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Realistic Superstring Models*

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ABSTRACT

I discuss the construction of realistic superstring standard–like models in the four dimensional free fermionic formulation. I discuss the massless spectrum of the superstring standard–like models and the texture of fermion mass matrices. These models suggest an explanation for the top quark mass hierarchy. At the cubic level of the superpotential only the top quark get a mass term. The lighter quarks and leptons obtain their mass terms from nonrenormalizable terms that are suppressed relative to the cubic order term. A numerical estimate yielded $m_t \sim 175-180\ GeV$. The suppression of the lightest generation masses results from the horizontal symmetries in the superstring models. The problems of neutrino masses, gauge coupling unification and hierarchical SUSY breaking are discussed. I argue that the realistic features of these models are due to the underlying $Z_2 \times Z_2$ orbifold, with standard embedding, at the free fermionic point in toroidal compactification space.

1. Introduction

The most fundamental problem in high energy physics is the nature of the mechanism responsible for breaking the electroweak symmetry and for generating the fermion masses. Two main school of thoughts were developed to address this problem. The first assumes that the origin of symmetry breaking is dynamic and that the scalars doublets are composite. The second, assumes the existence of fundamental scalar representations and tries to incorporate the Standard Model into a fundamental theory, which unifies the known interactions at a much higher scale. In view of LEP precision data, the second approach is more successful [1].

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Ultimately, future hadron colliders will determine the nature of the electroweak symmetry breaking mechanism.

If we accept the notion of fundamental scalar representations and unification, we must question what is the fundamental scale of unification. Slow evolution of the Standard Model gauge couplings and proton lifetime support a big desert scenario. It is very plausible that the fundamental scale of unification is the Planck scale, at which none of the known interactions can be neglected. Many of the observations at low energies will arise from a fundamental theory at the Planck scale. The most developed Planck scale theories, to date, are superstring theories. In this talk I discuss the construction of realistic superstring models and their phenomenological implications.

Initially it was hoped that the uniqueness of the heterotic string [2] in ten dimensions would lead to a unique heterotic string theory in four dimensions. However, soon thereafter it was realized that in four dimensions there is a large number of consistent theories. Viable models can be constructed by compactifying the extra dimensions on a Calabi–Yau manifold [3] or on an orbifold [4]. Alternatively, one can formulate consistent string theories by identifying the extra degrees of freedom as an internal conformal field theory in the form of free bosons [5], free fermions [6], or as a product of minimal models [7]. Thus the initial hope that consistency alone will determine the string vacuum did not materialize.

Further progress can be made by pursuing a dual approach. On the one hand we must study the theoretical aspects of superstring theory and understand its fundamental principles at the perturbative and nonperturbative levels. We then hope to learn how the true string vacuum is selected. Alternatively, we may try to construct realistic string models by imposing phenomenological constraints. The realistic string models may then be used as a testing ground to test our ideas on string theory and to study how Planck scale physics may determine the parameters of the Standard Model.

Two approaches can be pursued to connect superstring theory with the Standard Model. One is through a GUT model at an intermediate energy scale [8,9]. The second possibility is to derive the Standard Model directly from superstring theory [10–14]. Due to proton decay considerations the second possibility is preferred. Consider the dimension four operators, $\eta_1 u_L^C d_L^C d_L^C + \eta_2 d_L^C Q L$, that exist in the most general supersymmetric Standard Model. Unless η_1 and η_2 are highly suppressed, these dimension four operators mediate rapid proton decay. In the minimal supersymmetric Standard Model one imposes a discrete symmetry, R parity, that forbids these terms. In the context of superstring theories these discrete symmetries are usually not present. If $B-L$ is gauged as in $SO(10)$, these dimension four operators are forbidden by gauge invariance. However they may still be induced from the nonrenormalizable terms,

$$\eta_1 (u_L^C d_L^C d_L^C N_L^C)\Phi + \eta_2 (d_L^C Q L N_L^C)\Phi, \tag{1}$$

where Φ is a string of $SO(10)$ singlets that fixes the string selection rules and gets a VEV of $O(M_{Pl})$. N_L^C is the Standard Model singlet in the 16 of $SO(10)$. It is seen that the ratio $\langle N_L^c \rangle / M_{Pl}$ controls the rate of proton decay. Consequently, the VEV $\langle N_L^c \rangle$ has to be suppressed. In superstring GUT models, that have been constructed to date, $\langle N_L^c \rangle$ is used to break the GUT symmetry because there are no adjoint representations in the massless spectrum. Next, consider proton decay from dimension five operators. Dimension five operators are induced in SUSY GUT models by exchange of Higgsino color triplets [15]. Proton lifetime constraint requires that Higgsino color multiplets are sufficiently heavy, of the order of $10^{16}\ GeV$. Supersymmetric GUT models must admit some doublet–triplet splitting mechanism, which satisfies these requirements. Although, such a mechanism has been constructed in different supersymmetric GUT models, in general, further assumptions have to be made on the matter content and interactions of the supersymmetric GUT models. If the Standard Model gauge group is obtained directly at the string level then we can construct models in which the Higgsino color triplets are projected out from the massless spectrum by the GSO projections. Thus, the proton lifetime considerations motivate us to conjecture that in a realistic string model the Standard Model nonabelian gauge group must be obtained directly at the string level.

