Constraints on the unitarity triangle angle $\gamma$ from Dalitz plot analysis of $B^0 \rightarrow DK^+\pi^-$ decays

The LHCb collaboration

Abstract

The first study is presented of $CP$ violation with an amplitude analysis of the Dalitz plot of $B^0 \rightarrow DK^+\pi^-$ decays, with $D \rightarrow K^+\pi^-$, $K^+K^-$ and $\pi^+\pi^-$. The analysis is based on a data sample corresponding to 3.0 fb$^{-1}$ of pp collisions collected with the LHCb detector. No significant $CP$ violation effect is seen, and constraints are placed on the angle $\gamma$ of the unitarity triangle formed from elements of the Cabibbo-Kobayashi-Maskawa quark mixing matrix. Hadronic parameters associated with the $B^0 \rightarrow DK^*(892)^0$ decay are determined for the first time. These measurements can be used to improve the sensitivity to $\gamma$ of existing and future studies of the $B^0 \rightarrow DK^*(892)^0$ decay.


© CERN on behalf of the LHCb collaboration, licence [CC-BY-4.0](https://creativecommons.org/licenses/by/4.0/)

†Authors are listed at the end of this Letter.
One of the most important challenges of physics today is to understand the origin of the matter-antimatter asymmetry of the Universe. Within the Standard Model (SM) of particle physics, the CP symmetry between particles and antiparticles is broken only by the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix \([1,2]\). An important parameter in the CKM description of the SM flavour structure is \(\gamma \equiv \arg \left(-V_{ud}V_{ub}^{\ast} / (V_{cd}V_{cb}^{\ast})\right)\), one of the three angles of the unitarity triangle formed from CKM matrix elements \([3 – 5]\).

Since the SM cannot account for the baryon asymmetry of the Universe \([6]\) new sources of CP violation, that would show up as deviations from the SM, are expected. The precise determination of \(\gamma\) is necessary in order to be able to search for such small deviations.

The value of \(\gamma\) can be determined from the CP-violating interference between the two amplitudes in, for example, \(B^+ \rightarrow DK^+\) and charge-conjugate decays \([7–10]\). Here \(D\) denotes a superposition of the \(D^0\) and \(D^0\) states produced through \(b \rightarrow cW\) and \(b \rightarrow uW\) transitions (hereafter referred to as \(V_{cb}\) and \(V_{ub}\) amplitudes). This approach has negligible theoretical uncertainty \([11]\) but limited data samples are available experimentally. A similar method based on \(B^0 \rightarrow DK^+\pi^-\) decays has been proposed \([12,13]\) to help improve the precision. By studying the Dalitz plot (DP) \([14]\) distributions of \(B^0\) and \(B^0\) decays, interference between different contributions, like \(B^0 \rightarrow D_2^+(2460)^-K^+\) and \(B^0 \rightarrow DK^*(892)^0\), can be exploited to obtain additional sensitivity compared to the “quasi-two-body” analysis in which only the region of the DP dominated by the \(K^*(892)^0\) resonance is selected \([15–17]\).

This Letter describes the first study of CP violation with a DP analysis of \(B^0 \rightarrow DK^+\pi^-\) decays, with a sample corresponding to \(3.0 \pm 0.2 \text{ fb}^{-1}\) of \(pp\) collision data collected with the LHCb detector at centre-of-mass energies of 7 and 8 TeV. The inclusion of charge conjugate processes is implied throughout this Letter except where discussing asymmetries. The LHCb detector is a single-arm forward spectrometer covering the pseudorapidity range \(2 < \eta < 5\), described in detail in Refs. \([18,19]\). Simulated events are produced using the software described in Refs. \([20–25]\).

Candidate \(B^0 \rightarrow DK^+\pi^-\) decays are selected with the \(D\) meson decaying into the \(K^+\pi^-\), \(K^+K^-\) or \(\pi^+\pi^-\) final state. The selection is similar to that used for the DP analysis of \(B^0 \rightarrow D^0 K^+\pi^-\), \(\bar{D}^0 \rightarrow K^+\pi^-\) decays \([26]\) with the following modifications. Tighter requirements compared to Ref. \([26]\) are imposed to reject backgrounds from \(B\) decays without intermediate charmed mesons, and vetoes are applied to remove backgrounds specific to the \(D \rightarrow K^+K^-\) and \(\pi^+\pi^-\) samples. Candidates for each of the \(D\) decays are identified with separate boosted decision tree algorithms \([27]\). These are used, together with variables that describe the topology of the \(B\) decay, as inputs to neural network (NN) classifiers to separate \(B\) decays from combinatorial background.

The yields of signal and of several different backgrounds are determined from an extended maximum likelihood fit, in each mode, to the distributions of candidates in \(B\) candidate mass and NN output. Unbinned information on the \(B\) candidate mass is used, while each sample is divided into five bins of the NN output that contain a similar number of signal, and varying numbers of background, decays \([28,29]\).

In addition to \(B^0 \rightarrow DK^+\pi^-\) decays, components are included in the fit to account for \(B^0_s\) decays to the same final state, partially reconstructed \(B^0_{(s)} \rightarrow D^{(*)} K^\pm\pi^\mp\) backgrounds,
misidentified $B^0 \rightarrow D^{(*)}\pi^+\pi^-$, $B^0_{(s)} \rightarrow D^{(*)}K^+K^-$, $\Lambda^0_b \rightarrow D^{(*)}\bar{p}\pi^+$ and $\Lambda^0_b \rightarrow D^{(*)}\bar{p}K^+$ decays as well as combinatorial background. The modelling of the signal and background distributions in $B$ candidate mass is similar to that described in Ref. [26]. The sum of two Crystal Ball functions [30] is used for each of the correctly reconstructed $B$ decays, where the peak position and core width (i.e. the narrower of the two widths) are free parameters of the fit, while the $B^0_s-B^0$ mass difference is fixed to its known value [31]. The fraction of the signal function contained in the core and the relative width of the two components are constrained within uncertainties to their expectations and all other parameters are fixed to the values obtained in simulation, separately for each of the three $D$ samples. An exponential function is used to describe combinatorial background, with the shape parameter allowed to vary. Due to the loose NN output requirement it is necessary, in the $D \rightarrow K^+\pi^-$ sample, to account explicitly for partially combinatorial background where the final state $DK^+$ pair originates from a $B$ decay but is combined with a random pion; this is modelled with a non-parametric function. Non-parametric functions obtained from simulation based on known DP distributions [32 –38] are used to model the partially reconstructed and misidentified $B$ decays.

