Climate model experiments reveal that transient global warming is nearly proportional to cumulative carbon emissions on multi-decadal to centennial timescales\(^1-^5\). However, it is not quantitatively understood how this near linear dependence between warming and cumulative carbon emissions arises in transient climate simulations\(^6,^7\). Here, we present a theoretically-derived equation of the dependence of global warming on cumulative carbon emissions over time. For an atmosphere-ocean system, our analysis identifies a surface warming response to cumulative carbon emissions of 1.5±0.7 K for every 1,000 Pg of carbon emitted. This surface warming response is reduced by typically 10 to 20% by the end of the century and beyond. The climate response remains nearly constant on multi-decadal to centennial timescales as a result of partially-opposing effects of oceanic uptake of heat and carbon\(^8\). The resulting warming then becomes proportional to cumulative carbon emissions after many centuries, as noted earlier\(^9\). When we incorporate estimates of terrestrial carbon uptake\(^10\), the surface warming response is reduced to 1.1±0.5 K for every 1000 Pg of carbon emitted, but this modification is unlikely to significantly affect how the climate response changes over time. We suggest that our theoretical framework may be used to diagnose the global warming response in climate models and mechanistically understand the differences between their projections.

Warming of the Earth’s surface, \(\Delta T(t)\), depends on the increase in radiative forcing, \(R(t)\), from atmospheric CO\(_2\) minus the net heat flux into the Earth System, \(N(t)\)^11

\[
\Delta T(t) = \frac{R(t) - \varepsilon N(t)}{\lambda},
\]

where \(\Delta T\) is the change in global mean surface temperature relative to the pre-industrial era, \(\lambda\) is the equilibrium climate feedback parameter [\((\text{W m}^{-2}) \text{ K}^{-1}\)] or equivalently \(\lambda^{-1}\) is the climate sensitivity\(^12\), \(R(t)\) and \(N(t)\) are positive when downward and in W m\(^{-2}\). The net heat flux, \(N(t)\), is dominated by ocean heat uptake, since over 90% of \(N(t)\) passes into the ocean interior\(^13\). \(\varepsilon\) is the non-dimensional ocean heat uptake efficacy\(^11\), accounting for how ocean heat uptake may be more effective than radiative forcing in altering \(\Delta T\). The radiative forcing, \(R(t)\), taken to be at the top of the troposphere, is directly linked to atmospheric CO\(_2\) via a logarithmic relationship,\(^14\)
\[ R(t) = a \Delta \ln \text{CO}_2(t), \] (2)

where CO\(_2\) is measured as a mixing ratio (ppmv), \( a = 5.35 \text{ Wm}^{-2} \) is a CO\(_2\) radiative-forcing coefficient and \( \Delta \ln \text{CO}_2(t) \) represents \( \ln \text{CO}_2(t) - \ln \text{CO}_2(t_0) \) where \( t_0 \) is the preindustrial. An increase in cumulative carbon emissions naturally leads to a longterm increase in atmospheric CO\(_2\), radiative forcing and surface warming, which might be augmented by further warming from non-CO\(_2\) greenhouse gases or partly opposed by cooling from aerosols\(^{6,7,15}\). Focussing on the dominant effect of cumulative carbon emissions on global warming\(^{15}\), the relationship between the logarithmic change in atmospheric CO\(_2\), \( \Delta \ln \text{CO}_2(t) \), and cumulative carbon emissions over time must be found.

The rise in ln CO\(_2\) from cumulative emissions is affected by the uptake of anthropogenic carbon by the ocean and terrestrial carbon systems, which are comparable in magnitude in the present day\(^{16}\), but with much larger uncertainties for the terrestrial system\(^{17}\). To identify the different roles played by ocean and terrestrial carbon uptake, we first consider the surface warming response to carbon emissions in an atmosphere-ocean only system, and then assess the effect of incorporating estimates of the terrestrial system\(^{10}\). For a combined atmosphere-ocean system, atmospheric CO\(_2\) can be related to cumulative carbon emissions, \( I_{\text{em}} \) (PgC), by taking into account changes in the carbon inventories (Supplementary),

\[ \Delta \ln \text{CO}_2(t) = \frac{I_{\text{em}}(t) + I_{\text{Usat}}(t)}{I_B}, \] (3)

where the carbon undersaturation of the ocean, \( I_{\text{Usat}} \) (PgC), is defined by how much carbon the ocean needs to take up for an equilibrium to be reached with the atmosphere (Supplementary) and \( I_B = 3500 \pm 400 \) PgC is the buffered carbon inventory of the atmosphere and ocean\(^{9,18,19}\).