In view of the large number of, a priori, possible string models, trying to construct one realistic model may not seem very meaning full. It is very plausible that models with some realistic features may be constructed in different regions of the compactification space. What would then tell us why one is preferred over the other. However, not all the points in the compactification space are alike. String theory exhibits a new kind of symmetry, usually referred to as target space duality [25], which is a generalization of the $R \to 1/R$ duality in the case of S^1. At the self–dual point, $R_j = 1/R_j$, space–time symmetries are enhanced. For appropriate choices of the background fields the space–time symmetries are maximally enhanced. At the maximally symmetric point the internal degrees of freedom that are needed to cancel the conformal anomaly may be represented in terms of internal free fermions propagating on the string world–sheet. It is not outrageous to assume that if string theory has anything to do with nature the true string model will be located near this highly symmetric point. Thus, we are led to consider superstring standard–like models in the free fermionic formulation.

2. Superstring standard–like models

The superstring standard–like models are constructed in the free fermionic formulation. In the free fermionic formulation [6] of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. For the left–movers (world–sheet supersymmetric) one has the usual space–time fields X^μ, ψ^μ, $(\mu = 0, 1, 2, 3)$, and in addition the following eighteen real free fermion fields: χ^I, y^I, ω^I $(I = 1, \cdots, 6)$, transforming as the adjoint representation of $SU(2)^6$. The supercurrent is given in terms of these fields as follows

$$T_F(z) = \psi^\mu \partial_z X_\mu + \sum_{i=1}^{6} \chi^i y^i \omega^i.$$

For the right movers we have $\bar{X}^\mu$ and 44 real free fermion fields: $\bar{\phi}^a$, $a = 1, \cdots, 44$. Under parallel transport around a noncontractible loop the fermionic states pick up a phase. A model in this construction is defined by a set of basis vectors of boundary conditions for all world–sheet fermions. These basis vectors are constrained by the string consistency requirements (e.g. modular invariance) and completely determine the vacuum structure of the model. The physical spectrum is obtained by applying the generalized GSO projections. The low energy effective field theory is obtained by S–matrix elements between external states. The Yukawa couplings and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators. For a correlator to be nonvanishing all the symmetries of the model must be conserved. Thus, the boundary condition vectors determine the phenomenology of the models.

The first five vectors (including the vector **1**) in the basis consist of the NAHE* set

$$S = (\underbrace{1,\cdots,1}_{\psi^\mu,\chi^{1..6}},0,\cdots,0|0,\cdots,0). \tag{2a}$$

$$b_1 = (\underbrace{1,\cdots\cdots\cdots,1}_{\psi^\mu,\chi^{12},y^{3,\ldots,6},\bar{y}^{3,\ldots,6}},0,\cdots,0|\underbrace{1,\cdots,1}_{\bar{\psi}^{1,\ldots,5},\bar{\eta}^1},0,\cdots,0). \tag{2b}$$

$$b_2 = (\underbrace{1,\cdots\cdots\cdots\cdots\cdots,1}_{\psi^\mu,\chi^{34},y^{1,2},\omega^{5,6},\bar{y}^{1,2}\bar{\omega}^{5,6}},0,\cdots,0|\underbrace{1,\cdots,1}_{\bar{\psi}^{1,\ldots,5},\bar{\eta}^2},0,\cdots,0). \tag{2c}$$

$$b_3 = (\underbrace{1,\cdots\cdots\cdots\cdots\cdots,1}_{\psi^\mu,\chi^{56},\omega^{1,\cdots,4},\bar{\omega}^{1,\cdots,4}},0,\cdots,0|\underbrace{1,\cdots,1}_{\bar{\psi}^{1,\ldots,5},\bar{\eta}^3},0,\cdots,0). \tag{2d}$$

with the choice of generalized GSO projections $c\begin{pmatrix} b_i \\ b_j \end{pmatrix} = c\begin{pmatrix} b_i \\ S \end{pmatrix} = c\begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1$, and the others given by modular invariance.

The gauge group after the NAHE set is $SO(10) \times E_8 \times SO(6)^3$ with $N = 1$ space–time supersymmetry, and 48 spinorial 16 of $SO(10)$, sixteen from each sector b_1, b_2 and b_3. The NAHE set divides the internal world–sheet fermions in the following way: $\bar{\phi}^{1,\cdots,8}$ generate the hidden E_8 gauge group, $\bar{\psi}^{1,\cdots,5}$ generate the $SO(10)$ gauge group, and $\{\bar{y}^{3,\cdots,6},\bar{\eta}^1\}$, $\{\bar{y}^1,\bar{y}^2,\bar{\omega}^5,\bar{\omega}^6,\bar{\eta}^2\}$, $\{\bar{\omega}^{1,\cdots,4},\bar{\eta}^3\}$ generate the three horizontal $SO(6)^3$ symmetries. The left–moving $\{y,\omega\}$ states are divided to $\{y^{3,\cdots,6}\}$, $\{y^1,y^2,\omega^5,\omega^6\}$, $\{\omega^{1,\cdots,4}\}$ and χ^{12}, χ^{34}, χ^{56} generate the left–moving $N = 2$ world–sheet supersymmetry.

The internal fermionic states $\{y,\omega|\bar{y},\bar{\omega}\}$ correspond to the six left–moving and six right–moving compactified dimensions in a geometric formulation. This correspondence is illustrated by adding the vector X to the NAHE set, with periodic boundary conditions for the set $(\bar{\psi}^{1,\cdots,5},\bar{\eta}^{1,2,3})$ and antiperiodic boundary conditions for all other world–sheet fermions. This boundary condition vector extends

∗ This set was first constructed by Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) in the construction of the flipped $SU(5)$. *nahe*=pretty, in Hebrew.

the gauge symmetry to $E_6 \times U(1)^2 \times E_8 \times SO(4)^3$ with $N = 1$ supersymmetry and twenty-four chiral 27 of E_6. The same model is generated in the orbifold language [4] by moding out an $SO(12)$ lattice by a $Z_2 \times Z_2$ discrete symmetry with standard embedding. In the construction of the standard–like models beyond the NAHE set, the assignment of boundary conditions to the set of internal fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$ determines many of the properties of the low energy spectrum, such as the number of generations, the presence of Higgs doublets, Yukawa couplings, etc.