The fraction of signal decays in each NN output bin is allowed to vary freely in the fit; the correctly reconstructed $B^0_s$ decays and misidentified backgrounds are taken to have the same NN output distribution as signal. The fractions of combinatorial and partially reconstructed backgrounds in each NN output bin are each allowed to vary freely. All yields are free parameters of the fit, except those for misidentified backgrounds which are constrained within expectation relative to the signal yield, since the relative branching fractions [31] and misidentification probabilities [39] are well known.

The results of the fits are shown in Fig. 1, in which the NN output bins have been combined by weighting both the data and fit results by $S/(S+B)$, where $S$ ($B$) is the signal (background) yield in the signal region, defined as $\pm 2.5\sigma$ around the $B^0$ peak in each sample, where $\sigma$ is the core width of the signal shape. In this region there are in total $2840 \pm 70$ signal decays in the $D \rightarrow K^+\pi^-$ sample, whilst the corresponding values for the $D \rightarrow K^+K^-$ and $D \rightarrow \pi^+\pi^-$ samples are $339 \pm 22$ and $168 \pm 19$. A more detailed breakdown of the fit results can be found in Ref. [40]. Candidates within the signal region are used in the DP analysis. A simultaneous fit is performed to the samples with different $D$ decays by using the Jfit method [41] as implemented in the Laura++ package [42]. The likelihood function contains signal and background terms, with yields in each NN output bin fixed according to the results obtained previously. The NN output bin with the lowest $S/B$ value in the $D \rightarrow K^+\pi^-$ sample only is found not to contribute significantly to the sensitivity and is susceptible to mismodelling of the combinatorial background; it is therefore excluded from the subsequent analysis.

The signal probability function is derived from the isobar model obtained in Ref. [26], with amplitude

$$A \left(m^2(D\pi^-), m^2(K^+\pi^-)\right) = \sum_{j=1}^{N} c_j F_j \left(m^2(D\pi^-), m^2(K^+\pi^-)\right),$$

(1)

2
where $c_j$ are complex coefficients describing the relative contribution for each intermediate process, and the $F_j (m^2 (D\pi^-), m^2 (K^\pm \pi^-))$ terms describe the resonant dynamics through the lineshape, angular distribution and barrier factors. The sum is over amplitudes from the $D_0^+ (2400)^-, D_2^+(2460)^-, K^*(892)^0$, $K^*(1410)^0$ and $K_2^0 (1430)^0$ resonances as well as a $K^+\pi^- S$-wave component and both $S$-wave and $P$-wave nonresonant $D\pi^-$ amplitudes [26]. The masses and widths of $K^+\pi^-$ resonances are fixed, and those of $D\pi^-$ resonances are constrained within uncertainties to known values [26,31,34,43]. The values of the $c_j$ coefficients are allowed to vary in the fit, as are the shape parameters of the nonresonant amplitudes.

For the $D \to K^+\pi^-$ sample, only the $V_{cb}$ amplitude contributes and the signal probability function is given by Eq. (1). For the samples with $D \to K^+K^-$ and $\pi^+\pi^-$ decays, the $c_j$ terms are modified,

$$c_j \rightarrow \begin{cases} c_j & \text{for a } D\pi^- \text{ resonance,} \\ c_j [1 + x_{\pm,j} + iy_{\pm,j}] & \text{for a } K^+\pi^- \text{ resonance,} \end{cases}$$ (2)

with $x_{\pm,j} = r_{B,j} \cos (\delta_{B,j} \pm \gamma)$ and $y_{\pm,j} = r_{B,j} \sin (\delta_{B,j} \pm \gamma)$. Here $r_{B,j}$ and $\delta_{B,j}$ are the relative magnitude and strong (i.e. $CP$-conserving) phase of the $V_{ub}$ and $V_{cb}$ amplitudes for each $K^+\pi^-$ resonance $j$. In this analysis the $x_{\pm,j}$ and $y_{\pm,j}$ parameters are measured only for
The shapes of partially reconstructed and misidentified backgrounds are obtained from width parameters fixed to their known values \[31,44\] and magnitude constrained according to fit results onto \(\pi^+\pi^-\) for the samples with \(D \to K^+\pi^-\) (\(D \to K^+K^-\) or \(\pi^+\pi^-\)). The signal efficiency and backgrounds are modelled in the likelihood function, separately for each of the samples, following Refs. \[26,32,33\]. The CP violation effect is seen. The results of systematic uncertainties are expected to be reducible with larger data samples. The largest source of uncertainty in this category arises from lack of knowledge of the signal and background yields in the signal region, the efficiency breakdown of the systematic uncertainties is given in Ref. \[40\]. The leading sources of systematic uncertainty, such as production asymmetry \[46\] or removal of marginal components (the \(K^*(1410)^0\), \(K^*(1680)^0\), \(D_s^+(2760)^-\), \(D_s^0(2760)^-\) and \(D_{s1}^*(2573)^+\) resonances), and the use of alternative models for the \(K^+\pi^-\) S-wave and \(D\pi^-\) nonresonant amplitudes \[26\]. The possibilities of CP violation associated to the \(D_s^+(2700)^+\) amplitude, and of independent CP violation parameters in the two components of the \(K^+\pi^-\) S-wave amplitude \[45\], are also accounted for. The largest source of uncertainty in this category arises from changing the description of the \(K^+\pi^-\) S-wave. Other possible sources of systematic uncertainty, such as production asymmetry \[46\] or CP violation in the \(D \to K^+K^-\) and \(\pi^+\pi^-\) decays \[47,49\], are found to be negligible. A more detailed breakdown of the systematic uncertainties is given in Ref. \[40\]. The leading sources of systematic uncertainty are expected to be reducible with larger data samples.

The fit procedure is validated with ensembles of pseudoexperiments. In addition, samples of \(B_s^0 \to DK^-\pi^+\) decays are selected for each of the \(D\) decays and are used to test the fit with a model based on that of Refs. \[32,33\]. No significant CP violation effect is observed, consistent with the expectation that \(V_{ub}\) amplitudes are highly suppressed in this control channel.

