HEAT TRANSFER ENHANCEMENT IN MICRO-SCALE GEOMETRIES

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by

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Nomenclature

**Symbols**

- $a_R$: Rectangular channel width (m)
- $a$: Parameter of the Carreau-Yasuda model $^{228}$
- $A$: Cross-section area ($m^2$)
- $A_s$: Surface area ($m^2$)
- $a_T$: Shift factor (-)
- $b_R$: Rectangular channel depth (m)
- $c$: Polymer concentration (ppm)
- $C$: Specific heat capacity ($J/kg.K$)
- $c^*$: Critical overlap concentration (ppm)
- $De$: Deborah numbers (-)
- $D_H$: Hydraulic diameter (m)
- $E$: Enhancement efficiency (-)
- $e_f$: Relative pressure-drop (-)
- $e_{Nu}$: Enhancement of heat transfer (-)
- $f$: Darcy friction factor (-)
- $F$: Total normal force (N)
- $f_F$: Fanning friction factor (-)
- $fRe$: Friction factor-Reynolds number product (-)
- $G'$: Storage modulus (Pa)
- $G''$: Loss modulus (Pa)
- $Gz$: Graetz number (-)
- $\bar{h}$: Averaged heat transfer coefficient ($W/m^2.°C$)
- $h_x$: Local heat transfer coefficient ($W/m^2.°C$)
- $I$: Fluorescence intensity ($a.u.$)
- $I_{ref}$: Fluorescence intensity at the reference temperature $T_o$ ($a.u.$)
- $k$: Thermal conductivity of fluid ($W/m.°C$)
- $K$: Dean number (-)
- $K_{M-H}$: The coefficients of the Mark-Houwink formula in Eq. $3.14$ (-)
\(k_B\) Boltzmann’s constant \((J/K)\)
\(l\) Path-length \((m)\)
\(L\) Length of one serpentine unit \((m)\)
\(L_c\) Characteristic length \((m)\)
\(m\) Mass \((kg)\)
\(\dot{m}\) Mass flow rate \((kg/s)\)
\(M\) Geometry-dependent constant \((-)\)
\(M_w\) Molecular weight of polymer \((g/mol)\)
\(n\) Power law index in the Carreau-Yasuda model fits \([228]\)
\(N\) Number of units of the serpentine channel \((-)\)
\(N_A\) Avogadro’s constant \((mol)\)
\(N_1\) First normal-stress difference \((Pa)\)
\(N_2\) Second normal-stress difference \((Pa)\)
\(Nu\) Nusselt number \((-)\)
\(Nu_x\) Local Nusselt number \((-)\)
\(\bar{Nu}\) Mean Nusselt number \((-)\)
\(P\) Static pressure \((Pa)\)
\(Pr\) Prandtl numbers \((-)\)
\(q''\) Heat flux \((W/m^2)\)
\(Q\) Heat transfer rate \((W)\)
\(\dot{Q}\) Volume flow rate \((m^3/s)\)
\(r\) The radius of tube \((m)\)
\(R\) Universal gas constant \((8.314 \ J/mol.K)\)
\(R_c\) Curvature radius \((m)\)
\(R_{cone}\) Cone radius \((m)\)
\(Re\) Reynolds number \((-)\)
\(R_i\) Inner radius of curvature of the serpentine channel \((m)\)
\(R_o\) Outer radius of curvature of the serpentine channel \((m)\)
\(t\) Time \((s)\)
\(T\) Temperature \((^\circ C)\)
\(T_f\) Mean film temperature \((^\circ C)\)
\(T_q\) Torque \((N.m)\)
\(T_o\) Reference temperature \((^\circ C)\)
\(\bar{T}\) Mean temperature \((^\circ C)\)
\(T_{RMS}\) Root mean square temperature \((^\circ C)\)
\(u\) Velocity magnitude \((m/s)\)
\(\mathbf{u}\) velocity vector \((m/s)\)
\(U\) Velocity \((m/s)\)
\[ w \] Spanwise (transverse) velocity component \((m/s)\)

\[ W \] Width or depth of the square cross-section channel \((m)\)

\[ Wi \] Weissenberg number (-)

\[ Wi^* \] Modified Weissenberg number (-)

\[ x \] Position vector \((m)\)

\[ x_i \] independent variables (-)

\[ X_1, X_2, X_3 \] Locations along one serpentine unit \((m)\)

\[ x, y, z \] Cartesian coordinates (-)

**Greek symbols**

\[ \alpha \] Thermal diffusivity \((m^2/s)\)

\[ \Delta F \] The difference in the normal force due to inertia \((N)\)

\[ \Delta H \] Activation energy \((kJ/mol)\)

\[ \Delta P \] Pressure drop \((Pa)\)

\[ \Delta T_B \] Bulk fluid temperature difference \(\circ C\)

\[ \Delta T_{lm} \] Log-mean temperature difference \(\circ C\)

\[ \varepsilon \] Uncertainty (-)

\[ \varepsilon_R \] Uncertainty in the dependent parameter (-)

\[ \dot{\gamma} \] Shear rate \((1/s)\)

\[ \dot{\gamma}_{CH} \] Characteristic shear-rate \((1/s)\)

\[ \dot{\gamma}_r \] Reduced shear rate \((1/s)\)

\[ \eta \] Dynamic viscosity \((Pa.s)\)

\[ \eta_{CH} \] Characteristic shear viscosity \((Pa.s)\)

\[ \eta_i \] The viscosity modes in the generalised Maxwell model \((Pa.s)\)

\[ \eta_r \] Reduced shear viscosity \((Pa.s)\)

\[ \eta_s \] The solvent viscosity \((Pa.s)\)

\[ \eta_o \] Zero-shear-rate viscosity \((Pa.s)\)

\[ \eta_o(T) \] Zero-shear viscosity at a test temperature \(T\) \((Pa.s)\)

\[ \eta_o(T_o) \] Zero-shear viscosity at the reference temperature \(T_o\) \((Pa.s)\)

\[ [\eta] \] The intrinsic viscosity \((ml/g)\)

\[ \eta_\infty \] Infinite-shear-rate viscosity \((Pa.s)\)

\[ \lambda \] Fluid relaxation time \((s)\)

\[ \lambda_{CY} \] Time constant in Carreau-Yasuda model \((s)\)

\[ \lambda_i \] The relaxation time modes in the generalised Maxwell model \((s)\)

\[ \lambda_{Maxwell} \] The relaxation time from Maxwell model \((s)\)

\[ \lambda(T) \] The relaxation time at a test temperature \(T\) \((s)\)

\[ \lambda(T_o) \] The relaxation time at the reference temperature \(T_o\) \((s)\)

\[ \lambda_{Zimm} \] Zimm’s relaxation time \((s)\)

\[ \nu \] The solvent quality parameter (-)
\(\nu\)  
Momentum diffusivity \((m^2/s)\)

\(\rho\)  
Density \((kg/m^3)\)

\(\rho_o\)  
Density at the reference temperature \(T_o\) \((kg/m^3)\)

\(\sigma\)  
Shear stress \((Pa)\)

\(\tau_{xy}\)  
Shear stress \((Pa)\)

\(\tau_{xx}\)  
Normal stress \((Pa)\)

\(\tau_{yy}\)  
Normal stress \((Pa)\)

\(\theta\)  
Dimensional mean temperature \((-)\)

\(\omega\)  
Angular velocity (angular frequency) \((rad/s)\)

**Subscripts**

*Asym.*  
Asymmetric flow regime

*B*  
Bulk or mean

*c*  
Critical

*Exp.*  
The measured values of the \(G'\) and \(G''\) in Eq. 3.12

*in,C*  
Mean inlet cold

*in,H*  
Mean inlet hot

*max*  
Maximum

*m,i*  
Mean inlet

*min*  
Minimum

*Mod.*  
The predicted values of the \(G'\) and \(G''\) in Eq. 3.12

*m,o*  
Mean outlet

*Newt.*  
Newtonian

*N – Newt.*  
Non-Newtonian or viscoelastic

*Serpentine*  
Serpentine or curved channel

*Straight*  
Straight channel

*Sym.*  
Symmetric flow regime

*out*  
Mean outlet

*w*  
Wall
Declaration

I hereby declare that no portion of the work referred to in this PhD thesis entitled “Heat transfer enhancement in micro-scale geometries” has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

September, 2016

Waleed M. Abed
Abstract

Micro-geometries or “microfluidics” are commonly utilised in a widespread variety of applications such as, bioengineering devices, microelectronic devices, electronics cooling, chemical micro-reactors and mini or micro-heat exchangers. In the micro-scale systems (with “small” dimensions typically less than 1 millimeter), however, fluid mixing has been understood as one of the most fundamental and difficult-to-achieve issues because the flow of Newtonian fluids becomes increasingly controlled by viscous forces rather than inertia (as molecular diffusion is dominant at these small scales). As a consequence, the enhancement of convective heat transfer is problematic under these conditions (steady and laminar flow regime).

In this thesis, two different regimes of instabilities, namely “purely-inertial” and “purely-elastic”, have been adopted to enhance the convective heat transfer in the micro-scale geometries. Purely-inertial instability refers here to the secondary flow that arise in curved channels, also known as Dean flows, due to the centrifugal forces and also in crossed channels (cross-slot), symmetry-breaking bifurcations, which results in an axially-oriented spiral vortex along the outlet channels. While, purely-elastic instability is created in the flow of non-Newtonian viscoelastic fluids through curved channels due to the non-linear interaction between elastic stresses generated within the flowing viscoelastic solutions and the streamline curvature or through cross-slot device as a consequence of the planar extensional flow field (strong elongational flow) at the stagnation point. Fluid flow and convective heat transfer characteristics have been investigated experimentally and supporting numerical calculations for Newtonian flow within two different micro-geometries: a square cross-section serpentine microchannel and a square cross-section cross-slot micro-device. A group of Newtonian fluids, aqueous glycerine solutions and aqueous sucrose solutions, was utilised to carry out the experiments for purely-inertial flows whilst high-viscosity polymeric viscoelastic fluids, shear-thinning and approximately constant-viscosity Boger solutions, were used for the experiments to investigate purely-elastic instabilities.
For Newtonian fluids, the experimental results of the averaged Nusselt number show good agreement with the present numerical data where the deviations of the mean Nusselt number between the experimental results and numerical data are 10.7% and 8.6% with \( Pr = 1038 \) and 137, respectively. As a result of the existence of secondary flows in the flow fields of the serpentine microchannel geometry, the heat transfer performance of the serpentine microchannel is higher than that of the equivalent straight microchannel with the same cross-section over the entire range of Reynolds number (Dean number). Simultaneously, the relative pressure-drop losses in the serpentine microchannel rise with increasing Dean number. However, this increase is much smaller over the whole range of Dean number than the heat transfer enhancement. The maximum percentage increase of relative pressure-drop losses for both Prandtl numbers, 1038 and 137, is up to 2% and 39%, respectively, whereas the enhancement of heat transfer for \( Pr = 1038 \) and 137 increases by 56% and 158%, respectively.

For the viscoelastic solutions, the values of the product of friction factor-Reynolds number \( (fRe_{(N-Neut,)}) \) normalised by friction factor-Reynolds number product for laminar fully-developed Newtonian fluid flow \( (fRe_{(Neut,)}) \) in a straight square duct versus Weissenberg number indicated that, for \( Wi < 5 \), the normalised non-dimensional pressure drop is similar to that for a Newtonian fluid and with further increasing Weissenberg number \( (5 < Wi < 25) \) the purely-elastic instability develops and leads to an increase in the normalised pressure-drop. The increase in the normalised non-dimensional pressure-drop values for all viscoelastic solutions beyond \( Wi \approx 25 \) are much greater than the Newtonian limit, suggesting that the complexity of the elastic instabilities increase with increasing \( Wi \). Elastic turbulence generated in the flow of viscoelastic solutions is shown to enhance the convective heat transfer in the serpentine microchannel by approximately 200% for 50 ppm PAA in glycerine-based solution and 80 ppm PAA in sucrose-based solution and reaches up to 380% for 200 ppm PAA in glycerine-based solution and 500 ppm PAA in sucrose-based solution under creeping-flow conditions in comparison to that achieved by the equivalent Newtonian fluid flow at identical Graetz number.

For Newtonian fluids, a symmetry-breaking bifurcation occurs at the stagnation point in the cross-slot geometry when the Reynolds number increases beyond a certain critical value \( (\approx 40) \) \[107, 150\] and an axially-oriented spiral vortex is created along the outflow channels. Therefore, this spiral vortex promotes the mixing process in the cross-slot. This three-dimensional behaviour of the bifurcated flow enhances the heat transfer between the hot and cold fluids by convection along
the outflow channels. The normalised values of the root mean square temperature ($T_{RMS}$) collapse entirely for both cases (symmetry-imposed and cross-slot) along the outlet channel in the same pattern even with varying Prandtl number (1 - 100) in the case of Reynolds number lower than critical value ($Re < 40$) due to the poor mixing and diffusion-dominant heat transfer. For Reynolds numbers greater than the critical value ($Re > 40$), the normalised values of the $T_{RMS}$ for the cross-slot geometry show effectively enhanced heat transfer between the hot and cold streams for all Prandtl numbers. The heat transfer enhancement based on the ratio of $T_{RMS}$ between the symmetric and asymmetric flow regimes increases monotonically from approximately 1.6 to 3.7 with increasing Reynolds number from 40 to 100, respectively, at $Pr = 1$ due to the effect of the inertial instability.

For the flow of viscoelastic fluid (100 ppm PAA in 70%-glycerine-based solution), the isothermal dye visualisation at the centre of the cross-slot and also 20W downstream of the stagnation point at low Weissenberg number, $Wi = 5.8$, showed that the first weakly unstable elastic instability (symmetry-breaking bifurcation) between inflow streams is inadequate to strongly mix the viscoelastic fluid flowing through the cross-slot micro-geometry. Therefore, the temperature distribution shows two distinctly separated streams at very different temperatures in the centre of the cross-slot, even at the 20W distance downstream away from the stagnation point. As the Weissenberg number is further increased to $Wi = 118.5$ ($Re = 7.26$), the effect of strong unsteadiness on the quality of fluid mixing in the cross-slot is significantly increased. Thus, the instantaneous experimental temperature distribution of the 100 ppm PAA in 70%-glycerine-based solution is significantly modified towards a more uniform the temperature distribution between the hot and cold streams.
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*Waleed*....
Chapter 1

Introduction

1.1 Background and motivation for present study

The practical applications of micro-scale systems, such as bioengineering devices, micro-mixing, micro-rheology, microelectronic devices, cooling systems of computer chips, and mini or micro-heat exchangers have recently received a great deal of attention due to the development of fabrication technologies for these systems. In fact, micro-scale geometries or “microfluidics” refer to devices and methods for controlling and manipulating fluid flows with micrometer length scales (typically < 1 millimeter) [223]. The fundamental benefits of these micro-scale systems lead to several advantages compared to their traditional large-scale counterparts: (a) the possibility of fabricating individual and integrated devices with length scales on the order of tens and hundreds of microns make them more convenient for applications in the area of lab-on-a-chip, microelectromechanical systems (MEMS) devices for biological and chemical analysis. (b) Effective heat dissipation because of the large surface-to-volume ratio, potentially making micro-scale devices excellent tools for compact heat exchangers. (c) Comparing to traditional methods, micro-scale devices need a very small amount of sample to conduct the experiment with faster reactions.
Nevertheless, micro-scale geometries have often been associated with negligible effects of inertia due to their small dimensions. Traditionally, Newtonian fluid flows are assumed to occur at extremely small Reynolds number, $Re = \rho U L_c/\eta$, in these micro-geometries for example, for water with density, $\rho \sim 1000 \text{ kg/m}^3$ and viscosity, $\eta \sim 0.001 \text{ Pa.s}$, in a channel with a diameter of $L_c=100 \mu m$, the Reynolds number of the flow approaches 1 for a mean flow velocity, $U$ of $\sim 0.01 \text{ m/s}$. Thus, Newtonian fluids are difficult to be well mixed at short length and reasonable time because the flow in these micro-scale geometries is inherently steady and laminar. In this case, fluid mixing only relies on the effect of molecular diffusion due to the absence of inertial effects (lack of flow instability occurring by convection). Therefore, the use of conventional geometrical shape, e.g., straight microchannel, in these micro-devices requires more than several tens of centimeters to achieve the acceptable performance of mixing, which is unacceptable with these compact micro-devices. As a consequence, mixing at the micro-scale is difficult to achieve (as molecular diffusion is dominant) and thus the enhancement of convective heat transfer is problematic under these conditions.

However, enhancing heat transfer is considered one of the main motivating factors for these micro-scale systems. As the field of micro-devices continues to grow significantly, it is becoming increasingly important to enhance and improve the mechanisms of heat transfer and fluid flow within these compact micro-devices.

The scope of this study resides within the enhancement of heat transfer in micro-scale geometries. Therefore, there are two main points that provide motivation for this study: (a) various types of inertial effects are beginning to see utilisation in microfluidic systems recently. Secondary flows, also sometimes known as inertial instabilities [51], which arise depending on fluid inertia in the curvilinear [48, 49, 155, 216] or cross-shaped [92, 107, 108, 150] geometries, can be used for applications in enhanced mixing in the micro-geometries, and thereby enhancing convective heat transfer; (b) It has long been known that addition of polymers to working
fluctuations allows secondary flows and instabilities to be induced in mechanically driven flows [20, 120, 122], and a new kind of turbulence known as “elastic turbulence” in polymeric viscoelastic solutions has recently been discovered [76–78, 121, 187] which can be excited at vanishingly low Reynolds numbers.

A brief explanation about the concept and mechanism of inertial instabilities in Newtonian fluid flow and elastic instabilities in viscoelastic fluid flow is going to provide in this chapter.

1.2 Inertial instabilities in Newtonian fluid flow

Hydrodynamic instabilities in the flow of Newtonian fluids have received much attention of scientists and engineers since their discovery. In 1883, a well-known experimental investigation of turbulence transition in pipe flow was reported by Osborne Reynolds [176], who visualised flow by adding a dye stream to the center of water flowing through a pipe. Experimental observations demonstrated that when the velocity of water in the pipe is sufficiently low, the interface between the dyed and undyed water is smooth and straight representing that the flow is stable and laminar as shown in Figure 1.1 (a). For intermediate flow velocity, the laminar flow profile becomes unstable and a certain amplitude of perturbation is needed to drive the dyed water to mix up with the undyed water at a considerable distance from the pipe inlet (see Figure 1.1 (b)). While, when the flow velocity is sufficiently high, the steady-state water streams are disturbed and the dyed water mixes with the undyed water showing noticeable eddies in the flow as illustrated in Figure 1.1 (c). Essentially Reynolds captures the competition between the fluids inertia which tends to cause instabilities and the “smoothing” effects of internal friction of the fluid which is described by the viscous force (fluid viscosity effect). Therefore, in the turbulence flow regime, the flow patterns occur at high values of the Reynolds number, which is the dimensionless ratio between inertial force
and viscous force. Fluid inertia or momentum is responsible for many phenomena that happen everyday such as the irregular and chaotic patterns noticed in water flowing rapidly from a pipe, plumes emerging from a chimney, eddies in the wake of an airplane and many others [222].

![Sketches of laminar and unsteady flow regimes from Osborne Reynolds’ classic work in 1883. Dye added to the center of pipe flow allowed visualisation of flow regimes. (a) When the velocity of water is low enough, the interface between the dyed and undyed water is smooth and straight (stable and laminar flow regime). (b) As the velocity is increased to some point, a sudden transition to well mixed flow occurred downstream. (c) When the velocity is sufficiently high, the dyed water resolved into the surrounding water with distinct eddies (after [176]).](image)

**Figure 1.1:** Sketches of laminar and unsteady flow regimes from Osborne Reynolds’ classic work in 1883. Dye added to the center of pipe flow allowed visualisation of flow regimes. (a) When the velocity of water is low enough, the interface between the dyed and undyed water is smooth and straight (stable and laminar flow regime). (b) As the velocity is increased to some point, a sudden transition to well mixed flow occurred downstream. (c) When the velocity is sufficiently high, the dyed water resolved into the surrounding water with distinct eddies (after [176]).

Instabilities in fluids are traditionally described as being inertial in origin, with instabilities arising when fluid “particles” have sufficient momentum to cross streamlines thereby destroying laminar flow and setting up unsteady, chaotic, and eventually turbulent flow [62]. In micro-scale (microfluidic) systems, the nonlinearities provided by inertia are usually negligible due to the small length scale and consequently low Reynolds number. It would seem that the microfluidic flows are less
interesting because of the loss of nonlinearities. In fact, the rich physics and complex flow behaviours in microfluidic devices have recently drawn the attention of scientists from various disciplines in order to improve the hydrodynamic and thermal characteristics of these micro-systems \[51, 199\] using alternative techniques that depend on fluid inertia.

Secondary flows have been harnessed in microfluidic devices primarily for enhancing fluid mixing \[161\]. For fluid flows in curved pipes/channels, also known as Dean flows, a distinct feature is the appearance of so-called secondary flow \[48, 49\]. This secondary flow is generated as a result of centrifugal forces, which lead to the appearance of counter-rotating vortices in the cross-stream plane of the flow. This kind of secondary flow in curved microchannels has been employed to increase the “interfacial area” for diffusive mixing where most techniques used to promote mixing in microfluidic systems are depended on the concept of increasing the interfacial area \[51\]. The concept of “chaotic advection” is often utilised where fluid elements are stretched and folded to increase the interfacial area among these fluid particles to the highest level, and thereby, that leads to good mixing \[10, 11\]. Another alternative approach is to manipulate a microfluidic design for increasing the contact area between mixing fluids as well as the contact time to promote fluid mixing. Thus, micro-scale geometries are designed in a special way for example, cross-slot and T-shaped, that enables to increase the contact area or/and the contact time between fluids.

1.3 Elastic instabilities in viscoelastic fluid flow

The inertial hydrodynamic flow behaviour of Newtonian fluid can be substantially changed by adding a small amount of flexible high-molecular-weight polymers to a Newtonian solvent. Where, such fluids (polymeric solutions) are known as non-Newtonian viscoelastic fluids, which can exhibit both viscous and elastic
characteristics upon deformation [20]. Viscoelastic fluids are usually more viscous than simple Newtonian fluids due to the addition of high-molecular-weight polymers, therefore, inertial instabilities are often suppressed in the flow of these fluids. However, new instabilities (so-called purely-elastic instabilities) can arise even at very small Reynolds number as a consequence of nonlinearities associated with the elasticity of viscoelastic fluid [25, 76, 78, 120, 122, 135, 187, 192].

Purely-elastic flow instabilities of viscoelastic fluids containing flexible polymers have also been observed within different flow geometries for instance, swirling flow between parallel disks [76, 78, 171, 187], in serpentine or curvilinear channels [25, 77, 78, 127, 165, 208, 232, 233], in concentric cylinder devices [78, 103], in Dean flow [78, 122], lid-driven cavity flow [162], and in a cross-slot (planar extensional) flow [13, 80, 82, 84].

Elastic instabilities give rise to a different flow state in comparison to the base state. This can either be through a steady symmetry breaking bifurcation to a steady asymmetric state, such as in the cross-slot [13, 80, 82, 84] or the mixing-and-separating cell [2], or to a time dependent state such as in Taylor-Couette flow [78] or in the cross-slot at higher flow rate. At larger Weissenberg numbers than these first transitions, the flows can become increasingly complex until they become “turbulent”. This turbulence is quantified by investigating the spectra of certain flow quantities (such as Torque, pressure drop or velocity) where a broad band response is observed with a power-law decay at high frequencies.

