

**CURVES OF MARGINAL STABILITY IN N=2 SUPER-QCD\***

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Seiberg and Witten's solution<sup>1</sup> to  $N=2$   $SU(2)$  Yang-Mills with  $N_f=0$  flavors has a one-complex-dimensional Coulomb branch of degenerate vacua labeled by a coordinate  $u$ . The effective  $U(1)$  theory is described in terms of two functions  $a(u)$  and  $a_D(u)$ . The gauge coupling,  $\tau \equiv (\theta/2\pi) + i(4\pi^2/g^2)$  is given by  $\tau = da_D/da$ ; it must satisfy  $\text{Im}(\tau) > 0$ . The theory is governed by a dynamically generated strong-coupling scale which we set to 1.

The mass of a dyon hypermultiplet with electric and magnetic charges  $n_e$  and  $n_m$  is given by  $M = \sqrt{2}|n_m a_D(u) + n_e a(u)|$ . Whenever  $\text{Im}(a_D/a) = 0$ , any dyon becomes marginally unstable to decay into two or more other dyons (conserving electric and magnetic charges). This note presents a simple argument that determines the shape of the curve of marginal stability  $\text{Im}(a_D/a) = 0$ .

The effective theory has a duality group that acts on  $\begin{pmatrix} a_D \\ a \end{pmatrix}$  as a vector under  $SL(2, Z)$ , and on  $\tau$  in the usual way. Note that  $f(u) \equiv a_D/a$  transforms like  $\tau$ .

The  $U(1)$  effective theory breaks down at  $u = \pm 1$  and  $\infty$ , where a dyon hypermultiplet becomes massless. The  $SL(2, Z)$  monodromies around these points are  $ST^2S^{-1}$ ,  $(TS)T^2(TS)^{-1}$ , and  $-T^2$ . These matrices generate the group  $\Gamma(2) \subset SL(2, Z)$ .  $u(\tau)$  is a one-to-one map of a single fundamental domain of  $\Gamma(2)$  onto the complex plane, which has cusp points at  $\tau = 0, 1$ , and  $i\infty$ . These cusp points correspond to the three singularities in the  $u$ -plane, and are fixed points of the corresponding  $SL(2, Z)$  monodromies—see Fig. 1.

The range of the function  $f(u)$  is a subset of the complex plane with similar properties to the fundamental domain of  $\tau$ .  $\Gamma(2)$  acts identically on both the  $f$ -plane and the  $\tau$ -plane, and its generators fix the same 3 points. However, since  $\text{Im} f$  is not necessarily positive the range of  $f$  may extend below the real axis, unlike  $\tau$ . Indeed, since we know (from expanding the explicit expressions<sup>1</sup> for  $a$  and  $a_D$  around  $u = \pm 1$ ) that there are whole lines where  $f$  is real, it follows that  $f^{-1}$  must map an infinite number of  $\Gamma(2)$  domains, both above and below the real axis,

onto the  $u$ -plane.

There is only one possibility for the shape of the range of  $f(u)$ , due to the fact that the generators  $ST^2S^{-1}$  and  $(TS)T^2(TS)^{-1}$  are of infinite order, which implies that the opening angles of the corresponding cusps must also be of infinite order, *i.e.*, 0 or  $2\pi$ . An opening angle of 0 would correspond to a single fundamental domain of  $\Gamma(2)$ , which we have ruled out. Opening angles of  $2\pi$  correspond to the domain shown in Fig. 1, a full strip in the  $f$ -plane with one  $\Gamma(2)$  domain removed. It is easy to see that the monodromies for this region are correct. As a check, it is easily verified using the explicit expressions<sup>1</sup> that  $f(0) = -(i \pm 1)/2$ .

The curve of marginal stability is the image under  $f^{-1}$  of the interval  $[-1,1]$ , which is a simple closed curve in the  $u$ -plane (with  $f(-1) = \pm 1$  and  $f(+1) = 0$ ) as conjectured in Ref. 1. Outside of this curve are the images of the infinite number of  $\Gamma(2)$  domains between  $\text{Re}(f) = +1$  and  $-1$  and with  $\text{Im}(f) \geq 0$ . Inside the curve are the images of all but one of the  $\Gamma(2)$  domains with  $\text{Im}(f) < 0$ .

The curve  $\text{Im} f = 0$  has been shown by independent methods to be simple and closed.<sup>2</sup> Also, we have numerically computed it to be the curve shown in Fig. 1.

The methods presented here are easily extended to the massless  $N_f = 1, 2$ , and 3 cases.<sup>3</sup> For nonzero masses, as well as for  $N_c > 2$ , the curves of marginal stability become dense in moduli space.

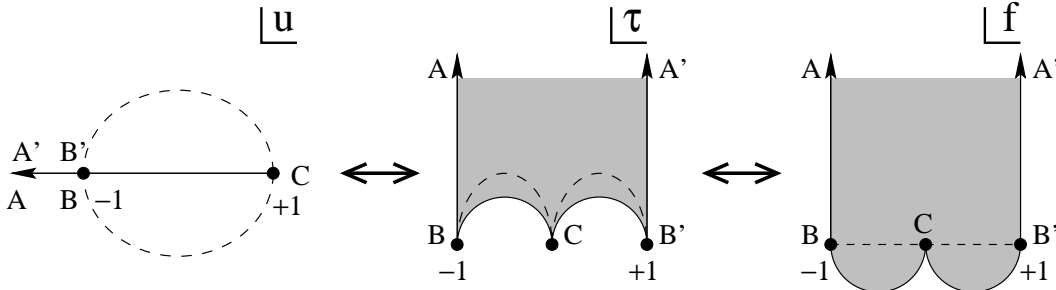


Fig. 1: The shaded regions are the images of the  $u$ -plane in the  $\tau$  and  $f$ -planes. The dashed lines are the images of  $\text{Im} f = 0$ .

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