Sources of systematic uncertainty on the \(x_\pm\) and \(y_\pm\) parameters can be divided into two categories: experimental and model uncertainties. The former includes effects due to knowledge of the signal and background yields in the signal region, the efficiency variation and background DP distributions, all of which are evaluated in similar ways to those described in Ref. \[26\]. Additionally, effects that may induce fake asymmetries are accounted for. The largest source of uncertainty in this category arises from lack of knowledge of the DP distribution for the \(B_s^0 \to D^+K^-\pi^+\) background.

Model uncertainties arise due to fixing parameters in the amplitude model, the addition or removal of marginal components (the \(K^*(1410)^0\), \(K^*(1680)^0\), \(D_s^+(2760)^-\), \(D_s^0(2760)^-\) and \(D_{s1}^*(2573)^+\) resonances), and the use of alternative models for the \(K^+\pi^-\) S-wave and \(D\pi^-\) nonresonant amplitudes \[26\]. The possibilities of CP violation associated to the \(D_s^+(2700)^+\) amplitude, and of independent CP violation parameters in the two components of the \(K^+\pi^-\) S-wave amplitude \[45\], are also accounted for. The largest source of uncertainty in this category arises from changing the description of the \(K^+\pi^-\) S-wave. Other possible sources of systematic uncertainty, such as production asymmetry \[46\] or CP violation in the \(D \to K^+K^-\) and \(\pi^+\pi^-\) decays \[47,49\], are found to be negligible. A more detailed breakdown of the systematic uncertainties is given in Ref. \[40\]. The leading sources of systematic uncertainty are expected to be reducible with larger data samples.
Figure 2: Dalitz plots for candidates in the $B$ candidate mass signal region in the $D \rightarrow K^+K^-$ and $\pi^+\pi^-$ samples for (a) $B^0$ and (b) $B^0$ candidates. Only candidates in the three purest NN bins are included. Background has not been subtracted, and therefore some contribution from $B_s \rightarrow D^*K^+\pi^-$ decays is expected at low $m(DK^+)$ (i.e. along the top right diagonal).

Figure 3: Projections of the $D \rightarrow K^+K^-$ and $\pi^+\pi^-$ samples and the fit result onto $m(K^\mp\pi^\pm)$ for (a) $B^0$ and (b) $B^0$ candidates. The data and the fit results in each NN output bin have been weighted according to $S/(S+B)$ and combined. The components are described in the legend.

for the $CP$ violation parameters associated with the $B^0 \rightarrow DK^*(892)^0$ decay are

$x_+ = 0.04 \pm 0.16 \pm 0.11$, \hspace{1cm} $y_+ = -0.47 \pm 0.28 \pm 0.22$, \hspace{1cm} $x_- = -0.02 \pm 0.13 \pm 0.14$, \hspace{1cm} $y_- = -0.35 \pm 0.26 \pm 0.41$,

where the uncertainties are statistical and systematic. The corresponding correlation matrices are given in Ref. [40].

The GammaCombo package [50] is used to evaluate constraints from these results on $\gamma$ and the hadronic parameters $r_B$ and $\delta_B$ associated with the $B^0 \rightarrow DK^*(892)^0$ decay. A
frequentist treatment referred to as the “plug-in” method, described in Refs. [51–53], is used. Figure 4 shows the results of likelihood scans for \( \gamma, r_B \) and \( \delta_B \). No value of \( \gamma \) is excluded at 95% confidence level (CL); the world-average value for \( \gamma \) [54,55] has a CL of 0.85.

The \( B^0 \to D K^* (892)^0 \) decay can also be used to determine parameters sensitive to \( \gamma \) with a quasi-two-body approach, as has been done with \( D \to K^+ K^- \), \( \pi^+ \pi^- \) [56], \( K^\pm \pi^\mp \), \( K^\pm \pi^\mp \pi^0 \), \( K^\pm \pi^\mp \pi^- \) [56,58] and \( D \to K^0_s \pi^+ \pi^- \) decays [59,60]. In the quasi-two-body analysis, the results depend on the effective hadronic parameters \( \kappa, \bar{r}_B \) and \( \bar{\delta}_B \), which are, respectively, the coherence factor and the relative magnitude and strong phase of the \( V_{ub} \) and \( V_{cb} \) amplitudes averaged over the selected region of phase space [17,40]. These parameters are calculated from the models for \( V_{cb} \) and \( V_{ub} \) amplitudes obtained from the fit for the \( K^* (892)^0 \) selection region \( |m(K^+ \pi^-) - m_{K^* (892)^0}| < 50 \text{ MeV}/c^2 \) and \( |\cos \theta_{K^*0}| > 0.4 \), where \( m_{K^* (892)^0} \) is the known value of the \( K^* (892)^0 \) mass [31] and \( \theta_{K^*0} \) is the \( K^*0 \) helicity angle, i.e. the angle between the \( K^+ \) and \( D \) directions in the \( K^+ \pi^- \) rest frame. To reduce correlations with the values for \( r_B \) and \( \delta_B \) determined from the DP analysis, the quantities \( \bar{R}_B = \bar{r}_B/r_B \) and \( \Delta \bar{\delta}_B = \bar{\delta}_B - \delta_B \) are calculated. The results are

\[
\kappa = 0.958^{+0.005+0.002}_{-0.010-0.045}, \quad \bar{R}_B = 1.02^{+0.03+0.06}_{-0.02-0.06}, \quad \Delta \bar{\delta}_B = 0.02^{+0.03+0.11}_{-0.02-0.06},
\]

where the uncertainties are statistical and systematic. The former are determined by varying the model parameters within their uncertainties from the fit. The largest source of systematic uncertainty arises from the treatment of \( CP \) violation in the \( K^+ \pi^- \) S-wave amplitude.

