iParker—A New Smart Car-Parking System Based on Dynamic Resource Allocation and Pricing

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Abstract—Parking in major cities, particularly with dense traffic, directly effects the traffic flow and people’s life. In this paper, we introduce a new smart parking system that is based on intelligent resource allocation, reservation, and pricing. The proposed system solves the current parking problems by offering guaranteed parking reservations with the lowest possible cost and searching time for drivers and the highest revenue and resource utilization for parking managers. New fair pricing policies are also proposed that can be implemented in practice. The new system is based on mathematical modeling using mixed-integer linear programming (MILP) with the objective of minimizing the total monetary cost for the drivers and maximizing the utilization of parking resources.

Index Terms—Dynamic pricing, dynamic resource allocation, mixed integer linear programming (MILP), reservation, smart car parking.

I. INTRODUCTION

Parking is an expensive process in terms of either money or the time and effort spent for the “free spot chasing.” Current studies reveal that a car is parked for 95 percent of its lifetime and only on the road for the other 5 percent [1]. If we take England in 2014 as an example, on average a car was driven for 361 hours a year according to the British National Travel Survey [2] yielding about 8404 hours in which a car would be parked. Now where would you park your car for these very long hours? Cruising for parking is naturally the first problem caused by the increase of car owners globally. On average, 30 percent of traffic is caused by drivers wandering around for parking spaces [3]. In 2006, a study in France revealed an estimation that 70 million hours were spent every year in France only in searching for parking which resulted in the loss of 700 million euros annually [4]. In 2011, a global parking survey by IBM [5] states that 20 minutes is spent on average in searching for a coveted spot. With these statistics, we can assume that a great portion of global pollution and fuel waste is related to cruising for parking.

Parking spaces are found to be more than plenty in some places and very rare to find in others. Pricing policies had played an important role in the overall parking availability for decades [6]. Here comes the important question: do we need to have more parking spaces or do we need better parking management? We believe it is the later and thus the motivation behind this work is about better parking management with fair and profitable pricing policies.

The work presented in this paper combines parking reservation and pricing models to overcome the parking problems. On the parking reservation side, Mouskos et al. [7] modeled the reservation process as a resource allocation problem. Their model is based on MILP with the objective of minimizing driver cost. Their model offers real time reservations with fixed pricing. Geng et al. [8] had extended their work by taking into account the user’s cost in terms of pricing and walking distance. In addition, they had expanded the model by adding fairness constraints and applying extensive simulations. Although the system proposed in [8] is very good, their model is still limited by being suitable only for short-term reservations and the parking revenue was not considered.

On the pricing side, Shoup et al. [3] introduced new concepts which led to the development of San Francisco Park (SFPark) [9] in San Francisco which aims to overcome the traffic congestion by dynamically changing prices based on sensor historical data. In SFPark, sensors are deployed on the asphalt to gather parking information that are stored in a database and processed weekly or monthly. According to historical data, the prices are increased and decreased proportional to the expected utilization. Although dynamically changing parking prices shall balance the supply and demand for parking and increase overall utilization, it is based on historical data and statistics which may not be accurate enough to have the proper effect.

In this paper, we present a new smart car parking system, named iParker, with static resource scheduling, dynamic resource allocation and pricing models, to optimize the parking system for both parking managers and drivers. The contributions of our work include: 1) increasing parking resource utilization, 2) increasing parking revenue, 3) improving parking experience of drivers by lowering cost, parking spot searching and walking times. Our work is different from the one in [8] where a dynamic resource allocation model was proposed. The main limitations of that model are that only reservation for limited period of time (e.g., few minutes) was allowed, fixed price was used and revenue was not taken into account and only a single choice of destination was considered. Whereas our model allows a driver to reserve a parking space for any time in the future, the revenue is considered and new pricing models are introduced. In addition, a parking solution with their individual journey planners is proposed. Our work in the pricing side also

Manuscript received April 29, 2015; revised July 14, 2015 and September 7, 2015; accepted September 21, 2015. Date of publication May 5, 2016; date of current version August 25, 2016. The Associate Editor for this paper was J. Cao.

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Digital Object Identifier 10.1109/TITS.2016.2531636

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differs from Shoup’s work as we propose a real-time dynamic pricing in parking and prove its effectiveness.

The rest of this paper is organized as follows: The related work is presented in Section II. In Section III, the overview, architecture and reaction scheme of the proposed system are presented. In Section IV, the smart resource allocation model is described and formulated. Extensive simulations and discussions are presented in Section V. Last, Section VI concludes this paper.

II. Related Work

For the past two decades, there have been numerous researches and investments in the car parking domain. Some of them had been deployed in practice like Parking Guidance and Information (PGI) systems [10]–[12]. PGI systems provide drivers with real time information on parking within controlled areas through variable message signs. They use deployed sensors mainly on the entrances and exits of parking areas to gather information about total occupancy. Other implementations typically use one sensor per one parking spot which has been seen in commercial shopping malls and in business districts to further utilize parking spaces and decrease searching time.