In the realistic free fermionic models the boundary condition vector X is replaced by the vector 2γ in which $\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\}$ are periodic and the remaining left– and right–moving fermionic states are antiperiodic. The set $\{1, S, 2\gamma, \xi_2\}$ generates a model with $N = 4$ space–time supersymmetry and $SO(12) \times SO(16) \times SO(16)$ gauge group. The b_1 and b_2 twist are applied to reduce the number of supersymmetries from $N = 4$ to $N = 1$ space–time supersymmetry. The gauge group is broken to $SO(4)^3 \times U(1)^3 \times SO(10) \times E_8$. The $U(1)$ combination $U(1) = U(1)_1 + U(1)_2 + U(1)_3$ has a non–vanishing trace and the trace of the two orthogonal combinations vanishes. The number of generations is still 24, eight from each sector b_1, b_2 and b_3 The chiral generations are now 16 of $SO(10)$ from the sectors b_j $(j = 1, 2, 3)$. The $10 + 1$ and the E_6 singlets from the sectors $b_j + X$ are replaced by vectorial 16 of the hidden $SO(16)$ gauge group from the sectors $b_j + 2\gamma$. As I will show below the structure of the sector $b_j + 2\gamma$ with respect to the sectors b_j plays an important role in the texture of fermion mass matrices.

The standard–like models are constructed by adding three additional vectors to the NAHE set [11,12,13,14]. The the $SO(10)$ symmetry is broken in two stages, first to $SO(6) \times SO(4)$ and next to $SU(3) \times SU(2) \times U(1)^2$. One example is presented in the table, where only the boundary conditions of the "compactified space" are shown. In the gauge sector $\alpha, \beta\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\cdots,8}\} = \{1^3, 0^5, 1^4, 0^4\}$ and $\gamma\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\cdots,8}\} = \{\frac{1}{2}^9, 0, 1^2, \frac{1}{2}^3, 0\}$ break the symmetry to $SU(3) \times SU(2) \times U(1)_{B-L} \times U(1)_{T_{3_R}} \times SU(5)_h \times SU(3)_h \times U(1)^2$. The choice of generalized GSO coefficients is: $c\begin{pmatrix} b_j \\ \alpha, \beta, \gamma \end{pmatrix} = -c\begin{pmatrix} \alpha \\ 1 \end{pmatrix} = c\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = -c\begin{pmatrix} \beta \\ 1 \end{pmatrix} = c\begin{pmatrix} \gamma \\ 1, \alpha \end{pmatrix} = -c\begin{pmatrix} \gamma \\ \beta \end{pmatrix} = -1$ (j=1,2,3), with the others specified by modular invariance and space–time supersymmetry.

3. The number of generations

Free fermionic models with the NAHE set correspond to $Z_2 \times Z_2$ orbifold at a special point in toroidal compactification space. At this point the internal compactified dimensions can be represented in terms of free world–sheet fermions. At this specific point the symmetries due to the compactified dimensions are enhanced from $U(1)^6$ to $SO(12)$. The enhancement is due both to compactification at the self–dual point $R_j = 1/R_j$ and due to specific values of the background fields. The structure of the $Z_2 \times Z_2$ orbifold, with standard embedding, at the specific point in compactification space is the root of the realistic properties of the free fermionic models. The first requirement from any superstring model is that the low energy spectrum contains a net chirality of three generations. A general $Z_2 \times Z_2$ orbifold would not produce three generation models. However, miraculously, at the most symmetric point in compactification space, three generations are obtained very naturally. The reason is that at this point the number of fixed points, in each twisted sector, can be simultaneously reduced to one fixed point, from each twisted sector. The three generations are then aligned along the three orthogonal complex planes of the $Z_2 \times Z_2$ orbifold. At the level of the NAHE set each sector b_1, b_2 and b_3 has eight generations. Three additional vectors are needed to reduce the number of generations to one generation from each sector b_1, b_2 and b_3. Each generation has horizontal symmetries that constrain the allowed interactions. Each generation has two gauged $U(1)$ symmetries $U(1)_{R_j}$ and $U(1)_{R_{j+3}}$. For every right–moving $U(1)$ symmetry there is a corresponding left–moving global $U(1)$ symmetry $U(1)_{L_j}$ and $U(1)_{L_{j+3}}$. Finally, each generation has two Ising model operators that are obtained by pairing a left–moving real fermion with a right–moving real fermion.