In summary, a data sample corresponding to 3.0 fb\(^{-1}\) of \( pp \) collisions collected with the LHCb detector has been used to measure, for the first time, parameters sensitive to the angle \( \gamma \) from a Dalitz plot analysis of \( B^0 \to D K^+ \pi^- \) decays. No significant \( CP \) violation effect is seen. The results are consistent with, and supersede, the results for \( A_d^{KK,\pi \pi} \) and \( R_d^{KK,\pi \pi} \) from Ref. [56]. Parameters that are needed to determine \( \gamma \) from quasi-two-body analyses of \( B^0 \to D K^* (892)^0 \) decays are measured. These results can be combined with current and future measurements with the \( B^0 \to D K^* (892)^0 \) channel to obtain stronger constraints on \( \gamma \).
Acknowledgements

We express our gratitude to our colleagues in the CERN accelerator departments for the excellent performance of the LHC. We thank the technical and administrative staff at the LHCb institutes. We acknowledge support from CERN and from the national agencies: CAPES, CNPq, FAPERJ and FINEP (Brazil); NSFC (China); CNRS/IN2P3 (France); BMBF, DFG and MPG (Germany); INFN (Italy); FOM and NWO (The Netherlands); MNiSW and NCN (Poland); MEN/IFA (Romania); MinES and FANO (Russia); MinECO (Spain); SNSF and SER (Switzerland); NASU (Ukraine); STFC (United Kingdom); NSF (USA). We acknowledge the computing resources that are provided by CERN, IN2P3 (France), KIT and DESY (Germany), INFN (Italy), SURF (The Netherlands), PIC (Spain), GridPP (United Kingdom), RRCKI and Yandex LLC (Russia), CSCS (Switzerland), IFIN-HH (Romania), CBPF (Brazil), PL-GRID (Poland) and OSC (USA). We are indebted to the communities behind the multiple open source software packages on which we depend. Individual groups or members have received support from AvH Foundation (Germany), EPLANET, Marie Skłodowska-Curie Actions and ERC (European Union), Conseil Général de Haute-Savoie, Labex ENIGMASS and OCEVU, Région Auvergne (France), RFBR and Yandex LLC (Russia), GVA, XuntaGal and GENCAT (Spain), The Royal Society, Royal Commission for the Exhibition of 1851 and the Leverhulme Trust (United Kingdom).

References


[7] M. Gronau and D. London, How to determine all the angles of the unitarity triangle from $B^0 \to D K_s^0$ and $B_s^0 \to D \phi$, Phys. Lett. B253 (1991) 483.


[27] LHCb collaboration, R. Aaij et al., First observations of $B^0 \rightarrow \bar{D}^0 f_0(980)$, JHEP 08 (2015) 005, arXiv:1505.01654.
[28] LHCb collaboration, R. Aaij et al., Search for the decay $B^0 \rightarrow \bar{D}^0 f_0(980)$, submitted to Phys. Rev. D.
[34] LHCb collaboration, R. Aaij et al., Dalitz plot analysis of $B^0 \rightarrow \bar{D}^0 \pi^+\pi^-$ decays, Phys. Rev. D92 (2015) 032002, arXiv:1505.01710.
[36] Belle collaboration, K. Abe et al., Study of $B^0 \rightarrow D^{(*)0} \pi^+\pi^-$ decays, arXiv:hep-ex/0412072.
[37] LHCb collaboration, R. Aaij et al., Observation of $B^0 \rightarrow \bar{D}^0 K^+K^-$ and evidence for $B_s^0 \rightarrow \bar{D}^0 K^+K^-$, Phys. Rev. Lett. 109 (2012) 131801, arXiv:1207.5991.
[38] LHCb collaboration, R. Aaij et al., Study of beauty baryon decays to $D^0\phi^-$ and $\Lambda_c^+h^-$ final states, Phys. Rev. D89 (2014) 032001, arXiv:1311.4823


[40] See supplemental material.


BaBar collaboration, B. Aubert *et al.*, *Constraints on the CKM angle γ in B0 → D0K∗0 and B0 → D0K∗0 from a Dalitz analysis of D0 and D0 decays to K±π±*, Phys. Rev. D79 (2009) 072003, arXiv:0805.2001.

Belle collaboration, K. Negishi *et al.*, *First model-independent Dalitz analysis of B0 → DK∗0, D → K±π± decay*, arXiv:1509.01098.
Supplemental material

A Mass fits

Projections of the fits to the $D \to K^+\pi^-$, $K^+K^-$ and $\pi^+\pi^-$ samples are shown with a logarithmic $y$-axis scale in Fig. 5, where all NN output bins have been combined after weighting according to $S/(S+B)$. Projections of the fits separated by NN output bin in each sample are shown in Figs. 6, 7 and 8. The fitted parameters obtained from all three data samples are reported in Table 1. The parameters $\mu(B)$, $\sigma(\text{core})$, $N(\text{core})/N(\text{total})$, $\sigma(\text{wide})/\sigma(\text{core})$ are, respectively, the peak position, the width of the Gaussian core, the fraction of the signal function contained in the core and the relative width of the two components of the $B^0$ signal shape. Quantities denoted $N$ are total yields of each fit component, while those denoted $f_{\text{signal}}^i$ are fractions of the signal in NN output bin $i$ (with similar notation for the fractions of the partially reconstructed and combinatorial backgrounds). The NN output bin labels 1–5 range from the bin with the lowest to highest value of $S/B$. The yields of each category inside the signal window defined as $\mu(B^0) \pm 2.5 \times \sigma(\text{core})$, which corresponds to 5246.6–5309.9 MeV/$c^2$, 5246.9–5310.5 MeV/$c^2$ and 5243.1–5312.3 MeV/$c^2$ in the $D \to K^+\pi^-$, $K^+K^-$ and $\pi^+\pi^-$ samples, are given in Tables 2, 3 and 4.