Most of the researches have focused on how to detect the occupancy state of parking spots [13]–[16]. However, those systems still have not solve all the problems. The competition for parking leads to higher traffic congestion where parking is monitored, leaving other parking resources vacant. Also this leads for the known phenomenon of “multiple cars chase same spot.” It is indeed essential to have the data on the occupancy state in parking areas but it is more important to efficiently utilize those data.

There are others researched parking reservation systems. For instance, Trusiewicz et al. [17] used Unstructured Supplementary Service Data (USSD) as communication medium between divers and parking reservation system. Although it is not free to use USSD for most of network operators, it is still a cheap and reliable technology to adopt in parking reservations. Inaba et al. [18] utilized RFID tags to store and update the reservations status and they discussed the difference between real-time and share-time reservations where the difference between them is that in share-time reservations, drivers must use the service in a known entry and exit time frame as they share the resource time. Whereas in real-time reservations, they are allowed to park for unlimited interval of time for being independent on other drivers. Wang et al. [19] had introduced a prototype for a distributed system at which there is one central processor which gathers the reservation requests and redirects them to the relevant local processors. Their system utilizes Blue-tooth and WiFi to detect the occupancy states inside parking lots, and notify the drivers with available spaces accordingly.

Short Message Services (SMS) reservations were presented in many research papers. For example, Hanif et al. [20] developed an embedded SMS reservation system using microcontroller, keypad, gate access control and a remote terminal unit (Micro-RTU). Micro-RTU is a standalone terminal with a processor and a GSM module to receive SMS and trigger I/O pins. Reservation over the internet was demonstrated in [21] by using a sensor network of ZigBee and pressure sensors to detect the occupancy state of parking spots. Reservations were allowed using a website. These reservation systems could reduce the overall parking problems indeed. However they mainly concentrated on the communication technologies or medium between drivers and the systems and rarely addressed a model or algorithm that efficiently manages the reservation in such a fashion that produces significant improvements.

Hashimoto et al. [22] proposed a reservation system that is auction based, at which there is an interval for the reservation process, drivers would need to register before the deadline at which the system will allocate the spaces based on the highest bid. In general, the auction based methods could lead to many fairness issues.

III. iParker—System Overview

Our new concept is to combine real time reservations (RTR) with share time reservations (STR), thus a driver can reserve a spot while heading to it (e.g., few minutes away) and also can reserve it at any time earlier (e.g., many days away). RTR are achieved by performing dynamic resource allocation which is similar to skills based routing in call centers. In the case of RTR, drivers are constantly allocated the best parking spots available until they reach their destinations. Whereas STR are achieved by performing static resource allocation that is based on time scheduling where a driver can explicitly choose the preferred resource and the time frame at which it will be occupied at anytime in the future. Different pricing policies for both types of reservations that are fair for drivers and parking managers are proposed in this paper. In addition, a dynamic pricing engine which periodically updates the parking prices based on real time resource utilization by occupancy and reservations and other events is introduced. iParker features the normal and disabled parking spots and drivers are given the freedom of choosing multiple destinations and the system will assign the optimal resources according to their chosen destinations and circumstances.

Throughout this paper, we will employ the term “parker” to refer to a driver or a car, “resource” to refer to a group of parking spots, “D-Type” to refer to a dynamic reservation, RTR or the type of a driver requesting a dynamic reservation and “S-Type” to refer to a static reservation, STR or the type of a driver requesting a static reservation.

A. Architecture and Reaction Scheme

iParker is a semi-distributed system as shown in Fig. 1: there are one central request center (CRC), one parking manager (PM) and multiple local smart allocation systems (SASs).

The CRC receives parkers’ requests, processes them and diverts them to the relevant local SAS. The request process is as following: parker chooses one to many destinations and if he/she is an S-type, preferred parking resource can be selected. Both types have to assign a weight parameter from 0-1 which reflects their desire between resource-destination proximity and resource price. Both types also set the maximum price and walking distance they can tolerate. For S-type, the spot
occupancy interval has to be defined. For D-type, the GPS coordinates are measured and attached to the request. Finally the parker identifications (i.e., driver and car license IDs) are accompanied with the request. CRC also responds back to all parkers with reservation offers and in the same fashion notifies the local SAS with parker’s response.

The PM is a central parking manager who is an interface among parking authorities, parking resource managers, SASs and local pricing engines. Parking authorities can use the PM to manually update the relevant pricing engine or data centre. For instance, to fix pricing values for certain parking resource or update the data centre with upcoming events near a relevant resource.

Below we describe the main components of a local smart allocation system:

- **Pricing Engine**—Pricing engines are small applications that run a pricing model on web-servers. The duties of a pricing engine are to fetch parking utilization data and updates from parking authorities every predefined time interval and to set the new parking prices accordingly. The engine runs independent on the SAS, calculates the new prices and updates the data centre.

- **Sensors**—Every resource is occupied with a spot occupancy detection system. Ideally this system must provide accurate data on the utilization of the parking resource, deployed either indoors or outdoors. The detection system is normally composed of a wireless/wired sensor network that can provide occupation state of every parking spot, or alternatively composed of counter sensors at the entrance and exit of parking lots that is only capable of providing total utilization value. The later method can only work in controlled environments, therefore we prefer to use sensor networks and a central processor that updates the data centre with the utilization values.