Recent studies of viscoelastic flows have exploited these purely-elastic instabilities to improve the performance of microfluidic devices. The small length scales in these microfluidic devices enable large shear rates, and thus high Weissenberg number, $Wi = \lambda \dot{\gamma}$, where $\lambda$ is the relaxation time (a characteristic time scale) of the viscoelastic fluid and $\dot{\gamma}$ is a characteristic shear rate of the flow, at extremely low Reynolds number. Therefore, purely-elastic instabilities can be induced in the micro-scale devices via viscoelastic flows with vanishingly small Reynolds number.
Several studies have been devoted to harness purely-elastic instabilities for enhancing microfluidic mixing for instance, a nonlinear fluid resistor [75], a bistable flip-flop memory element [75], a flow rectifier with anisotropic resistance [75, 197], curved microchannel [129, 165], cross-slot micro-cell [13, 56, 82, 83, 164, 198] and another prospective application of these instabilities is to enhancement of mixing at lab-on-a-chip length scales [77, 78].

In this thesis, attention will be limited to study the enhancement of convective heat transfer in the serpentine microchannel and cross-slot micro-geometry by exploiting the purely-elastic instabilities (followed by elastic turbulence at higher flow rates) occurring in these geometries using viscoelastic fluids.

### 1.3.1 Elastic instabilities in curvilinear flow

Curvilinear streamlines are considered the most important ingredient in order to create elastic instabilities in viscoelastic fluid flows [76–78, 122, 192]. In general, elastic stresses generated by the polymer molecules in a flow of viscoelastic fluids are considered the main source of nonlinearity even in the absence of inertia [122, 192]. Viscoelastic fluids are capable of storing stresses, which depend on the deformation history. The degree of polymer molecular stretching can be characterised by the dimensionless Weissenberg number, \( Wi \), which represents the ratio between the elastic forces and the viscous forces [120, 121, 135]. This aspect of nonlinearity of mechanical properties of viscoelastic fluid was observed about two decades ago [122, 192], when purely-elastic instabilities were experimentally identified in curvilinear shear flows [77, 78, 122, 192]. The mechanism of elastic instabilities involves “normal stresses”, which are generated by the stretching of polymer molecules along the curvilinear streamlines, leading to elastic forces (like that produced in a stretched rubber band) [20, 76, 120]. In the curvilinear streamlines, the polymer molecules press inward from all directions, causing the
generation of a radial pressure gradient or “hoop stress”. These elastic stresses are anisotropic and mainly characterised by the difference in the normal stresses along the flow motion and the velocity gradient directions (called the first normal-stress difference) [20, 120]. If the radial pressure gradient is sufficiently large, it can drive an instability leading to a more complex flow called “elastic turbulence”. This complex combination between normal stresses and curvilinear streamlines is deemed a distinguishing feature of elastic instabilities [20, 120, 122, 192].

It has been found that curved streamlines are a necessary condition for generating infinitesimal perturbations to be amplified by the normal stress imbalances in viscoelastic flows [122, 135, 162]. The onset of elastic instabilities in curved flows has been attributed to a balance between normal stresses and streamline curvature and Mckinley and co-workers [139, 162] documented a simple dimensionless criterion, the so-called Pakdel-Mckinley condition, that should be exceeded for the onset of purely-elastic instability in the flow of viscoelastic fluid. The critical conditions required for the purely-elastic instability onset in the curvilinear flow of viscoelastic fluid are expressed as:

$$\left[ \frac{\tau_{11}}{\eta \dot{\gamma}} \frac{\lambda u}{R_c} \right] \geq M_{\text{crit}}^2$$

(1.1)

where $R_c$, $u$ and $\dot{\gamma}$ are the local streamline radius of curvature, velocity magnitude and shear rate, respectively. $\tau_{11}$ represents the local streamwise normal-stress and $\eta$ is the shear viscosity. $M$ is a geometry-dependent constant which typically ranges between 1 and 6. In Eq. 1.1 $(\lambda u)$ represents the characteristic length over which perturbation decays and $(1/R_c)$ is the typical curvature of the curved streamlines. $(\eta \dot{\gamma})$ is the shear stress $(\tau_{\text{shear}})$. The term $\left[ \frac{\tau_{11}}{\eta \dot{\gamma}} \right]$ in this dimensionless criterion represents rheological information, which refers to the comparison between normal stresses and shear stresses, (the local Weissenberg number, $Wi = \frac{\lambda u}{\tau_{\text{shear}}} [99]$).

While, the term $\left[ \frac{\lambda u}{R_c} \right]$ is a geometric part and compares a typical distance (the characteristic length) over which a polymer relaxes to the radius of curvature.
Therefore, this dimensionless Pakdel-Mckinley criterion shows that even at small Weissenberg number \((Wi \approx 1)\), an instability may appear as long as the streamlines are sufficiently curved \([148]\) and the flow will undergo a transition to elastic turbulence when Weissenberg number grows \([76, 148]\). Zilz et al. \([232, 233]\) also illustrated the geometrical scaling for the purely-elastic instability onset in a serpentine channel by adopting the dimensionless Pakdel-Mckinley criterion to the specific flow geometry.

1.3.2 Elastic instabilities in extensional flow

Elastic instabilities can also arise in extensional viscoelastic flow (stagnation point flows). For example, elastic instabilities have been documented in flow geometries involving stagnation points such as, opposed jet \([37, 152]\), cross-slot \([11, 13, 42, 56, 79, 83, 84, 158, 164, 169, 170, 189]\), two-roll mill \([156]\) and four-roll mill \([23, 156]\) devices. Attention in this section focuses on the purely-elastic instability occurring in a cross-slot arrangement.

The simplicity of the configuration and the ease of the control of the cross-slot geometry have motivated an extensive range of applications in several scientific fields for enhancing mixing of fluids \([67]\). The standard configuration of a cross-slot comprises four channels joining orthogonally at the same point with two opposed inlets and outlets to generate a stagnation point at the centre of symmetry as illustrated in Figures 4.10 and 7.1. The cross-slot geometry generates a region of high extensional deformation along the inlet/outlet symmetry plane. Extension in complex flows (i.e., viscoelastic fluid flows) is normally created around a stagnation point where the symmetry of cross-slot design provides very well-defined boundary conditions \([189]\). Along a line through the stagnation point, a narrow region of viscoelastic fluid forms with high polymer stress extending downstream.
from the stagnation point where the flow velocity is zero and the shear strain (velocity gradient) is finite [84, 198]. The onset of elastic instabilities arises in the flow of viscoelastic fluids due to the planar elongational flow field (strong extensional flow), which is generated at the stagnation point. This leads to a nonlinear increase in the first normal-stress difference (the extensional viscosity) with the flow rate and it promotes the appearance of elastic instabilities. Moreover, as the flow velocity is approximately zero at the stagnation point, the residence time of the polymer molecules is adequate (in theory infinite) to reach a steady-state elongation allowing the estimation of the steady-state extensional viscosity using, for example, birefringence measurements [79, 80, 83, 84, 163, 198].

A consensus can be drawn about the occurrence and development of elastic instabilities for the stagnation point viscoelastic flows from the previous experimental [13, 56, 79, 84, 164] and numerical [1, 6, 19, 42, 43, 158, 169, 170] studies. As the flow rate of viscoelastic fluids increases equally from the opposing inlet channels (and thereby Weissenberg number increases), the flow patterns in the cross-slot geometry evolve from a stable symmetric (Newtonian-like) flow to a stable asymmetric (symmetry-breaking bifurcation) flow and eventually to an unsteady asymmetric (time-dependent) flow and the direction of asymmetry flips between two outflow channels with time. From the experimental observations [13, 56, 164], particle-tracking images in the field of time-dependent flow reveal that the existence of vortical structures around the stagnation point. A physical explanation behind these purely-elastic flow instabilities was believed based on a centrifugal-type instability as a result of the distortion of the velocity field as the flow turned into the upper and lower channels. These fluctuations in the flow field arise because of the elastic compressive stresses produced by the two opposing inflow streams [37, 152, 177]. The direct transition from a steady symmetric flow to an unsteady state (time-dependent) can occur immediately in the case of certain conditions (aspect ratio of channels, viscoelastic properties, etc.).
1.4 Aim and objectives of PhD project

Although the principles of hydrodynamic characteristics in the micro-scale geometries have been numerically studied extensively, for few experimental investigations are available for validating the numerical studies especially for heat transfer problems. The technical difficulties because of the small length-scales (hundreds of micrometers and smaller) of these geometries are considered the main obstructions that have delayed the progress towards understanding the thermo-hydraulic behaviour of micro-scale systems experimentally. The main aim of this thesis is to investigate the enhancement of convective heat transfer in the micro-scale geometries using two different flow regimes of instabilities namely: purely-inertial and purely-elastic. To accomplish this aim successfully, the specific objectives are formulated in this thesis as follows:

1. A comprehensive literature review in the field of fluid flow and heat transfer in both large-scale and micro-scale geometries involving curved/serpentine channel and cross-slot geometry is carried out in order to understand the current state-of-the-art and plan the thesis contributions in context.

2. The purely-inertial flow instability occurring as a result of both Dean’s vortices and chaotic advection in serpentine microchannel flow is harnessed for enhancing heat transfer in Newtonian fluids. Therefore, the thermo-hydrodynamic behaviour of the serpentine microchannel is experimentally and numerically investigated up to moderate values of Reynolds number in this study.

3. The detailed experimental measurements and numerical simulations of the symmetry-breaking bifurcations (the spiral vortex flow instability) in cross-slot micro-geometry are performed on Newtonian fluid flows with a wide range of Reynolds number in order to improve convective heat transfer.
4. It is necessary to understand the potential of using elastic turbulence as a potential approach to enhance heat transfer in micro-scale geometries. Thus, systematic experiments are conducted to exploit the purely-elastic shear flow instabilities of viscoelastic fluids in the serpentine microchannel for promoting the performance of convective heat transfer.

5. According to the literature review, no work has been reported hitherto addressing the effect of stagnation-point purely-elastic flow instabilities of the viscoelastic fluid in the cross-slot micro-geometry on heat transfer. Therefore, the experimental investigation of convective heat transfer in the cross-slot micro-geometry is conducted using viscoelastic fluids to understand the fundamental mechanisms responsible for the heat transfer enhancement.

1.5 Thesis outline

This thesis consists of nine chapters, which represent the scope of the current work, organised as follows:

**Chapter 1:** The background and motivation for present study contained within this Introduction have highlighted the hydrodynamic fluid flow behaviour in the micro-scale geometries. Also, this chapter discussed the potential of both inertial instabilities in Newtonian fluid flow and elastic instabilities in viscoelastic fluid flows. The aim and objectives of this PhD project and outline of the thesis have also been reported.

**Chapter 2:** Literature review provides an overview of hydrodynamic flow behaviour in two different geometries, curved/serpentine channel and cross-slot geometry, and the associated heat transfer performance. The relevant and available studies for both Newtonian and viscoelastic fluids of the serpentine microchannel
will be introduced and explained with emphasis on the purely-inertial and purely-elastic mechanisms. While, an overview of the flow behaviour for Newtonian and viscoelastic fluids in the cross-slot micro-geometry will be presented and the proposed mechanism of purely-inertial and purely-elastic in the stagnation-point flow will be introduced.

**Chapter 3:** The rheological properties of viscoelastic fluids used in the present study will be presented and explained individually where the reader is given a fairly detailed introduction to the subject to elucidate the various rheological characterisation techniques and the fundamental differences between Newtonian and non-Newtonian fluids as well as a brief description of non-Newtonian fluid classification. Additionally, thermophysical properties measurements for the working fluids used will be introduced and described.

**Chapter 4:** The experimental testing facility used for the present study for both selected geometries (serpentine microchannel and cross-slot cell) will be presented together with the experimental equipment, techniques and data analysis procedures. Also, the main non-dimensional parameters used in this thesis will be provided. Finally, an estimation of the associated uncertainties in the experimental measurements is provided.

**Chapter 5:** The experimental and numerical results for Newtonian fluid flow and heat transfer in the serpentine microchannel geometry will be presented in this chapter. The three-dimensional numerical simulation of the square cross-section serpentine microchannel will be introduced and described using the commercial CFD software package Fluent to help elucidate the physical mechanisms responsible for the heat transfer enhancement. Moreover, the behaviour of chaotic advection in the serpentine microchannel will be addressed here as well.

**Chapter 6:** This chapter describes the experimental results of a detailed investigation into the flow and convective heat transfer of Newtonian fluids (mixtures
of aqueous solutions of glycerine-based and sucrose-based) and viscoelastic fluids (both shear-thinning and Boger solutions) through a square cross-section serpentine microchannel in a regime of elastic turbulence.

Chapter 7: The possibility of utilising the mixing of Newtonian fluids in the cross-slot micro-geometries to promote convective heat transfer at low or moderate values of Reynolds numbers will be elucidated in this chapter. Meanwhile, three-dimensional numerical simulations of the cross-slot geometry will be presented to clarify the effect of the purely-inertial flow instability on the heat transfer in the cross-slot micro-geometry. In addition, the transition behaviour between steady symmetric flow and steady asymmetric flow in the square cross-slot geometry will be described.

Chapter 8: This chapter investigates the effect of extensional flow instabilities in a square cross-slot micro-geometry on heat transfer of a 100 ppm polyacrylamide in a 70% glycerine/30% water solvent with the absence of inertial effects.

Chapter 9: The key conclusions that have been documented throughout the thesis will be given, along with some suggestions for future work.
Chapter 2

Literature review

In the current study, two representative examples are selected as case studies to investigate the enhancement of heat transfer in micro-scale geometries: a serpentine microchannel geometry and a cross-slot micro-geometry. In this chapter, the literature survey has been outlined in the following manner: the relevant and available studies for both Newtonian and viscoelastic fluids of the serpentine microchannel geometry (Section 2.1) and a review of the flow behaviour for Newtonian and viscoelastic fluids in the cross-slot micro-geometry (Section 2.2). As there is no previous literature about heat transfer in the cross-slot micro-geometry (almost bereft), the literature survey for this section is, by necessity, biased towards the isothermal flow behaviour of Newtonian and viscoelastic fluids.

2.1 Serpentine channel geometry

Flow patterns in curved tubes have important applications to problems in engineering and technology. The flow of Newtonian and viscoelastic fluids through curved ducts is of particular interest as, for Newtonian fluid flow in curved ducts, the generation of a secondary flow occurs due to unbalanced centrifugal forces
whereas, for viscoelastic fluid flow, the inherent non-linearity of the fluid can also lead to secondary flow even in absence of inertia. In this section a comprehensive review on flows of Newtonian and viscoelastic fluids and heat transfer characteristics occurring in curved ducts will be presented.

2.1.1 Newtonian fluid flows and heat transfer characteristics

Flow characteristics in curved ducts

In a curved pipe, fluid motion is not parallel to the axis of curvature, different from the flow in straight pipe, due to the existence of a secondary motion [60]. When fluid flows through curved pipes, the more rapidly flowing central parts of the flow are forced outwards from the center of curvature by centrifugal action whereas the slower parts along the wall are forced inwards where the pressure is lower, and so-called “secondary flow” occurs in a perpendicular plane to the main flow direction [49]. Due to the existence of curvature in the axial flow direction, centrifugal force is generated. A secondary flow induced by the centrifugal force has an ability to enhance fluid mixing thereby promote the heat transfer rate. The flow structure of such secondary flows, which are called “Dean’s vortices” after Dean [49], is illustrated in Figure 2.1.

In 1928, Dean [49] achieved the first analytical solution for fully-developed laminar fluid flow within curved tubes of circular cross-section. This investigation showed that a pair of counter-rotating vortices is formed in the cross-stream plane of the flow due to an imbalance between centrifugal forces and the radial gradients of pressure forces. As a consequence of this pioneering study, these vortices are usually called “Dean’s vortices”. Dean described the features of these vortices
by defining a dimensionless number $K$ (known as the Dean number, defined in Section 4.3).

Since the seminal work of Dean \cite{48, 49}, a large number of analytical, experimental and numerical studies has been performed on Newtonian fluid flow in curved pipes and ducts with different geometrical parameters (curvature ratio and aspect ratio), with attention paid to Dean vortices over a wide range of Dean numbers as mentioned in the reviews of Naphon et al. \cite{155} and Vashisth et al. \cite{216}.

Some of these studies \cite{33, 116, 195, 203} addressed the influence of the curvature ratio parameter, $R_c/r$ (where $R_c$ is the radius of curvature and $r$ is the radius of tube), on the intensity of secondary flows (in the form of a pair of counter-rotating vortices) in the curved tubes. The experimental and numerical investigations by Fellouah et al. \cite{63} for Dean instability (centrifugal instability) of laminar flow in 180° curved channels have been performed by adopting the radial gradient of the axial velocity as the precise criterion for elucidating Dean instability. The influences of both curvature ratio (5.5 - 20) and aspect ratio, $= a_R/b_R$ where $a_R$ and $b_R$ are the width and depth of the rectangular channel, respectively, (0.5 - 12) on the conditions of flow development were also studied. The results revealed that the onset of Dean instability for Newtonian fluids (the critical value of the Dean number) decreases with the increasing duct curvature ratio whereas the variation
of the critical Dean number with duct aspect ratio is less regular.

Sugiyama et al. [203] visualised the patterns of secondary flow in a fully-developed laminar flow region within curved rectangular channels with curvature ratio ranging from 5 to 8 and aspect ratio ranging from 0.5 to 2.5. From flow visualisation, it was observed that the secondary flow vortices change with increasing Dean number as shown in Figure 2.2. For the flow patterns in the case of aspect ratio 1 and curvature ratio 8, only one pair of secondary flow vortices appear in the laminar stable region at low Dean number (see Figure 2.2 (a)). As the Dean number increases, a retarded layer occurs near the outer wall (see Figure 2.2 (b and c)) and with a further increase of the Dean number, two pairs of additional secondary flow vortices are set up (see Figure 2.2 (d and e)).

![Flow patterns](image)

(a) $K = 93, \text{Re} = 265$
(b) $K = 139, \text{Re} = 393$
(c) $K = 183, \text{Re} = 518$
(d) $K = 383, \text{Re} = 1083$
(e) $K = 527, \text{Re} = 1489$

**Figure 2.2**: The behaviour of flow patterns in the channel of aspect ratio 1 and curvature ratio 8 (after [203]).

The analytical solution of a fully-developed, steady, isothermal, incompressible,
laminar flow of a Newtonian liquid within a toroidal-type, coiled-tube geometry, was determined by Soeberg [194] using a symmetry technique of the secondary-flow field. The analytical results detected that for $K < 16$, the secondary flow affected the shape of the harmonics of the axial velocity. For $K > 16$, the harmonics altered shape and amplitude of the velocity profiles. The velocity profile at the centre became flatter as the Dean number increased. For $K > 100$, the Coriolis force affected the stability of the laminar-flow field, moving the point of transition to turbulent flow. The flow stability of two-vortex and four-vortex solutions through a slightly curved circular cross-section tube was analysed numerically by Yanase et al. [226] using the Fourier-Chebyshev spectral method for the range of Dean numbers from 96 to 10,000. Their results showed that the two-vortex solution was stable in response to any small disturbances, whilst the four-vortex solution was unstable to asymmetric disturbances. Eventually, the four-vortex flow turns into a two-vortex flow.

The numerical solutions of Joseph et al. [104], Cheng et al. [32], Ghia and Sokhey [73] and Dennis and Ng [50] aim to specify the critical Dean number (beyond this critical value the additional counter-rotating vortices appear in the cross-section plane) in curved duct flow. A wide range of Dean number (0.8 - 307.8 [104], 5 - 715 [32], 55 - 350 [73] and 96 - 5000 [50]) and different geometrical parameters (square cross-section [104], rectangular cross-section with aspect ratios 0.5, 1, 2 and 5 [32], rectangular cross-section with curvature ratios 3 - 100 [73] and circular cross-section [50]) were simulated. The numerical results revealed that there are two regimes of secondary flow of the fully-developed laminar flow in curved ducts. Below the critical Dean number (100 [104] and 143 [73] for aspect ratio 1), the expected pattern of secondary flow appears with twin counter-rotating vortices. Above the critical Dean number, an additional counter-rotating pair of vortices appears near the central outer region of the curved duct in addition to the familiar secondary flow to form four vortices in the cross-section direction of the axial flow.
Also, it is found that the critical Dean number is dependent on the aspect ratio [32]. This phenomenon of Dean’s instability for curved duct flow beyond the critical Dean number was consistent with the experimental velocity measurements using Laser Doppler Anemometry (LDA) by Hille et al. [86].

Secondary instability and transitions of flow in a curved duct of square cross-section were investigated through a series of both experimental and numerical studies by Mees et al. [140–142]. Their results confirmed that a 6-cell secondary flow pattern was noticed early in the flow development for a Dean number above 350 where a first small pair of Dean’s vortices splits and two new vortices are formed in between, leading to formation of two pairs of Dean’s vortices due to the primary centrifugal instability. Eventually, the 6-cell flow breaks down symmetrically into a 2-cell state. This secondary flow behaviour is very similar to that observed experimentally by Sugiyama et al. [203] in a curved duct with aspect ratio 2. The experimental flow visualization of the secondary flow was in very good agreement with the numerical simulations based on the steady three-dimensional Navier-Stokes equations.

Very recently, experimental investigation and numerical simulation were simultaneously carried out by Li et al. [130] for understanding the 3D flow development in rectangular cross-section curved ducts with continuously changing curvature ratio in the range of Reynolds number from $2.4 \times 10^4$ to $1.4 \times 10^5$. The axial velocity development in the horizontal mid-plane and patterns of secondary flow at different aspect ratios (0.4, 1 and 2.3) were experimentally analysed using Particle Image Velocimetry (PIV) technique. Their results revealed that complex changes in the flow pattern in terms of onset, development and vanishing of different Dean vortex types were observed. These complex changes are dependent on the flow and geometric parameters such as Reynolds number (Dean number), curvature and aspect ratios.
Chaotic advection

In this part, the effect of the spatial Dean’s vortex evolution on the fluid flow and heat transfer in curved/ wavy channels will be addressed based on the principle of chaotic advection by alternating superposition of different vortex patterns dating back to Aref’s theoretical model of blinking vortex in the early 1980s [10]. Aref’s classic mixing strategy depended on that when elements of moving fluid change position chaotically, these elements are exponentially stretched and folded, enhancing mixing. Thus, for cases where the quantity and the location of the Dean vortices change significantly along the flow direction, this could lead to so-called “chaotic advection”. Chaotic advection can be thought of as an intermediate state that lies between laminar and turbulent advection [11] and is one of the most efficient approaches to promote fluid mixing in curved or wavy microchannels at low Reynolds number. The phenomenon of chaotic advection was observed in planar-based channel laminar flow together with Dean vortices in different geometrical shapes, such as B-shaped channels [123], C-shaped channels [123], V-shaped channels [123], meandering channel [101, 188], twisted curved channels [124, 166], three-dimensional serpentine microchannels [133], bent coil configurations [118], periodic wavy channels [204, 206] and periodic zigzag channels [230, 231].

Experiments on hydrodynamic and heat transfer behaviour of the flow in a twisted curved channel (a 90° twisted curved channel of square cross-section of 40mm sides and curvature ratio= 5.5 as shown in Figure 2.3 (a)) were conducted in a water tunnel by Peerhossaini and co-workers [124, 166] and also in two heat exchanger coils (one helical and the other of chaotic geometry see Figure 2.3 (b and c), both had a 90° curved tube of circular cross-section of 23mm inner radius) tested in a heat exchanger test facility [166]. For the twisted curved channel, machined out of a block of transparent Plexiglas, the velocity patterns of the flow was measured by a Laser-Doppler Velocimeter (LDV) and the flow field was visualised using
the technique of Laser-Induced Fluorescence (LIF) over a range of Dean number 141-530. The flow visualisation results illustrated that this kind of flow is indeed chaotic advection thus heat transfer is enhanced due to the chaotic trajectories generated in the flow.