Figure 5: Results of fits to $DK^+\pi^-$ candidates in the (a) $D \to K^+\pi^-$, (b) $D \to K^+K^-$ and (c) $D \to \pi^+\pi^-$ samples. The data and the fit results in each NN output bin have been weighted according to $S/(S+B)$ and combined.
Figure 6: Results of the fit to $DK^+\pi^-$, $D \to K^+\pi^-$ candidates shown separately in the five bins of the neural network output variable. The bins are shown, from (a)-(e), in order of increasing $S/B$. The components are as indicated in the legend. The vertical dotted lines in (a) show the signal window used for the fit to the Dalitz plot.
Figure 7: Results of the fit to $DK^+\pi^-$, $D \rightarrow K^+K^-$ candidates shown separately in the five bins of the neural network output variable. The bins are shown, from (a)-(e), in order of increasing $S/B$. The components are as indicated in the legend. The vertical dotted lines in (a) show the signal window used for the fit to the Dalitz plot.
Figure 8: Results of the fit to $D K^+ \pi^-$, $D \to \pi^+ \pi^-$ candidates shown separately in the five bins of the neural network output variable. The bins are shown, from (a)-(e), in order of increasing $S/B$. The components are as indicated in the legend. The vertical dotted lines in (a) show the signal window used for the fit to the Dalitz plot.
Table 1: Results for the freely varied parameters obtained from the fits to the four data samples. Entries where no number is given are fixed to zero. Fractions marked (*) are not varied in the fit, and give the difference between unity and the sum of the other fractions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( D \to K^{+}\pi^- )</th>
<th>( D \to K^+K^- )</th>
<th>( D \to \pi^+\pi^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu(B) ) (MeV/c^2)</td>
<td>5278.3 ± 0.4</td>
<td>5278.7 ± 0.5</td>
<td>5277.7 ± 1.0</td>
</tr>
<tr>
<td>( \sigma) (MeV/c^2)</td>
<td>12.7 ± 0.4</td>
<td>12.7 ± 0.5</td>
<td>13.9 ± 0.8</td>
</tr>
<tr>
<td>( N/\sigma ) (total)</td>
<td>0.787 ± 0.017</td>
<td>0.798 ± 0.018</td>
<td>0.797 ± 0.018</td>
</tr>
<tr>
<td>( \sigma) (core)</td>
<td>1.80 ± 0.05</td>
<td>1.75 ± 0.05</td>
<td>1.76 ± 0.05</td>
</tr>
<tr>
<td>Exp. slope (c^2/GeV)</td>
<td>-1.84 ± 0.13</td>
<td>-1.05 ± 0.19</td>
<td>-1.35 ± 0.26</td>
</tr>
<tr>
<td>( N(B^0 \to DK\pi) )</td>
<td>3125 ± 79</td>
<td>418 ± 27</td>
<td>185 ± 21</td>
</tr>
<tr>
<td>( N(B^0 \to DK\pi) )</td>
<td>146 ± 27</td>
<td>1014 ± 41</td>
<td>429 ± 28</td>
</tr>
<tr>
<td>( N(\text{comb. bkgd.}) )</td>
<td>5694 ± 529</td>
<td>2092 ± 95</td>
<td>1288 ± 86</td>
</tr>
<tr>
<td>( N(B \to D^{(*)}K + X) )</td>
<td>2648 ± 454</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( N(B^0 \to D^*K\pi) )</td>
<td>3028 ± 115</td>
<td>543 ± 48</td>
<td>183 ± 33</td>
</tr>
<tr>
<td>( N(B^0 \to D^*K\pi) )</td>
<td>—</td>
<td>1493 ± 77</td>
<td>639 ± 52</td>
</tr>
<tr>
<td>( N(B^0 \to D^{(*)}\pi\pi) )</td>
<td>783 ± 67</td>
<td>146 ± 17</td>
<td>72 ± 11</td>
</tr>
<tr>
<td>( N(A_0^0 \to D^{(*)}p\pi) )</td>
<td>—</td>
<td>241 ± 47</td>
<td>118 ± 26</td>
</tr>
<tr>
<td>( N(A_0^0 \to D^{(*)}pK) )</td>
<td>416 ± 64</td>
<td>34 ± 9</td>
<td>17 ± 5</td>
</tr>
<tr>
<td>( N(B^0 \to D^{(*)}KK) )</td>
<td>371 ± 51</td>
<td>64 ± 15</td>
<td>33 ± 8</td>
</tr>
<tr>
<td>( N(B_s^0 \to D^{(*)}KK) )</td>
<td>171 ± 47</td>
<td>25 ± 11</td>
<td>14 ± 6</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\begin{array}{l|rrr}
\text{Parameter} & D \to K^{+}\pi^- & D \to K^+K^- & D \to \pi^+\pi^- \\
\hline
f_1^{\text{signal}} & 0.210 ± 0.012 & 0.187 ± 0.017 & 0.214 ± 0.029 \\
f_2^{\text{signal}} & 0.192 ± 0.008 & 0.186 ± 0.011 & 0.184 ± 0.019 \\
f_3^{\text{signal}} & 0.206 ± 0.008 & 0.201 ± 0.012 & 0.225 ± 0.019 \\
f_4^{\text{signal}} & 0.201 ± 0.007 & 0.215 ± 0.012 & 0.193 ± 0.018 \\
\hline
\end{array}
\end{align*}
\]
Table 2: Yields in the signal window of the fit components in the five NN output bins for the $D \rightarrow K^+\pi^-$ sample. The last column indicates whether or not each component is explicitly modelled in the Dalitz plot fit.

<table>
<thead>
<tr>
<th>Component</th>
<th>Yield</th>
<th>Included?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow DK^+\pi^-$</td>
<td>597</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0_s \rightarrow DK^+\pi^-$</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>comb. bkgd.</td>
<td>540</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^{(*)}K^+X^-$</td>
<td>305</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^*K^+\pi^-$</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D^{(*)}K^+\pi^-$</td>
<td>19</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{(*)}K^+K^-$</td>
<td>8</td>
<td>No</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D^{(*)}K^+K^-$</td>
<td>10</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 3: Yields in the signal window of the fit components in the five NN output bins for the $D \rightarrow K^+K^-$ sample. The last column indicates whether or not each component is explicitly modelled in the Dalitz plot fit.

<table>
<thead>
<tr>
<th>Component</th>
<th>Yield</th>
<th>Included?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow DK^+\pi^-$</td>
<td>70</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0_s \rightarrow DK^+\pi^-$</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>comb. bkgd.</td>
<td>173</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^*K^+\pi^-$</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D^*K^+\pi^-$</td>
<td>19</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{(*)}K^+\pi^-$</td>
<td>4</td>
<td>Yes</td>
</tr>
<tr>
<td>$A^0 \rightarrow D^{(*)}p\pi^-$</td>
<td>11</td>
<td>Yes</td>
</tr>
<tr>
<td>$\Lambda^0 \rightarrow D^{(*)}K^+\pi^-$</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{(*)}K^+K^-$</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D^{(*)}K^+K^-$</td>
<td>1</td>
<td>No</td>
</tr>
</tbody>
</table>
Table 4: Yields in the signal window of the fit components in the five NN output bins for the $D \rightarrow \pi^+\pi^-$ sample. The last column indicates whether or not each component is explicitly modelled in the Dalitz plot fit.