- **Data Centre**—Holds all the information from all iParker components and store them in a structured data container. It’s consisted of a pricing table which contains the up to date information on pricing per resource per minute, utilization table which holds the utilization data, and finally authority table which stores other parameters that is set by parking authorities (e.g., events related). A Data Centre is also responsible for updating multiple types of virtual message signs and public devices of up to date pricing information and parking availability.

- **Smart Allocation Centre**—A web service that runs a sophisticated MILP model that optimally and fairly assigns/reserves parking resources to the parkers. The assignment is based on key variables that are not limited to driver constraints, current resource utilization, up to date pricing information and events occurrences. The centre provides non stop parking reservation service to the parkers and is described in details in the next section.

- **Virtual Message Sign (VMS)**—Updates parkers/public with up to date pricing and parking availability information. This is achieved by deploying numerous numbers of VMS panels across cities especially around on-street parking areas. For off-street parking lots, one VMS panel at the entrance is sufficient to inform arrivals of updated information. It is important to mention that a parker will only pay according to the price rate fixed in the reservation offer. If the parker is not using the service, he/she will pay according to the price rate displayed at the time of his/her parking. VMS is specially and critically important for non smart-phone.

IV. SMART RESOURCE ALLOCATION

The problem addressed in this study combines the real time and share time reservation systems. Real time reservations are typically independent on the amount a parker will consume in a parking space, i.e., a parker can spend as much time as he/she needs without affecting the rest of the parkers. On the other hand, share time reservations are dependent on the exact spot occupancy and spot leave times. Share time reservations are generally modeled as birth-death stochastic processes. In our model, dynamic reservations are real time and static reservations are share time.
The objective of our MILP model is to minimize the total monetary cost for parkers and ultimately maximize the total resource utilization to obtain the maximum revenue for parking managers. We will formulate our model based on the queuing model in Fig. 2. There are N resources in which every resource \( j \) is split into \( P_1 \) spots (number of normal parking spots for dynamic reservations), \( P_2 \) spots (number of normal parking spots for static reservations) \( P_3 \) and \( P_4 \) (similar to \( P_1 \) and \( P_2 \) but for disabled people). The running time of the smart allocation centre is discretized into small time periods. We will denote each time period as a decision point \( K \). All parkers arrive to the allocation center randomly and independently joins the relevant WAIT queue. At each decision point, the allocation centre will allocate resources to dynamic and static parkers and move them to the relevant RESERVE queue. Parkers in the dynamic reserve queue (DRESERVE) will get re-allocated a better parking spot (if available) after each decision point until they reach a defined zone defined as their first destination. Parkers in the static wait queue (SWAIT) will only get allocated once and then move to the static reserve queue (SRESERVE). When parkers arrive to their resources, they will be moved to the occupy queue and then \( P_1, P_2, P_3 \) or \( P_4 \) will be decremented by the number of parkers. When parkers leave the parking spaces, they will be completely removed from the system and again the count of spaces would be incremented. To fully maximize the utilization of resources, we will initialize the system by making 50% of the resources to be “dynamic” and 50% to be “static.” Then we will follow the strategy in Fig. 3: when parkers in DWAIT queue reach their destination and fail to get allocated for the reason that \( P_1 \) or \( P_3 \) became zero (i.e., no free parking spaces for dynamic reservers), the system automatically diverts them to SWAIT queue, such that they get a chance for allocation for the “static” resources.

A. Problem Formulation

At each decision point \( K \), we will define the state of the smart allocation system \( A(K) \) and the state of parkers \( X_i(K) \) as follows (see Table I for the list of definitions):

\[
A(K) = \{DW(K), SW(K), DR(K), SR(K), Z(K)\} \tag{1}
\]

\[
X_i(K) = \{l_i(K), \psi_i(K), u_i(K), \omega_i(K), F_i(K)\} \tag{2}
\]

The key input of the allocation system is the feasible resources \( F_i(K) \) that each parker is eligible for. To formulate this, we define some major attributes for parkers. 1) \( M_i \) is the maximum total price that parker \( i \) can afford to pay. 2) \( D_i \) is the maximum total walking distance that parker \( i \) can tolerate. 3) \( \varphi_i \) is the type of parking spot that parker \( i \) needs. 4) \( \beta_i \) is the type of reservation. 5) \( P_{ij}(K), \bar{P}_{ij} \) and \( g_i(K) \) are parameters that describe the state of parking resources and are explained later.
Below we define some key binary variables:

\[ \varphi_i = \begin{cases} 1, & \text{if } i \text{ is requesting a normal resource} \\ 0, & \text{if } i \text{ is requesting a disabled resource} \end{cases} \]

\[ \beta_i = \begin{cases} 1, & \text{if } i \text{ is requesting a dynamic reservation} \\ 0, & \text{if } i \text{ is requesting a static reservation} \end{cases} \]

\[ \psi_i(K) = \begin{cases} j, & \text{if } i \text{ reserved the resource } j \\ 0, & \text{otherwise} \end{cases} \]