Table 1. A three generations $SU(3) \times SU(2) \times U(1)^2$ model [13].

	y^3y^6, $y^4\bar{y}^4$, $y^5\bar{y}^5$, $\bar{y}^3\bar{y}^6$	$y^1\omega^6$, $y^2\bar{y}^2$, $\omega^5\bar{\omega}^5$, $\bar{y}^1\bar{\omega}^6$	$\omega^1\omega^3$, $\omega^2\bar{\omega}^2$, $\omega^4\bar{\omega}^4$, $\bar{\omega}^1\bar{\omega}^3$
α	1, 1, 1, 0	1, 1, 1, 0	1, 1, 1, 0
β	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0
γ	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1

4. Higgs doublets

Higgs doublets in the standard–like models are obtained from two distinct sectors. The first type are obtained from the Neveu–Schwarz sector, which produces three pairs of electroweak doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$. The Neveu–Schwarz sector corresponds to the untwisted sector of the orbifold models. Each pair of Higgs doublets can couple at tree level only to the states from the sector b_j. This results from the horizontal symmetries, $U(1)_j$, $(1,2,3)$ and is a reflection of the structure of the $Z_2 \times Z_2$ twisting. There is a stringy doublet–triplet splitting mechanism that projects out the color triplets and leaves the electroweak doublets in the spectrum. Thus, the superstring standard–like models resolve the GUT hierarchy problem. The second type of Higgs doublets are obtained from the vector combination $b_1 + b_2 + \alpha + \beta$. The states in this sector are obtained by acting on the vacuum with a single fermionic oscillator and transform only under the observable sector.

In addition to electroweak doublets and color triplets the Neveu–Schwarz sector and the sector $b_1 + b_2 + \alpha + \beta$ produce singlets of $SO(10) \times E_8$. These singlets play an important role in the phenomenology of the superstring standard–like models. The VEVs of the $SO(10)$ singlet fields in the massless spectrum of the superstring models determine the light Higgs representations and generate the fermion mass hierarchy.

The Neveu–Schwarz sector and the sector $b_1+b_2+\alpha+\beta$ produce four [12] or five [13] pairs of electroweak doublets. Several pairs receive heavy mass from the VEVs of Standard Model singlets in the massless spectrum. At the cubic level there are two pairs of electroweak doublets. The light Higgs doublets are combinations of (h_1, h_2, h_{45}) and $(\bar{h}_1, \bar{h}_2, \bar{h}_{45})$. The Higgs doublets h_3 and $\bar{h}_3$ obtain a large mass from $SO(10)$ singlet VEVs. This results from requiring F–flatness of the cubic level superpotential [20]. The absence of h_3 and $\bar{h}_3$ from the light eigenstates results in b_3 being identified with the lightest generation. At the nonrenormalizable level one additional pair receives a superheavy mass and one pair remains light to give masses to the fermions at the electroweak scale. Requiring F–flatness imposes that the light Higgs representations are $\bar{h}_1$ or $\bar{h}_2$ and h_{45}.

5. The sectors $b_j + 2\gamma$

As mentioned above the realistic free fermionic models contain massless states from the sectors $b_j+2\gamma$ ($j=1,2,3$). These states arise due to the $Z_2 \times Z_2$ twisting on a gauge lattice with $SO(16)\times SO(16)$ rather than $E_8 \times E_8$. Thus, the realistic free fermionic models correspond to $(2,0)$ rather than $(2,2)$ compactification. The sectors $b_j+2\gamma$ produce the vectorial 16 representation of the hidden $SO(16)$ gauge group, decomposed under the final hidden gauge group. The number of 16 of the hidden $SO(16)$ is equal to the number of 16 of the observable $SO(10)$ gauge group. The horizontal charges in the sectors $b_j+2\gamma$ are similar to the ones in the sectors b_j. The VEVs of the states from the sectors $b_j+2\gamma$ are responsible for generating texture zeroes in the fermion mass matrices.

The massless spectrum described until now results from the $Z_2 \times Z_2$ twist with standard embedding. Therefore, it generally holds for all the free fermionic models that are based on $Z_2 \times Z_2$ orbifold with standard embedding. In addition to the states from the sectors mentioned above there are massless sectors that arise due to the sectors that correspond to Wilson line breaking. The states in these sectors usually do not have the standard $SO(10)$ embedding. In particular the weak hypercharge and $U(1)_{Z'}$ charge usually differs from the standard $SO(10)$ assignment.

6. Top quark mass hierarchy

Trilinear and nonrenormalizable contributions to the superpotential are obtained by calculating correlators between vertex operators [16]

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \tag{3}$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators. The non–vanishing terms are obtained by applying the rules of Ref. [16]. The cubic level Yukawa couplings for the quarks and leptons are determined by the boundary conditions in the vector γ according to the following rule [14,17]

$$\Delta_j = |\gamma(U(1)_{L_{j+3}}) - \gamma(U(1)_{R_{j+3}})| = 0, 1 \qquad (j = 1, 2, 3) \tag{4a}$$

$$\Delta_j = 0 \rightarrow d_j Q_j h_j + e_j L_j h_j; \tag{4b}$$

$$\Delta_j = 1 \rightarrow u_j Q_j \bar{h}_j + N_j L_j \bar{h}_j, \tag{4c}$$

where $\gamma(U(1)_{R_{j+3}})$, $\gamma(U(1)_{L_{j+3}})$ are the boundary conditions of the world–sheet fermionic currents that generate the $U(1)_{R_{j+3}}$, $U(1)_{L_{j+3}}$ symmetries.

The superstring standard–like models contain an anomalous $U(1)$ gauge symmetry. The anomalous $U(1)$ generates a Fayet–Iliopoulos term by the VEV of the dilaton field that breaks supersymmetry and destabilizes the vacuum [19]. Supersymmetry is restored by giving VEVs to Standard Model singlets in the massless spectrum of the superstring models. However, as the charge of these singlets must have $Q_A < 0$ to cancel the anomalous $U(1)$ D–term equation, in many models a phenomenologically realistic solution does not exist. In fact a very restricted class of standard–like models with $\Delta_j = 1$ for $j = 1, 2, 3$, were found to admit a solution to the F and D flatness constraints. Consequently, the only models that were found to admit a solution are models which have tree level Yukawa couplings only for $+\frac{2}{3}$ charged quarks.