The goal is to identify how the surface warming responds to cumulative carbon emissions, extending previous empirical diagnostics from climate models\(^{1,5,7,8}\). By combining equations (1) and (2) with our new relationship (3), we now provide a time-dependent equation defining the climate response to cumulative carbon emissions for an atmosphere-ocean system,

\[ \Delta T(t) = \frac{a}{\lambda I_B} \left( 1 - \frac{\varepsilon N(t)}{R(t)} \right) \left( 1 + \frac{I_{\text{Usat}}(t)}{I_{\text{em}}(t)} \right) I_{\text{em}}(t). \] (4)

Exploiting equation (4), the current surface warming is then linked to cumulative carbon emissions by a proportionality factor of 1.5±0.7 K for every cumulative 1000 PgC emitted into the atmosphere-ocean system (Fig. 1a) based upon present-day estimates of surface warming and how anthropogenic carbon is partitioned between the atmosphere and ocean\(^{6,7}\) (Methods). This proportionality factor, \( \Delta T(t)/I_{\text{em}}(t) \), is the same as the metric used to define the transient climate response to cumulative carbon emissions (TCRE), taken when CO\(_2\) reaches double its preindustrial value\(^1\). Our TCRE estimate is consistent with estimates from intermediate Earth system models\(^{1,20}\) of 1.5 K per 1000 PgC with a range of 1.0 to 2.1 K per 1000 PgC and from more recent CMIP5 simulations\(^{5,16}\) with a range of 0.8 to 2.5 K per 1000 PgC.

Our theory suggests that many centuries to millennia after emissions cease, the surface warming asymptotes to being simply proportional to cumulative emissions,
\[ \Delta T_{eq} = \left( \frac{a}{\lambda I_B} \right) I_{em} \] (ref 9) (Fig. 1c, dashed line), as both \( N(t) \) and \( I_{Usat}(t) \) approach zero in equation (4) as the ocean approaches a thermal and carbon equilibrium with the atmosphere. For this long-term limit, the surface warming response is typically 1.2 K per 1000 PgC (ref 9) with a range of 0.6 to 1.9 K per 1000 PgC based upon the uncertainty in climate sensitivity\(^1\). Hence, our analysis suggests that the surface warming response to cumulative carbon emissions remains broadly stable over time for a coupled atmosphere-ocean without climate-carbon feedbacks, only decreasing by about 20% from the present day to many centuries in the future.

We now test equation (4), and our prediction for a limited temporal variation in the surface warming response to cumulative carbon emissions, using a coupled atmosphere-ocean model of intermediate complexity designed for the Earth System (GENIE;\(^2\) Methods). Equation (4) allows the surface warming to be accurately related to the effects of cumulative carbon emissions (Fig. 1a,b) as long as the effects of ocean heat uptake, \( \varepsilon N(t) \), and ocean carbon undersaturation, \( I_{Usat}(t) \) (Fig. 2a,b), are accounted for; the discrepancy between the model response and our theory is less than 0.25 K for a range of emission scenarios (Fig. 1b), compared with a warming signal reaching 4 K. The proportionality of surface warming to cumulative carbon emissions remains relatively stable over time (Fig. 1a). The model also reveals that the long-term equilibrium warming is proportional to cumulative emissions after many centuries (Fig. 1c) consistent with our theory, but with slightly enhanced warming further increasing atmospheric CO\(_2\) due to carbon-climate feedbacks, such as CO\(_2\) solubility decreasing with higher temperatures (Fig. 2c,d; Supplementary Figure 1).