By defining \( L_j \) as the location of the resource \( j \), the remaining distance \( d_{ij}(K) \) and driving time \( \tau_{ij}(K) \) at time \( K \) between parker \( i \) and resource \( j \) can be estimated as follows:

\[ d_{ij}(K) = ||l_i(K) - L_j|| \]

\[ \tau_{ij}(K) = \frac{d_{ij}(K)}{V_i(K)}. \]

Now it is possible to set a pricing scheme for parker \( i \)'s first attribute \( M_{ij}(K) \) as a function of \( \tau_{ij}(K), \psi_i(K), T_i \) (the total occupancy time) and \( C_j(K) \) (the current price per hour for occupying resource \( j \)). As shown in (7), we charge the dynamic parker a reservation fee that is equivalent to his/her total reservation time. On the other hand, we charge the static parker a flat reservation fee equivalent to the price of an occupancy hour

\[ M_{ij}(K) = \begin{cases} \left( \frac{C_i(K)}{60} \right)(\psi_i(K) + T_i + \tau_{ij}(K)), & \text{if } \beta_i = 1 \\ \left( \frac{C_i(K)}{60} \right)T_i + C_j(K), & \text{if } \beta_i = 0. \end{cases} \] (7)

For both static and dynamic parkers, we allow them to choose multiple destinations. \( \mathcal{D}_i = \{ d_1, d_2, \ldots, d_N \} \) is the set of locations of the destinations \( \mathcal{D}_i = \{ 1, 2, \ldots, n_d \} \) chosen by parker \( i \) with \( d_1 \) being the first destination. A parker can choose up to \( n_d \) destination. Parker \( i \)'s second attribute \( D_{ij}(K) \) can now be formulated to express the total traveling time on foot. Equation (8) allows the allocation system to identify the nearest resource \( j \) to parker \( i \) according to his/her chosen destinations \( \mathcal{D}_i \)

\[ D_{ij}(K) = \sum_{n \in \mathcal{D}_i} ||d_{n_1} - L_j||. \] (8)

Now we can calculate the total cost function \( J_{ij}(K) \) that we will minimize for parker \( i \) according to the weight \( sw_i = [0 - 1] \). If parker \( i \) wants the cheapest resource, then \( sw_i = 1 \) is the choice. If parker \( i \) is only interested in the best spot in terms of walking distance, then \( sw_i = 0 \) is the choice. A value 0.5 is also possible for a combination between price and proximity

\[ J_{ij}(K) = sw_i \frac{M_{ij}(K)}{M_i} + (1 - sw_i) \frac{D_{ij}(K)}{D_i}. \] (9)

Remark: We have employed the grouping spot technique in this model to save computational power, such that a resource \( j \) may have \( N \) number of spots. For example, \( \mathbb{P}1_j = 1, \mathbb{P}2_j = 2, \mathbb{P}3_j = 3 \) and \( \mathbb{P}1_j = 4 \) means that in resource \( j \), there are 1 normal-dynamic, 2 normal-static, 3 disabled-dynamic and 4 disabled-static free spots.

For dynamic parkers, the allocation system must be fed with data from parking sensors in real-time. Therefore we define \( P_{ij}(K) \) as the number of free dynamic spots in resource \( j \) that is compatible for parker \( i \)'s parking type. We also define \( g_i(K) \) as the set of resources that contains at least 1 free parking spot. \( g_i(K) \) will be equal to either the set of free normal parking resources denoted as \( \sigma(K) \) or the set of free disabled parking resources denoted as \( \bar{\sigma}(K) \)

\[ P_{ij}(K) = \varphi_i \mathbb{P}1_j(K) + (1 - \varphi_i) \mathbb{P}3_j(K) \] (10)

\[ \sigma(K) = \{ j : j \in \text{Resources}, \mathbb{P}1_j(K) > 0 \} \]

\[ \bar{\sigma}(K) = \{ j : j \in \text{Resources}, \mathbb{P}3_j(K) > 0 \} \]

\[ g_i(K) = \begin{cases} \sigma(K), & \text{if } \varphi_i = 1 \\ \bar{\sigma}(K), & \text{if } \varphi_i = 0. \end{cases} \] (11)

\( \Pi_i(K) \) can now be defined as the set of feasible resources that can be allocated to dynamic parkers. \( \Pi_i(K) \) is determined