<table>
<thead>
<tr>
<th>Component</th>
<th>bin 1</th>
<th>bin 2</th>
<th>bin 3</th>
<th>bin 4</th>
<th>bin 5</th>
<th>Included?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow DK^+\pi^-$</td>
<td>36</td>
<td>31</td>
<td>38</td>
<td>32</td>
<td>31</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0_s \rightarrow DK^+\pi^-$</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>comb. bkgd.</td>
<td>119</td>
<td>17</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^*K^+\pi^-$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{(*)}\pi^+\pi^-$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>Yes</td>
</tr>
<tr>
<td>$A_0^0 \rightarrow D^{(*)}p\pi^-$</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>$A_0^0 \rightarrow D^{(*)}K^+\bar{p}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>$B^0 \rightarrow D^{(*)}K^+K^-$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D^{(*)}K^+K^-$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>No</td>
</tr>
</tbody>
</table>
B Uncertainties

Experimental and model uncertainties are summarised in Tables 5 and 6. The sources of experimental systematic uncertainty are from the signal and background yields in the signal window \((S/B)\), the variation of the efficiency \((\epsilon)\) across the Dalitz plot, the background Dalitz plot distributions \((B\,DP)\), fit bias, asymmetry between \(B^0\) and \(\bar{B}^0\) candidates in the background yields \((B\,asym.)\) as well as asymmetries in the background Dalitz plot distributions \((B\,DP\,asym.)\) and in \(\epsilon\). The sources of model uncertainty are due to fixed parameters in the model \((fixed\,pars.)\), the use of alternative models for the \(K^+\pi^-\,S\)-wave and \(D\pi\) nonresonant amplitudes \((alt.\,mod.)\) and the variation of the treatment of \(CP\) violation effects in the \(D_{s1}(2700)^+\) \((D_{s}^{**}\,CPV)\) and \(K^+\pi^-\,S\)-wave \((K\pi_{S-wave}\,CPV)\) components. The total uncertainties are obtained by combining all sources in quadrature. The statistical and systematic correlation matrices are given in Table 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty</th>
<th>(S/B)</th>
<th>(\epsilon)</th>
<th>(B,DP)</th>
<th>fit bias</th>
<th>(B,asym.)</th>
<th>(B,DP,asym.)</th>
<th>(\epsilon,asym.)</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_+)</td>
<td></td>
<td>0.010</td>
<td>0.035</td>
<td>0.046</td>
<td>0.021</td>
<td>0.007</td>
<td>0.049</td>
<td>0.000</td>
<td>0.079</td>
</tr>
<tr>
<td>(x_-)</td>
<td></td>
<td>0.026</td>
<td>0.028</td>
<td>0.063</td>
<td>0.019</td>
<td>0.010</td>
<td>0.045</td>
<td>0.001</td>
<td>0.089</td>
</tr>
<tr>
<td>(y_+)</td>
<td></td>
<td>0.019</td>
<td>0.042</td>
<td>0.122</td>
<td>0.066</td>
<td>0.017</td>
<td>0.027</td>
<td>0.000</td>
<td>0.149</td>
</tr>
<tr>
<td>(y_-)</td>
<td></td>
<td>0.024</td>
<td>0.022</td>
<td>0.054</td>
<td>0.035</td>
<td>0.018</td>
<td>0.071</td>
<td>0.000</td>
<td>0.103</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty</th>
<th>fixed pars.</th>
<th>add/rem.</th>
<th>alt. mod.</th>
<th>(D_{s}^{**},CPV)</th>
<th>(K\pi_{S-wave},CPV)</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_+)</td>
<td></td>
<td>0.027</td>
<td>0.028</td>
<td>0.068</td>
<td>0.008</td>
<td>0.003</td>
<td>0.079</td>
</tr>
<tr>
<td>(x_-)</td>
<td></td>
<td>0.030</td>
<td>0.034</td>
<td>0.076</td>
<td>0.056</td>
<td>0.022</td>
<td>0.107</td>
</tr>
<tr>
<td>(y_+)</td>
<td></td>
<td>0.075</td>
<td>0.061</td>
<td>0.131</td>
<td>0.012</td>
<td>0.047</td>
<td>0.170</td>
</tr>
<tr>
<td>(y_-)</td>
<td></td>
<td>0.040</td>
<td>0.066</td>
<td>0.255</td>
<td>0.286</td>
<td>0.064</td>
<td>0.396</td>
</tr>
</tbody>
</table>

Table 5: Experimental systematic uncertainties.

Table 6: Model uncertainties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty</th>
<th>(x_+)</th>
<th>(y_+)</th>
<th>(x_-)</th>
<th>(y_-)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>0.34</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td></td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>0.05</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>0.25</td>
<td>0.29</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(x_+)</th>
<th>(y_+)</th>
<th>(x_-)</th>
<th>(y_-)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00</td>
<td></td>
<td>0.87</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.29</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.37</td>
<td>0.41</td>
<td>0.73</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 7: Correlation matrices associated to the (left) statistical and (right) systematic uncertainties of the \(CP\) violation parameters associated with the \(B^0\rightarrow DK^*(892)^0\) decay.
C Results

The results, with statistical uncertainties only, for the complex coefficients $c_j$ are given in Table 8. Due to the changes in the selection requirements, the overlap between the $D \rightarrow K^+\pi^-$ sample and the dataset used in Ref. [26] is only around 60%, and the results are found to be consistent.

The results for $(x_+, y_+)$ and $(x_-, y_-)$ are shown as contours in Fig. 9. Invariant mass projections onto $m(D^0\pi^-)$ and $m(D^0K^+)$ are shown in Fig. 10. There is no evident $CP$ violation.