This result suggests an explanation for the top quark mass hierarchy relative to the lighter quarks and leptons. At the cubic level only the top quark gets a mass term and the mass terms for the lighter quarks and leptons are obtained from nonrenormalizable terms. To study this scenario we have to examine the nonrenormalizable contributions to the doublet Higgs mass matrix and to the fermion mass matrices [20,21].

At the cubic level there are two pairs of electroweak doublets. At the nonrenormalizable level one additional pair receives a superheavy mass and one pair remains light to give masses to the fermions at the electroweak scale. Requiring F–flatness imposes that the light Higgs representations are $\bar{h}_1$ or $\bar{h}_2$ and h_{45}.

The nonrenormalizable fermion mass terms of order N are of the form $cgf_i f_j h\phi^{N-3}$ or $cgf_i f_j \bar{h}\phi^{N-3}$, where c is a calculable coefficient, g is the gauge coupling at the unification scale, f_i, f_j are the fermions from the sectors b_1, b_2 and

b_3, h and $\bar{h}$ are the light Higgs doublets, and ϕ^{N-3} is a string of Standard Model singlets that get a VEV and produce a suppression factor $(\langle\phi\rangle/M)^{N-3}$ relative to the cubic level terms. Several scales contribute to the generalized VEVs. The leading one is the scale of VEVs that are used to cancel the "anomalous" $U(1)$ D–term equation. The next scale is generated by Hidden sector condensates. Finally, there is a scale which is related to the breaking of $U(1)_{Z'}$, $\Lambda_{Z'}$. Examination of the higher order nonrenormalizable terms reveals that $\Lambda_{Z'}$ has to be suppressed relative to the other two scales.

At the cubic level only the top quark gets a nonvanishing mass term. Therefore only the top quark mass is characterized by the electroweak scale. The remaining quarks and leptons obtain their mass terms from nonrenormalizable terms. The cubic and nonrenormalizable terms in the superpotential are obtained by calculating correlators between the vertex operators. The top quark Yukawa coupling is generically given by

$$g\sqrt{2} \tag{5}$$

where g is the gauge coupling at the unification scale. In the model of Ref. [13], bottom quark and tau lepton mass terms are obtained at the quartic order,

$$W_4 = \{d^c_{L_1} Q_1 h'_{45} \Phi_1 + e^c_{L_1} L_1 h'_{45} \Phi_1 + d^c_{L_2} Q_2 h'_{45} \bar{\Phi}_2 + e^c_{L_2} L_2 h'_{45} \bar{\Phi}_2\}. \tag{6}$$

The VEVs of Φ are obtained from the cancelation of the anomalous D–term equation. The coefficient of the quartic order mass terms were calculated by calculating the quartic order correlators and the one dimensional integral was evaluated numerically. Thus after inserting the VEV of $\bar{\Phi}_2$ the effective bottom quark and tau lepton Yukawa couplings are given by [13],

$$\lambda_b = \lambda_\tau = 0.35 g^3. \tag{7}$$

They are suppressed relative to the top Yukawa by

$$\frac{\lambda_b}{\lambda_t} = \frac{0.35 g^3}{g\sqrt{2}} \sim \frac{1}{8}. \tag{8}$$

To evaluate the top quark mass, the three Yukawa couplings are run to the low energy scale by using the MSSM RGEs. The bottom mass is then used to calculate

$\tan\beta$ and the top quark mass is found to be [13],

$$m_t \sim 175 - 180 GeV. \tag{9}$$

The fact that the top Yukawa is found near a fixed point suggests that this is in fact a good prediction of the superstring standard–like models. By varying $\lambda_t \sim 0.5 - 1.5$ at the unification scale, it is found that λ_t is always $O(1)$ at the electroweak scale.

7. Fermion mass matrices

An analysis of fermion mass terms up to order $N = 8$ revealed the general texture of fermion mass matrices in these models. The sectors b_1 and b_2 produce the two heavy generations. Their mass terms are suppressed by singlet VEVs that are used in the cancellation of the anomalous $U(1)$ D–term equation. The sector b_3 produces the lightest generation. The diagonal mass terms for the states from b_3 can only be generated by VEVs that break $U(1)_{Z'}$. This is due to the horizontal $U(1)$ charges and because the Higgs pair h_3 and $\bar{h}_3$ necessarily gets a Planck scale mass [20]. The suppression of the lightest generation mass terms is seen to be a result of the structure of the vectors α and β with respect to the sectors b_1, b_2 and b_3. The mixing between the generations is obtained from exchange of states from the sectors $b_j + 2\gamma$. The general texture of the fermion mass matrices in the superstring standard–like models is of the following form,

$$M_U \sim \begin{pmatrix} \epsilon, a, b \\ \tilde{a}, A, c \\ \tilde{b}, \tilde{c}, \lambda_t \end{pmatrix}; \;\; M_D \sim \begin{pmatrix} \epsilon, d, e \\ \tilde{d}, B, f \\ \tilde{e}, \tilde{f}, C \end{pmatrix}; \;\; M_E \sim \begin{pmatrix} \epsilon, g, h \\ \tilde{g}, D, i \\ \tilde{h}, \tilde{i}, E \end{pmatrix},$$

where $\epsilon \sim (\Lambda_{Z'}/M)^2$. The diagonal terms in capital letters represent leading terms that are suppressed by singlet VEVs, and $\lambda_t = O(1)$. The mixing terms are generated by hidden sector states from the sectors $b_j + 2\gamma$ and are represented by small letters. They are proportional to $(\langle TT \rangle / M^2)$.