We now apply our time-dependent equation (4) to understand why the warming remains nearly proportional to cumulative carbon emissions. The surface warming response to cumulative carbon emissions (or the TCRE), \( \Delta T/I_{em} \), can be interpreted as the sensitivity of surface warming to radiative forcing, \( \Delta T/R = (1-\varepsilon N/R) \lambda \), multiplied by the sensitivity of radiative forcing to cumulative carbon emissions, \( R/I_{em} = (a/I_B)(1+I_{Usat}/I_{em}) \). The sensitivity of surface warming to radiative forcing (Fig. 3a) is initially reduced by ocean heat uptake from equation (1), but then later increases as ocean heat uptake diminishes, such that the term \( (1-\varepsilon N/R) \) in equation (4) increases toward 1. Meanwhile, the sensitivity of radiative forcing to cumulative emissions (Fig. 3b) is initially increased by excess CO\(_2\) in the atmosphere due to ocean carbon undersaturation, \( I_{Usat} \) (Fig. 2b), in equation (3), but then declines through ocean uptake of carbon, such that \( I_{Usat} \) decreases to zero and the term \( (1+I_{Usat}/I_{em}) \) in equation (4) decreases towards 1. The net result of these combined responses is a decrease in the surface warming response to cumulative carbon emissions, \( \Delta T/I_{em} \), by between 8% and 22% from 2011 to 2100 (Fig. 3c), broadly consistent with the modelled decrease of 6% to 25% (Fig. 3c,d). Thus, the climate effect of ocean heat and carbon uptake in equation (4) is to partially compensate each other on a centennial timescale\(^8\).

The modelled variation of the climate response to cumulative emissions is relatively insensitive to the details of the emission scenario\(^2\) (Fig. 3a-c). This relatively weak dependence is in accord with how the normalised time-dependent terms \( (1-\varepsilon N/R) \) and \( (1+I_{Usat}/I_{em}) \) in equation (4) only vary by 8% and 10% respectively across the six emission scenarios at 2100 (Fig. 3a,b), although the sensitivity in TCRE to emission scenario is larger in another Earth System model\(^2\).

Ocean uptake of heat and carbon leads to broadly-opposing climate responses due to the contrasting trends in the sensitivities of the warming to radiative forcing (Fig. 3a) and
radiative forcing to cumulative carbon emissions (Fig. 3b). The ocean sequestering of heat and carbon are both achieved in a similar manner: there is a relatively rapid drawdown of heat and carbon from the atmosphere into the surface mixed layer on annual to decadal timescales (Fig. 4a); a subsequent ventilation of the main thermocline and upper ocean over several decades to a century\(^{25}\) (Fig. 4b); and a slower ventilation of the deep ocean over many centuries or even millennia\(^{26}\) (Fig. 4c). The ocean thermal and carbon responses can differ though, over several decades to centuries, such as by declining heat uptake leading to a long-term warming after emissions cease\(^{27}\) or circulation changes modifying the regional response\(^{28}\), for example, a drift in the Atlantic meridional overturning alters the thermal uptake more than the carbon uptake (Supplementary Figures 2 & 3). These differences in thermal and carbon response are partly reflected in the nature of their terms in equation (4): the heat uptake term \((1 - \varepsilon N / R)\) is more sensitive in depending on the instantaneous ocean heat flux and its efficacy\(^{11,27}\), whereas the carbon uptake term \((1 + I_{usat} / I_{em})\) depends on the cumulative ocean carbon uptake.

On longer timescales of several thousand years, the model sensitivity of the warming to cumulative carbon emissions is greater than the theory by between 0.3 to 0.4 K for every 1000 PgC emitted (Fig. 3c,d). This offset is again due to ocean carbon-climate feedbacks\(^{10,29}\) (Fig. 2c,d), which can be further modified by sediment interactions\(^{30}\) (Supplementary Figure 1).