![Diagram](image1)

(a) Twisted curved channel  (b) Helical heat exchanger coil  (c) Chaotic heat exchanger coil

**Figure 2.3:** (a) Schematic diagram of the twisted curved channel, (b) representation of the helical heat exchanger coil and (c) representation of the chaotic heat exchanger coil (after [166]).

Jiang et al. [101] performed an experimental and numerical study to improve fluid mixing via curved stream flow in a square curved channel (four circular arcs connected with two straight inlet and outlet sections). A new technique of chaotic fluid mixing was displayed involving alternately changing flow patterns along the four circular arcs, which are joined together to configure the curved mixer. The range of Dean number for the experiments and numerical simulations varied from 35 to 351 and from 10 to 200, respectively, for $Pr = 7$. The results showed, for Dean number = 10, no chaotic mixing occurs, whereas at Dean number around 140, the chaotic mixing occurs because a qualitative change of the flow patterns happens for Dean number above a critical Dean number ($\sim 140$).

A set of numerical [205, 206] and experimental [204] investigations has systematically been carried out by Sui et al. [204] to study fully-developed laminar flow and heat transfer in periodic wavy rectangular microchannels. These investigations studied the behaviour of fluid flow and heat transfer in the periodic wavy (ten wavy units) microchannels for water ($Pr = 7$) and Reynolds numbers ranging from 100
to 800 under constant wall temperature or constant heat flux conditions considering the impact of chaotic advection along the axial flow direction. The numerical simulations \cite{205, 206} confirmed that the number and the location of the Dean vortices, which evolve temporally and spatially with the increase in the Reynolds number, undergo extreme changes (very complex patterns of Dean vortices) along the axial flow direction. The spatial evolution of the flow patterns was caused by the changing direction of the centrifugal force along the wavy microchannel with curvature changing sign. Also, the numerical calculations indicated that there is a reasonable agreement with experimental results in terms of heat transfer enhancement and pressure-drop losses \cite{204}. The symmetric secondary flow patterns may develop in the axial flow direction, thereby causing chaotic advection along the flow direction. This chaotic advection can noticeably augment the convective fluid mixing and the rate of heat transfer. Therefore, the performance of heat transfer is significantly increased comparative to that of straight microchannels whereas the increase in pressure-drop losses of wavy microchannels can be much smaller than the enhancement of heat transfer.

The most recent work that relevant to present study is that of Zheng et al. \cite{230, 231} who performed two numerical studies simulating periodic zigzag channels with square cross-section to study steady laminar heat transfer \cite{230} and semi-circular cross-section to study transient laminar heat transfer \cite{231}. In the steady laminar heat transfer regime, the entire computational domain involved up to fourteen repeating zigzag units with connected inlet and outlet sections for $Pr = 1$ and Reynolds number ranging from 50 to 400 whilst it consisted of seven repeating zigzag units with the inlet and outlet sections for $Pr = 0.7, 1, 6.13, 20$ and Reynolds number ranging from 400 to 800 in the case of transient laminar heat transfer. The results showed that in the square zigzag channel the fluid flow field is fully-developed periodic laminar flow for $Re < 200$ and converts then to an unsteady flow at $Re > 400$ because the secondary flow vortices alter significantly
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their locations along the axial direction [230]. Over a sufficient period of time, significant enhancement of heat transfer can be observed in the transient flow regime, which is accompanied by an increase in pressure-drop, compared with a straight channel [231]. Both numerical investigations, steady and transient, concluded that chaotic advection is generated in simple planar-based zigzag channels.

**Heat transfer characteristics in curved ducts**

Although the thermo-hydrodynamic behaviour in curved/ wavy microchannels has been extensively investigated in a number of numerical studies, experimental studies are very limited because of the technical difficulties of obtaining precise measurements in micro-scale flows. Several other studies have been accomplished for convective heat transfer in large-scale curved/ wavy ducts with different characteristics, operating, or boundary conditions.

The behaviour of laminar and turbulent flow and heat transfer in a circular cross-section coils over the range of Reynolds numbers from 12 to 65000 for two curvature ratios, 17 and 104, has been experimentally studied by Seban and McLaughlin [190]. The results for the laminar flow of oil and the turbulent flow of water in tube coils revealed that the wall temperature varied peripherally but could not be precisely evaluated for the heat conducted in the wall. It was also found that the thermal entrance length was much shorter than that required for straight tubes. The empirical correlation of heat transfer coefficient for laminar flow was consistent with the theoretical data.

Mori and Nakayama [145,147] investigated the mechanism of forced convective heat transfer in laminar flow with constant heat flux [145], turbulent flow with constant heat flux [146] and laminar and turbulent flow with constant wall temperature for three different Prandtl numbers, $Pr$ (defined in Section 4.3), (air, water and oil) [147] in circular curved pipes experimentally and theoretically. They
obtained the first theoretical heat transfer solutions by boundary layer methods for the fully-developed temperature field. The results showed that Nusselt number is found to be noticeably influenced by the secondary flow by curvature. A correlation of the mean Nusselt number, $\overline{Nu}$, as a function of curvature ratio, Dean number and Prandtl number was proposed for laminar and turbulent flow under the conditions of constant heat flux and constant wall temperature for circular curved pipes. Cheng and Akiyama also addressed numerically laminar forced convective heat transfer in rectangular channels with constant heat flux for different aspect ratios, 0.2, 0.5, 1, 2 and 5, and Dean number ranged from 150 to 1000. The numerical results in terms of $fRe$ and $Nu$ for $Pr = 0.73$ illustrated that the ratio of $(fRe)_{c}/(fRe)_{s}$ (where $f$ is the friction factor defined in Section 4.3 and $Nu_{c}/Nu_{s}$ is 1.8 and 2.2, respectively, where the subscripts $c$ and $s$ denote “curved” and “straight” channel, respectively. Later, they also investigated numerically convective heat transfer in the thermal entrance region for fully-developed laminar flow in curved tubes under the condition of uniform wall heat flux for Dean numbers ranging from 0 to 100. This numerical investigation is considered an extension of the classical Graetz problem in straight tubes to curved pipes. The impact of secondary flow on developing temperature field through the horizontal and vertical planes in the thermal entrance region was analysed over a range of Prandtl numbers 0.1, 0.7, 10 and 500. The influence of Prandtl number is to shorten the length of thermal entrance zone and the temperature field evolves rather rapidly with large Prandtl number. While, the effect of Dean number is much more appreciable at high Prandtl numbers than at low Prandtl number.

Dravid et al. documented a numerical solution for entrance-region heat transfer in curved tubes using the fully-developed velocity profiles developed by Mori and Nakayama. This numerical study involved three different wall conditions, constant wall temperature, constant wall heat flux and axially uniform wall
heat flux with peripherally uniform wall temperature, in the laminar flow regime with a single Dean number of 225. The numerical solution predicted that the local heat transfer coefficient is an oscillatory function of increasing axial downstream distance (does not decrease continuously as in a straight tube). This oscillation arises in the helically coiled tube because of the fact that even at high Dean numbers the fluid core is not well-mixed. Experimental measurements of the local Nusselt number were also performed using an electrically heated coil. The experimental results confirmed the early convergence to the fully-developed region first discovered by Seban and McLaughlin [190].

Kalb and Seader [109, 110] studied numerically the enhancement of laminar heat transfer in circular cross-section curved tubes under constant wall heat flux conditions. They observed that the ratio between heat transfer enhancement and pressure-drop losses is significantly increased in curved tubes over a Dean number range from 1 to 1200. The numerical results were presented with Prandtl number ranging from 0.005 to 1600 and curvature ratio parameter was varied from 10 to 100. Thangam and Hur [209] carried out numerical simulations of laminar flow in rectangular curved ducts with varying curvature ratio and Dean number. The results indicated that the structural features of secondary flow in the curved channel evolved into a double-pair of counter-rotating vortices for low aspect ratio ducts and roll cells (additional weak counter-rotating vortices) for ducts of high aspect ratio based on the curvature ratio and Dean number. Hwang and Chao [93] conducted a numerical investigation to study the characteristics of laminar convective heat transfer in a square curved duct under the condition of constant wall temperature. The numerical solutions clarified that the main fluid flow involves either a single-pair of vortices or a double-pair of vortices for $Pr = 0.7$ and 7 depending on the value of Dean number (ranging from 0 to $10^6$) and curvature ratios (ranging from 5 to $\infty$). For $K < 114$ the flow pattern possesses a single-pair of counter-rotating vortices, and with increasing Dean number ($K > 143$) the flow
Pattern evolves to a double-pair of secondary flow vortices in the cross-section plane. While, for $114 < K < 143$ the flow pattern depends on the initial flow pattern utilised. This kind of the secondary flow patterns changes not only the magnitude of Nusselt number but also the slope of Nusselt number.

Facão and Oliveira [61] modelled the fluid flow and heat transfer in a rectangular curved duct using the commercial CFD software package Fluent over a wide range of Dean number (23-2013). This numerical investigation revealed that the value of $\overline{Nu}$ increases up to 10 times for the curved duct relative to a straight duct, because of secondary vortices forming where the flow patterns change from one pair of vortices at low $K$ to three pairs at $K = 131$. Chandratilleke et al. [28,29] studied numerically characteristics of secondary flow and convective heat transfer for fully-developed steady laminar flow through a curved rectangular duct externally heated on the outer wall by a uniform heat flux over the Dean numbers in the range 20 to 500. The numerical prediction revealed that the Nusselt number for a curved duct with aspect ratio ranging from 1 to 8 is $20\% - 70\%$ higher than a straight duct [29]. Their numerical simulation adopted two approaches to capture the onset of hydrodynamic instability and resulting Dean vortices [28]: one based on assigned helicity (vortex structures) onset and the other on adverse pressure gradient at the outer duct wall. Both approaches were readily integrated into a computational scheme using the finite volume-based CFD package Fluent for accurate and reliable identification of onset of Dean’s vortices. It was found that the adverse pressure gradient method, over-predicts the onset of instability.

Later, Nadim and Chandratilleke [154] simulated the immiscible behaviour of two non-mixing fluids flowing within both rectangular and circular curved channels. The effect of centrifugal forces arising due to the channel curvature on the phase distribution and flow patterns is able to promote the interactions between the fluid phases and vortex structures of the immiscible flow in curved channels. Also, the process of convective heat transfer was significantly affected by the intensity
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of secondary flow and the non-uniform phase property distribution. Manglik et al. [136] carried out 3D numerical simulations for laminar forced convection heat transfer in periodic wavy-plate-fin channels under the conditions of constant heat flux and uniform wall temperature over a Reynolds number range from 10 to 1000. The results showed that pairs of symmetric counter-rotating vortices appear in the spanwise direction of the wavy-plate-fin channels. These symmetric vortices lead to a significant increase in the enhancement of convective heat transfer and pressure-drop losses compared to a straight duct.

Fletcher and co-workers have recently conducted a series of numerical simulations [70, 71, 181–183] for investigating the enhancement of heat transfer and fluid flow characteristics in periodic serpentine (wavy) channels under the conditions of constant heat flux and constant wall temperature with different cross-section shapes such as, circular cross-section [70, 181, 183], square cross-section [70, 71], and semi-circle cross-section [70, 182, 183]. The calculations revealed that the convective heat transfer is significantly enhanced within periodic serpentine channels compared to straight channels that have the same cross-section and path-length with a comparatively small rise in the pressure-drop losses. The amount of heat transfer enhancement, for example, for water ($Pr = 6.13$) ranges from two and a half to three-fold, dependent on the geometric shape of the cross-section. It was also confirmed that the symmetric pair of counter-rotating Dean vortices evolves to a more complex flow pattern when the fluid flows through the bends.

Experimental measurements and numerical simulations of pressure drop and heat transfer with a rectangular cross-section serpentine channel comprising of 10 units in series were quantitatively performed by Karale et al. [112] for understanding the influence of different geometrical parameters of serpentine channels (bend angle, straight length before bend, and curvature) on heat transfer enhancement. The comparison between the experimental and numerical results indicated that the pressure drop in terms of friction factor and Nusselt number for serpentine
channel is higher than a straight channel where the serpentine channel pressure drop and Nusselt number ranged approximately from 1.2 times and 1.9 times, respectively, at $Re = 130$ to 1.9 times and 4.32 times, respectively, at $Re = 666$. The influence of the geometrical parameters (the corrugation angle, the curvature radius, the developed length, the hydraulic diameter and the aspect ratio) of a wavy “corrugated” channel on heat transfer, pressure drops, mixing performances and residence time distribution have been investigated both experimentally and numerically by Anxionnaz-Minvielle et al. [9] to interpret the thermo-hydraulic mechanisms and the flow behaviour over a Reynolds number range from 200 to 2100. The main conclusion concerned the relevance of the Dean number to predict the corrugated flow behaviour. Thus, for a given Reynolds number, the hydraulic diameter increases, which induces a curvature radius decrease and has a strong effect on the Dean number. Nusselt numbers and Darcy fiction factor are also directly linked to the Dean number and remain constant during the scale-up process from 2 to 4mm providing that the aspect ratio is equal to 1.

Most recently, Dai et al. [44, 45] carried out an experimental study to investigate characteristics of laminar flow and heat transfer of water flowing within semi-circular cross-section (diameter 2mm) wavy (zigzag or sinusoidal) channels for Reynolds numbers ranging from 50 to 900. Three-dimensional experimental measurements of the complex flow in these wavy channels using scanning-based micro-PIV visualisation and 3D reconstruction techniques were able to provide a way to better understand the complex flow behaviour and evolution of the complex flow comprising the identification and localisation of vortical structures. In order to understand and interpret the complex thermal behaviour, heat transfer simulations were also developed using the ANSYS CFX package, employed to analyse the three-dimensional heat transfer characteristics of the experimental test sections. Experimental and numerical findings showed that a significant enhancement of heat transfer up to a factor of four in mean Nusselt number is achieved
in wavy channels compared with the equivalent straight channel, whereas an increased pressure-drop loss is also observed. The experimental time-resolved velocity measurements \cite{[46]} revealed a transition from a steady flow to a time-dependent oscillatory flow in the zigzag channel having a few dominant frequencies and subsequently to a complex transient flow. From water flow visualisation, it was found that the flow transition from steady to unsteady flow occurs at a relatively low Reynolds number ($Re \sim 215$) in the laminar regime in the zigzag channel compared with that in a straight channel \cite{[16]}.

Although, as the foregoing discussion has made clear, there have been a significant number of studies investigating heat transfer in curved pipes/channels, the majority of these studies are restricted to purely numerical approaches and to rather low Prandtl numbers. Therefore the thermo-hydrodynamic behaviour of serpentine microchannels requires a more specific and detailed assessment, which is experimentally and numerically addressed in this thesis (see Chapter 5) to help elucidate the physical mechanisms responsible for the heat transfer enhancement.

The outstanding feature of fluid flow and convective heat transfer in curved-shape pipes/channels from previous studies is that, the mean heat-transfer rate is larger than that in a straight pipe/channel at the same flow rate and geometrical shape (cross-section area and path-length) due to the additional mixing by the secondary flow (centrifugal forces cause secondary fluid motion which gives rise to increased heat transfer rates).

### 2.1.2 Viscoelastic fluid flows and heat transfer characteristics

The majority of previous studies for fluid flow behaviour and heat transfer in curved tubes/channels are confined to Newtonian fluids. Very few attempts have been made to understand the flow behaviour of non-Newtonian fluids in curved
tubes, in spite of the important applications of these non-Newtonian fluids in the field of polymers, biochemical and biomedical areas.

Jones [102] was the first who presented the theoretical solution for the flow of a non-Newtonian fluid in a coiled tube and developed a successive approximation solution by adopting Oldroyd-B model. Walters and co-workers [17, 212, 213] reported a series of analytical solutions [212, 213], which were based on the extension of Dean’s analysis [48, 49], and experimental study [17] for viscoelastic fluid flows in curved pipes of circular [17, 212] and elliptical [213] cross-sections. Their findings indicated that the general behaviour of the motion of viscoelastic fluid is similar to that of Newtonian liquids, where elements of the viscoelastic fluid moving along the curved pipe in two sets ofspirals separated by the central plane. Elasticity of the viscoelastic fluid, however, could significantly influence the pitch of these spirals.

Rajshekhran et al. [174] investigated experimentally the flow behaviour and heat transfer of aqueous CMC solutions in coiled tubes (internal diameter, 6.4 - 12.7 mm) with a variation in curvature ratio (4.5 - 32). Their experimental results indicated that the pressure drop in terms of friction factor and heat transfer coefficients for shear-thinning but relatively inelastic CMC solutions in coiled tubes are higher than those for straight pipes and are influenced by the curvature ratio. Later on, the experimental study of heat transfer to Newtonian (water) and viscoelastic (aqueous polyacrylamide solution) liquids during laminar flow in helical coils was carried out by Oliver and Asghar [159]. This study found that heat transfer coefficients are higher than those attainable in straight tubes under comparable flow conditions. For Dean numbers less than 20, heat transfer is up to 80% higher than for a corresponding straight tube, whereas the pressure gradients are almost identical.

As the use of micro-scale devices witnessed a significant increase in a variety of
applications across many scientific fields as described in Chapter [1] it has become necessary to improve the flow behaviour and heat transfer performance of these devices [199]. However, micro-geometries have usually been associated with negligible inertial effect because of the predominantly laminar flow due to the small scale of dimensions ($\leq 1000\mu m$) for these geometries [51]. Consequently, the process of mixing fluid in the micro-geometries relies on molecular diffusion, which is extremely slow compared with convective mixing. Thus, one of the proposed approaches to enhance heat transfer in these conduction-dominated regimes is to use viscoelastic fluids in order to introduce non-linear effects and promote the appearance of instabilities even at low Reynolds number [20, 34, 120]. The flow behaviour of viscoelastic fluids can be significantly different from those of their Newtonian counterparts [20, 143, 144]. The most important elastic property of viscoelastic fluids is, probably, the dependence of mechanical stresses on the history of the flow where these stresses do not immediately return to zero when the flow stops, but rather decay with some relaxation time [20, 26]. Viscoelastic fluid flows in such geometries have been seen to exhibit “turbulent-like” characteristics such as chaotic and randomly fluctuating fluid motion across a broad range of spatial and temporal scales and have led this to be called “elastic turbulence” [25, 76, 78, 121, 122, 128, 148, 187, 192]. Elastic instabilities can occur even at extremely low Reynolds number (inertial effects are negligible) these types of instabilities driven completely by elasticity are commonly called as “purely-elastic” or “inertia-less” instabilities [76, 78, 121, 122, 192].

In fact, the concept of elastic instability appeared as a well-known phenomenon for viscoelastic fluid flows in the 1990s. Larson et al. [122] reported theoretical and experimental results that show that purely-elastic instabilities can arise in viscoelastic fluid flows by a coupling of the first normal-stress difference and streamline curvature. This seminal work was followed by a series of experimental investigations that used different geometries containing curved streamlines
to study elastic instabilities in low Reynolds number fluid flows. This included swirling flow between parallel disks [76, 78, 171, 187], in serpentine or wavy channels [25, 77, 78, 127, 129, 165, 208, 232, 233] and in concentric cylinder devices [18, 78]. Elastic instabilities and resulting non-linear interactions between elastic stresses generated within the flowing high-molecular-weight polymer solutions and the streamline curvature are “purely-elastic” in nature, driven by the elastic (normal) stresses developed in the flow and occur at Reynolds numbers far removed from the usual turbulence observed for Newtonian fluids which is, of course, inertial in nature. As a result of turbulent motion at small scales, elastic turbulence has been proposed as an efficient technique for mixing within very low Reynolds flows, such as in curved microchannel flows.

Experimental work by Groisman and Steinberg [77] has pioneered an understanding for the mechanisms of elastic turbulence in a serpentine channel. In their experiments they considered a viscoelastic flow in a 3 mm square cross-section serpentine channel consisting of a sequence of 60 smoothly connected half-rings with inner and outer radii 3 mm and 6 mm, respectively, using a dilute solution of 80 ppm high-molecular-weight polyacrylamide ($M_w \sim 18 \times 10^6$ g/mol) in a viscous sucrose syrup as the working fluid. A turbulent flow regime was observed for low values of the Reynolds number with irregular flow patterns developing which led to efficient mixing between the dyed and undyed streams in their curved channel flow. Typically, viscoelastic fluid flows with negligible inertia ($Re < 1$) that have been stretched along curved streamlines can undergo a series of flow transitions from viscometric laminar flow, to clearly chaotic flow, and eventually to fully developed “elastic turbulence”. From their experiments they concluded that elastic instabilities of highly-elastic fluids can be exploited to augment fluid mixing in serpentine or curved channels. This paper was followed by several studies motivated by the seminal work of Groisman and Steinberg [77, 78] which found that viscoelastic instabilities can be created in micro-devices as pointed out in a review
paper by Li et al. [128].

An experimental investigation was carried out by Burghelea et al. [25] to study the possibility of generating a chaotic flow in a 100 $\mu m$ square curvilinear microchannel, which was configured from connecting 40 identical half-rings with each other. Microscopic Particle Image Velocimetry ($\mu$-PIV) was utilised to measure the flow velocity in the microchannel. A mixture of 80 ppm polyacrylamide ($M_w \sim 1.8 \times 10^7$ g/mol) in two sucrose-based Newtonian solvents was used to prepare low and high viscosity viscoelastic solutions. The flow visualisation revealed that a nonlinear transition occurs in the viscoelastic flow at $Re = 1.7 \times 10^{-2}$, $Wi = 1.4$ and $Re = 8 \times 10^{-5}$, $Wi = 3.5$ for low and high viscosity solutions, respectively. Furthermore, the fluctuating effects of the secondary flow occur usually in strongly extensional flows which are led to grow the elastic stresses.

The mechanism of a purely-elastic flow instability for a dilute Boger fluid (66 ppm PAA in a sucrose-based Newtonian solvent) within a (85 x 60 $\mu m$) rectangular cross-section microchannel having a zigzag path (curved streamlines) has been quantitatively investigated by Pathak et al. [165] using Rhodamine-B dye as a fluorescent probe for flow visualisation. For Newtonian fluid streams with inertia effects absent ($Re << 1$), mixing occurs by purely-molecular diffusion and the degree of mixing decreases with an increase of flow rate because of decreasing residence time ($= l/U_B$, where $l$ is the channel path-length and $U_B$ is the average flow velocity) in the channel. In contrast, for viscoelastic fluid streams with $Re << 1$ and $Wi > 1$, enhancement of mixing is noticeably observed in the channel by exciting viscoelastic flow instability. The experimental results confirmed that curved streamlines and first normal-stress differences are important conditions for the formation of elastic instabilities in the micro-scale geometries.

Three different configurations of microchannels with special designs possessed a rectangular cross-section of flow passage (60 x 300 $\mu m$, 60 x 100 $\mu m$ and 75 x 200
have been used by Li et al. [127] in order to create chaotic fluid motions using aqueous CTAC solutions with weight concentration of 200 ppm and 1000 ppm. The experimental results showed that the typical laminar flow structures were noted for low-Reynolds-number Newtonian fluid flow in which curvilinear streamlines exist in these chosen microchannels. While the flow of viscoelastic fluid (CTAC solutions) exhibited irregular and asymmetrical flow structures in space as well as time-dependent. This chaotic flow behaviour was considered to be created as a consequence of the interaction between viscoelasticity of the CTAC solution and the curvilinear streamlines in these microchannels flow. The experimental observation of the mixing flow of CTAC solution in a curvilinear channel displayed a great enhancement effect for all special designs of the microchannels.