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<td>Decision point in time.</td>
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<tr>
<td>( DW(K) )</td>
<td>Set of parkers in DWAIT queue.</td>
</tr>
<tr>
<td>( SW(K) )</td>
<td>Set of parkers in SWAIT queue.</td>
</tr>
<tr>
<td>( DR(K) )</td>
<td>Set of parkers in DRESERVE queue.</td>
</tr>
<tr>
<td>( SR(K) )</td>
<td>Set of parkers in SRESERVE queue.</td>
</tr>
<tr>
<td>( Z(K) )</td>
<td>Set describing the # of vacant spots in all resources.</td>
</tr>
<tr>
<td>( P1 )</td>
<td># of normal parking spots for dynamic reservations.</td>
</tr>
<tr>
<td>( P2 )</td>
<td># of normal parking spots for static reservations.</td>
</tr>
<tr>
<td>( P3 )</td>
<td># of disabled parking spots for dynamic reservations.</td>
</tr>
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<td>( P4 )</td>
<td># of disabled parking spots for static reservations.</td>
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<tr>
<td>( \psi_i )</td>
<td>Location of resource ( j ).</td>
</tr>
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<td>( C_j(K) )</td>
<td>Price per hour for occupying resource ( j ).</td>
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<td>( \phi_i )</td>
<td>Binary variable describes if parker ( i ) is allocated resource ( j ).</td>
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<td>( l_i(K) )</td>
<td>Location of parker ( i ).</td>
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<tr>
<td>( M_i )</td>
<td>Maximum total price that parker ( i ) can afford to pay.</td>
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<tr>
<td>( \bar{\theta} )</td>
<td>Occupancy interval starting time.</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Occupancy interval ending time.</td>
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<td>Set of feasible resources for dynamic parkers.</td>
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<td>Set of feasible resources for any parker ( i ) of any type.</td>
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</table>
by filtering all the resources to match with parker $i$’s highest boundaries on price and proximity ($M_i$ and $D_i$). If resource $j$ is from the same type (normal/disabled) that parker $i$ demands, there are free parking spots of that kind and boundary requirements are met, $j$ will be added to $\Pi_i$ at decision time $K$

$$\Pi_i(K) = \{ j : M_{ij}(K) \leq M_i, \ D_{ij}(K) \leq D_i, \ P_{ij}(K) > 0, \ j \in g_i(K) \}.$$ \hfill (12)

We will employ different approach to define the feasible resources for static parkers. As we mentioned earlier that static reservations are share-time reservation system. Thus the allocation system will allocate resources to static parkers according to the availability of free parking spots at the occupancy starting time for parker $i$ and of course according to parker’s requirements.

We define $\bar{P}_{ij}$ as the total number of static spots in resource $j$ that is compatible for parker $i$’s parking type, i.e.,

$$\bar{P}_{ij} = \varphi_i P_{2j} + (1 - \varphi_i) P_{4j}$$ \hfill (13)

$\Theta_{ij}(K)$ is then computed, which is a binary variable equal to 1 if there is a conflict in occupancy intervals of parkers $i$ and $v$ in $SW(K)$ and $SR(K)$ on a resource $j$ of the same kind. Occupancy interval starts at $\theta$ and ends at $\vartheta$,\n
$$\Theta_{ij}(K) = \begin{cases} 1, & \text{if } ((\theta_{ij} \geq \theta_{v}) \land (\theta_{ij} \leq \vartheta_{ij})) \land (\varphi_i = \varphi_v) \land (\beta_i = \beta_v) \land (i \neq v) \\ 0, & \text{otherwise.} \end{cases}$$

Then we define matrix $E(K) = [\Theta_{ij}(K)]$ and introduce the key array $conflict_{ij}(K)$ which allows the resource allocation for static parkers such that

$$conflict_{ij}(K) = \sum_{i \in SW(K), j \in Resources} E(K).$$ \hfill (14)

Based on $conflict_{ij}(K)$ and parker requirements, it is possible to compute the feasible resources for static parkers $\Phi_i(K)$ as follows:

$$\Phi_i(K) = \{ j : M_{ij}(K) \leq M_i, \ D_{ij}(K) \leq D_i, \ conflict_{ij}(K) < \bar{P}_{ij}(K), \ j \in Resources \}.$$ \hfill (15)

We will combine all parkers together (dynamic and static) in one objective function. This can be achieved by introducing $F_i(K)$ as the set of feasible resources for each parker $i$ of any type

$$F_i(K) = \begin{cases} \Pi_i(K), & \text{if } \beta_i = 1 \\ \Phi_i(K), & \text{if } \beta_i = 0. \end{cases}$$ \hfill (16)

### B. Objective Function

From the parker’s point of view, iParker minimizes the overall parker cost in terms of price and proximity. From the parking managers’ point of view, iParker maximizes the resource utilization and total revenue. We introduce the binary decision variable $x_{ij}(K)$ such that

$$x_{ij}(K) = \begin{cases} 1, & \text{if parker } i \text{ is assigned resource } j \\ 0, & \text{otherwise} \end{cases}$$ \hfill (17)

and define matrix $X(K) = [x_{ij}(K)]$. Now we can formulate the objective function and constraints for our problem that is solved at each decision point $K$:

$$\text{minimize} \ \sum_{i \in DW(K) \cup DR(K) \cup F_i(K)} \sum_{j \in F_i(K)} x_{ij}(K) \cdot J_{ij}(K)$$

$$+ \sum_{i \in DW(K) \cup SW(K)} \left(1 - \sum_{j \in F_i(K)} x_{ij}(K) \right)$$ \hfill (18)

$$\text{s.t.:} \ \sum_{j \in F_i(K)} x_{ij}(K) \leq 1 \ \forall i \in DW(K) \cup SW(K)$$ \hfill (19)