The real climate system includes the terrestrial system, as well as the atmosphere and ocean. The ocean still plays the dominant role in the uptake of heat\(^{13}\), but the present-day carbon uptake by the terrestrial system is a similar order of magnitude to that by the ocean\(^{16}\). Now consider the effect of the terrestrial system: extending equation (3) for \(\Delta \ln CO_2\) to include the effect of a change in the terrestrial uptake of anthropogenic carbon (Supplementary) and again combining with equations (1) and (2) provides an equation for the surface warming response to cumulative carbon emissions,

\[
\Delta T(t) = \frac{a}{\lambda I_B} \left(1 - \frac{\varepsilon N(t)}{R(t)}\right) \left(1 + \frac{I_{usat}(t)}{I_{em}(t)} - \frac{\Delta I_{ter}(t)}{I_{em}(t)}\right) I_{em}(t),
\]

where \(\Delta I_{ter}\) (PgC) represents the change terrestrial carbon storage since the preindustrial and \(I_{em}\) is now the cumulative carbon emitted into the combined atmosphere-ocean-terrestrial system, making \(\Delta I_{ter}/I_{em}\) the fraction of cumulative carbon emissions taken up by the land. Exploiting equation (5) and present-day estimates of cumulative terrestrial carbon uptake, \(\Delta I_{ter}\), the proportionality of surface warming to cumulative carbon emissions (the TCRE), \(\Delta T/\Delta I_{em}\), reduces to 1.1±0.5 K per 1000 PgC for an atmosphere-ocean-terrestrial system, a decrease in the TCRE of 0.4 K per 1000 PgC due to increased terrestrial drawdown of carbon (Methods). Our estimate of the TCRE remains within the IPCC likely range\(^{5,7}\) of 0.8 to 2.5 K per 1000 PgC. While the terrestrial system is then important in determining the value of the TCRE, the trend in the TCRE might still be limited in time, since a model-intercomparison study\(^{10}\) suggests that \(\Delta I_{ter}/I_{em}\) only decreases from the present-day value of 0.28±0.17 by typically -0.01 to -0.14 by 2100 and, hence, leads to the TCRE changing by less than 20% from 2011 to 2100 (Methods). However, our view of how the TCRE is controlled over time might be modified by additional climate forcing from short-lived climate agents\(^{15}\) or non-linear feedbacks\(^{16}\), such as release of methane from direct emissions, marine hydrates or permafrost.
Our study emphasizes how transient global warming is proportional to cumulative carbon emissions through the partial compensation between the ocean and terrestrial uptake of heat and excess carbon, which is expected to apply on multi-decadal to centennial timescales. The theoretical framework may be used to diagnose existing climate models and understand their differences, as well as explore the global warming response over a wider parameter regime than usually investigated by extrapolating from existing simulations by climate models. In terms of wider policy implications, our theory reiterates a simple message: the more cumulative carbon emissions are allowed to increase, the more global surface warming will also increase.

Methods

Evaluating the present day warming response for the coupled atmosphere-ocean system

For the atmosphere-ocean only system, $I_{em}$ is estimated as the sum of the anthropogenic increases in atmospheric CO$_2$, equal to 240±10 PgC in 2011\textsuperscript{10}, and ocean carbon\textsuperscript{16}, equal to 155±30 PgC, then giving $I_{em} = 395±32$ PgC in 2011 assuming normal uncertainty distributions. Present-day ocean carbon undersaturation, $I_{USat}$, is estimated as the difference between the eventual uptake of ocean carbon in a model with atmospheric CO$_2$ fixed at a mixing ratio of 391ppm at 2011 and the real ocean carbon uptake in 2011. The eventual ocean carbon uptake with CO$_2$ fixed at 391ppm is 952 PgC in the GENIE model\textsuperscript{22}, integrated without climate feedbacks for 25000 years to reach a steady state. Therefore, $I_{USat}$ in 2011 is estimated as 952-(155±30) PgC = 797±30 PgC.