8. Quark flavor mixing

In Ref. [21] it was shown that if the states from the sectors $b_j + 2\gamma$ obtain VEVs in the application of the DSW mechanism, then a Cabibbo angle of the correct order of magnitude can be obtained in the superstring standard–like models. For one specific choice of singlet VEVs that solve the cubic level F and D constraints the down mass matrix M_D is given by

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2\bar{V}_3\Phi_{45}}{M^3} & 0 \\ \frac{V_2\bar{V}_3\Phi_{45}\xi_1}{M^4} & \frac{\bar{\Phi}_2^-\xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+\xi_2}{M^2} \end{pmatrix} v_2, \qquad (10)$$

where $v_2 = \langle h_{45}\rangle$ and we have used $\frac{1}{2}g\sqrt{2\alpha'} = \sqrt{8\pi}/M_{Pl}$, to define $M \equiv M_{Pl}/2\sqrt{8\pi} \approx 1.2\times10^{18}GeV$ [16]. The undetermined VEVs of $\bar{\Phi}_{13}$ and ξ_2 are used to fix m_b and m_s such that $\langle\xi_1\rangle \sim M$. We also take $tan\beta = v_1/v_2 \sim 1.5$. Substituting the values of the VEVs above and diagonalizing M_D by a biunitary transformation we obtain the Cabibbo mixing matrix

$$|V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (11)$$

Since the running from the scale M down to the weak scale does not affect the Cabibbo angle by much [22], we conclude that realistic mixing of the correct order of magnitude can be obtained in this scenario. The analysis was extended to show that reasonable values for the entire CKM matrix parameters can be obtained for appropriate flat F and D solutions. For one specific solution the up and down quark mass matrices take the form

$$M_u \sim \begin{pmatrix} \epsilon & \frac{V_3\bar{V}_2\Phi_{45}\bar{\Phi}_3^+}{M^4} & 0 \\ \frac{V_3\bar{V}_2\Phi_{45}\bar{\Phi}_2^+}{M^4} & \frac{\bar{\Phi}_i^-\bar{\Phi}_i^+}{M^2} & \frac{V_1\bar{V}_2\Phi_{45}\bar{\Phi}_2^+}{M^4} \\ 0 & \frac{V_1\bar{V}_2\Phi_{45}\bar{\Phi}_1^+}{M^4} & 1 \end{pmatrix} v_1, \qquad (12)$$

and

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_3\bar{V}_2\Phi_{45}}{M^3} & 0 \\ \frac{V_3\bar{V}_2\Phi_{45}\xi_1}{M^4} & \frac{\bar{\Phi}_2^-\xi_1}{M^2} & \frac{V_1\bar{V}_2\Phi_{45}\xi_i}{M^4} \\ 0 & \frac{V_1\bar{V}_2\Phi_{45}\xi_i}{M^4} & \frac{\Phi_1^+\xi_2}{M^2} \end{pmatrix} v_2, \qquad (13)$$

with v_1, v_2 and M as before. The up and down quark mass matrices are diagonalized by bi–unitary transformations

$$U_L M_u U_R^\dagger = D_u \equiv \mathrm{diag}(m_u, m_c, m_t), \qquad (14a)$$

$$D_L M_d D_R^\dagger = D_d \equiv \mathrm{diag}(m_d, m_s, m_b), \tag{14b}$$

with the CKM mixing matrix given by

$$V = U_L D_L^\dagger. \tag{15}$$

The VEVs of ξ_1 and ξ_2 are fixed to be $\langle \xi_1 \rangle \sim M/12$ and $\langle \xi_2 \rangle \sim M/4$ by the masses m_s and m_b respectively. Substituting the VEVs and diagonalizing M_u and M_d by a bi–unitary transformation, we obtain the mixing matrix

$$|V| \sim \begin{pmatrix} 0.98 & 0.205 & 0.002 \\ 0.205 & 0.98 & 0.012 \\ 0.0004 & 0.012 & 0.99 \end{pmatrix}. \tag{16}$$

The texture and hierarchy of the mass terms in Eqs. (11–12) arise due to the set of singlet VEVs in Eqs. (29). The zeroes in the 13 and 31 entries of the mass matrices are protected to all orders of nonrenormalizable terms. To obtain a non–vanishing contribution to these entries either V_1 and $\bar{V}_3$ or V_3 and $\bar{V}_1$ must obtain a VEV simultaneously. Thus, there is a residual horizontal symmetry that protects these vanishing terms. The 11 entry in the mass matrices, e.g. the diagonal mass terms for the lightest generation states, can only be obtained from VEVs that break $U(1)_{Z'}$ [18]. We assume that $U(1)_{Z'}$ is broken at an intermediate energy scale that is suppressed relative to the scale of scalar VEVs [20]. In Ref. [26] we showed that $U(1)_{Z'}$ is broken by hidden sector matter condensates at $\Lambda_{Z'} \leq 10^{14} GeV$. Consequently, we have taken $\epsilon \leq (\Lambda_{Z'}/M)^2 \sim 10^{-8}$.

Texture zeroes in the fermion mass matrices are obtained if the VEVs of some states from the sectors $b_j + 2\gamma$ vanish. These texture zeroes are protected by the symmetries of the string models to all order of nonrenormalizable terms [21]. For example in the above mass matrices the 13 and 31 vanish because $\{V_1, V_3\}$ get a VEV but $\bar{V}_1$ and $\bar{V}_3$ do not. Therefore these mass matrix terms cannot be formed because they would not be invariant under all the string symmetries. Other textures are possible for other choices of VEVs for the states from the sectors $b_j + 2\gamma$.