Figure 11 shows the two-dimensional 68% confidence level for each pair of observables from $\gamma$, $r_B$ and $\delta_B$. No region is excluded at 95% CL.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Real part</th>
<th>Imaginary part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{*}(892)^0$</td>
<td>-0.07 ± 0.10</td>
<td>-1.19 ± 0.04</td>
</tr>
<tr>
<td>$K^{*}(1410)^0$</td>
<td>0.16 ± 0.04</td>
<td>0.21 ± 0.06</td>
</tr>
<tr>
<td>$K_0^*(1430)^0$</td>
<td>0.40 ± 0.08</td>
<td>0.67 ± 0.06</td>
</tr>
<tr>
<td>Nonresonant $K\pi$ S-wave</td>
<td>0.37 ± 0.07</td>
<td>0.69 ± 0.07</td>
</tr>
<tr>
<td>$K_2^{*}(1430)^0$</td>
<td>-0.01 ± 0.06</td>
<td>-0.48 ± 0.04</td>
</tr>
<tr>
<td>$D_0^*(2400)^-$</td>
<td>-1.10 ± 0.05</td>
<td>-0.18 ± 0.07</td>
</tr>
<tr>
<td>$D_2^*(2460)^-$</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Nonresonant $D\pi$ S-wave</td>
<td>-0.44 ± 0.06</td>
<td>0.02 ± 0.07</td>
</tr>
<tr>
<td>Nonresonant $D\pi$ P-wave</td>
<td>-0.61 ± 0.05</td>
<td>-0.08 ± 0.06</td>
</tr>
<tr>
<td>$D_{s1}^*(2700)^+$</td>
<td>0.57 ± 0.05</td>
<td>-0.09 ± 0.19</td>
</tr>
</tbody>
</table>
Figure 9: Contours at 68\% CL for the (blue) \((x_+, y_+)\) and (red) \((x_-, y_-)\) parameters associated with the \(B^0 \to DK^{*}(892)^0\) decay, with statistical uncertainties only. The central values are marked by a circle and a cross, respectively.
Figure 10: Projections of the $D \rightarrow K^+K^-$ and $\pi^+\pi^-$ samples and the fit result onto (a,b) $m(D\pi^\mp)$ and (c,d) $m(DK^\mp)$ for (a,c) $B^0$ and (b,d) $B^0$ candidates. The data and the fit results in each NN output bin have been weighted according to $S/(S+B)$ and combined. The components are described in the legend. The projections onto $m(K^\mp\pi^\mp)$ are given in Fig. 3.
Figure 11: Confidence level contours for (a) $\gamma$ and $r_B$, (b) $\gamma$ and $\delta_B$ and (c) $r_B$ and $\delta_B$. The shaded regions are allowed at 68% CL.
D Quasi-two-body parameters

In the quasi-two-body analyses of $B^0 \rightarrow DK^*(892)^0$ decays, the following parameters are defined \cite{17}:

$$\kappa = \left| \frac{\int |A_{cb}(p)A_{ub}(p)| \exp \{i\delta(p)\} \, dp}{\sqrt{\int |A_{cb}(p)|^2 \, dp \int |A_{ub}(p)|^2 \, dp}} \right|, \quad (3)$$

$$\bar{r}_B = \left( \frac{\int |A_{ub}(p)|^2 \, dp}{\int |A_{cb}(p)|^2 \, dp} \right)^{1/2}, \quad (4)$$

$$\bar{\delta}_B = \arg \left( \frac{\int |A_{cb}(p)A_{ub}(p)| \exp \{i\delta(p)\} \, dp}{\sqrt{\int |A_{cb}(p)|^2 \, dp \int |A_{ub}(p)|^2 \, dp}} \right), \quad (5)$$

where all the integrations are over the part of the phase space $p$ inside the used $K^*(892)^0$ selection window. In these equations, $|A_{cb}(p)|$ and $|A_{ub}(p)|$ refer to the magnitudes of the total $V_{cb}$ and $V_{ub}$ amplitudes, and $\delta(p)$ is their relative strong phase. In terms of the parameters used in this analysis,

$$|A_{cb}(p)| = \left| \sum_j c_j F_j(p) \right|, \quad (6)$$

$$|A_{ub}(p)| = \left| \sum_j c_j r_{B,j} \exp \{i\delta_{B,j}\} F_j(p) \right|, \quad (7)$$

$$\delta(p) = \arg \left( \frac{\sum_j c_j r_{B,j} \exp \{i\delta_{B,j}\} F_j(p)}{\sum_j c_j F_j(p)} \right), \quad (8)$$

where the $r_{B,j}$, $\delta_{B,j}$ values are allowed to differ for each $K^+\pi^-$ resonance, and $r_{B,j} = 0$ for $D\pi^-$ resonances. (The $r_B$, $\delta_B$ notation without the $j$ subscript is retained for the parameters associated with the $B^0 \rightarrow DK^*(892)^0$ decay.) In the limit that there is no amplitude (either resonant or nonresonant) contributing within the $K^*(892)^0$ selection window other than those associated with the $B^0 \rightarrow DK^*(892)^0$ decay, one finds $|A_{ub}(p)| \rightarrow r_B |A_{ub}(p)|$ and $\delta(p) \rightarrow \delta_B$, and hence $\kappa \rightarrow 1$, $\bar{r}_B \rightarrow r_B$ and $\bar{\delta}_B \rightarrow \delta_B$. In order to reduce correlations between $\bar{r}_B$ and $r_B$ and between $\bar{\delta}_B$ and $\delta_B$, it is convenient to introduce the parameters

$$\bar{R}_B = \frac{\bar{r}_B}{r_B}, \quad (9)$$

$$\Delta \bar{\delta}_B = \bar{\delta}_B - \delta_B, \quad (10)$$

which are obtained by replacing all $r_{B,j}$ by $r_{B,j}/r_B$ and all $\delta_{B,j}$ by $\delta_{B,j} - \delta_B$ in Eqs. (6)–(8).

These quantities are determined from the results of the Dalitz plot analysis. An alternative fit is performed with $x_{\pm,j} + iy_{\pm,j}$, defined in Eq. (2), replaced by $r_{B,j} \exp \{i (\delta_{B,j} \pm \gamma)\}$.
The results of this fit are consistent with the values for $\gamma$, $r_B$ and $\delta_B$ obtained from the fitted $x_\pm$ and $y_\pm$, and are used to evaluate $|A_{cb}(p)|$, $|A_{ub}(p)|$ and $\delta(p)$ at many points inside the selection window and thereby to determine $\kappa$, $\bar{R}_B$ and $\Delta\bar{\delta}_B$. The procedure is repeated many times with both $V_{cb}$ and $V_{ub}$ amplitude model parameters varied within their statistical uncertainties from the fit, leading to the distributions shown in Fig. 12. Since the transformations from the fitted model parameters to the quasi-two-body parameters are highly non-linear, the reported central values correspond to the peak positions of these distributions, while positive and negative uncertainties are obtained by incrementally including the most probable values until 68% of all entries are covered.