Calculating the present-day sensitivity of warming due to emissions from individual terms in equation (4) is problematic, because there is a wide range in the estimates of climate sensitivity\textsuperscript{12} $\lambda^I$ of 0.5 to 1.2 (Wm$^{-2}$)$^{-1}$ or with a revised lower bound\textsuperscript{25} of 0.375 (Wm$^{-2}$)$^{-1}$. $N$ is difficult to assess due to decadal changes in ocean heat content, and $R$ has a large uncertainty from sources other than well-mixed greenhouse gases. From the recent IPCC 5\textsuperscript{th} Assessment Report,\textsuperscript{6} the warming, $\Delta T$, ascribed to the change in all well-mixed greenhouse gases from 1951 to 2011 is 0.9±0.4 K and the radiative forcing from all well-mixed greenhouse gases, $R$, is 2.83±0.27 W m$^{-2}$ in 2011. Their ratio suggests $\Delta T/R= (1-\epsilon N/R)/\lambda$ of 0.32±0.14 K (W m$^{-2}$)$^{-1}$ in equation (1), assuming that warming prior to 1951 was negligible. Our present day estimates of the warming response to cumulative emissions (TCRE), $\Delta T/I_{em} = (a(I_{USat} I_{em} - \Delta I_{USat}/I_{em}))$ is 1.5±0.7 K per 1000 PgC, equation (4), based on estimates for 2011 and using $\sigma=5.35$Wm$^{-2}$ and $I_{em}=3500±400$ PgC, representing the range of evaluated model values of $I_{em}$ (refs 18, 19).

Evaluating the warming response for the coupled atmosphere-ocean-terrestrial system

The anthropogenic cumulative carbon emission into the atmosphere-ocean-terrestrial system, $I_{em}$, is estimated as 545±85 PgC in 2011\textsuperscript{10}, while the cumulative carbon uptake by the terrestrial system is $\Delta I_{ter}=150±90$ PgC, giving $\Delta I_{ter}/I_{em}=0.28±0.17$. This estimate, combined with $I_{USat}$ analysis above, results in a value for the term $(1+I_{USat}/I_{em} - \Delta I_{USat}/I_{em})=2.2±0.6$. The warming response to cumulative emissions (TCRE), $\Delta T/I_{em} = (a(I_{USat} I_{em} - \Delta I_{USat}/I_{em}))$, is 1.1±0.5K per 1000PgC, equation (5), using estimates for 2011. Based on a recent model-intercomparison project\textsuperscript{10}, 8 of the 11 coupled models of the atmosphere-ocean-terrestrial system simulate $\Delta I_{em}/I_{em}$ values ranging from 0.27 to 0.14 in 2100, a change of only -0.01 to -0.14 from the present-day best estimate of 0.28±0.17. This reduction in $\Delta I_{em}/I_{em}$ leads to the TCRE for an atmosphere-ocean-terrestrial system to change by +10% to -21% from 2011 to 2100, based upon combining with GENIE values in equation (5).

GENIE model formulation and analysis of output

The GENIE Earth System model\textsuperscript{22,30} is configured as a coarse-resolution atmosphere-ocean system, containing coupled circulation and biogeochemistry with 16 ocean layers and 36x36 equal-area grid elements over the globe. This version of GENIE\textsuperscript{30} includes climate feedbacks, in which increased CO$_2$ is allowed to heat the system, but with sediment interactions disabled. An additional model integration is included without climate feedbacks in order to diagnose their effect. Both model configurations are forced to reproduce historical CO$_2$ concentrations to 2010 and then forced until 2100, either with Representative Concentration Pathways\textsuperscript{6} or SRES emissions\textsuperscript{22}. The model integrations forced by SRES emissions are continued to year 5000 with the annual emission rate reduced to zero between 2100 and 2150.
In the GENIE model, the climate sensitivity $\lambda^{-1}$ is 0.78 K(Wm$^{-2}$)$^{-1}$, $I_B$ is 3500 PgC, and a slightly larger value of $\alpha$=5.77 Wm$^{-2}$ is employed. The efficacy of ocean heat uptake, $\varepsilon$, is diagnosed from knowing $R(t)$, $N(t)$ and $\Delta T(t)$ in equation (1); in our theoretical analysis of GENIE output we apply the average GENIE value over six emission scenarios$^{23}$ for the 21$^{st}$ century of $\varepsilon$=1.071±0.008, although other climate models$^{11,27,28}$ reveal larger values of typically 1.3 to 1.4.