Tatsumi et al. [208] visualised and measured quantitatively flow velocity for both Newtonian fluid (64.4% sucrose-based solution) and viscoelastic fluid (100 ppm PAA in 64.4% sucrose-based solution) in a (84 × 60 µm) rectangular cross-section serpentine microchannel consisting of 14 periodical circular curve units using μ-PIV measurement. The measurement of fluorescent visualisation revealed that the mixing performance for the viscoelastic solution was enhanced significantly compared with the Newtonian solution even under very low Reynolds number regime (Re = 1.8 × 10^{-4} - 3.6 × 10^{-3}) due to the combination of the viscoelastic fluid properties (Wi = 17.6 - 352) and the curvilinear shape of the microchannel.

Zilz et al. [232] described the geometric scaling of the onset of a purely-elastic flow instability in serpentine channels of rectangular cross-section through the combination of experimental, numerical and theoretical investigations. Experimental measurements carried out in serpentine microchannels with varying radii of curvature and channel dimensions (width and height) using solutions of the flexible polymer polyethylene oxide (PEO) with molecular weight $M_w = 2 \times 10^6$ g/mol at a concentration of 125 ppm (w/w) in different water/glycerol mixtures. Whilst three-dimensional time-resolved numerical simulations were performed using the
upper-convected Maxwell (UCM) model in identical microchannel geometries. The experimental and numerical approaches were confirmed with a theoretical analysis based on the Pakdel-McKinley criterion \cite{162}. The results showed that the instability is driven by the curvilinear streamlines generated by the flow geometry in combination with the elasticity of the fluid. Additionally, the instability is time-dependent. After understanding the scaling behaviour of the onset of a purely-elastic flow instability in the serpentine microchannels, they proposed subsequently to use the serpentine microchannel as a rheometric micro-device that is able to quantify low fluid relaxation times ($< 1\text{ms}$) \cite{233}. Casanellas et al. \cite{27} investigated both experimentally and numerically the impact of fluid shear-thinning on the onset of purely-elastic flow instabilities in microfluidic serpentine channels using solutions of polyethylene oxide ($M_w = 4 \times 10^6 \text{g/mol}$) in a mixture of water-glycerol (75% - 25% in weight) solvent with different levels of shear-thinning. This investigation found that shear-thinning has a stabilising effect on the critical flow rate.

Very recently, the three-dimensional flow structures of a 1000 ppm of CTAC/NaSal solution in a (100 $\times$ 60 $\mu$m) rectangular cross-section curved microchannel comprising of 32 pairs of identical half-circle units using confocal $\mu$-PIV were investigated by Li et al. \cite{129} to observe the detailed flow evolution. The 3D flow structure was monitored by scanning the flow layer-by-layer in the depth direction of the curved microchannel using a fast-moving piezo stage with fluorescent beads as tracer particles. The experimental measurements revealed that the internal flow streams oscillate and are typically twisted. Additionally, the flow direction alters based on geometric curvature. The velocity features of the viscoelastic flow field within the curved microchannel exhibit the features of elastic turbulence.

We could not find any papers dealing explicitly with convective heat transfer in curved/serpentine microchannels using non-Newtonian fluid within a regime of
elastic turbulence. The single paper that investigates the heat transfer of viscoelastic fluids (100 ppm and 200 ppm PAM in sucrose-based solutions) in a (200 µm) square cross-section serpentine microchannel consisting of 45 periodical curved segments is the work of Li et al. [126]. Resistance thermometry of TitaniumPlatinum (TiPt) films was adopted for measuring wall temperatures. The experiments of fluid flow and heat transfer characteristics for both viscoelastic fluids (polymeric solutions) and Newtonian fluid (sucrose solution) were conducted at very-low-Re flow (0.02 ≤ Re ≤ 0.06) under thermal boundary conditions as: the bottom wall heated (constant wall heat flux) and other side walls held at constant temperature. Their experimental results show that the viscoelastic fluid has better heat transfer performance in the microchannel in comparison with that of the Newtonian fluid. They believed that this enhancement in heat transfer could be attributed to the irregular motion induced by the fluid viscoelasticity in the curved microchannel under creeping-flow condition. However, their results are clearly preliminary.

The behaviour of heat transfer in an axisymmetric swirling (von Karman) flow was experimentally studied by Traore et al. [215] within a regime of elastic turbulence. They studied the role of elastic turbulence in the von Karman flow with a cooled lower stationary wall and compared the temperature fluctuations with those of a low diffusivity tracer. PIV measurements for both isothermal and non-isothermal flows revealed no significant effect of the heat transfer process on the flow topology using 100 ppm polyacrylamide in a 65% (wt) aqueous solution of sucrose. Also, it was found that within an elastic turbulent flow regime the efficiency of heat transfer can locally increase up to four-fold as compared to the purely conductive state of the Newtonian fluid flow under the same conditions.

From the previous works presented above a consensus can be drawn about certain aspects of elastic turbulence within curved or serpentine channels.

1. In investigations of elastic instabilities, researchers have used several types
of viscoelastic fluids including, for example, high-molecular-weight polyacrylamide (PAA) \cite{25, 77, 78, 165, 208} and polyethylene-oxide (PEO) \cite{27, 232} in highly viscous (sucrose-based or glycerine-based) Newtonian solvents. Also, the viscoelastic fluids that were most often utilised in these experimental studies are dilute solutions where interaction between polymer molecules does not occur because the concentration of the flexible high-molecular-weight polymers is generally lower than the critical overlap concentration.

2. Purely-elastic instabilities can arise in shear flows from gradients in elastic stresses with curvilinear streamlines \cite{27, 76, 78, 121, 122, 165, 192, 208, 232}. Therefore, purely-elastic flow instabilities and elastic turbulence have mostly been observed only in geometries involving streamline curvature where these instabilities are assumed to be driven by the “hoop stress”, which originates from the normal stress differences.

3. Purely-elastic instabilities of viscoelastic flows can occur at a certain critical value of the Weissenberg number, which for viscoelastic fluids plays the same role of the Reynolds number in Newtonian fluids for creating the nonlinearities that cause unstable flow \cite{121, 187}. Therefore, elastic turbulence, or “turbulence without inertia” commented by Larson \cite{121}, in curved or serpentine channels is dependent on the dynamic state only driven by the nonlinear elastic stresses at vanishingly small Reynolds number, with Weissenberg number exceeding a critical value.

Although, the purely-elastic instabilities and elastic turbulence in viscoelastic fluids have been the subject of many theoretical and experimental studies during the past two decades, which are reviewed in \cite{122, 128, 192}, no study is found in the previous literature that addresses the effectiveness of elastic turbulence to enhance heat transfer in the curved microchannels, which is the main focus in this thesis with the aim to fill the gap of knowledge in this research area. An
experimental investigation is performed to study the enhancement of convective heat transfer by employing the interaction of large elastic stresses created by the shearing motion within the fluid flow with streamline curvature of the serpentine microchannel leads initially to a purely-elastic instability and then the generation of elastic turbulence (see Chapter 6).

2.2 Cross-slot geometry

The isothermal flow behaviour in the cross-slot geometry, which is a common geometric shape in microfluidic applications, has been thoroughly studied using both Newtonian and non-Newtonian fluids. The simplicity of the configuration and the ease of the control of the cross-slot geometry have motivated an extensive range of applications in several scientific fields for enhancing mixing of fluids [67]. The cross-slot micro-geometry has been used for inducing enhanced mixing at micro-scale applications [67, 85, 151, 199], which can also be utilised to distinguish purely-elastic instabilities of viscoelastic solutions in a planar extensional flow [13, 80–82, 84]. However, no previous publications can be found dealing with characteristics of convective heat transfer for both Newtonian and viscoelastic fluids within the cross-slot micro-geometry, although there is a wide range of papers in the field of flow behaviour in the cross-slot geometry which will be classified as follows.

2.2.1 Newtonian fluid flow in cross-slots

In the early 1990’s, Kalashnikov and Tsiklauri [107, 108] investigated experimentally the features of stability of the planar laminar flow in a rectangular cross-section crossed channels using both Newtonian (water) [107] and non-Newtonian (polyethylene-oxide in water) [108] fluids. For the Newtonian fluid, they observed
that the flow at small Reynolds number is stable, laminar and two-dimensional (coincides with the vertical symmetry plane of the exit channels). While, the flow beyond a certain critical value of Reynolds number (55 and 43 for increasing and decreasing Reynolds number, respectively) in the cross-slot cell becomes unstable and three-dimensional. This kind of instability occurs in the cross-slot flow because a vertical series of fan-shaped vortices arises in the central transverse plane of the outflow channels as shown in Figure 2.4. For the non-Newtonian fluid, the results indicated that the elastic properties of viscoelastic fluid quantitatively affect the three-dimensional flow in the crossed channels.

Figure 2.4: The schematic picture of the streamlines that occur with variation of the Reynolds number in the crossed channels. (a) and (b) show the streamlines, which form a distinctive pattern that is repeated with period $2s$, in the vertical symmetry plane of the flow. The dotted lines indicate to the location of the transverse section of the outflow channels. (c) illustrates schematically certain features of the flow in plan where the transverse scales of the flow are increased (after [107]).
This paper was followed by several experimental and numerical investigations on flows with an internal stagnation point, such as those generated in cross-slot or T-junctions geometries, to study the possibility of mixing fluids in microfluidic systems under strong extensional deformations. Three different regimes of laminar flow (stratified flow, vortex flow and engulfment flow) were observed in a micro T-shaped mixer depending on the Reynolds number in the outflow mixing channel \cite{22, 58, 117, 196}. Experimental analysis and modelling of micro-mixing in a T-shaped micromixer were also conducted by Hoffmann et al. \cite{87}. The experimental measurements using micro Particle Image Velocimetry (\(\mu\)-PIV) and Laser Induced Fluorescence (\(\mu\)-LIF) techniques illustrate that in the “engulfment” flow regime the improvement of mixing performance can be achieved because the two inlet streams are forced together into a spiral structure with increasing Reynolds number \((Re > 180)\).

In the cross-slot micro-geometries the fluid flow around the stagnation point undergoes strong shear and therefore enhances the mixing of fluids. Wong et al. \cite{224} simulated three-dimensional liquid flow in a cross-shaped micromixer incorporated with static mixing elements for characterising the impacts of static mixing elements on the performance of mixing fluid using a commercial CFD software package. In this cross-shaped micromixer, 3D numerical simulations were studied as three inlet channels and one main outlet channel with two static mixing elements. The numerical result showed that the incorporation of static mixing elements within the cross-shaped micromixer in the outflow channel can significantly improve the mixing performance due to the generation of eddies behind these mixing elements. Consequently, the swirling effect of these eddies boosts the contact area between the two incoming fluid streams and reduces the diffusion distance for achieving homogeneous mixing.

The Taguchi method, which is regarded as one of the most powerful tools based in statistical methods for experimental optimisation, and numerical simulations in
an obstacle micromixer with three mixing units were employed by Shih and Chung [193] for investigating the effect of four geometrical factors (gap ratio, number of mixing units, baffle width and chamber ratio) on sensitivity of mixing fluid over a wide range of Reynolds number (0.1 \( \leq Re \leq 40 \)) [193]. The results pointed out that the performance of the micromixer is significantly affected by the gap ratio (width/height) and Reynolds number. Uniform mixing is achieved at lower \( Re \) flow (\( Re \leq 0.5 \)) by a diffusion-enhanced mechanism (molecular diffusion) while mixing enhancement occurs at higher \( Re \) flow (\( Re \geq 20 \)) by a convection-enhanced mechanism (vortex agitation).

Two-dimensional numerical simulations were conducted by Srisamran and Devahastin [200] to investigate the mixing behaviour of Newtonian (water) and shear-thinning fluids (CMC solutions) using a cell with two crossed microchannels. The governing equations were solved using finite element-based commercial software FEMLAB version 3.0 with the range of Reynolds numbers based on the inlet slot width from 10 to 200 and the mixing index (the ratio between the standard deviation of the cross-section fluid temperature and the temperature difference of the inlet streams) of the fluid streams in the range of 0.62 - 1. The numerical findings depicted that, with the increase of the Reynolds number, recirculation and preferential zones appear and the diameter of these vortices enlarges with an increase in \( Re \).

Huchet et al. [92] studied experimentally the flow pattern behaviour and mass transfer using an electrochemical method in a microdevice composed of crossing microchannels. Two hydraulic diameters (500, 833 \( \mu m \)) and two flow patterns (crossing-flow, impinging-flow) were used to compare their influence on the mixing performance and mass transfer in the crossed microchannels. The experimental results obtained in the laminar regime are consistent with CFD simulations using the commercial CFD solver Fluent version 6.2 for Reynolds numbers ranging between 100 and 400. The experimental and numerical results confirmed that
the occurrence of hydrodynamic instabilities and three-dimensional flow in the outflow channels can promote mixing processes. Also, the disturbance created by the intersection of flow is more significant in the impinging-flow than in the crossing-flow configuration. Therefore, the impinging-flow in crossing rectangular microchannels seems to be an attractive geometry compared to crossing-flow.

The characteristics of fluid flow and mass transfer within $400\mu m$ square cross-section T-shaped and cross-shaped micromixers have been numerically and experimentally investigated by Mouheb and co-workers [150, 151] to compare the impact of geometric configuration and the degree of freedom for the fluid flow in these micro-systems on mixing performance. In the experiments, Particle Image Velocimetry (PIV) combined with electrochemical methods [151] and micro-Laser Induced Fluorescence ($\mu$-LIF) [150] were utilised in order to compare the effectiveness of both micromixers (T-shaped and cross-shaped) for the range of Reynolds numbers from 25 and 50 to 100 and 200 for T-shaped and cross-shaped, respectively. In addition, 3D numerical simulations were performed for studying hydrodynamics and concentration profiles in T-shaped and cross-shaped micromixers using the commercial CFD solver Fluent version 6.2. The experimental and numerical results revealed, in terms of the concentration profiles, the mixing enhancement in the cross-shaped micromixer can occur at low flow rates compared to the T-shape. This enhancement in mixing in the cross-shaped micromixer is attributed to the stronger vortex stretching and high shear rate. Also, the cross-shaped micromixer induces a greater degree of freedom for the flowing fluid and a smaller pressure drop than the T-shaped micromixer due to the existence of two outflow streams.

Oliveira et al. [157] carried out a systematic numerical and experimental investigation on Newtonian fluid flow through a $(100\mu m)$ square cross-section cross-slot geometry comprising three entrances and one exit in order to study the occurrence of divergent streamlines and the onset and enhancement of symmetrical
wall-detached recirculations at low Reynolds numbers. From the good agreement between the experimental observations and the numerical simulations, they confirmed the influence of the geometric parameters and of inertia on the fluid flow patterns. The appearance of the central recirculations relies non-monotonically on the relative width of the entrance channels and inertia promotes the appearance of the free vortices. Also, the results showed that the Newtonian fluid flow remains symmetric relative to the horizontal centreline for a wide range of flow conditions ($Re \leq 113$).

The numerical investigation of Poole et al. [173] concentrated on a new bifurcation phenomenon through a two-dimensional cross-slot, or cross-channel, geometry. The governing equations (conservation of mass and momentum) were discretised in space by integration over the set of control volumes (the finite volume method) forming the computational mesh. The numerical data showed that the flow remains steady and symmetric at low Reynolds numbers and identical regions of standing recirculation attached to the four corners increase linearly in size with $Re$. In contrast, beyond a critical Reynolds number ($= 1490 \pm 10$), the unstable symmetrical solution was evolved to a pair of steady asymmetric solutions.

Very recently Haward et al. [85] elucidated experimentally and numerically the mechanism of fluid mixing in cross-slot geometries with a range of aspect ratios (depth/width), 0.4 - 3.87. The results revealed that fluid flow is steady and symmetric (the interface between fluid streams is sharp and vertical) at low Reynolds numbers. However, as Reynolds numbers increases above a critical value, which is dependent on the cross-slot aspect ratio ($20 \leq Re \leq 100$), the laminar flow experiences a steady symmetry-breaking bifurcation and a spiral vortex structure is created abruptly about the central axis of the outflow channels as shown in Figure 2.5. This spiral instability, which is described by a Landau model analogous to that used near equilibrium tricritical points, can be harnessed to boost fluid mixing in the cross-slot geometry at modest Reynolds numbers.
In spite of the great progress in the understanding of the hydrodynamic behaviour (symmetry-breaking bifurcation) of Newtonian fluids occurring in the micro-scale cross-slot device as mentioned in the literature above, the characteristics of convective heat transfer in the cross-slot geometry still need to be explored. In this thesis, Chapter 7 is dedicated to study experimentally and numerically the symmetry-breaking bifurcation of Newtonian fluids in the cross-slot micro-geometries in order to promote convective heat transfer at low or moderate values of Reynolds numbers.
2.2.2 Viscoelastic fluid flow in cross-slots

A combination of experimental investigations and numerical simulations have been carried out by Schoonen et al. [189] in order to study the extensional rheology of polymeric solutions in the (10 mm × 20 mm) cross-slot geometry with (20 mm radius) rounded corners. In the experiments, Laser Doppler anemometry (LDA) and flow induced birefringence (FIB) were utilised for measuring velocity field and axial integrated stresses, respectively. The numerical simulations were separated into two parts: the 3D velocity field was obtained from finite element calculations with the viscous Carreau model and the viscoelastic stresses were determined with a four mode Giesekus and Phan-Thien-Tanner (PTT) model using a streamline integration technique. The local velocities and stresses obtained from numerical simulations illustrate a good agreement with those from experimental measurements. This was followed by several experimental studies and numerical simulations to study the appearance of a purely-elastic instability of extensional flow through the cross-slot geometry.

Arratia et al. [13] studied experimentally the flow behaviour of a dilute polymeric solution (200 ppm PAA in 97%-glycerine aqueous solution) at low Reynolds number (\(Re < 10^{-2}\)) in a cross-slot microchannel (rectangular cross-section with 650 µm width and 500 µm depth). They observed that, for Newtonian fluid (99% glycerine) at low Reynolds number (\(Re = 7 \times 10^{-3}\)), the flow between dyed and undyed fluids is symmetric and stable (sharp interface edge between the dyed and undyed streams). For the dilute polymeric solution, two symmetry-breaking instability modes were distinguished in the cross-slot flow above a critical value of Deborah number. Firstly, the symmetry flow pattern becomes asymmetric but remains steady (the interface between the dyed and undyed stream is deformed (wavy)). In the second instability, the flow switches to a time-dependent flow and the direction of asymmetry flips between two outflow channels with time due to
the existence of vortical structures around the stagnation point.

Pathak and Hudson [164] evaluated the flow-induced birefringence behaviour and transmittance bands of micellar solutions under extension (planar elongational flow) in a microfluidic cross-slot ($530 \times 281 \mu m$) using micro-particle image velocimetry ($\mu$-PIV). The experimental findings revealed that, for very small $Wi$, the birefringence is linear. While, with increasing $Wi$, a sharp birefringence band, which forms along the outflow channels, appears and the velocity field becomes asymmetric with negligible inertia (Reynolds numbers smaller than $10^{-2}$). A series of experiments were carried out by Dubash et al. [56] to investigate elastic instability for wormlike micellar solutions flow through a cross-slot micro-device (rectangular cross-section, $500 \times 250 \mu m$) using birefringence and PIV measurements. Four different concentrations of a CTAB-NaSal aqueous solution were examined, two solutions were strongly viscoelastic and the other two were weakly viscoelastic. The experimental results showed that, with increasing flowrate, the elastic instability for the highly viscoelastic solutions transitions from a stable symmetric flow to a stable asymmetric (symmetry-breaking) flow and eventually to an unsteady asymmetric flow. They also noticed, lip vortices form along the walls of the inflow channels. While the elastic instability for the weakly viscoelastic solutions switches directly from a stable symmetric flow to an unsteady flow.

Haward and co-workers [79–84] performed a group of detailed experiments on the cross-slot flow behaviour using various types of micellar solutions such as, atactic Polystyrene (aPS) [83], Cetylpyridinium Chloride (CPyCl) and Sodium Salicylate (NaSal) [80, 82], Poly(ethylene oxide) (PEO) [81, 84] and Hyaluronic Acid (HA) [79], with aspect ratios (depth/width), of $5$ [80, 83] and $10.5$ [79, 84]. The experimental measurements showed the flow field for a viscous Newtonian fluid in the cross-slot device remains symmetric and stable up to moderate values of Reynolds number. On the other hand, the flow field of viscoelastic solutions is stable and symmetric at low strain rate values where the birefringent strand of the polymeric
solutions extends along the outflow streamline from the stagnation point. With increasing flow rate (moderate deformation rates), the flow becomes increasingly asymmetric, but remains steady. This asymmetric flow is characterised by a dividing streamline and birefringent strand connecting diagonally opposite corners of the cross-slot. As the nominal deformation rate is increased further, the asymmetric flow evolves eventually to be time-dependent. Haward and co-workers believed that the effects of shear localisation, which can occur with micellar solutions, may cause these purely-elastic instabilities in cross-slot flows [80, 82].

Several numerical studies have qualitatively addressed the viscoelastic fluid flow behaviour through the cross-slot geometry to illustrate the conditions that can affect the onset of purely-elastic instabilities. The two-dimensional cross-slot flow behaviour reported by Arratia et al. [13] has been numerically simulated by Poole et al. [170] using constant viscosity models (an upper-convected Maxwell (UCM) fluid) under inertialess flow conditions (i.e., $Re$ tending to 0). The qualitative results of the numerical simulation were in a good agreement with the experimental results [13]. In the same year, Poole et al. [169] demonstrated numerically the impact of the aspect ratio of the microfluidic cross-slot geometry using an Oldroyd-B model and a simplified Phan-Thien-Tanner (PTT) model. The effect of the aspect ratio on the three-dimensional cross-slot flow was achieved under creeping-flow conditions by varying the depth of the crossed-channel from a quasi-Hele Shaw flow arrangement (low aspect ratio) up to large aspect ratios (quasi-two dimensional flow). An efficient design of the shape of the dilute viscoelastic flow in the cross-slot geometry was numerically studied by Alves [6] for achieving optimal performance using a finite-volume viscoelastic code coupled with the CONDOR optimizer. This optimal shape of the chamber is capable for producing ideal planar extensional flow along the flow centrelines [84]. Xi and Graham [225] elucidated numerically the presence of an oscillatory instability of viscoelastic flow in a two-dimensional cross-slot geometry under the condition of negligible inertial effect.
The data from numerical simulations explained that the mechanism of instability for steady-state viscoelastic stagnation-point flow arises by the non-linear interaction between the base flow with extensional stresses and their steep gradients in the stagnation-point region.

A numerical investigation was also implemented by Rocha et al. [177] for simulating fluid flows in cross-slots using a finite-volume method (FENE-CR and FENE-P models) and predicting a transition to steady asymmetry at gradually lower Deborah numbers, $De$ (which represents the ratio between the relaxation time of the fluid and a characteristic time scale of the flow usually used to characterise the degree of elasticity), with increasing values of the extensibility parameter and polymer concentration. The influence of corner sharpness was insignificant up to a radius of curvature of 50% of the channel width. Additionally, Becherer et al. [19] undertook an analytical study to investigate the main characteristics of singular solutions strands in a cross-channel due to the finite extensibility. Oliveira et al. [158] implemented a numerical simulation of the flow of upper-convected Maxwell (UCM) and Phan-Thien-Tanner (PTT) fluid models in a flow-focusing cross-slot device under creeping-flow conditions. The purpose of this numerical investigation was to assess the effects of Deborah number, velocity ratio (the ratio of the inlet average velocities in the side streams to the average velocity in the central inlet stream, ranging from 1 to 500) and width ratio (the relative width of the entrance branches) on the onset of flow asymmetries and instabilities by manipulation of velocity and width ratios of the three inlet channels. The numerical data revealed that purely-elastic instabilities are shown to occur as the Deborah number is increased above a critical threshold with entirely negligible inertial effects. Two types of instability were distinguished from two-dimensional numerical simulations: steady and asymmetric flow and unsteady flow, oscillating in time, as a consequence of increasing Deborah number (high deformation rates). The flow develops directly from steady symmetric to unsteady (oscillating in time) without
passing through the intermediate flow regime (steady asymmetric flow) at very low values of the velocity ratio or width ratio, because the normal stresses are not adequately high to trigger this intermediate transition.