9. Neutrino masses

A seesaw type neutrino mass matrix can be constructed from analysis of non-renormalizable terms and for specific choices of singlet VEVs [26]. The neutrino seesaw mass matrix takes the general form The neutrino mass matrix therefore takes the following form for each generation in the basis (ν_L, N^C, Φ)

$$\begin{pmatrix} 0 & km_u & 0 \\ km_u & 0 & m_\chi \\ 0 & m_\chi & m_\phi \end{pmatrix} , \tag{17}$$

with $m_\chi \sim \left(\frac{\Lambda_{Z'}}{M}\right)^3 \left(\frac{\langle\phi\rangle}{M}\right)^n M$ and $m_\phi \sim \left(\frac{\Lambda_{Z'}}{M}\right)^4 \left(\frac{\langle\phi\rangle}{M}\right)^m M$. n and m are the orders at which the terms are obtained. The mass eigenstates are mainly ν, N and ϕ with a small mixing and with the eigenvalues

$$m_\nu \sim m_\phi \left(\frac{km_u}{m_\chi}\right)^2 \qquad m_N, M_\phi \sim m_\chi \tag{18}$$

The constant k gives the effects of Yukawa coupling renormalization. The seesaw scale m_χ is determined by the $U(1)_{Z'}$ breaking scale and by the order at which the nonrenormalizable seesaw terms are obtained. In Ref. [26] the $U(1)_{Z'} \sim 10^{14}\ GeV$ breaking scale was obtained from condensates of the hidden $SU(5)$ gauge group with nontrivial $U(1)_{Z'}$ charges. The order of nonrenormalizable terms that contribute to the seesaw terms in the neutrino mass matrix depends highly on the choice of flat flat directions. Neutrino masses that are in agreement with experimental constraints can be obtained. A novel feature of the superstring seesaw mechanism is that although the $U(1)_{Z'}$ breaking scale may be large (e.g. $\Lambda_{Z'} \approx 10^{14} GeV$) the effective see-saw scale can be much smaller.

9. Gauge coupling unification

While LEP results indicate that the gauge coupling in the minimal supersymmetric Standard Model unify at $10^{16} GeV$, superstring theory predicts that the unification scale is at $10^{18} GeV$. The superstring standard–like models may resolve this problem due to the existence of color triplets and electroweak doublets from exotic sectors that arise from the additional vectors α, β and γ. These exotic states carry fractional charges and do not fit into standard $SO(10)$ representations.

Therefore, they contribute less to the evolution of the $U(1)_Y$ beta function than standard $SO(10)$ multiplets. For example in Ref. [23] representations with the following beta function coefficients, in a $SU(3)\times SU(2)\times U(1)_Y$ basis, were found

$$b_{D_1,\bar{D}_1,D_2,\bar{D}_2}=\begin{pmatrix}\frac{1}{2}\\ 0\\ \frac{1}{5}\end{pmatrix};b_{D_3,\bar{D}_3}=\begin{pmatrix}\frac{1}{2}\\ 0\\ \frac{1}{20}\end{pmatrix};b_{\ell,\bar{\ell}}=\begin{pmatrix}0\\ \frac{1}{2}\\ 0\end{pmatrix}.$$

The standard–like models predict $\sin^2\theta_W=3/8$ at the unification scale due to the embedding of the weak hypercharge in $SO(10)$. In Ref. [23], I showed that provided that the additional exotic color triplets and electroweak doublets exist at the appropriate scales, the scale of gauge coupling unification is pushed to $10^{18}GeV$, with the correct value of $\sin^2\theta_W$ at low energies.

11. Hierarchical SUSY breaking

In Ref. [24] we address the following question: Given a supersymmetric string vacuum at the Planck scale, is it possible to obtain hierarchical supersymmetry breaking in the observable sector? A supersymmetric string vacuum is obtained by finding solutions to the cubic level F and D constraints. We take a gauge coupling in agreement with gauge coupling unification, thus taking a fixed value for the dilaton VEV. We then investigate the role of nonrenormalizable terms and strong hidden sector dynamics. The hidden sector contains two non–Abelian hidden gauge groups, $SU(5)\times SU(3)$, with matter in vector–like representations. The hidden $SU(3)$ group is broken near the Planck scale. We analyze the dynamics of the hidden $SU(5)$ group. The $SU(5)$ hidden matter mass matrix is given by

$$\mathcal{M}=\begin{pmatrix}0 & C_1 & 0\\ B_1 & A_2 & C_2\\ 0 & C_3 & A_1\end{pmatrix}, \tag{19}$$

where A,B,C arise from nonrenormalizable terms of orders $N=5,8,7$ respectively and are given by

$$A_1=\frac{\langle\Phi_{45}\bar{\Phi}_1^-\xi_2\rangle}{M^2},\qquad A_2=\frac{\langle\Phi_{45}\Phi_2^+\xi_1\rangle}{M^2}, \tag{20a,b}$$

$$B_1=\frac{\langle V_3\bar{V}_2\Phi_{45}\Phi_{45}\bar{\Phi}_{13}\xi_1\rangle}{M^5}, \tag{20c}$$

$$C_1 = \frac{\langle V_3 \bar{V}_2 \Phi_{45} \Phi_{45} \bar{\Phi}_{13} \rangle}{M^4}, \quad C_2 = \frac{\langle V_1 \bar{V}_2 \Phi_{45} \Phi_{45} \xi_1 \rangle}{M^4}, \qquad (20d,e)$$

$$C_3 = \frac{\langle V_1 \bar{V}_2 \Phi_{45} \Phi_{45} \xi_2 \rangle}{M^4} \quad . \qquad (20f)$$