$I_{Usat}(t)$ is diagnosed from the model configuration without climate feedbacks, since the derivation of (5) (Supplementary) assumes constant marine biological drawdown and carbon storage. All other quantities are diagnosed using the default model configuration including climate feedbacks, in which marine biological carbon drawdown is allowed to alter. $I_{Usat}$ is evaluated from the difference between the preformed and saturated DIC integrated over the globe, $I_{Usat}(t) = -V \overline{\text{DIC}_{pre}}(t) = -V \left( \overline{\text{DIC}_{pre}}(t) - \overline{\text{DIC}_{sat}}(t) \right)$, where $\overline{\text{DIC}_{pre}}(t)$ is the preformed DIC and $\overline{\text{DIC}_{sat}}(t)$ is the saturated DIC with respect to instantaneous atmospheric CO$_2$(t) and based upon global-mean ocean preformed titration alkalinity, temperature and salinity; the model includes additional preformed tracers for titration alkalinity and DIC, which are fixed to their respective tracer values in the surface ocean, but are conserved in the ocean interior. The heat flux, $N$, is diagnosed from the rate of change in ocean heat content, based upon the rate of change in global-mean ocean temperature change multiplied by the global-mean ocean heat capacity$^9$. 
References


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P.G. and R.G.W. provided the theory with P.G. deriving the equations for the transient adjustment. A.R. conducted the supporting numerical modelling with GENIE. P.G. and R.G.W. led the writing of this study, and contributed equally, and A.R. provided comments on the manuscript.

Competing Financial Interests

The authors declare no competing financial interests.

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Figure 1. Global surface warming, $\Delta T$ (K), versus cumulative carbon emissions, $I_{em}$ (PgC): (a) Surface warming over the 21st century versus cumulative emissions based on our theory, equation (4) with inputs from our coupled atmosphere-ocean model of intermediate complexity (GENIE) from year 2010 to 2100 for either IPCC concentration pathways or emission scenarios; (b) The discrepancy in surface warming direct from model output of $\Delta T$ and $\Delta I_{em}$ minus the theory (a) is less than 0.2 K; (c) surface warming evaluated after many centuries up to 5000 years (dots) from our equilibrium theory (dashed line) and our model output forced by the emission scenarios (the theory uses model values of $a$, $\lambda$ and $I_B$ using equation (4) with $N=0$ and $I_{Usat}=0$).
Figure 2. Cumulative carbon emissions, cumulative ocean carbon undersaturation and $\Delta \ln CO_2$ over time in our coupled atmosphere-ocean model (GENIE) for six 21st century emission scenarios\textsuperscript{23}: (a) Cumulative emissions, $I_{em}$ (1000PgC); (b) Ocean undersaturation, $I_{Usat}$ (1000PgC), where $I_{Usat}$ is diagnosed in the model configuration without climate feedbacks permitted; (c) $\Delta \ln CO_2$ calculated from equation (3), assuming no climate feedbacks altering the ocean carbon cycle (the equivalent radiative heat flux, $\Delta \ln CO_2(t)$ ranges from +2 to +5 Wm$^{-2}$ at year 5000); (d) The error in $\Delta \ln CO_2$ from the model minus the theory, equation (3), for the model configurations without climate feedbacks (solid lines) and with climate feedbacks (dashed lines), where climate feedbacks lead to a consistent slight increase in $CO_2$. 
Figure 3. Thermal and carbon response to cumulative carbon emissions over 3000 years for six emissions scenarios in our coupled atmosphere-ocean model (marked in box): (a) The sensitivity of surface warming to radiative forcing diagnosed from theory, $\Delta T/R = (1-\varepsilon N/R)\lambda$, increases over time (left-hand axis) as the entire ocean approaches a thermal equilibrium and the normalised heat uptake, $N/R$, goes to zero (right-hand axis); (b) The sensitivity of radiative forcing to cumulative emissions diagnosed from theory, $R/I_{em} = (a/I_B)(1+I_{Usat}/I_{em})$, decreases over time (left-hand axis) as the entire ocean approaches a carbon equilibrium and the normalised ocean carbon undersaturation, $I_{Usat}/I_{em}$, goes to zero (right-hand axis); (c) The surface warming response to cumulative emissions (TCRE) diagnosed from theory (solid lines), $\Delta T/I_{em} = (a/(\lambda I_B))(1-\varepsilon N/R)(1+I_{Usat}/I_{em})$, decreases only slowly over time due to the competing effects of the ocean thermal and carbon responses, expressed in $(1-\varepsilon N/R)(1+I_{Usat}/I_{em})$ (right-hand axis); (d) The error in the TCRE from (c)-(a). The model warming response is slightly larger than the theory due to positive carbon-climate feedbacks in the model, such as ocean solubility-$\text{CO}_2$ feedback and ocean biological feedback to changes in nutrient supply and circulation.
Figure 4. A schematic depiction of the ocean thermal and carbon response to anthropogenic carbon emissions (left and right panels): (a) the initial response, then after (b) the upper ocean and (c) the deep ocean approach an equilibrium with the atmosphere. The ocean is depicted as a two layer system: warm waters with low dissolved inorganic carbon (DIC) within the thermocline, overlying high DIC in the cold, deep ocean. Surface waters take up heat and excess CO₂ from the atmosphere, then are physically transferred via ventilation pathways (grey arrows, nominally for the Atlantic). In (a) carbon emissions lead to radiative forcing $R$ inducing surface warming $\Delta T$ and an ocean heat uptake, $N$, as well as an ocean uptake of CO₂. After several decades in (b), much of the upper ocean approaches an equilibrium, so that ocean uptake only continues at high latitudes. Eventually after many centuries in (c), the deep ocean also approaches an equilibrium, so that the ocean uptake of heat and CO₂ then ceases.
Supplementary Information:

Derivation of carbon partitioning and ocean carbon undersaturation

Consider an atmosphere-ocean system at steady state containing $M\text{CO}_2$ amount of CO$_2$ in the atmosphere, $I_{ter}$ amount of carbon in the terrestrial biosphere and $V\text{DIC}$ amount of Dissolved Inorganic Carbon (DIC) in the ocean; here $M$ is the molar volume of the atmosphere, $V$ is the volume of the ocean and an overbar indicates an average over the entire ocean, all three terms in PgC. Carbon is emitted into the atmosphere and cumulative emissions reaches $I_{em}(t)$ at time $t$. A perturbation carbon inventory equation for the atmosphere, ocean and terrestrial system is written as,

$$M\delta\text{CO}_2(t) + V\delta\text{DIC}(t) + \delta I_{ter} = I_{em}(t), \quad (S1)$$

where $\delta$ represents a small change from the preindustrial. Assuming that the global-mean ocean concentrations of DIC due to remineralised biological soft tissue and CaCO$_3$ remain constant, the change in global-mean DIC is split into two components,

$$\delta\text{DIC}(t) = \delta\text{DIC}_{sat}(t) + \delta\text{DIC}_{res}(t), \quad (S2)$$

where $\text{DIC}_{sat}(t)$ is the saturated DIC concentration of a water parcel if brought into carbon equilibrium with the atmospheric CO$_2$ mixing ratio at time $t$, and $\text{DIC}_{res}$ is the residual DIC concentration, defined as total DIC minus the sum of $\text{DIC}_{sat}$ and the DIC from soft tissue and calcium carbonate remineralisation. $\text{DIC}_{res}$ can be positive or negative. Here, $\text{DIC}_{sat}$ and $\text{DIC}_{res}$ are non-conservative tracers in the ocean interior, since $\text{DIC}_{sat}$ is defined here in terms of the instantaneous CO$_2(t)$ and is re-calculated across the whole ocean whenever atmospheric CO$_2$ changes. Combining equations (S1) and (S2) gives,

$$M\delta\text{CO}_2 + V\delta\text{DIC}_{sat}(t) = I_{em}(t) - V\delta\text{DIC}_{res}(t) - \delta I_{ter}(t). \quad (S3)$$

This perturbation carbon balance is re-written, defining the left-hand side of equation (S3) in terms of the buffered carbon inventory, $I_B$ (ref 18, 19),

$$\left(M\text{CO}_2 + \frac{V\text{DIC}_{sat}(t)}{B}\right)\delta\ln\text{CO}_2(t) = I_B\delta\ln\text{CO}_2(t) = I_{em}(t) - V\delta\text{DIC}_{res}(t) - \delta I_{ter}(t), \quad (S4)$$
where the Revelle buffer factor $B \equiv \left( \frac{\delta \text{CO}_2/\text{CO}_2}{\delta \text{DIC}_{\text{sat}}/\text{DIC}_{\text{sat}}} \right)$ and