Another numerical study presented by Afonso et al. [1] to simulate purely-elastic instabilities in three-dimensional cross-slot geometries used the UCM and PTT models. Two different flow configurations corresponding to uniaxial extension and biaxial extension were analysed, and the influences of extensional flow type and Deborah number on flow dynamics near the interior stagnation point were made. The numerical results showed that the flow is symmetric relative to the centreline at low values of $De$ whereas, for $De$ above a critical value, the flow becomes asymmetric, while remaining steady. Subsequently, the flow becomes unsteady above an even higher onset value of the $De$ due to the normal-stress values for components aligned with the outflow channels (strong viscoelastic effects in uniaxial extension).

The effects of both the aspect ratio of the crossed channels and the rheological properties of viscoelastic fluids on the extensional flow patterns in a cross-slot micro-geometry have been experimentally studied under negligible inertial flow conditions by Sousa et al. [198]. Generally, a series of purely-elastic flow transitions were observed in the viscoelastic fluid flow from symmetric flow to steady asymmetric flow and eventually to time-dependent flow depending on the Weissenberg number. The experimental results indicated that the direct transition of viscoelastic fluid flow from a steady symmetric condition to an unsteady state (time-dependent flow instability) occurs immediately as long as the channel aspect ratio is less than 0.5.

Very recently, two-dimensional cross-slot geometry was proposed as an interesting and useful candidate for a numerical benchmark case of viscoelastic fluid flows by Cruz et al. [42] utilising three different viscoelastic models. The numerical data
obtained from two-dimensional simulations sees, as others have, that the creeping steady symmetric flow undergoes a bifurcation to a steady asymmetric flow followed by a second transition to time-dependent flow for all examined models. Afterwards, they also conducted three-dimensional numerical simulations using upper-convected Maxwell and Phan-Thien-Tanner constitutive equations to investigate the mechanism of purely-elastic steady bifurcation and transition to time-dependent flow in the planar cross-slot steady flow with a wide range of aspect ratios, \(0.01 \rightarrow \infty\) \[43\]. Viscoelastic creeping-flow evolves directly from a steady symmetric flow to a time-dependent flow in the shallow-channel regime (aspect ratio \(<0.5\)). In contrast, in the deep-channel regime (aspect ratio \(>0.5\)), the viscoelastic creeping-flow transitions firstly from a steady symmetric to a steady asymmetric flow and at higher \(De\), develop to unsteady time-dependent flow.

As the foregoing has made clear, most previous experimental and numerical studies mainly focus on descriptions of the viscoelastic fluid flow in the cross-slot micro-geometry. No literature can be found dealing with the heat transfer behaviour in this micro-scale geometry. In the present study (Chapter 8 of this thesis), an experimental investigation is carried out to study the convective heat transfer in the cross-slot micro-geometry in order to understand the influence of viscoelastic fluid flow behaviour on heat transfer.
Chapter 3

Working fluid preparation and rheological characteristics

The main attention of this chapter is focused on the rheological characteristics of viscoelastic fluids that flow through micro-scale geometries. The background Section 3.1 gives definition of the rheology concept and the fundamental differences between Newtonian and non-Newtonian fluids as well as a brief description of a non-Newtonian fluid classification. Section 3.2 provides a detailed explanation about the selection and the preparation of the selected Newtonian and viscoelastic fluids. Section 3.3 focuses on the rheometer tests such as steady-shear tests and oscillatory-shear tests with suitable modelling for these tests. Section 3.4 describes the determination of critical overlap concentration whereas the principles of relaxation time measurements are reported in Section 3.5 using various experimental and theoretical procedures. Section 3.6 addresses the effect of temperature on rheological properties such as shear viscosity and relaxation time. Finally, thermophysical properties measurements for the working solutions used in the current study are described in the Section 3.7.
Chapter 3. Working fluid preparation and rheological characteristics

3.1 Background

The term “Rheology”, which was coined at the founding of the American Society of Rheology in 1929, is the science that deals with the study of flow and deformation of solids and fluids under the effect of an applied force. Since then, the science of rheology has attracted the attention of scientists and engineers in several fields comprising chemical engineering, physics, material science and biology.

In a general concept, fluids are divided into two major categories namely, Newtonian and non-Newtonian. Newtonian fluids, which are named after Sir Issac Newton (1642-1726), obey Newton’s law of viscous resistance as:

\[ \tau = \eta \dot{\gamma} \] (3.1)

where \( \tau \) is the shear stress and \( \dot{\gamma} \) is the shear rate. The coefficient \( \eta \) is defined as a dynamic viscosity of the fluid. The Newtonian fluids (e.g., air, water, glycerine, oil, honey, alcohol and etc.) possess a constant viscosity at all shear rates, in other words, the viscosity of the fluid remains constant with the variation of the shear rates. For such Newtonian fluids, the shear stresses are linearly dependent on the shear rate by a content of proportionality known shear viscosity.

While, fluids in which the value of viscosity is not constant are defined as non-Newtonian fluids. This kind of fluids deviates from Newton’s law of viscosity, and exhibits variable viscosity. There are several categories of these non-Newtonian fluids, which are classified depending on the relationship between shear stress and shear rate. The non-Newtonian fluids (e.g., polymeric solutions, colloidal suspensions, blood, chocolate, paint and cosmetics) can be conveniently categorised into three general classes:

1. Time-independent fluids are those for which the relation between the value of shear stress and the value of shear rate is a unique but non-linear function of
the instantaneous shear stress at that point. These fluids are variously called as “purely viscous”, “inelastic” or “generalised Newtonian fluids, GNF”. The time-independent non-Newtonian fluids can be also subdivided according to the flow curves between the shear stress and the shear rate as shown in Figure 3.1 as: Bingham plastics, pseudoplastic fluids (shear thinning), dilatant fluids (shear thickening).

2. Time-dependent fluids have more complex relationship between the shear stress and the shear rate. In this kind of non-Newtonian fluids, the shear rate depends not only on the shear stress (kinematic history) but also on the previous shear stress rate history (shearing time) of the fluid. Time-dependent fluids can be subdivided into two types, thixotropic fluids and rheopectic fluids, based on whether the shear stress increases or decreases in time at a given shear rate and under a constant temperature.

3. Viscoelastic fluids exhibit characteristics of both solid-like (elastic) behaviour and fluid-like (viscous) behaviour, and show partial recovery upon the removal of the deformable shear stress. The rheological characteristics of such a fluid at any instant will be a function of the recent history of the fluid and can’t be described by only relationships between shear stress and shear rate, but will need to include of the time derivative of both quantities.

It is worth mentioning that the aforementioned classification strategy for the non-Newtonian fluids is entirely arbitrary because the most non-Newtonian fluids usually display a combination of two types of these features under appropriate conditions for example, it is common for some polymeric solutions to exhibit simultaneously a combination of time-independent (shear-thinning) and viscoelastic behaviour at certain circumstances. The behaviour of non-Newtonian fluids is now dealt with in more detail. Generally, the behaviour of non-Newtonian fluids is modelled using rheological characteristics and correlations of shear stress and
shear rate where many rheological models for different non-Newtonian fluids are available in the previous literature. The measurement of rheological properties can be routinely implemented from bulk sample deformation using a rheometer. The main attention here is conducted to the rheological characteristics of flow systems of interest in studies of viscoelastic fluid flow through micro-scale geometries.

### 3.2 Fluid selection and working fluid preparation

#### 3.2.1 Newtonian fluids

Experiments with Newtonian fluids are usually used in order to emphasis the validity of the present experimental arrangements and measuring system and also to give a control to which the viscoelastic fluids data can be compared. Newtonian fluids were initially utilised in the current experimental investigation because they
have a constant viscosity and are inelastic. In order to investigate the effect of elastic turbulence on the enhancement of convective heat transfer within micro-scale geometries, serpentine microchannel and cross-slot micro-geometry, the impact of inertia was neglected (maintaining low Reynolds number) for achieving creeping flow in these systems \cite{76,78,233}. Thus, Newtonian fluids with a relatively high viscosity were required. Glycerine and viscous sucrose syrup were selected as Newtonian fluids.

Glycerine is considered one of the most widely utilised ingredients in pharmaceuticals, cosmetics, food industries, paper manufacture and rubber industry \cite{106,207}. Glycerine supplied by ReAgent Chemical Services, which is virtually nontoxic, is a viscous Newtonian liquid and it is very stable under most circumstances. Physically, it is a clear, almost colourless, viscous, high boiling point (290°C at the atmospheric pressure) and a good solvent \cite{15}. The thermophysical properties of glycerine are provided in Table 3.1. Glycerine does not degrade biologically though at high concentrations it absorbs moisture from the ambience. The moisture absorption was avoided as much as possible by storing the prepared solutions in the sealed plastic bottles. Sucrose that is also chosen for preparing other type of Newtonian solution was biochemical grade and supplied by ACROS Organics. The physical properties of sucrose are solid white, odourless, molecular weight 342.29, melting point 190 – 192°C and solubility in water at 15°C 1970 g/l.

The experimental Newtonian results for the serpentine microchannel geometry were produced using two types of Newtonian fluids namely, an aqueous glycerine solution and an aqueous sucrose solution. The first type of Newtonian fluid was a mixture of 10% distilled water and 90% glycerine by weight (hereafter W/GLY mixture). While, the second Newtonian fluid was an aqueous solution of 65% sucrose and 1% sodium chloride (NaCl) in distilled water all by weight (hereafter W/SUC solution). The Newtonian fluid that has been utilized in the cross-slot micro-geometry measurements was only one concentration of glycerine, 70%
(w/w), mixed with 30% distilled water (hereafter W/70GLY mixture).

**Table 3.1:** The thermophysical properties of pure glycerine at 20°C [15].

<table>
<thead>
<tr>
<th>Specific gravity</th>
<th>Dynamic viscosity</th>
<th>Specific heat capacity</th>
<th>Thermal conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Pa.s</td>
<td>J/kg.K</td>
<td>W/m.K</td>
</tr>
<tr>
<td>1.2636</td>
<td>1.499</td>
<td>2426.25</td>
<td>0.28931</td>
</tr>
</tbody>
</table>

### 3.2.2 Viscoelastic fluids

As is well known, the addition of small amounts of polymer to a solvent produces a viscoelastic fluid possessing both viscous and elastic characteristics [20]. Polyacrylamide (PAA), which is one of the most extensively utilised polymer, is completely miscible in water (water-soluble) [72]. Polyacrylamide is widely used in many industries and modern technologies for instance, water treatment, paper manufacture, mining, oil recovery, hair care products (i.e. shampoo and conditioner), thickener and suspending agent and turbulent drag-reduction agent [227]. PAA, which is a high-molecular-weight flexible polymer, was selected in this study due to its high elasticity. Also, it has been previously used by many researchers for investigating the phenomenon of elastic instability and turbulence (see for example [26, 76–78, 187, 208]). In addition, PAA has more distinct elastic properties compared with other polymers (xanthan gum and carboxymethylcellulose (CMC)) because its molecular structure is very flexible [219].

In the current work, two groups of polymeric viscoelastic fluids (shear-thinning solutions and constant-viscosity elastic solutions, usually called Boger fluids [21]) have been used to study the influences of both viscoelasticity and shear-thinning viscosity individually on the characteristics of convective heat transfer and fluid flow within serpentine microchannel geometry. The polyacrylamide used during this work was supplied by Polysciences, Inc. with a molecular weight of $M_w \sim 1.8 \times 10^7$ g/mol. The effects of shear-thinning viscosity are investigated by dissolving
Table 3.2: An overview of the composition of the working fluids.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>PAA concentration ppm</th>
<th>Solvent ((X + H_2O))</th>
<th>NaCl %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Newtonian and shear-thinning solutions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W/GLY</td>
<td>-</td>
<td>Glycerine 90</td>
<td>-</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>50</td>
<td>Glycerine 90</td>
<td>-</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>100</td>
<td>Glycerine 90</td>
<td>-</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>200</td>
<td>Glycerine 90</td>
<td>-</td>
</tr>
<tr>
<td>W/70GLY</td>
<td>-</td>
<td>Glycerine 70</td>
<td>-</td>
</tr>
<tr>
<td>100-W/70GLY</td>
<td>100</td>
<td>Glycerine 70</td>
<td>-</td>
</tr>
<tr>
<td><strong>Newtonian and Boger solutions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W/SUC</td>
<td>-</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>80</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>120</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>500</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
</tbody>
</table>

small amounts of PAA in an aqueous glycerine solution (W/GLY solvent) whilst the influences of viscoelasticity are studied by adding small amounts of PAA to an aqueous sucrose solution (W/SUC solvent). Three shear-thinning solutions containing 50 ppm, 100 ppm and 200 ppm concentration by weight of the PAA dissolved in the W/GLY solvent were prepared. These shear-thinning solutions are henceforth referred to as 50-W/GLY, 100-W/GLY and 200-W/GLY, respectively. While, the Boger solutions were produced by dissolving 80 ppm, 120 ppm and 500 ppm concentration by weight of the same polymer in the W/SUC solvent. these Boger solutions are hereafter referred to as 80-W/SUC, 120-W/SUC and 500-W/SUC respectively. In addition, the viscoelastic fluid used for the experimental measurements in the cross-slot micro-geometry was also shear-thinning solution and consists of 100 ppm PAA in a 70% glycerine/ 30% water solvent (W/70GLY) that is henceforth referred to as 100-W/70GLY. Table 3.2 provides an overview of the composition of the studied solutions.
3.2.3 The preparation of working solutions

All viscoelastic solutions were made with mass ratio of the ingredients. The following scenario was applied to prepare the shear-thinning solutions (50-W/GLY, 100-W/GLY and 200-W/GLY). The polyacrylamide powder was weighted into a measuring flask using a Denver Instrument P-114 precision balance, which has an uncertainty $\pm 1\,mg$ in the range of load from $0.01\,g$ to $120\,g$. This certain amount of PAA powder was first deposited in the distilled water. After 5-10 min. of absorbing at rest, the mixture of PAA and water is gently mixed using 4 cm magnetic stirrer at low rotational speed ($60\,rpm$) for 24 hrs at ambient temperature ($\sim 22^\circ C$) to avoid PAA degradation. The respective amount of glycerine was added to the PAA-water mixture. The entire solution (glycerine + PAA-water) was then mixed again using electrical stirrer (Stuart SS10) at approximately $60\,rpm$ for another period of 24 hrs to obtain homogeneous viscoelastic solutions. During mixing process, cover beaker (graduated glass beaker) with a plastic sheet (cling film) was used to avoid evaporation and the prepared solution was then put in the sealed plastic bottle to be ready for use. The rheological properties (e.g., viscosity) of this prepared sample were measured by taking one sample from top and one sample from bottom the batch.

By adopting the same protocol mentioned above, the viscoelastic fluid 100 ppm PAA in a 70% glycerine/30% water solvent (100-W/70GLY) was prepared for the experimental measurements in the cross-slot micro-geometry. 0.1 g of the temperature-sensitive fluorescent dye (Rhodamine-B) was dissolved in the 1 l of 100 ppm PAA solution to obtain the temperature-sensitive viscoelastic solution. All rheological properties were normally measured with and without addition the temperature-sensitive dye to ensure that the rheological properties of viscoelastic solution don’t change with adding the fluorescent dye (Rhodamine-B).
The Boger solutions (80-W/SUC, 120-W/SUC and 500-W/SUC) were made according to the procedure of Schiamberg et al. [187]. These Boger solutions were prepared by adding the respective proportions of 65% mass fraction sucrose and 1% mass fraction sodium chloride (NaCl) to the distilled water. Where, the viscosity of the solutions was controlled by adding sucrose to obtain viscous solvent. Meanwhile, sodium chloride was added for fixing ionic contents within the polymeric solution and increase solubility. All these ingredients were stirred by rotating the container at 350 rpm for 24 hrs. In this stage, this solution was regarded as a Newtonian fluid (an aqueous sucrose solution, W/SUC). The PAA was slowly deposited into the homogeneous sucrose solution, and gently mixed by a magnetic stirrer at 60 rpm for another period of 48 hrs at ambient temperature (∼22°C). In order to avoid the growth of bacteria, all the measurements with these Boger solutions had to be performed shortly after their preparation.

3.3 Rheometer tests

One of the most common and well-established flow experiments, which is usually employed to extract the fluid parameters for both Newtonian and viscoelastic fluids, is the rotational steady-shear experiment. The polymeric viscoelastic solutions have been characterised by means of different types of rheometric experiments for instance, steady-shear measurements and small-amplitude oscillatory-shear measurements (SAOS). Multiple independent measurements of fresh samples were conducted to quantify the quality of the experimental data. The rheological characteristics comprise the shear viscosity measurements, the first normal-stress difference and the storage modulus and the loss modulus.

All rheological measurements for Newtonian (solvents) and viscoelastic solutions
were conducted using a TA Instruments Rheolyst AR1000N controlled-stress rheometer (shown in Figure 3.2) with an acrylic cone-and-plate geometry (60 mm diameter, 2° cone angle) with an uncertainty in viscosity of ±2% [59]. The manufacturers software (Rheology Advantage Instrumental Control V5.5.0) was utilised in conjunction with the rheometer to collect the experimental data. The cone-and-plate geometry consists of a stationary plate and a rotating cone, which is placed directly above the stationary plate as shown the schematic diagram in Figure 3.3. The sample of working fluids is positioned on the stationary plate and the cone is lowered to trap the tested sample between the plate and the cone. The stationary plate is a Peltier plate, which employs the Peltier effect to set the sample temperature. The temperature controller accuracy of the TA rheometer was within ±0.1°C using a Techne TE10A thermostatic bath. The tip of the cone is slightly truncated (61µm) so that the tip can’t become worn or damage the plate and there is a small gap between the plate and the cone to adjust for this truncation. The major advantage of the cone-and-plate geometry is that the shear rate is uniform throughout the sample [10].

Figure 3.2: Photograph of TA Instrument Rheolyst AR 1000N controlled-stress rheometer.
3.3.1 Steady-shear tests

In the current study, the steady-shear mode was adopted for measuring the shear viscosity against shear stress variation for the Newtonian and viscoelastic solutions and also to measure the first normal-stress difference. The steady-shear mode measurements were performed at a wide range of temperatures to cover the widest possible range of temperature for the experimental measurements. The controlled-stress rheometer applies the calculated torque, $T_q$, (and hence shear stress) to the sample of working fluid by rotating the cone and the resulting shear rate can be estimated by measuring the angular velocity, $\omega$, of the cone and the shear viscosity can be determined using,

$$\eta = \frac{\sigma}{\dot{\gamma}}$$

(3.2)

where $\sigma$ is the shear stress, which is a function of the torque. $\dot{\gamma}$ is the shear rate, which is a function of the angular velocity. Measured values of the shear viscosity were sampled every 15 s by the rheometer and the steady-state value is reached when three consecutive measurements are within 3% (3% is an arbitrary selected value). Steady-shear measurements were carried out over a range of shear stress from 0.1 Pa to 100 Pa for specifying the solution properties.
The variation of shear viscosity for the selected fluids (Newtonian and viscoelastic solutions) versus shear rate is illustrated in Figures 3.4, 3.5 and 3.6 at 20°C. The viscosity of Newtonian solutions (W/GLY, W/SUC and W/70GLY) remains a constant (see Figures 3.4, 3.5 and 3.6) no matter the variation of shear rate (i.e. the Newtonian viscosity is independent of the shear rate [20, 222]). Figure 3.4 shows the variation of shear viscosity with shear rate for the shear-thinning solutions (50-W/GLY, 100-W/GLY and 200-W/GLY) whereas Figure 3.5 exhibits the shear viscosity for the Boger solutions (80-W/SUC, 120-W/SUC and 500-W/SUC) against shear rate. As can be seen from Figures 3.4 and 3.5, the shear viscosity shows an increased dependence on shear rate with increasing the concentrations of polymer. Also, Figures 3.4 and 3.5 show that viscoelastic solutions, shear-thinning and Boger solutions, exhibit shear-rate dependent viscosity. However, the PAA-W/GLY solutions possess significantly greater shear-thinning effects than the PAA-W/SUC solutions. The experimental measurements show that the 80-W/SUC, 120-W/SUC and 500-W/SUC solutions possess a shear viscosity that is approximately constant. Where, this range of shear rates is common in the serpentine microchannel experiments. Figure 3.6 displays the shear viscosity variation versus shear rate for the glycerine-based 100 ppm PAA solution (100-W/70GLY) which also exhibits shear-rate dependent viscosity and shear-thinning effect. In order to confirm that the addition of the temperature-sensitive fluorescent dye (Rhodamine-B) does not affect the rheological properties of viscoelastic solution the shear viscosity data presents with and without addition the fluorescent dye to the 100 ppm PAA solution. Figure 3.6 can emphasise that there is no alteration in the values of shear viscosity.
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Figure 3.4: Shear viscosity versus shear rate for PAA-W/GLY solutions at 20 °C. Black lines are Carreau-Yasuda fits with parameters shown in Table 3.4.

Figure 3.5: Shear viscosity versus shear rate for PAA-W/SUC solutions at 20 °C. Black lines are Carreau-Yasuda fits with parameters shown in Table 3.4.
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In the flow of viscoelastic fluids, the normal stress components, which represent here first \( (N_1) \) and second \( (N_2) \) normal-stress differences, are non-zero values. Therefore the viscoelastic fluid microstructure becomes anisotropic in a flow process. Usually, the first normal-stress difference has a positive value. This value is much larger than the second normal-stress difference numerically (i.e. \( N_2 \approx 0.1N_1 \)). According to Barnes et al. [10], the molecules of polymer in the polymeric solutions may approximately occupy spherical shape at rest, but deform to an ellipsoidal shape in the direction of shear flow. The internal restoring forces, which are larger along the molecules’ major axis, try to return the molecules to their original spherical shape. These total restoring forces are greater in the direction normal to the shear deformation flow and they give rise to an increase in the first and second normal-stress differences. There are many natural phenomena related with the effects of first normal-stress differences in viscoelastic flows for
example, the Weissenberg or rod-climbing effect, die swell and elastic instabilities [16].