Sources of systematic uncertainty are accounted for by evaluating their effects on the quasi-two-body parameters. The dominant sources are from the use of an alternative description of the $K^+\pi^-$ S-wave, and from changing the treatment of $CP$ violation in the $D^{*+}_{s1}(2700)$ component and the $K^+\pi^-$ S-wave. Most systematic uncertainties are symmetrised for consistency with the rest of the analysis, but asymmetric systematic uncertainties are reported on $\kappa$ since it is $\leq 1$ by definition.
LHCb collaboration

16 Sezione INFN di Cagliari, Cagliari, Italy
17 Sezione INFN di Ferrara, Ferrara, Italy
18 Sezione INFN di Firenze, Firenze, Italy
19 Laboratori Nazionali dell’INFN di Frascati, Frascati, Italy
20 Sezione INFN di Genova, Genova, Italy
21 Sezione INFN di Milano Bicocca, Milano, Italy
22 Sezione INFN di Milano, Milano, Italy
23 Sezione INFN di Padova, Padova, Italy
24 Sezione INFN di Pisa, Pisa, Italy
25 Sezione INFN di Roma Tor Vergata, Roma, Italy
26 Sezione INFN di Roma La Sapienza, Roma, Italy
27 Henryk Niewodniczanski Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland
28 AGH - University of Science and Technology, Faculty of Physics and Applied Computer Science, Kraków, Poland
29 National Center for Nuclear Research (NCBJ), Warsaw, Poland
30 Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania
31 Petersburg Nuclear Physics Institute (PNPI), Gatchina, Russia
32 Institute of Theoretical and Experimental Physics (ITEP), Moscow, Russia
33 Institute of Nuclear Physics, Moscow State University (SINP MSU), Moscow, Russia
34 Institute for Nuclear Research of the Russian Academy of Sciences (INR RAN), Moscow, Russia
35 Budker Institute of Nuclear Physics (SB RAS) and Novosibirsk State University, Novosibirsk, Russia
36 Institute for High Energy Physics (IHEP), Protvino, Russia
37 Universitat de Barcelona, Barcelona, Spain
38 Universidad de Santiago de Compostela, Santiago de Compostela, Spain
39 European Organization for Nuclear Research (CERN), Geneva, Switzerland
40 Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland
41 Physik-Institut, Universität Zürich, Zürich, Switzerland
42 Nikhef National Institute for Subatomic Physics, Amsterdam, The Netherlands
43 Nikhef National Institute for Subatomic Physics and VU University Amsterdam, Amsterdam, The Netherlands
44 NSC Kharkiv Institute of Physics and Technology (NSC KIPT), Kharkiv, Ukraine
45 Institute for Nuclear Research of the National Academy of Sciences (KINR), Kyiv, Ukraine
46 University of Birmingham, Birmingham, United Kingdom
47 H.H. Wills Physics Laboratory, University of Bristol, Bristol, United Kingdom
48 Cavendish Laboratory, University of Cambridge, Cambridge, United Kingdom
49 Department of Physics, University of Warwick, Coventry, United Kingdom
50 STFC Rutherford Appleton Laboratory, Didcot, United Kingdom
51 School of Physics and Astronomy, University of Edinburgh, Edinburgh, United Kingdom
52 School of Physics and Astronomy, University of Glasgow, Glasgow, United Kingdom
53 Oliver Lodge Laboratory, University of Liverpool, Liverpool, United Kingdom
54 Imperial College London, London, United Kingdom
55 School of Physics and Astronomy, University of Manchester, Manchester, United Kingdom
56 Department of Physics, University of Oxford, Oxford, United Kingdom
57 Massachusetts Institute of Technology, Cambridge, MA, United States
58 University of Cincinnati, Cincinnati, OH, United States
59 University of Maryland, College Park, MD, United States
60 Syracuse University, Syracuse, NY, United States
61 Pontificia Universidade Católica do Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil, associated to 2
62 University of Chinese Academy of Sciences, Beijing, China, associated to 3
63 Institute of Particle Physics, Central China Normal University, Wuhan, Hubei, China, associated to 3
64 Departamento de Fisica, Universidad Nacional de Colombia, Bogota, Colombia, associated to 8
65 Institut für Physik, Universität Rostock, Rostock, Germany, associated to 12
66 National Research Centre Kurchatov Institute, Moscow, Russia, associated to
67 Yandex School of Data Analysis, Moscow, Russia, associated to
68 Instituto de Fisica Corpuscular (IFIC), Universitat de Valencia-CSIC, Valencia, Spain, associated to
69 Van Swinderen Institute, University of Groningen, Groningen, The Netherlands, associated to

a Universidade Federal do Triângulo Mineiro (UFTM), Uberaba-MG, Brazil
b Laboratoire Leprince-Ringuet, Palaiseau, France
c P.N. Lebedev Physical Institute, Russian Academy of Science (LPI RAS), Moscow, Russia
d Università di Bari, Bari, Italy
e Università di Bologna, Bologna, Italy
f Università di Cagliari, Cagliari, Italy
g Università di Ferrara, Ferrara, Italy
h Università di Urbino, Urbino, Italy
i Università di Modena e Reggio Emilia, Modena, Italy
j Università di Genova, Genova, Italy
k Università di Milano Bicocca, Milano, Italy
l Università di Roma Tor Vergata, Roma, Italy
m Università di Roma La Sapienza, Roma, Italy
n Università della Basilicata, Potenza, Italy
o AGH - University of Science and Technology, Faculty of Computer Science, Electronics and Telecommunications, Kraków, Poland
p LIFAELS, La Salle, Universitat Ramon Llull, Barcelona, Spain
q Hanoi University of Science, Hanoi, Viet Nam
r Università di Padova, Padova, Italy
s Università di Pisa, Pisa, Italy
t Scuola Normale Superiore, Pisa, Italy
u Università degli Studi di Milano, Milano, Italy

† Deceased