$$\sum_{i \in DW(K) \cup DR(K) \cdot \varphi_i = 1} x_{ij}(K) \leq \varphi_1(K) \ \forall j \in \sigma(K)$$ \hfill (20)

$$\sum_{i \in DW(K) \cup DR(K) \cdot \varphi_i = 0} x_{ij}(K) \leq \varphi_3(K) \ \forall j \in \sigma(K)$$ \hfill (21)

$$\sum_{j \in F_i(K)} x_{ij}(K) = 1 \ \forall i \in DR(K)$$ \hfill (22)

$$\left(\sum_{n \in F_i(K)} x_{in}(K) \right) - x_{mj}(K) \geq 0 \ \forall i, m \in DW(K), \ j \in F_i(K)$$ \hfill (23)

The objective function in (18) can be split into 2 parts, $\sum_{i \in DW(K) \cup DR(K)} \sum_{j \in F_i(K)} x_{ij}(K) \cdot J_{ij}(K)$ and $\sum_{i \in DW(K) \cup SW(K)} \left(1 - \sum_{j \in F_i(K)} x_{ij}(K) \right)$. In this problem we minimize the objective function and this will have two effects according to the mentioned parts. The first part aims to minimize the total monetary cost in (9) for all parkers in $DW(K)$ and $DR(K)$, such that parker $i$ will be assigned the resource $j$ in his/her feasible resources $F_i$ with the least $J_{ij}$. Note here that we did not include the parkers from $SW(K)$ nor $SR(K)$; because static parkers do get allocated only once and for the $j$ of their choice. If we did not add the second part to the equation, the system will not allocate any parkers by setting all $x_{ij}(K)$ to zero. Therefore we introduce the second part to allocate as much parkers as possible in the $DW(K)$ and $SW(K)$. Resource allocation will be maximized because $J_{ij}$ by its definition is less than 1, thus adding a cost of 1 to the objective function is satisfactory enough to guarantee maximum resource allocation.
The constraints in this problem can be described as follows:

- **Capacity**—1) constraint (19) ensures that all parkers in $DW(K)$ and $SW(K)$ cannot be assigned more than one resource. Also it indicates that those parkers might not get allocated, i.e., $x_{ij}(K) = 0$. 2) constraints (20) and (21) indicate that the sum of numbers of dynamic parkers who are going to get reservations (parkers in $DW(K)$) and parkers who already got reservations earlier (parkers in $DR(K)$) must be less than the total unoccupied spots at time $K$.

- **Reservation guarantee**—Constraint (22) guarantees that every parker $i$ in $DR(K)$ must retain their allocation. Note here that parkers in $SR(K)$ are not mentioned. This is because static parkers (by the definition of the objective function) get allocated once and they do not enter the allocation system again once allocated and therefore their reservation guarantee is also true.

- **Cost guarantee**—All kinds of reservation systems must commit to the offer or quotation they supply to the customer, and this is what constraint (24) will achieve. The system will record all the parkers’ cost $J$ at every decision time $K$ and it will ensure that it does not reallocate any parker to a resource $j$ with cost higher than $J(K - 1)$. Also note that $SR(K)$ is not mentioned in the constraint; because a static parker will choose the preferred resource $j$ that he/she wants to occupy. Thus the value of $J$ of that parker will never change.

- **Fairness**—Constraint (23) indicates that if parker $i$ is located nearer to his/her feasible resources $F_i(K)$ as compared to parker $m$ such that $\tau_{mj}(K) > \tau_{ij}(K)$ and parkers $i$ and $m$ are requesting the same parking type, then a priority of allocation would be given to parker $i$ such that $x_{mj}(K)$ must be set to 0 if $x_{ij}(K) = 0$.

To further maximize the resource utilization by “dynamic” parkers, constraint (25) is introduced where we define $t_0(K)$ to be the threshold at which a parker must be $t_0(K)$ further away from destination in order to be eligible for dynamic allocation. $t_0(K)$ is set dynamically according to the real-time resource utilization following the rules in Table II.

### C. Dynamic Pricing Engine

We also examine the effect of dynamically changing the prices of occupying spots in real time fashion based on real-time utilization data of each resource $j$ rather than changing them every couple of days or months based on historical data. The dynamic pricing engine will operate every predefined minutes to update prices according to the rules in Table II.

Table II will be utilized as following: if a resource $j = 1$ at a given time $K = 1$ with all parking spots free and an original price $C_0$, the spot price at this time would be set to $C_1(K) = 0.25 * C_0$, because utilization is found to be 0%. Now if Utilization increased to 60%, the engine will increase the price to the full original price $C_0$. Similarly when the utilization further increases, the price increases till its maximum (200% of the original price) and when the utilization falls, the price will drop to its minimum (25% of the original price).

The motivation behind dynamic pricing is to introduce a fair balance of utilization and revenue across all parking resources which in turn will assist in reducing overall traffic congestion. It is important to note that pricing change would only take effect on parkers in WAIT queue. Which means that a parker who already got a reservation or occupied a spot will be paying at the same rate that was fixed for him/her at the time of reservation/occupation.