The first normal-stress difference, \( N_1 \), can be estimated from the total normal force for a cone-and-plate geometry using [218],

\[
N_1 = \frac{2F}{\pi R_{Cone}^2}
\]

(3.3)

where \( F \) is the total normal force and \( R_{Cone} \) is the cone radius. The experimental values of normal force for all viscoelastic solutions may be slightly lower than the true values because of inertia effects, which is known as the “negative normal stress effect” [16] and the experimental values of normal force were corrected by adding the inertial contributions using [218],

\[
\Delta F = \frac{3\pi \rho \omega^2 R_{Cone}^4}{40}
\]

(3.4)

where \( \Delta F \) is the difference in the normal force owing to inertia, \( \rho \) is the density of the sample and \( \omega \) is the angular velocity. Figures 3.7 and 3.8 show the experimental results of the first normal-stress difference, \( (N_1 = \tau_{xx} - \tau_{yy}) \), for PAA-W/GLY and PAA-W/SUC solutions, respectively, where the first normal-stress difference is deemed as a measure of the elastic stresses exist in these viscoelastic solutions. It can be observed from Figures 3.7 and 3.8 that the first normal-stress difference increases gradually with an increase in the shear rate for each viscoelastic solution. The results, shown in Figures 3.7 and 3.8, reveal that the first normal-stress difference of the 200-W/GLY and 500-W/SUC solutions were greater than that of the low polymer concentration solutions. Thus, 200-W/GLY and 500-W/SUC solutions were much more elastic than that with low polymer concentration solutions. The data of the first normal-stress difference was fitted with a power-law between shear rates \( \dot{\gamma} > 10 \text{ (1/s)} \) because of the limited experimental resolution (below the sensitivity of the rheometer) and the shear rates \( \dot{\gamma} < 250 \text{ (1/s)} \) where
the shear flow became unstable owing to viscoelastic instabilities, with 20 s of equilibration time at each value of shear rate and over several trials (at least four runs for each sample). The power-law fits to the first normal-stress difference data are plotted for each viscoelastic solution (the black solid lines) in Figures 3.7 and 3.8 and the fitting parameters are listed in Table 3.3. The temperature was kept constant at 20°C by using a Peltier plate to set the temperature of the solution sample to within ±0.1°C.

Table 3.3: Power-law parameters for the first normal-stress difference variation.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>Range of $\dot{\gamma}$ (1/s)</th>
<th>$N_1 = A \times \dot{\gamma}^b$</th>
<th>$A$</th>
<th>$b$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shear-thinning solutions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>11-185</td>
<td>0.214</td>
<td>1.407</td>
<td>0.980</td>
<td></td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>10-145</td>
<td>0.783</td>
<td>1.437</td>
<td>0.992</td>
<td></td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>10-126</td>
<td>1.779</td>
<td>1.539</td>
<td>0.995</td>
<td></td>
</tr>
<tr>
<td><strong>Boger solutions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>11-240</td>
<td>0.026</td>
<td>1.775</td>
<td>0.977</td>
<td></td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>11-240</td>
<td>0.095</td>
<td>1.817</td>
<td>0.990</td>
<td></td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>11-160</td>
<td>0.925</td>
<td>1.875</td>
<td>0.990</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 3. *Working fluid preparation and rheological characteristics*

Figure 3.7: First normal-stress difference against shear rate for PAA-W/GLY solutions at 20 °C. Solid lines represent a power-law fit to the normal-stress data.

Figure 3.8: First normal-stress difference against shear rate for PAA-W/SUC solutions at 20 °C. Solid lines represent a power-law fit to the normal-stress data.
3.3.2 Carreau-Yasuda model fits

In this research, the experimental results of shear viscosity show the fluids exhibit shear-thinning. It has been found that a decrease in shear viscosity with an increase in shear rate. The molecular structure for shear-thinning solutions undergoes a change with increasing shear rate and the molecules attempt to arrange themselves in the flow direction due to a loss of viscosity. Three distinct regimes can be observed from the shear flow curve of the shear-thinning solutions namely, the lower-shear-rate regime, the shear-thinning regime and the infinite-shear-rate regime \[16, 34\]. In the lower-shear-rate regime, the viscoelastic solution behaves as a Newtonian fluid with an approximately constant shear-viscosity (known in this regime as the zero-shear-rate viscosity, \(\eta_0\)). While in the shear-thinning regime, shear viscosity declines continuously with increasing shear rate. Finally, in the infinite-shear-rate regime, the shear viscosity is independent of the shear rate and the viscosity of the solution behaves as a Newtonian fluid. The shear viscosity in this regime is called the infinite-shear-rate viscosity, \(\eta_\infty\).

Various mathematical models have been reported to describe the behaviour of the shear viscosity with shear rate for such shear-thinning solutions, such as the power law \[16\] and Cross \[41\] amongst many others. The Carreau-Yasuda model \[228\] is usually utilised to model the entire range of shear-thinning fluids with asymptotic viscosities at zero and infinite shear rates. This rheological model is a generalisation of the Newton’s law of viscosity and describes with more precision the variation of shear viscosity with shear rate. Therefore, the Carreau-Yasuda model was adopted to fit the experimental shear viscosity data shown as solid lines in Figures 3.4, 3.5 and 3.6

\[
\eta_{CY} = \eta_\infty + \frac{(\eta_0 - \eta_\infty)}{[1 + (\dot{\gamma}\lambda_{CY})^{\alpha}]^\beta}
\]  

(3.5)
where $\eta_o$, $\eta_\infty$ are zero-shear-rate viscosity and infinite-shear-rate viscosity, respectively, $\lambda_{CY}$ is a constant that represents the inverse shear rate at the onset of shear-thinning, $a$ is a fitting parameter introduced by Yasuda et al. [228] and $n$ is a power law index. The main difference between the Carreau-Yasuda model and other models (e.g., the power law relation) is that the Carreau-Yasuda formula includes five parameters ($\eta_o$, $\eta_\infty$, $\lambda_{CY}$, $n$, $a$) to describe the rheological behaviour of the shear-thinning fluids. All fitting parameters for the Carreau-Yasuda model, which are tabulated in Table 3.4, have been determined using the least-squares-fitting method outlined in Escudier et al. [59] in essence minimisation of the standard deviation:

$$\sum_{i=1}^{N} \left[ 1 - \frac{\eta_{Exp}}{\eta_{CY}} \right]^2$$  \hspace{1cm} (3.6)

where $N$ is the number of data points. $\eta_{Exp.}$ is the measured value of shear viscosity and $\eta_{CY}$ is the viscosity obtained from equation 3.5. This type of error determination was more suitable than other traditional types of linear least square fit because it guarantees that the high and low shear rate values have the same level of effect on the determination of the fit.

**Table 3.4:** Fitting parameters of the Carreau-Yasuda model with the non-dimensional overlap concentration for all viscoelastic solutions used at 20 $^\circ$C.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>$\eta_o$</th>
<th>$\eta_\infty$</th>
<th>$\lambda_{CY}$</th>
<th>$a$</th>
<th>$n$</th>
<th>$c/c^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Pa.s}$</td>
<td>$\text{Pa.s}$</td>
<td>$s$</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Newtonian and shear-thinning solutions</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W/GLY</td>
<td>0.208</td>
<td>0.220</td>
<td>19.517</td>
<td>1.372</td>
<td>0.501</td>
<td>0.154</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>1.145</td>
<td>0.220</td>
<td>19.517</td>
<td>1.372</td>
<td>0.501</td>
<td>0.154</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>2.221</td>
<td>0.253</td>
<td>22.122</td>
<td>1.763</td>
<td>0.595</td>
<td>0.307</td>
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<tr>
<td>200-W/GLY</td>
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<td>0.304</td>
<td>25.399</td>
<td>1.205</td>
<td>0.647</td>
<td>0.615</td>
</tr>
<tr>
<td>W/70GLY</td>
<td>0.0226</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>100-W/70GLY</td>
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<td>0.030</td>
<td>29.572</td>
<td>1.436</td>
<td>0.525</td>
<td>0.313</td>
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<td><em>Newtonian and Boger solutions</em></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>W/SUC</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>80-W/SUC</td>
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<td>0.186</td>
<td>0.564</td>
<td>0.364</td>
<td>0.421</td>
<td>0.178</td>
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<td>0.261</td>
<td>0.490</td>
<td>0.434</td>
<td>0.645</td>
<td>0.267</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>0.769</td>
<td>0.348</td>
<td>1.238</td>
<td>0.274</td>
<td>0.490</td>
<td>1.111</td>
</tr>
</tbody>
</table>
3.3.3 Small-amplitude oscillatory-shear tests

The linear viscoelastic response of polymeric solutions has been investigated through oscillatory-shear flow. Small-amplitude oscillatory-shear stress (SAOS) test is used to measure the storage modulus, $G'$, which measures the energy of elastic storage and the state of the structured materials, and the loss modulus, $G''$, which represents viscous dissipation or loss of energy [16, 20, 34, 120]. Therefore, SAOS test is able to provide useful rheological characterisation of viscoelastic fluids such as details about the energy storage and the energy dissipation in the viscoelastic fluid flow and the relaxation time. The linear viscoelastic response is quantified by measuring the elastic storage modulus and the viscous loss modulus which for a single mode material is,

$$G' = \frac{\eta \lambda \omega^2}{1 + (\lambda \omega)^2}$$  \hspace{1cm} (3.7)

$$G'' = \frac{\eta \omega}{1 + (\lambda \omega)^2}$$  \hspace{1cm} (3.8)

where $\eta$ is the dynamic or shear viscosity, $\omega$ is the angular frequency and $\lambda$ is a relaxation time. Therefore, for Newtonian fluids (or any inelastic fluids) the relaxation time is equal to zero and this leads to $G' = 0$ and $G'' = \eta \omega$. A frequency sweep is carried out within the linear viscoelastic region (the oscillation frequency is changed while the oscillatory shear stress is kept constant at a low value such that the results are independent of the precise stress value applied). In the linear viscoelastic regime, the storage and the loss moduli are independent of the magnitude of stresses applied and are only functions of the frequency. The 60mm diameter, 2° cone angle acrylic cone geometry was also utilised for providing the oscillation results in order to minimise the effects of inertia. In addition, prior to each oscillatory test the rheometer was “mapped” to minimise the very small variations in behaviour throughout one revolution of the rheometer shaft. Also, a stress sweep was firstly performed at 0.1 Hz to determine the linear viscoelastic region before each frequency sweep test.
The viscoelastic properties in terms of the storage modulus, \( G' \), and the loss modulus, \( G'' \), against angular frequency for the shear-thinning solutions, the Boger solutions and 100-W/70GLY solution used in the current work are displayed in Figures 3.9, 3.10 and 3.11 respectively. As can be seen from Figures 3.9, 3.10 and 3.11, \( G' \) and \( G'' \) both increase as the concentration of PAA increases.

![Figure 3.9](image-url)

**Figure 3.9:** Storage and loss moduli versus angular frequency for the shear-thinning (PAA-W/GLY) solutions at 20 °C. Solid lines show the best fit of the oscillatory-shear data to a multimode Maxwell model [137].
Angular frequency, $\omega$ (rad/s) vs. Storage, $G'$ and loss, $G''$ moduli (Pa) for various solutions:

(a) 80-W/SUC solution
(b) 120-W/SUC solution
(c) 500-W/SUC solution

**Figure 3.10:** Storage and loss moduli versus angular frequency for the Boger (PAA-W/SUC) solutions at 20 °C. Solid lines show the best fit of the oscillatory-shear data to a multimode Maxwell model [137].

**Figure 3.11:** Storage and loss moduli versus angular frequency for the 100-W/70GLY solution at 20 °C. Solid lines show the best fit of the oscillatory-shear data to a multimode Maxwell model [137].
3.3.4 Maxwell’s model fits

The earliest model of a linear viscoelastic fluid was reported by Maxwell in 1867 [137]. The Maxwell model is commonly depicted as a series combination of an elastic behaviour (Hookean spring of rigidity) and a viscous behaviour (Newtonian dashpot of viscosity) [16, 20, 34]. Maxwell’s model assumes an elastic solid in linear series with a viscous liquid to obtain a simple viscoelastic composite fluid see Figure 3.12. Thus, fluid behaviour with both, viscosity and elasticity can be mathematically described using the Maxwell relation as [16, 20, 34, 149]:

\[ \tau + \lambda \frac{\partial \tau}{\partial t} = \eta_0 \dot{\gamma} \]  

(3.9)

where \( \tau \) is the extra stress tensor, \( \lambda \) is the fluid relaxation time, \( t \) is time, \( \eta_0 \) is the low-shear viscosity and \( \dot{\gamma} \) is the rate of strain tensor. Molecules (coils) of polymer suspended in Newtonian solvent add elasticity to the viscous solution when polymer molecules are stretched. The viscoelastic behaviour of such viscoelastic fluids at low shear frequency often follows the Maxwell model where variations of the storage modulus and the loss modulus as a function of oscillatory frequency. The formations of the storage and the loss moduli (Eq. 3.7 and Eq. 3.8) can be simply described to the generalised Maxwell model as:

\[ G' = \sum_{i}^{N} \frac{\eta_i \lambda_i \omega^2}{1 + (\lambda_i \omega)^2} \]  

(3.10)

\[ G'' = \sum_{i}^{N} \frac{\eta_i \omega}{1 + (\lambda_i \omega)^2} \]  

(3.11)

where \( \lambda \) and \( \eta \) are the multimode Maxwell model parameters, which are obtained by fitting the experimental data of SAOS measurements. The parameter \( \lambda_i \) represents the relaxation time modes of the viscoelastic fluids and \( \eta_i \) refers to the viscosity modes of the viscoelastic fluid. The multimode Maxwell model has more
than one adjustable nonlinear parameter and can accurately describe the experimental data in SAOS. The storage modulus ($G'$) and the loss modulus ($G''$) in this state are related to the relaxation times and viscosities of the individual Maxwell elements. The form of the fitting curves for the storage and loss moduli is useful to deduce quantitative information about the relaxation times in the viscoelastic fluids. By trial and error with initial values of $\lambda_i$ and $\eta_i$ can provide a reasonable fit at low frequencies. Relaxation times are added and $G'$ and $G''$ moduli estimated and modified until the whole curve is fit. Approximately any set of $G'$ and $G''$ moduli data can be properly fit with an appropriate number of relaxation times selected to reduce the residuals to acceptable limits. A refined fit can be obtained using a nonlinear fitting programme. Microsoft Solver in Excel was employed to implement a nonlinear regression of the SAOS data in this study. In order to minimise the variance between the measured and modelled values of the storage and the loss moduli when using a nonlinear regression to fit data a simple sum of squared differences between the model and the experimental data was applied as an objective function as given below [149]:

$$\sum_{i=1}^{N} \left\{ \left(1 - \frac{G'_{\text{Mod.}}}{G'_{\text{Exp.}}} \right)^2 + \left(1 - \frac{G''_{\text{Mod.}}}{G''_{\text{Exp.}}} \right)^2 \right\}$$

(3.12)

where $N$ is the number of fitted experimental points and the subscripts $\text{Mod.}$ and $\text{Exp.}$ refer to the predicted and measured values of the $G'$ and $G''$, respectively. The experimental data of SAOS measurements for all working solutions at 20°C are fitted within a multimode Maxwell model fit shown as solid lines in Figures 3.9, 3.10, and 3.11. The model curves of $G'$ and $G''$ moduli are simultaneously fitted to a series of Maxwell elements (three modes). Also, a fit was deemed sufficient if each residual was less than $10^{-6}$. Moreover, there are many other linear viscoelastic models, e.g. Jeffreys model [100], have been reported. These linear viscoelastic models were formed from different combination of linear viscous and
elastic elements and their general features do not differ from those of the Maxwell model, even if more complicated expressions are derived for the relaxation modulus 16, 20, 34, 120.

Figure 3.12: A model of polymeric fluids is the Maxwell model that assumes an elastic solid (Hookean spring) in linear series with a viscous liquid (dashpot) to obtain a simple viscoelastic composite fluid 24.

3.4 Determining the critical overlap concentration

The definition of critical overlap concentration, \( c^* \), which corresponds to the approximate concentration when the polymer coils in solution begin to overlap with each other 119, 213, is usually used to classify the polymeric viscoelastic solutions as either dilute, semi-dilute or concentrated. The polymer molecules are individually well separated from one another in the solvent and are independently free to move in the dilute regime. When shear is applied, the rheological property of the solution is deemed to be the sum of the individual contributions of the molecules. Beyond the critical concentration, \( c^* \), the independent rotational and translational motions of the polymer molecules are restricted due to entanglement
formation with other molecules. Therefore, the polymeric viscoelastic solutions with concentrations below $c^*$ are said to be dilute (the molecules of the polymer are spaced so far apart that they do not interact with each other) whereas in the semi-dilute range the polymer concentration in the solution is larger than $c^*$ meaning the molecules of polymer in the solution overlap. In fact, to be classified as truly dilute a more stringent convention that $c << c^*$ is sometimes used [38].

The critical overlap concentration of viscoelastic solutions can be estimated by determining the zero-shear viscosity, $\eta_o$, using the Carreau-Yasuda model fit (as explained earlier in Section 3.3.2) for various concentrations of the PAA polymer used in the current work [178]. The zero-shear viscosity is then plotted in log-log form versus a wide range of PAA concentrations. Generally, two power-law ranges become apparent from the plotted data between the zero-shear viscosity ($\eta_o$) and the PAA concentration ($c$). These two ranges of the power-law are referred to the dilute range and the semi-dilute/concentrated range. The intersection point between these two ranges is known as the critical overlap concentration ($c^*$) [178][119]. Therefore, the viscoelastic solutions can be characterised as dilute solutions ($c << c^*$) or semi-dilute solutions ($c \simeq c^*$) depending on the value of $c^*$.

These power-law correlations for the dilute regime and semi-dilute regime can be described by fitting the experimental data as:

$$\eta_o \propto c^k$$

(3.13)

Basically, the scope of the exponent value of $k$ is comparatively narrow in the dilute region. While, the exponent value of $k$ is relatively wide in the semi-dilute solutions [119]. The strong dependence of the zero-shear viscosity on the concentration of polymer is well justified via the dominant role played by the intermolecular interactions with neighboring molecules in both regimes. These interactions with neighboring molecules proliferate as the polymer concentration increases [17][74][119].
Solvents that are used to produce the viscoelastic solutions can be divided into two main types depending on the solvent quality: poor and good solvents [68]. In poor solvents, polymer molecules tend to stick to polymer segments of neighbouring molecules because they have a relatively low affinity with the solvent whilst, the interpolymer interaction in the good solvents is directly prevented because the contacts between the polymer molecules and the solvent are perfect. Thus, there is a strong dependence of the zero-shear viscosity on concentration by the intermolecular interactions with neighbouring molecules in the polymeric solutions. On the other hand, as the performance of hydrodynamic and heat transfer is dependent on the viscous and elastic nature of the polymeric viscoelastic solution, the solvent effect on the rheological properties of a viscoelastic fluid is significantly important. Following the earlier investigation by Cho et al. [36] who measured the steady-shear viscosity of a 1000 ppm PAA (Separan AP-273) aqueous solution using several solvents: distilled water, tap water, tap water with acid (4% NaCl), and tap water with salt (100 ppm NaOH). The addition of 100 ppm NaOH to the tap water results in a 100% increase of the viscosity in the low-shear-rate range. In contrast, the addition of 4% NaCl to the tap water decreases the shear viscosity of the PAA solution over the whole range of shear rate by a factor of 4 to 25 depending on the shear rate. It is noticeable that similar observations were conducted with aqueous solutions of polyethylene oxide (PEO). From the above findings together with those of other researchers who utilised distilled water as a solvent [35, 131], the rheological properties of polymeric solutions may be modified by changing the chemistry of the solvent.

Fourteen samples of the PAA-W/GLY solution (with concentrations from 15 ppm to 3000 ppm (w/w)) have been prepared by adopting the same preparation methodology previously described in Section 3.2.3 to determine the critical overlap concentration. Also, eleven concentrations from 25 ppm to 1500 ppm (w/w) of the PAA-W/SUC solution were used to cover both dilute and semi-dilute solutions
and to specify the critical overlap concentration. The experimental shear viscosity data for the PAA-W/GLY solutions and for the PAA-W/SUC solutions against shear rate is presented in Figure 3.13 and Figure 3.14, respectively. This experimental data is well represented using the Carreau-Yasuda model that can describe the whole shear-thinning behaviour of the viscoelastic fluids \[228\]. The values of the fitting parameters in the Carreau-Yasuda model for both PAA-W/GLY and PAA-W/SUC solutions are listed in Table 3.5 and Table 3.6, respectively, and the corresponding shear-flow curves are also plotted as solid lines in Figures 3.13 and 3.14. Figures 3.15 and 3.16 exhibit the variation in zero-shear-rate viscosity versus polymer concentration. As can be seen from Figures 3.15 and 3.16, two obvious power-law ranges are indicated to the dilute range and the semi-dilute range. The critical overlap concentration (the approximate turning point from the diluted range to the semi-diluted range) for the PAA-W/GLY solutions over a range of fourteen concentrations is approximately 325 ppm at 20°C where this value agrees well with values available in previous publications \[12\]. Whilst for the PAA-W/SUC solutions, over a range of eleven concentrations, \(c^*\) is around 450 ppm at 20°C (see Figure 3.16).

The critical overlap concentration \( (c^*) \) for the 100 ppm PAA in a 70%-aqueous glycerine solution was estimated through determination of the intrinsic viscosity \([\eta]\) using the Mark-Houwink scaling relation \[185\]:

\[
[\eta] = K_{M-H} M_w^m
\]  

(3.14)

where \(M_w\) is the molecular weight of polymer \((1.8 \times 10^7 \text{ g/mol})\) and \(K_{M-H}\) and \(m\) are the coefficients of the Mark-Houwink formula. These Mark-Houwink coefficients listed in Table 3.7 were estimated by Izyumnikov et al. \[98\] and Klein and Conrad \[114\] from the experimental data for polyacrylamide in water at 25°C \[153\]. The values of the Mark-Houwink coefficients for polyacrylamide in water were adopted because of the lack of available experimental data regarding
Table 3.5: Parameters of Carreau-Yasuda model fits for the PAA-W/GLY solutions (shear viscosity measured at 20°C).

<table>
<thead>
<tr>
<th>ppm</th>
<th>(c/w/w)</th>
<th>(\eta_0)</th>
<th>(\eta_\infty)</th>
<th>(\lambda_{CY})</th>
<th>(a)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.208</td>
<td>0.1858</td>
<td>8.9852</td>
<td>2.2374</td>
<td>0.4139</td>
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<td>25</td>
<td>0.7788</td>
<td>0.1971</td>
<td>7.6331</td>
<td>0.8330</td>
<td>0.4222</td>
<td></td>
</tr>
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<td>50</td>
<td>1.1452</td>
<td>0.2201</td>
<td>19.517</td>
<td>1.3717</td>
<td>0.5015</td>
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<td>75</td>
<td>1.4957</td>
<td>0.2438</td>
<td>22.574</td>
<td>2.4903</td>
<td>0.4411</td>
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</tr>
<tr>
<td>100</td>
<td>2.2213</td>
<td>0.2537</td>
<td>22.122</td>
<td>1.7633</td>
<td>0.5952</td>
<td></td>
</tr>
<tr>
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<td>25.399</td>
<td>1.2054</td>
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<td>0.3414</td>
<td>5.9527</td>
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<td>901.14</td>
<td>3.4906</td>
<td>0.6680</td>
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Table 3.6: Parameters of Carreau-Yasuda model fits for the PAA-W/SUC solutions (shear viscosity measured at 20°C).

<table>
<thead>
<tr>
<th>ppm</th>
<th>(c/w/w)</th>
<th>(\eta_0)</th>
<th>(\eta_\infty)</th>
<th>(\lambda_{CY})</th>
<th>(a)</th>
<th>(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solvent</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.160</td>
<td>0.1575</td>
<td>0.4378</td>
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</tr>
<tr>
<td>100</td>
<td>0.3150</td>
<td>0.2070</td>
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<td>0.5794</td>
<td>0.5456</td>
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<td>0.3696</td>
<td>0.2617</td>
<td>0.4901</td>
<td>0.4341</td>
<td>0.6451</td>
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</tr>
<tr>
<td>150</td>
<td>0.4155</td>
<td>0.2806</td>
<td>0.6938</td>
<td>0.6959</td>
<td>0.8171</td>
<td></td>
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<tr>
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</tr>
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<td>0.4903</td>
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<tr>
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</tr>
<tr>
<td>1500</td>
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<td>1.5578</td>
<td>0.5759</td>
<td>0.8296</td>
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</tbody>
</table>

The glycerine content effect on the coefficients in the Mark-Houwink correlation for polyacrylamide. The critical overlap concentration can be evaluated from a knowledge of the intrinsic viscosity as \[20, 214\]:

\[
c^* \sim \frac{1}{[\eta]} \quad (3.15)
\]
The value of critical overlap concentration for the PAA in a 70%-aqueous glycerine solution (100-W/70GLY) is approximately 320 ppm. Given this estimation of the critical overlap concentration ($c^*$), the 100-W/70GLY solution is considered a dilute polymer solution. In addition, concentrations of working fluids for both PAA-W/GLY solutions and PAA-W/SUC solutions can be regarded as dilute solutions with the exception of 500-W/SUC, which can be considered as a semi-dilute solution ($c/c^* \sim 1.1$). The non-dimensional concentration, $c/c^*$, for all working fluids is provided in Table 3.4.