$\delta \ln \text{CO}_2(t) = \ln \text{CO}_2(t) - \ln \text{CO}_2(t_0) = \delta \text{CO}_2(t)/\text{CO}_2(t)$, where $t_0$ is the time before emissions began, such that $I_{\text{em}}(t_0)=0$. The buffered carbon inventory, $I_B$, is nearly constant for perturbations on the right-hand side of equation (S4) of less than 4000 PgC (ref. 18, 19), allowing (S4) to be written for perturbations up to this size as

$$I_B \Delta \ln \text{CO}_2(t) = I_{\text{em}}(t) - V \Delta \overline{\text{DIC}_{\text{res}}}(t) - \Delta I_{\text{ter}}(t),$$  \hspace{1cm} (S5)

where $\Delta$ represents the change from the preindustrial. Since the system is initially at steady state with low global-mean DIC$_{\text{res}}$, the value of DIC$_{\text{res}}$ at time $t$ approximates the change since preindustrial,

$$\Delta \overline{\text{DIC}_{\text{res}}}(t) = \overline{\text{DIC}_{\text{res}}}(t) - \overline{\text{DIC}_{\text{res}}}(t_0) \approx \overline{\text{DIC}_{\text{res}}}(t),$$

giving,

$$\Delta \ln \text{CO}_2(t) = \frac{I_{\text{em}}(t) + I_{\text{usat}}(t) - \Delta I_{\text{ter}}(t)}{I_B},$$  \hspace{1cm} (S6)

where we define the carbon undersaturation of the ocean as $I_{\text{usat}}(t) = -V \overline{\text{DIC}_{\text{res}}}(t)$; equation (S6) is then employed in the derivation of equation (5). For an atmosphere-ocean only system, $\Delta I_{\text{ter}} = 0$ and leads to (S6) simplifying to equation (3),

$$\Delta \ln \text{CO}_2(t) = \frac{I_{\text{em}}(t) + I_{\text{usat}}(t)}{I_B}. $$  \hspace{1cm} (S7)
Supplementary Figure 1. The sensitivity of warming to cumulative emissions over time for six emissions scenarios, diagnosed from theory and direct model output for two configurations of the coupled atmosphere-ocean model (GENIE, model detail in methods): (a) $\Delta T_{/\text{I}em}$ in K (PgC)$^{-1}$ for the theory (as in Fig. 2c); (b) $\Delta T_{/\text{I}em}$ over time for the default coupled model configured with climate feedbacks on, but without interactive sediments; (c) The error in $\Delta T_{/\text{I}em}$ in the default coupled model with climate feedbacks and without sediments minus theory [panel (b) minus panel (a)]; (d) $\Delta T_{/\text{I}em}$ over time for the coupled model with climate feedbacks on and interactive CaCO$_3$ sediment interactions turned on; and (e) The error in $\Delta T_{/\text{I}em}$ over time for the coupled model with both climate feedbacks and interactive CaCO$_3$ sediment interactions minus theory, [panel (d) minus panel (a)]. The effects of climate feedbacks and CaCO$_3$ sediment interactions do not change $\Delta T_{/\text{I}em}$ significantly over the coming centuries from our theory based on equation (4).
Supplementary Figure 2. The GENIE Earth system model projection of the rate of change in temperature; model details in Methods. The B1 IMAGE SRES CO$_2$ emissions scenario is chosen in order to minimize the distorting effect of AMOC weakening. There is a zonal average of the Pacific basin on the left-hand side and a zonal average for the Atlantic on the right-hand side. Rates of change are calculated as the difference between annual average for the year marked in the bottom left-hand corner of each panel, compared to the annual average of the proceeding year. The color scale for temperature is based on equal increments in log$_{10}$ space to illustrate the response on different time-scales and regions of the ocean.
Supplementary Figure 3. The GENIE Earth system model projection of the rate of change in DIC; details as in Figure S2. Rates of change are calculated as the difference between annual average for the year marked in the bottom left-hand corner of each panel, compared to the annual average of the proceeding year. The color scale for DIC is based on equal increments in log$_{10}$ space to illustrate the response on different time-scales and regions of the ocean.