In order to realize this in practice and for drivers who will not use iParker, VMSs should be deployed nearby the parking resources to show the latest pricing information and the time at which the next pricing update would be made.

### D. Algorithm and Implementation

The software used to solve the MILP problem is IBM ILOG CPLEX (CPLEX). In order to evaluate the system effectiveness, sets of data are first randomized to represent the data of parkers, resources, destinations and pricing. Using Microsoft Excel, the parkers arrivals are generated following Poisson distribution and the rest of parameters are generated following exponential distribution. A database is then created to store the random data and act as the storage node for the CPLEX program. The CPLEX program that is discussed in Fig. 4 inputs the random data generated earlier and updates the database after the parkers’ allocation.

### V. Results and Discussions

**Performance Metrics**: from the point of parker’s view, smart parking should cost less (either in terms of money or walking distance or searching time or all). On the other hand, from the point of parking managers’ view, smart parking should provide
the highest resource utilization and generate the highest revenue. Thus we define the following main performance metrics:

- Total Utilization—is the total average resource utilization and we denote it as $U_{avg}$. We also break it down in the simulation results to parking ($U_P$), reservation ($U_R$), normal-parker ($U_{Normal\_avg}$) and disabled-parker ($U_{Disabled\_avg}$) utilization.
- Revenue—is the revenue generated and we break it to on-street and off-street revenues in the simulations.
- Searching Time—is the average time spent by a parker from the time of reaching their destination to the time of physically occupying it.
- Total Cost—is the average total cost incurred to a parker who ultimately had occupied a parking resource and can be formulated as
  \[
  \text{TotalCost} = \left(\frac{1}{3}\right) \left(\frac{2}{\text{sw}_i \left(M_{ij}(K)/M_j\right)} + \left(1 - \text{sw}_i\right) \left(D_{ij}(K)/D_i\right)\right) + \left(\frac{\text{SearchingTime}_i(K)}{\text{SearchingTime}_{max}}\right)
  \]
- Wandering—is the ratio of parkers who arrived to their destination, however they could not find or get allocated an available parking resource.

By adding “—on” or “—off” to any of the metrics, we denote to on-street and off-street respectively. Also note that “D” refers to “Dynamic” and “S” refers to “Static.”

Simulations Setup: In this section, iParker system is denoted as Smart-Parking (SP), guided system as (G) and nonguided as (NG). G is modeled to be a smart parking system but without reservations and it is described as follows: parkers know about the real-time availability of parking resources, their pricing and their proximity to their targeted destinations. Parkers in G will minimize their cost exactly as in SP. As for NG, parkers do not have any information about parking resources availability nor price information. In NG system, parkers will search for a free tolerated parking resource in an increasing radius method until they occupy it. The values in Table III will be used in all simulations (see Fig. 5 for the simulation case study environment).

Scalability: MILP problems are NP-Hard and the time consumption of problem solving is highly proportional to the problem complexity. In addition, the static allocation part of our problem gets more time consuming as the parkers reserve resources in the late future. In our problem, it is critical to obtain a solution at each decision point in a reasonable time interval. Therefore, the following strategies are considered to reduce the size of the problem and are adopted in the next simulations:
1) Grouping: The number of resources can be very much reduced by grouping resources together, such that a resource will contain several parking spaces (e.g., a parking lot or a street). Similarly, destinations that are close can be grouped. 2) Area splitting: If the number of resources and destinations are still very large, the area can be split to a number of sub-areas where a problem will be solved for each. 3) Reservations control: The dynamic reservations can be limited by discriminating users who are very far away from their destinations. Whereas the static reservations can be limited by reducing the time frame at which a parker can reserve a spot (e.g., 1 week).

### A. Simulations Results I: Uniform Arrival Rate

The results in Table IV prove the concepts behind our system. From the parking managers’ point of view, the total average utilization increases by 21% which represents 16% and 14% increase in revenue as compared with non-guided and guided systems, respectively. Although the utilization by parking (denoted by UP) is higher in other systems compared to SP, the total utilization by SP is the highest due to the introduction of dynamic and static reservations. The effect of dynamically varying $t_0(K)$ can be seen in the results as the U-on of SP in dense traffic is lower than that of G and NG. We expected this to occur as by definition of (25). When the parking spaces are close to being saturated, $t_0(K)$ approaches zero. This happens more quickly for on-street parking in dense traffic because they
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Fig. 6. Performance metrics using variant arrival rate.

TABLE V
PERFORMANCE METRICS WITH DIFFERENT DYNAMIC-STATIC RESOURCE RATIOS

<table>
<thead>
<tr>
<th>DR:SR</th>
<th>10:90</th>
<th>30:70</th>
<th>50:50</th>
<th>70:30</th>
<th>90:10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>Revenue</td>
<td>0.50</td>
<td>0.53</td>
<td>0.55</td>
<td>0.56</td>
<td>0.54</td>
</tr>
<tr>
<td>Utilization</td>
<td>0.45</td>
<td>0.47</td>
<td>0.48</td>
<td>0.49</td>
<td>0.48</td>
</tr>
</tbody>
</table>

are much cheaper than off-street parking, therefore the SP system allocates the incoming arrivals to off-street parking till the on-street utilization decreases. This however can be considered as an advantage because it has introduced a good balance of utilization between different parking resources which in turn balances the general traffic flow.