Table 3.7: Coefficients of Mark-Houwink correlation for polyacrylamide in water at 25°C.

<table>
<thead>
<tr>
<th>References</th>
<th>$M_w \times 10^6$ (g/mole)</th>
<th>$K_{M-H}$ (ml/g)</th>
<th>$m$</th>
<th>$[\eta]$ (ml/g)</th>
<th>$c^*$ (ppm)</th>
</tr>
</thead>
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<tr>
<td>Izyumnikov et al. [98]</td>
<td>0.43-10</td>
<td>0.00742</td>
<td>0.775</td>
<td>3113.4</td>
<td>321</td>
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<tr>
<td>Klein and Conrad [114]</td>
<td>0.5-6</td>
<td>0.00490</td>
<td>0.800</td>
<td>3121.8</td>
<td>320</td>
</tr>
</tbody>
</table>

Figure 3.13: Variation of shear viscosity with shear rate for various PAA concentrations in W/GLY solvent at 20 °C. Solid lines are CarreauYasuda fits with parameters shown in Table 3.5.
### Chapter 3. Working fluid preparation and rheological characteristics

#### Table 3.6: Coefficients of Carreau-Yasuda model

<table>
<thead>
<tr>
<th>Polymer concentration, $c$ (w/w ppm)</th>
<th>Zero-shear viscosity, $\eta_0$ (Pa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 ppm PAA</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>50 ppm PAA</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>80 ppm PAA</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>100 ppm PAA</td>
<td>$10^{0}$</td>
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<tr>
<td>120 ppm PAA</td>
<td>$10^{1}$</td>
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</tr>
<tr>
<td>1000 ppm PAA</td>
<td>$10^{6}$</td>
</tr>
<tr>
<td>1500 ppm PAA</td>
<td>$10^{7}$</td>
</tr>
</tbody>
</table>

#### Figure 3.14: Variation of shear viscosity with shear rate for various PAA concentrations in W/SUC solvent at 20°C. Solid lines are Carreau-Yasuda fits with parameters shown in Table 3.6.

#### Figure 3.15: Extrapolated zero-shear viscosity versus PAA concentration for PAA-W/GLY solutions at 20°C.
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Figure 3.16: Extrapolated zero-shear viscosity versus PAA concentration for PAA-W/SUC solutions at 20°C.

3.5 Relaxation time measurements

Relaxation time, $\lambda$, which is a crucial parameter for describing viscoelastic fluid behaviour, indicates the characteristic time required for the polymer coils to relax from a deformed state to their equilibrium configuration. A simple expression for the shear relaxation time from rheological data of the small-amplitude oscillatory-shear test is [16, 20]:

$$\lambda = \frac{G'}{G'' \omega}$$  \hspace{1cm} (3.16)

where $\lambda$ is the shear relaxation time and $\omega$ is the angular frequency of oscillation. $G'$ and $G''$ are the storage modulus and the loss modulus, respectively. It can also be concluded from Eqs. 3.7 and 3.8 that the relaxation time of the viscoelastic fluid can be obtained from the inverse of the angular frequency where the storage modulus and the loss modulus crossover in the flow transition regime [120].
Figures 3.17, 3.18 and 3.19 show the frequency/shear-rate dependent polymer relaxation time from the measurements of the small amplitude oscillatory-shear tests for PAA-W/GLY solutions, PAA-W/SUC solutions and 100-W/70GLY solution, respectively, at 20°C. The values of the longest relaxation time, \( \lambda_o \), which are estimated in the limit of the angular velocity approaching to zero (\( \lambda_o = \lim_{\omega \to 0} \left( \frac{G'}{G''} \right) \)), are summarised in Table 3.8 for all working fluids. The slight concentration dependence of the relaxation time suggests that the highest concentration solutions (e.g. 500-W/SUC) are semi-dilute in agreement with the critical overlap concentration estimate in Section 3.4. Moreover, another procedure has been carried out to estimate the mode-averaged longest relaxation time by fitting the experimental data of the storage and loss moduli to a multimode Maxwell model fit using the method explained in Section 3.3.4 to give \[ \bar{\lambda}_{Maxwell} = \frac{\sum_{i=1}^{N} \lambda_i \eta_i}{\sum_{i=1}^{N} \eta_i} \] (3.17)

where \( \lambda_i \) is the relaxation time of the viscoelastic fluid (the relaxation modes) and \( \eta_i \) is the viscosity of the viscoelastic fluid (the viscous modes) where three modes were typically required for a satisfactory fit. The estimated values of the longest relaxation time for the selected viscoelastic solutions are also listed in Table 3.8 and it can be seen that they are very similar to \( \lambda_o \).

Within this framework, a shear-rate dependent relaxation time from the first normal-stress difference \( (N_1) \) data was evaluated by taking \[ \lambda = \frac{N_1}{2\eta^2} \] (3.18)

Figure 3.20 and Figure 3.21 show the shear-rate dependent relaxation time from the data of the first normal-stress difference for PAA-W/GLY and PAA-W/SUC solutions, respectively, at 20°C. Generally, estimating a relaxation time from the first normal-stress difference \( (N_1) \) data over the limited shear-rate range available
produces data broadly in agreement with the values estimated over the same frequency range in the small-amplitude oscillatory-shear (see Figures 3.17 and 3.18). It was not possible to quantify the limit of relaxation time in the limit of the shear rate tending to zero \( \lambda = \frac{1}{2} \lim_{\dot{\gamma} \to 0} \left( \frac{N_1}{\eta^2} \right) \) because the measured values of the first normal-stress difference are very close to the resolution (too close to the limits to be reliable) of the AR1000N controlled-stress rheometer.

Another alternative, the relaxation time of a dilute polymeric solution can also be evaluated according to Zimm's theory as [20, 120, 214]:

\[
\lambda_{\text{Zimm}} = F \frac{[\eta] M_w \eta_s}{N_A k_B T} \tag{3.19}
\]

where \( N_A \) is the Avogadro's constant, \( k_B \) is the Boltzmann’s constant, \( T \) is the absolute temperature, \( M_w \) is the molecular weight of polymer, \( \eta_s \) is the solvent viscosity and \([\eta]\) the intrinsic viscosity which can be estimated from Eq. 3.15 to be 3077 and 2222 \( \text{ml/g} \) for PAA-W/GLY and PAA-W/SUC, respectively, using the experimentally-determined values of critical overlap concentration. The prefactor \( F \) is defined by the Riemann Zeta relationship to be [179, 214]:

\[
F = \sum_{i=1}^{\infty} \left( \frac{1}{i^3} \right) \tag{3.20}
\]

where \( \nu \) is the solvent quality parameter, which has the limiting values ranging from 0.5 to 0.6 depending on solvent quality [53]. The solvent quality parameter can be obtained for a good solvent from the exponent \( (m) \) in the Mark-Houwink correlation for the intrinsic viscosity of the PAA solutions scaling as [98, 114]:

\[
[\eta] = K_{M-H} M_w^{(3\nu - 1)} \tag{3.21}
\]

where \( M_w \) is the molecular weight of polymer (PAA), \( K_{M-H} \) is the coefficient of the Mark-Houwink formula (see its values in Table 3.7) and here the term \((3\nu - 1)\)
is equal to the exponent \((m)\) in the Mark-Houwink correlation as listed in Table 3.7. Therefore, the solvent quality parameter value is \(\sim 0.6\). Thus, the front factor \((F)\) is approximately equal to 0.5313 where this precise value can be determined from a detailed eigenvalue calculation \[160\]. Therefore, the estimated relaxation times for PAA-W/GLY and PAA-W/SUC are \(2.5 s\) and \(1.4 s\), respectively. The theoretical Zimm relaxation times obtained can be compared to those determined from the \(SAOS\) measured in the cone-and-plate rotational rheometer (see Table 3.8), showing good agreement.

The shear relaxation time obtained from the small-amplitude oscillatory-shear data will be used in all following discussion and calculations of Weissenberg number to avoid any confusion. The justification for this use is because the flow is shear-dominated in the serpentine microchannel. Therefore, the use of a relaxation time based on shear is the most appropriate, ideally a shear-rate dependent relaxation time from first normal-stress difference \((N_1)\) is probably the most appropriate but the data is too limited in extent and reliability.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Working solutions} & \lambda_o & \lambda_{Maxwell} & \lambda_{Zimm} \\
& s & s & s \\
\hline
\text{Shear-thinning solutions} \\
50-W/GLY & 1.25 & 1.22 & 2.5 \\
100-W/GLY & 2.11 & 2.24 & 2.5 \\
200-W/GLY & 5.64 & 5.49 & 2.5 \\
100-W/70GLY & 0.675 & 0.436 & 0.277 \\
\hline
\text{Boger solutions} \\
80-W/SUC & 1.12 & 0.87 & 1.4 \\
120-W/SUC & 1.71 & 1.63 & 1.4 \\
500-W/SUC & 4.03 & 3.87 & 1.4 \\
\hline
\end{array}
\]

Table 3.8: Relaxation time data for all viscoelastic solutions used in the current work at 20°C.
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Figure 3.17: Relaxation time obtained from small-amplitude oscillatory-shear stress measurements for PAA-W/GLY solutions at 20 °C.

Figure 3.18: Relaxation time obtained from small-amplitude oscillatory-shear stress measurements for PAA-W/SUC solutions at 20 °C.
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Figure 3.19: Relaxation time obtained from small-amplitude oscillatory-shear stress measurements for 100-W/70GLY solution at 20 °C.

Figure 3.20: Relaxation time obtained from first normal-stress difference data ($\lambda = \frac{N_1}{\eta^2}$) for PAA-W/GLY solutions at 20 °C.
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3.6 Effect of temperature on rheological properties

In this section, the rheological properties (e.g., viscosity and relaxation time) of the viscoelastic solutions and solvents were quantified over a range of different temperatures in order to determine the effect of temperature change on these rheological properties. Figures 3.22, 3.23 and 3.24 illustrate the effect of temperature change on the shear viscosity of PAA-W/GLY, PAA-W/SUC and 100-W/70GLY solutions versus shear rate, respectively. It can be observed that the shear viscosity decreases as the temperature rises from 20°C (293K) to 30°C (303K) for PAA-W/GLY and PAA-W/SUC solutions and from 20°C (293K) to 44°C (317K) for 100-W/70GLY solution. All viscoelastic solutions reveal a decrease in the zero-shear viscosity with increasing temperature as seen in Figures 3.22, 3.23 and 3.24.
A large variation is found in the values of shear viscosity at lower shear rates whereas a slight difference is observed for the shear viscosity values at higher shear rates. This demonstrates that the effect of temperature is noticeable at lower shear rates when the structure starts to distort.

**Figure 3.22:** The variation of shear viscosity for the shear-thinning (PAA-W/GLY) solutions versus shear rate at different temperatures. Solid lines are Carreau-Yasuda fits [228].
Figure 3.23: The variation of shear viscosity for the Boger (PAA-W/SUC) solutions versus shear rate at different temperatures. Solid lines are Carreau-Yasuda fits [228].

Figure 3.24: The variation of shear viscosity for the 100-W/70GLY solution versus shear rate at different temperatures. Solid lines are Carreau-Yasuda fits [228].
The curves of shear viscosity against shear rate possess similar behaviour and shape at the different temperatures. This similarity can be exploited by combining the shear viscosity data taken at different temperatures into one master curve using the “method of reduced variables” \cite{20}. The time-temperature superposition principle has been applied to obtain temperature independent master curves of shear viscosity as a function of shear rate \cite{64}. Where, time-temperature superposition works for dilute solutions, as long as there are no large temperature-dependent changes in the solvent quality \cite{7}. The shear viscosity data obtained at a set of temperatures is horizontally and vertically shifted until the results superimpose on that of a given reference temperature, $T_o$. The degree of shift required is defined as the shift factor ($a_T$), which can be estimated from zero-shear viscosities as \cite{20}:

$$a_T = \frac{\eta_o(T)T_o\rho_o}{\eta_o(T_o)T\rho}$$  \hspace{1cm} (3.22)

where $\eta_o(T)$ is the zero-shear viscosity at a test temperature $T$, $\eta_o(T_o)$ is the zero-shear viscosity at the reference temperature $T_o$, $\rho$ and $\rho_o$ are the density at a test temperature and the reference temperature, respectively. The temperature dependence of density ($T_o\rho_o/T\rho$) in Eq. 3.22 is approximately unity and changes very little over the relatively narrow range of temperature covered (20°C - 30°C) and (20°C - 44°C), therefore this dependence of density was neglected in this data treatment \cite{20,120}. The reduced variables procedure predicts that the master curve of shear viscosity as a function of shear rate is constructed by plotting the reduced shear viscosity, $\eta_r$, versus reduced shear rate, $\dot{\gamma}_r$, which are defined by,

$$\eta_r = \frac{\eta(\dot{\gamma}, T)}{a_T}$$  \hspace{1cm} (3.23)

$$\dot{\gamma}_r = a_T\dot{\gamma}$$  \hspace{1cm} (3.24)

The reduced shear viscosity ($\eta_r$) versus the reduced shear rate ($\dot{\gamma}_r$), which were performed at different temperatures, are presented in Figures 3.25, 3.26 and 3.27.
for PAA-W/GLY, PAA-W/SUC and 100-W/70GLY solutions, respectively, at a reference temperature of $T_o = 293K$. The values of the shift factor are also tabulated in Table 3.9 for PAA-W/GLY and PAA-W/SUC and for solution 100-W/70GLY in Table 3.10. Since the shift factor is lower than unity ($a_T < 1$) at temperatures above the reference temperature ($T > T_o$) and greater than unity ($a_T > 1$) at temperatures below the reference temperature ($T < T_o$), the master curve typically covers a much wider range of effective shear rates than are covered by experiments at any single temperature. Therefore, the time-temperature shifting is exceptionally useful in practical applications because it allows predicting the fluid response for time scale either longer or shorter than can measure by conventional rheometer. The Carreau-Yasuda model [228] was also adopted to fit the data of the master curve (reduced shear viscosity ($\eta_r$) against reduced shear rate ($\dot{\gamma}_r$)) for all working solutions shown as black solid lines in Figures 3.25, 3.26 and 3.27. The temperature dependence of shift factor ($a_T$) was appropriately described using Arrhenius relationship, which is commonly expressed as [20]:

$$a_T = \text{Exp}\left(\frac{\Delta H}{R} \left( \frac{1}{T} - \frac{1}{T_o} \right) \right)$$

(3.25)

where $\Delta H$ is known as the “activation energy for flow” [64], $R$ is the universal gas constant (8.314 J/mol.K) and $T_o$ is the reference temperature. The shift factors ($a_T$) for each viscoelastic solution are plotted in a semi-log plot as functions of the reciprocal temperature function ($T^{-1} - T_o^{-1}$) with a reference temperature ($T_o = 293K$) as shown in Figure 3.28. The shift factor for each of the three solvents (W/GLY, W/SUC and W/70GLY) is also estimated by measuring the viscosity across the temperature range used in the experiments (293 K - 303 K) for the W/GLY and W/SUC solvents and (293 K - 317 K) for the W/70GLY solvent as listed in Table 3.9 and Table 3.10 and fitted to an exponential Arrhenius-type fit (see Figure 3.28). The Arrhenius equation (Eq. 3.25) predicts a linear relationship that accurately characterises the thermorheological behaviour of a wide variety of
viscoelastic solutions close to the reference temperature $T_o$. The slope of this line determines the value ($\Delta H/R$), which is also known as the “thermal sensitivity” [20]. The values of the thermal sensitivity ($\Delta H/R$) for each viscoelastic solution are given in Table 3.11. The experimental findings in terms of the thermal sensitivity reveal that the glycerine-based viscoelastic solutions have a stronger temperature-dependence than the sucrose-based solutions (see Table 3.11).

Table 3.9: Values of the shift factor ($a_T$) used to describe temperature dependence for the shear-thinning (PAA-W/GLY) solutions and the Boger (PAA-W/SUC) solutions with a reference temperature $T_o = 20^\circ C$.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>Shift factor ($a_T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20°C</td>
</tr>
<tr>
<td>Newtonian and shear-thinning solutions</td>
<td></td>
</tr>
<tr>
<td>W/GLY</td>
<td>1</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>1</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>1</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>1</td>
</tr>
<tr>
<td>Newtonian and Boger solutions</td>
<td></td>
</tr>
<tr>
<td>W/SUC</td>
<td>1</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>1</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>1</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.10: Values of the shift factor ($a_T$) used to describe temperature dependence for the solvent (W/70GLY) and the 100-W/70GLY solution with a reference temperature $T_o = 20^\circ C$.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>Shift factor ($a_T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20°C</td>
</tr>
<tr>
<td>W/70GLY</td>
<td>1</td>
</tr>
<tr>
<td>100-W/70GLY</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 3.11: Flow activation energy for all viscoelastic solutions at $T_0 = 20^\circ C$ obtained from experimental measurements of shear viscosity.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>$\Delta H/R$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAA-W/GLY</td>
<td>5290</td>
</tr>
<tr>
<td>PAA-W/SUC</td>
<td>3140</td>
</tr>
<tr>
<td>100-W/70GLY</td>
<td>4070</td>
</tr>
</tbody>
</table>

Figure 3.25: Master curves for the shear viscosity as a function of shear rate for the shear-thinning (PAA-W/GLY) solutions. Data taken at the indicated temperatures were shifted to a reference temperature (20°C).
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**Reduced shear rate, \( \gamma_r \) (1/s)**

**Reduced shear viscosity, \( \eta_r \) (Pa.s)**

10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}

0.2 0.4 0.6 0.8 1

80-W/SUC at 20°C
80-W/SUC at 26°C
80-W/SUC at 28°C
80-W/SUC at 30°C

Carreau-Yasuda fits

Figure 3.26: Master curves for the shear viscosity as a function of shear rate for the Boger (PAA-W/SUC) solutions. Data taken at the indicated temperatures were shifted to a reference temperature (20°C).

10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}

100-W/70GLY at 20°C
100-W/70GLY at 24°C
100-W/70GLY at 28°C
100-W/70GLY at 32°C
100-W/70GLY at 36°C
100-W/70GLY at 40°C
100-W/70GLY at 44°C

Carreau-Yasuda fits

Figure 3.27: Master curves for the shear viscosity as a function of shear rate for the 100-W/70GLY solution. Data taken at the indicated temperatures were shifted to a reference temperature (20°C).
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The shear relaxation time obtained from the small-amplitude oscillatory-shear measurements was evaluated across the temperature range used in the experiments as illustrated in Figures 3.29, 3.30 and 3.31 for all selected working solutions. The relaxation time can be specified in terms of a time-temperature shift factor (time scale shift factor) at a reference temperature by [20, 120]:

\[ a_T = \frac{\lambda(T)}{\lambda(T_o)} \]  

\[ (3.26) \]
where $\lambda(T_o)$ is the relaxation time at a reference temperature $T_o$ (where the shift factor ($a_T$) is equal to unity at the reference temperature) and $\lambda(T)$ is the relaxation time at temperature $T$. Values of the time-temperature shift factor determined by Eq. 3.26 and the relaxation time are listed in Tables 3.12, 3.13 and 3.14 for PAA-W/GLY, PAA-W/SUC and 100-W/70GLY solutions, respectively. Arrhenius formula (Eq. 3.25) is employed to describe the time-temperature shift factor in terms of the activation energy ($\Delta H$). In order to estimate the thermal sensitivity for each of the working solutions the shift factor ($a_T$) is plotted versus the reciprocal of temperature where the slope of the curve represents the thermal sensitivity as depicted in Figure 3.32. The flow activation energies obtained from shear viscosity data and relaxation time data are almost identical as would be expected. However, the slight fluctuations of the activation energy values may be attributed to the accuracy of rheological measurements (see Tables 3.11 and 3.15). Additionally, the flow activation energies obtained from shear viscosity values at different temperatures are estimated with solvent values whilst they are evaluated from values of the relaxation time only for viscoelastic solutions.

**Table 3.12**: Relaxation time and shift factor values obtained from experimental measurements of SAOS for the PAA-W/GLY solutions with a reference temperature, $T_o = 20^\circ C$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Temperature ($^\circ C$)</th>
<th>20</th>
<th>24</th>
<th>26</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>50-W/GLY</strong></td>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>1.25</td>
<td>0.88</td>
<td>0.74</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>Shift factor, $a_T$</td>
<td>1.000</td>
<td>0.7072</td>
<td>0.5958</td>
<td>0.4971</td>
</tr>
<tr>
<td><strong>100-W/GLY</strong></td>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>2.11</td>
<td>1.61</td>
<td>1.57</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>Shift factor, $a_T$</td>
<td>1.000</td>
<td>0.7668</td>
<td>0.7473</td>
<td>0.6587</td>
</tr>
<tr>
<td><strong>200-W/GLY</strong></td>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>5.64</td>
<td>5.09</td>
<td>4.78</td>
<td>4.57</td>
</tr>
<tr>
<td></td>
<td>Shift factor, $a_T$</td>
<td>1.000</td>
<td>0.9021</td>
<td>0.8486</td>
<td>0.8101</td>
</tr>
</tbody>
</table>
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#### Table 3.13: Relaxation time and shift factor values obtained from experimental measurements of SAOS for the PAA-W/SUC solutions with a reference temperature, $T_o = 20^\circ C$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Temperature ($^\circ C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td><strong>80-W/SUC</strong></td>
<td></td>
</tr>
<tr>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>1.12</td>
</tr>
<tr>
<td>Shift factor, $a_T$</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>120-W/SUC</strong></td>
<td></td>
</tr>
<tr>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>1.71</td>
</tr>
<tr>
<td>Shift factor, $a_T$</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>500-W/SUC</strong></td>
<td></td>
</tr>
<tr>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>4.03</td>
</tr>
<tr>
<td>Shift factor, $a_T$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

#### Table 3.14: Relaxation time and shift factor values obtained from experimental measurements of SAOS for the 100-W/70GLY solution with a reference temperature, $T_o = 20^\circ C$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Temperature ($^\circ C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Relaxation time, $\lambda_o$ (s)</td>
<td>0.675</td>
</tr>
<tr>
<td>Shift factor, $a_T$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

#### Table 3.15: Flow activation energy for all viscoelastic solutions at $T_o = 20^\circ C$ obtained from experimental measurements of shear relaxation time.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>$\Delta H/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAA-W/GLY</td>
<td>4500</td>
</tr>
<tr>
<td>PAA-W/SUC</td>
<td>3200</td>
</tr>
<tr>
<td>100-W/70GLY</td>
<td>4700</td>
</tr>
</tbody>
</table>
Figure 3.29: The variation of relaxation time for the shear-thinning (PAA-W/GLY) solutions versus angular frequency at different temperatures.
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Figure 3.30: The variation of relaxation time for the Boger (PAA-W/SUC) solutions versus angular frequency at different temperatures.

