ship, granary and storehouse. The measurement can be real-time and the shape of the cavity can be arbitrary (complex shape has better field uniformity). After the measurement system is calibrated, only \( \tau \) and \( S \)-parameters need to be measured. Measured volume values in the frequency range of 3 GHz \( \sim \) 4 GHz are summarised in Table 1.

It is interesting to note that, although the proposed method does not depend on the \( Q \) factor of the cavity. Results from a small \( Q \) cavity tend to have a larger standard deviation. This is easy to understand, when the \( Q \) factor reduces, the environment degrades from RIMP to free space, and the transmission coefficient becomes highly dependent on the antenna patterns, orientations, and the distance between antennas.

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