On the other hand, from the parkers’ point of view, SP proven to offer parkers the lowest combined cost as compared to G and NG. For instance, in dense traffic, the total cost is reduced by 28% and 10% as compared to NG and G respectively. Although the cost in terms of money in SP due to reservations is higher than other systems, the overallarker satisfaction is higher when using SP. This is clearly shown in the results for dense traffic where the searching time is decreased in SP by about 25 minutes as compared to NG and 11 minutes as compared to G. The increase in searching time in G in dense traffic is mainly due to the phenomenon of “multiple cars chasing the same spot.” The dramatic increase of searching time in NG is due to the fact that they search for available spaces blindly. Also SP has the lowest cost because of the zero wandering time (the time spent when a parker arrives destination and finds no available parking space). The wandering ratio for parkers in NG increases by about 20% in dense traffic as compared to normal traffic and by about 16% in G.

B. Simulations Results II: Variant Arrival Rate

The merits of iParker are more visualized in Fig. 6 where we have performed another simulations with variant arrival rates from high rate in the morning and in the evening and low rate in the afternoon. The second simulation results agree with the first simulation results in terms of maximizing the utilization parking resources, increasing revenue and minimizing parkers’ cost.

However there are a few things to note here: 1) the revenue at 7 PM for SP is greater than 1. This could normally happen when the parking resource is near to be fully occupied and taking into account that the fees for reservation is added on top. 2) the total cost of parkers is not reduced in SP as compared to G in the times with low arrival rate. This is because, in G, the cost is minimized exactly like in SP, and when the arrival rate is low, the probability of wandering in G is close to zero and therefore the searching time is minimal. This however can be solved by reducing the reservation fees at the times of low arrival rate. 3) the searching time in SP is about constant throughout the day, which confirms that our model does not allow wandering for users and thus decreases the overall traffic congestion.

We compared our system with that of [8] and it is shown in Fig. 6 that our model yields on average about 5% more utilization, 40% less searching time and 18% more revenue. The increase in utilization and revenue is clearly because our model allows static reservations and also the reservation time threshold for dynamic reservations is not fixed as in [8]. Finally an increase of 4% in the total parker cost is seen and this is because in [8], they do not charge fees for reservations. These conclude that our model does outperform the one in [8].
Most of the parameters used in the simulations are set to be dynamic and are not fixed except for the parameter that sets the ratio between the resources that are configured for dynamic reservations to the static ones (DR:SR). Table V shows the main performance metrics with different DR:SR under variant arrival rate. The table shows DR:SR\textsuperscript{17} = 70:30 having the best results. The reason is that the arrival rate is set to 70:30 for parkers as shown in Table III. However the changes observed are negligible. This proves the good efficiency of the dynamic-static interface discussed earlier. Therefore the DR : SR = 50:50 will be reasonable to be the default setting as the parkers’ choices between dynamic and static reservations in real world will not be constant.

C. Simulations Results III: Fixed Pricing vs. Dynamic Pricing

In this section, we explore the effect of dynamically varying the resource pricing according to real time utilization measure using the scheme in Table II and we present the results in Fig. 7. It is observed as expected that by continuously changing the prices of resources, we can control and limit the utilization of those resources. Furthermore, these changes results in a fair balance of utilization between parking resources which in turn assist in reducing the overall traffic congestion caused by parking. The average utilization of the parking resource 1 is higher than that of other resources when using fixed pricing. On the other hand, a significant change occurs when using dynamic pricing, such that the average utilization of the 3 parking resources is close to identical.

VI. CONCLUSION AND FUTURE WORK

In this paper we have proposed iParker, a new smart parking system which is based on MILP model that yields optimal solution for dynamically and statically allocating parking resources to parkers—providing flexible reservation options. The new concepts introduced in this paper are the combination of real-time reservations with share-time reservations, dynamically performing system decisions (reservation time constraints and pricing) according to real-time utilization information, and offering the drivers the choice of choosing multiple destinations and reservation type. We also have proposed pricing policies for both static and dynamic reservations that maximize the profit from parking. Extensive simulation results indicate that the proposed system significantly cuts the total effective cost for all parkers by as much as 28%, maximizes the total utilization by up to 21% and increases the total revenue for parking management up to 16% as compared to the non-guided parking system. Finally we proposed a dynamic pricing scheme and by integrating it to iParker’s model, we found by simulations that it balances the utilization across all the parking resources and thus assist in eliminating the overall traffic congestion caused by parking.

Currently, the research focuses on a new parking sensing infrastructure and an indoor navigation service for car parking. In the future, we aim to evaluate our system using real-time data and greater number of resources and destinations. In addition, a scalability analysis is to be performed to examine the efficiency of the proposed scalability techniques. Last, it would also be useful to simulate different parking arrival scenarios in real life.
REFERENCES


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