Taking generically $\langle \phi \rangle \sim gM/4\pi \sim M/10$ we obtain $A_i \sim 10^{15}\ GeV$, $B_i \sim 10^{12}\ GeV$, and $C_i \sim 10^{13}\ GeV$. From Eqs. (19–20) we observe that to insure a nonsingular hidden matter mass matrix, we must require $C_1 \neq 0$ and $B_1 \neq 0$. This imposes $\bar{V}_3 \neq 0$ and $V_2 \neq 0$. Thus, the nonvanishing VEVs that generates the Cabibbo mixing also guarantee the stability of the supersymmetric vacuum. The gaugino and matter condensates are given by the well known expressions for supersymmetric $SU(N)$ with matter in $N + \bar{N}$ representations [27],

$$\frac{1}{32\pi^2} \langle \lambda\lambda \rangle = \Lambda^3 \left(det \frac{\mathcal{M}}{\Lambda} \right)^{1/N} , \qquad (21a)$$

$$\Pi_{ij} = \langle \bar{T}_i T_j \rangle = \frac{1}{32\pi^2} \langle \lambda\lambda \rangle \mathcal{M}_{ij}{}^{-1}, \qquad (21b)$$

where $\langle \lambda\lambda \rangle$, $\mathcal{M}$ and Λ are the hidden gaugino condensate, the hidden matter mass matrix and the $SU(5)$ condensation scale, respectively. Modular invariant generalization of Eqs. (20a,b) for the string case were derived in Ref. [28]. The nonrenormalizable terms can be put in modular invariant form by following the procedure outlined in Ref. [29]. Approximating the Dedekind η function by $\eta(\hat{T}) \approx e^{-\pi\hat{T}/12}(1 - e^{-2\pi\hat{T}})$ we verified that the calculation using the modular invariant expression from Ref. [28] (with $\langle \hat{T} \rangle \approx M$) differ from the results using Eq. (20), by at most an order of magnitude. The hidden $SU(5)$ matter mass matrix is nonsingular for specific F and D flat solutions. In Ref. [24] a specific cubic level F and D flat solution was found. The gravitino mass due to the gaugino and matter condensates was estimated to be of the order $1 - 10\ TeV$. The new aspect of our scenario for supersymmetry breaking is the following. As long as only states from the Neveu–Schwarz sector or the sector $b_1 + b_2 + \alpha + \beta$ receive VEVs in the application of the DSW mechanism then one can find exact flat directions at the cubic level of the superpotential. These flat directions will be exact and will not be spoiled by nonrenormalizable terms. The states from the Neveu–Schwarz sector and the sector $b_1 + b_2 + \alpha + \beta$ correspond to untwisted and twisted moduli. However, once some hidden sector matter states obtain a nonvanishing VEV, the cubic level flat directions are no longer exact. Supersymmetry is broken by the inclusion of nonrenormalizable terms. Hidden sector strong dynamics at an intermediate scale may then be responsible for generating the hierarchy in the usual fashion.

12. conclusion

The Standard Model is in agreement with all current experiments. Furthermore, present day experiments seem to support the big desert scenario and the notion of unification. The Planck scale is the ultimate scale of unification at which none of the known interactions can be ignored. Many properties of the Standard Model will arise from the fundamental Planck scale theory. Superstring theory stands out as the only known theory that can consistently unify gravity with the gauge interactions. The heterotic string is the only string theory that can produce realistic phenomenology. Its consistency requires twenty–six critical dimensions in the bosonic sector and ten critical dimensions in the supersymmetric sector. In the bosonic sector sixteen degrees of freedom are compactified on a flat torus and produce the observable and hidden gauge degrees of freedom. Six degrees of freedom from the bosonic sector, combined with six degrees of freedom from the supersymmetric sector, are compactified on a Calabi–Yau manifold or on an orbifold. String theory exhibits a new kind of symmetry: "target space duality". At the self–dual point, the compactified degrees of freedom can be represented in terms of free world–sheet fermions. At this point space–time symmetries are maximally enhanced. The most realistic superstring models constructed to date were constructed at this point in the compactification space. The underlying structure of the $Z_2 \times Z_2$ orbifold at the free fermionic point in toroidal compactification space is the origin of the realistic nature of free fermionic models. We believe that if string unification is relevant in nature, then the underlying structure of the $Z_2 \times Z_2$ orbifold at the free fermionic point in toroidal compactification space will be intrinsic to the eventual "true" heterotic string model. Thus, it makes sense, in our opinion, to try to build realistic models specifically at this point in the huge compactification space.

The superstring standard–like models contain in their massless spectrum all the necessary states to obtain realistic low energy phenomenology. They resolve the problems of proton decay through dimension four and five operators that are endemic to other superstring and GUT models. The existence of only three generations with standard $SO(10)$ embedding is understood to arise naturally from $Z_2 \times Z_2$ twisting at the free fermionic point in compactification space. Better understanding of the correspondence with other superstring formulations will provide further insight into the realistic properties of these models. In this context it is especially interesting to try to understand the significance of the self–dual point in the compactification space. Finally, the free fermionic standard-like models provide a highly constrained and phenomenologically realistic laboratory to study how the Planck scale may determine the parameters of the Standard Model.

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