Figure 3.31: The variation of relaxation time for the 100-W/70GLY solution versus angular frequency at different temperatures.
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3.7 Thermophysical property measurements

Thermophysical properties of selected fluids for both the Newtonian and viscoelastic solutions are presented in this section. The thermophysical properties such as, density, viscosity, the specific heat and thermal conductivity were measured in order to quantify the enhancement of convection heat transfer in the micro-scale.
geometries. Instruments and equipment used with the measurement procedure for each property are described in detail in the following subsections:

### 3.7.1 Density

A portable density meter (DMA 35 N, Anton Paar) was utilised for measuring the density \( (\rho) \) of the working fluids as shown in Figure 3.33. Its robust and lightweight design allows using on-site with a wide range of work environments for instance, food and beverage industry, pharmacy and chemistry and petrochemistry. This instrument has a quoted accuracy of \( \pm 0.001 g/cm^3 \) with the measuring range of density, temperature and viscosity of 0 to 3 g/cm\(^3\), 0 to 50 °C and 0 to 1000 mPa.s, respectively. The averaged data of density (at least ten trials for each sample) is tabulated in the Table 3.16 at 20°C for all working fluids. The density measurements for all working fluids reveal that the variation in density across the temperature range utilised in the experiments (20°C − 45°C) is less than 1%.

Figure 3.33: Photograph of density meter (DMA 35 N, Anton Paar).
3.7.2 Viscosity

As mentioned previously in Section 3.3, all viscosity measurements were conducted using a TA Instruments Rheolyst AR1000N controlled-stress rheometer connected with an acrylic cone-and-plate geometry (60 mm diameter, 2° cone angle) with a precision in viscosity of ±2%. The shear viscosity data for the various PAA concentrations are shown in Figures 3.4, 3.5, and 3.6 together with the corresponding Carreau-Yasuda model fits [228] at 20°C. The viscosity of Newtonian solutions (W/GLY, W/SUC and 70%-glycerine aqueous solution) is independent of the shear rate and remains a constant no matter the variation of shear rate. In contrast, the viscoelastic solutions exhibit shear-rate dependent viscosity. However, the PAA-W/GLY solutions possess significantly greater shear-thinning effects than the PAA-W/SUC solutions.

3.7.3 Specific heat capacity

Differential scanning calorimetry (DSC) - Model V24.11 DSC Q2000, TA Instruments, USA - was used to measure the specific heat capacity ($C$) by the modulated method to obtain a measurement of heat capacity for all Newtonian and viscoelastic solutions that utilised in the present research. The DSC was fully computer-controlled with rapid energy compensation and equipped with automatic data analysis software to calculate heat capacity from the heat flow data. Accuracy and reproducibility of heat capacity measurements on the DSC were first validated with the standard samples of sapphire, which are run under the same conditions that are subsequently used for all samples. Sapphire is considered an excellent reference material used for calibrating the DSC for the specific heat capacity measurements [211]. Figure 3.34 shows the experimental results of a sapphire sample (with sample mass 22.6 mg and heating rate 2°C/min.) in the range
from 742.1 to 874.2 \( J/kg^\circ C \) together with standard sapphire data \[52\] for temperatures from 10 to 83.3 \( ^\circ C \). Comparison with standard sapphire data illustrates that the accuracy of the experimental sapphire measurements was within \( \pm 0.5\% \).

Figures 3.35 and 3.36 display the experimental specific heat capacity results for both PAA-W/GLY and PAA-W/SUC solutions in the range of temperatures between 10 and 83.3 \( ^\circ C \). The experimental values of the specific heat capacity for all selected solutions at 20\( ^\circ C \) are also listed in Table 3.16. The data representation has been plotted as an average from at least three runs for each sample. The experimental results indicate that the specific heat capacities increase gradually with increasing temperature for all viscoelastic solutions. However, the specific heat capacities of PAA-W/SUC solutions are slightly greater than the values of PAA-W/GLY solutions over the entire temperature range. The data in Figures 3.35 and 3.36 show that there is a slight effect from the different polymer concentrations in the chosen viscoelastic solutions on the values of specific heat capacity but no clear trends are apparent and the differences are within the repeatability of the technique for identical fluids. Thus we conclude, for the fluids used here, the addition of polymers to a Newtonian solvent does not alter its specific heat capacity in agreement with previous studies in the literature \[34\].
**Figure 3.34:** The experimental data of a standard sapphire sample with standard sapphire data [52].

**Figure 3.35:** Typical variation of specific heat capacity for PAA-W/GLY solutions against temperature.
3.7.4 Thermal conductivity

The thermal conductivity \((k)\) of the samples W/GLY, 200-W/GLY, W/SUC and 500-W/SUC has been measured using the Fox-50 device (conductivity range from 0.1 to 10 \(W/m\circ C\)), which is a commercial instrument manufactured by LaserComp Thermal Conductivity Instrument. Measurements of the thermal conductivity are taken from the thermal contact resistance via the guarded heat flow meter technique. Pyrex was selected to calibrate the Fox-50 device because it is close to the expected value of the thermal conductivities. The specimen size is 63\(mm\) diameter and 25\(mm\) thick. The experiments were conducted with a temperature difference between the hot and cold plates at 10\(\circ C\) at two different mean temperatures (10 \(\circ C\) and 40 \(\circ C\)). The average values of the thermal conductivity (at least three runs for each sample) for the selected samples were then determined. The results suggest that the addition of small amounts of polymers has a negligible effect on
the values of the thermal conductivities as any small variation in the results is within the uncertainty and repeatability of the measurement (±10%). Therefore, the current results are consistent with the results of Lee et al. [125], who measured thermal conductivities of various viscoelastic fluids at four different temperatures (20°C - 50°C) using a conventional thermal conductivity cell. They demonstrated that the addition of polymer up to 10000 ppm (w/w) to Newtonian solvents does not change values of thermal conductivity for these resulting solutions. Therefore, it is possible to use the thermal conductivity values of aqueous glycerine solutions [15] for all PAA-W/GLY solutions and sucrose solution values [14, 220] for PAA-W/SUC solutions of a corresponding temperature (see data in Table 3.16).

Table 3.16: Thermal properties for all Newtonian and viscoelastic solutions that utilised in the current research.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( C ) (J/kg°C)</th>
<th>( k ) (W/m°C)</th>
<th>Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measured</strong></td>
<td><strong>Measured</strong></td>
<td><strong>Ref.</strong></td>
<td><strong>Ref.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Newtonian and shear-thinning solutions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W/GLY</td>
<td>1233</td>
<td>2477</td>
<td>0.305</td>
<td>0.302 [15]</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>1234</td>
<td>2492</td>
<td></td>
<td>1379</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>1236</td>
<td>2496</td>
<td></td>
<td>1829</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>1237</td>
<td>2454</td>
<td>0.285</td>
<td></td>
</tr>
<tr>
<td><strong>Newtonian and Boger solutions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W/SUC</td>
<td>1311</td>
<td>2606</td>
<td>2655 [14]</td>
<td>0.368</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>1315</td>
<td>2561</td>
<td></td>
<td>1086</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>1315</td>
<td>2538</td>
<td></td>
<td>1253</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>1318</td>
<td>2611</td>
<td>0.365</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4

Experimental arrangement and operational procedure

The main motivation of the practical experiments in the micro-scale geometries is to provide detailed quantitative information on the heat transfer enhancement in both purely-inertial and purely-elastic flow regimes. Therefore, this chapter is separated into two main parts. Section 4.1 contains the description of the experimental facility and test section including the serpentine microchannel geometry. While, Section 4.2 involves a description of the experimental setup and test section of the cross-slot geometry. Also, the operational procedures for the experimental data collection for both geometries are presented. The main non-dimensional parameters are provided in Section 4.3. This is followed by an estimation of the associated uncertainties in the experimental measurements in Section 4.4.
4.1 Serpentine microchannel geometry

4.1.1 Test rig arrangement

The experimental investigation of the flow and convection heat transfer characteristics for both Newtonian and viscoelastic fluids is performed using the experimental setup illustrated in Figure 4.1. This rig consists of a square cross-section wavy “serpentine” microchannel which is mounted on a plastic frame (PVC) that includes $20 \times 15 \times 20\text{mm}$ reservoirs at either end of the microchannel to install pressure tapings and place thermocouples as shown in Figure 4.2. The fluid is pumped through the serpentine microchannel using a pressure vessel where the amount of driven liquid flows between an upstream reservoir and a downstream reservoir and then to a collection container where it can be weighed to determine the flow rate. The capacity of the cylindrical pressure vessel ($115\text{mm}$ diameter and $175\text{mm}$ length), which is made of stainless steel, is $\sim 1$ liter. This pressure vessel is connected to a compressed air supply, which is used for controlling the working fluid discharged through the serpentine microchannel. Flexible plastic tubes (Tygon tube, outer diameter $6\text{mm}$) were used to connect the test section with the pressure vessel and also for discharging fluid out to the collection container. A
Denver Instrument TP-1502 precision balance, which has an uncertainty $\pm 2mg$ in the range of load from 0.01g to 1500g, was used to measure the flow rate by weighing the amount of collected fluid with time (every 60s, four times from 60s to 240s at each applied pressure). This open-flow system allows the experiments to be run with repeatable and well-controlled initial conditions, and easy collecting of extensive data. It is worth mentioning that syringe pumps were not used as the large pressure drops required were not attainable and the fluid pumping time was also lower than the chosen method. The volume of fluid required to reach steady-state condition was also more than the volume of available syringes.

![Figure 4.2: Isometric view of the experimental facility for the serpentine microchannel geometry.](image)

### 4.1.2 Serpentine microchannel test-section configuration

According to the comprehensive information provided in the literature review [77, 78, 208, 232], the curvilinear geometry is usually one of the necessary conditions that have to be provided for generating elastic instabilities in flow of viscoelastic
fluids. Also, potential applications of the serpentine microchannel in different fields such as bioengineering devices, microelectronic devices, cooling systems of computer chips, and mini or micro-heat exchangers have recently received a great deal of attention due to the development of fabrication technologies for these serpentine/ curved microchannels.

The serpentine microchannel was machined into a \((20\, \text{mm} \times 84\, \text{mm})\) piece of copper \((385\, \text{W/m.K})\) using Computer Numeric Control (CNC) machining and included 20 half-loops with inner and outer radii of 1 and 2 \(\text{mm}\), respectively, as shown in Figure 4.3. The serpentine microchannel has a negligible surface roughness of less than 5\(\mu\text{m}\) as confirmed by 3D vertical laser-scanning microscope (Enhanced Vertical Scanning Interferometry EVSI, Veeco Instruments Inc.). The serpentine geometry was smoothly joined to inlet and outlet straight channel sections on both sides to ensure fully-developed flow in the entrance of the serpentine section. The total length of the microchannel is \(77 \pm 0.1\, \text{mm}\) whilst the path-length along the centreline of the serpentine microchannel is equal to \(114.25 \pm 0.1\, \text{mm}\). The microchannel was enclosed by an \(12\, \text{mm}\)-thick upper wall fabricated from a flat and smooth piece of polyvinyl chloride PVC \((0.25\, \text{W/m.K})\) and assembled with screws on the microchannel top. The microchannel had a square cross-section with a depth and width of \(1.075 \pm 0.01\, \text{mm}\) with dimensionless radius \((R_i/W = 1)\) where \(R_i\) is the inner radius of curvature of the serpentine channel and \(W\) is the width of the channel: see details in Figure 4.4. The dimensions of the serpentine microchannel were measured using images captured on a Nikon EPIPHOT-TME (Nikon test and laboratory equipment) inverted microscope, which is connected to a sensitive camera (Infinity 2). These images are subsequently analysed using ImageJ software, which was utilised to convert the dimensions of the images from pixel to millimetre (in the \(100\times\) magnification setting, 1230 \text{pixels} = 1 \text{mm}) after magnification. Twenty images were captured in each direction of the channel.
width (surface plane), horizontal and vertical, along the serpentine microchannel (see samples of these images in Figure 4.5) in order to minimise any random measurement error. The copper side walls and the bottom wall of the serpentine channel are considered to be isothermal whereas the insulating properties of the PVC ensured an adiabatic boundary condition on the upper wall. The whole facil-

**Figure 4.3:** Plan views of the serpentine microchannel. (a) Photograph. (b) Schematic diagram.

**Figure 4.4:** Schematic diagrams of the serpentine microchannel. (a) Detailed view. (b) Cross-section (A-A).
ity was placed in a Techne TE-10A water bath continuously-stirred and maintained at a temperature of 30°C to achieve constant temperature boundary conditions to the copper walls of the serpentine microchannel.

![Sample images](image)

**Figure 4.5:** Samples of 100× microscopic images captured at different locations of horizontal and vertical directions of the channel width (surface plane) along the serpentine microchannel.

### 4.1.3 Pressure-drop measurements

The measurements of pressure-drop, $\Delta P$, along the microchannel are obtained via two pressure taps using a wet-wet Validyne DP15-26 differential pressure transducer (DP15, Validyne Engineering) as shown in Figure 4.6. These pressure taps were placed on the upper wall of an upstream reservoir and a downstream reservoir. A 2mm internal diameter stainless steel tube was used to fabricate the pressure taps connected with flexible Tygon tube (internal diameter 3mm). The pressure transducer utilised two different diaphragms (Model-3-34 (0-22 kPa) and Model-3-46 (0-350 kPa)) to cover the full working range of $0 < \Delta P < 200 \text{ kPa}$. The voltage output of each of the diaphragms was periodically calibrated for the differential pressure range of $0 < \Delta P < 20 \text{ kPa}$ and the other range of $0 < \Delta P < 200 \text{ kPa}$ using a high-accuracy differential pressure transducer, MKS Baratron by MKS Instruments Inc. USA, and both are stated to be accurate to ±0.25% full-scale by the manufacturer. The relation between applied pressure drop and the DC voltage
output of the transducer for each of the diaphragms is linear as shown in Figure 4.7. The voltage output for the pressure transducer, which was digitised using a Validyne CD223 digital transducer indicator (CD223, Validyne Engineering), was sampled by an analogue-to-digital converter (LabJack U12) at 100 Hz for 60 s after reaching steady-state conditions (approximately 30 min. per flowrate). In addition, the pressure transducer reading under the no-flow condition was recorded at the beginning and end of each measurement to account for any drift and subsequent readings were accordingly corrected with the zero flowrate readings. Measurement of the pressure in the reservoirs meant that issues related to hole-pressure error were minimised but resulted in any fluctuation information being severely damped: thus our pressure-drop data is restricted to mean values [16, 25, 218].
Chapter 4. Experimental arrangement and operational procedure

Voltage, \( V \) (volts)
Pressure, \( P \) (kPa)

Least Squares Fit

\[
P = 19.336 V + 0.2856
\]

\( R^2 = 0.9999 \)

(a) High differential pressure range \((0 < \Delta P < 200 \text{kPa})\)

(b) Low differential pressure range \((0 < \Delta P < 20 \text{kPa})\)

**Figure 4.7:** Calibration of the pressure transducer.
4.1.4 Temperature measurements

The inlet and outlet fluid temperatures were quantified using sheathed chromel-alumel (K-type) thermocouples (KMQSS, Omega) with a 0.5\text{mm} diameter probe positioned in the upstream reservoir and the downstream reservoir of the test section. Further, four K-type thermocouples were also embedded at axial locations along the side walls in both sides at a distance of 1\text{mm} from the wall of the serpentine microchannel to measure the wall temperature of the serpentine microchannel as shown in Figure 4.4 (b). The large thermal conductivity of the copper ensured that, although the “wall” temperature was actually measured 1\text{mm} away, the error associated with this assumption was very small. These thermocouples were connected to a computer via a data acquisition system (LabJack U6). The readings of all these thermocouples were electronically recorded over a period of 60\text{s} at a frequency of 100\text{Hz} once steady-state conditions had been obtained. The thermocouples, which had an accuracy of approximately ±1\text{°C}, were calibrated against a mercury thermometer of certified accuracy (±0.1\text{°C}) in the range of temperature from 0\text{°C} to 80\text{°C}. An accuracy of ±0.5\text{°C} was found in the temperature calibration experiments. The pre-calibrated values of these thermocouples are plotted against calibrator reference temperature (mercury thermometer readings) in Figure 4.8.
4.1.5 Experimental procedure

The experimental procedure employed to implement the practical experiments is described in detail in this section. The experimental setup has been designed and configured to investigate the behaviour of both Newtonian and viscoelastic fluids and convective heat transfer in the serpentine microchannel geometry. The experiments of convective heat transfer were performed with thermal boundary conditions such that the upper wall of the serpentine microchannel was insulated (adiabatic) and the side walls were maintained at constant temperature.

After preparing and measuring the rheological properties of the working fluids, which involve the Newtonian and viscoelastic fluids as already described in Chapter 3, the fluid was supplied to the pressure vessel and then pumped into the test section between the upstream reservoir and the downstream reservoir. Adequate time, which was 30 min. per flowrate, was required to obtain a steady-state
conditions before collecting any experimental data. The flowrate, which was varied by adjusting the pressure regulator, was manually measured by weighing the amount of collected fluid using an accurate digital balance with time (every 60s, four times from 60s to 240s). Meanwhile, the pressure-drop across the test section was monitored using the Validyne DP15-26 differential pressure transducer. To avoid any drift in the pressure measurements, the pressure reading at no-flow rate was taken when there were no residual air bubbles in the pressure transducer by bleeding fluid through purge valves (see Figure 4.6). Total measured pressure-drop across the test section during the experiment includes the pressure loss due to the flow in the connecting tubes, fitting and the inlet and exit losses. The pressure losses from the connecting tubes and fitting were directly measured at low and high flow rate when there was no test section with the conditions identical to the measuring case in order to avoid any effects of hydrostatic pressure. As for the losses of pressure-drop resulting from the inlet and exit are usually estimated from the traditional relationships used in micro-scale geometries [105, 111, 167, 201]. These minor pressure-drops were found to be less than 5% of the total measured pressure-drop. The inlet and exit temperatures of fluid flow were measured in the reservoirs of the test section using two pairs of temperature sensors (thermocouples). The surface temperature of the serpentine microchannel was also quantified using four K-type thermocouples. The temperature of the water bath, which was used to achieve a constant wall temperature for the serpentine microchannel, was set at a desired temperature (30°C) and the entire facility of the test section was then placed inside it. Temperature and pressure drop data of the flowing fluids were electronically measured and sampled using data acquisition systems. It is well known that the fluid properties vary with temperature through the longitudinal direction of the test section which can certainly affect the heat transfer rate. Thus, the thermophysical properties of the working fluids were evaluated at the mean film fluid temperature (the average fluid temperature between the inlet and exit of the microchannel) [97, 191]. Furthermore, for viscoelastic fluids, the
characteristic shear-viscosity was evaluated at a characteristic shear rate from the Carreau-Yasuda model fit \[228\] to the steady-shear viscosity measurements at the mean film temperature.

Various fluid flow and heat transfer data sets were collected corresponding to each separate setting of flowrate and temperature to observe the influence of flowrate on Darcy friction factor and the effect of flowrate on pressure-drop and Nusselt number.

4.2 Cross-slot micro-geometry

4.2.1 Test rig arrangement

A schematic diagram of the experimental facility, which has been used to investigate the characteristics of heat transfer and fluid flow in the cross-slot geometry, is shown in Figure 4.9. Flow-through the microfluidic cross-slot system was driven under controlled flow rate conditions using two identical pressure vessels, which are made of stainless steel with 115 mm diameter and 175 mm length. These pressure vessels are connected to a compressed air supply for driving the working fluids through the cross-slot micro-geometry. One of the pressure vessels is fitted with a temperature controller (ESM-4430, EMKO), which has an accuracy ±0.25% of full scale for the temperature sensor, for utilisation with an electrical heater for accurately controlling the temperature of the working fluid. While the second pressure vessel is maintained at room temperature. Fluid was pumped into one pair of opposing channels (inflow) and out of the second pair (outflow), and a stagnation point is formed at the geometrical centre of the cross-slot (when the flow remains symmetric). The next section of the flow loop consists of flowmeters and ball valves used to control and measure the flow rate. Data acquisition system is utilised to sample measurements of the pressure transducer and thermocouples.
4.2.2 Cross-slot micro-geometry configuration

The cross-slot test section was designed and configured to yield both high accuracy measurements and to monitor flow behaviour and temperature distribution. Thus, the cross-slot cell consists of two layers: a bottom plastic layer including the cross-slot structure, and a top borosilicate window for optical access with fluidic connections. The cross-slot cell is illustrated assembled and disassembled in Figure 4.10. The cross-slot micro-geometry was micro-machined into a piece of high density polyoxymethylene, also commercially known as ACETAL (density: 1.41 g/cm$^3$, melting point: 175 $^\circ$C), using CNC machining, which enables accurate three-dimensional shapes to be obtained. The special features of ACETAL are high stiffness, high thermal resistance, low friction and excellent dimensional stability [94].

The cross-slot geometry was enclosed by a 6.5mm-thick upper layer fabricated from borosilicate glass (Edmund Optics) to maintain an adiabatic boundary condition and to visualise flow structure and patterns of temperature distribution. The cross-slot device has a square cross-section microchannel width of $W = 519 \mu m$ ($\pm 10\mu m$), providing an aspect ratio ($depth/width$) = 1. Relevant dimensions
Dimensions of the cross-slot geometry were measured using a Nikon EPIPHOT-TME (Nikon test and laboratory equipment) inverted microscope equipped with a 100× objective lens. Measurements were made at different positions along each channel of the cross-slot geometry by pixel counting. Where, dimensions in pixels were calibrated using a standard dimension for 100× magnification provided by the microscope image acquisition software. The measurement accuracy of this procedure was found to be ±10µm (see samples of some measured images in Figure 4.12). Surface roughness of the cross-slot micro-geometry was measured using the same image processing approach by a Nikon EPIPHOT-TME inverted microscope (described above in Section 4.1.2). The roughness of the cross-slot surfaces was approximately evaluated to be less than the accuracy of the image processing.
technique. Consequently, the roughness influence of cross-slot surfaces on the fully-developed flow can be neglected [4]. The total length of the inlet and outlet channels is 20 mm ($40W$) so that the flow was fully-developed before entering the stagnation region for all flow conditions investigated [57, 173]. The $x$ and $y$ axes with the stagnation point taken as the origin of coordinates were defined as shown in Figure 4.11 and the $z$-axis normal to the page. Four cylindrical holes (with diameter and depth of 10 mm) at the end of each crossed channel of the cross-slot were drilled and tapped so that this allows thermocouples to be located and pressure tapings to be installed and also positioning the pipelines (flexible Tygon tube, outer diameter 6 mm) for the flowing fluid. The pressure taps (2 mm internal diameter) were installed in the bottom of the cylindrical holes in order to measure the pressure-drop between the inflow and outflow channels.

![Figure 4.11: Detailed dimensions for the cross-slot micro-geometry. Inset: a field image illustrating the two opposed inlets, the two opposed outlets and the place of stagnation point.](image)
Also, two thermocouples (0.5mm diameter) were placed in each cylindrical hole for quantifying fluid temperatures at the inlets and outlets of the cross-slot geometry (see Figure 4.10). These pressure taps and thermocouples were sealed by epoxy in order to prevent any leakages from the cross-slot micro-device. The two main sections of the cross-slot cell were held together by (6 mm) screws.

![Image](image1.png)

**Figure 4.12:** Samples of 100× microscopic images captured at different random positions (stagnation point and along each crossed channel) of the cross-slot geometry.

### 4.2.3 Pressure-drop measurements

A diaphragm type differential pressure transducer (DP15-26, Validyne Engineering) with ±0.25% of full scale accuracy was employed for measuring the pressure differential between the inlet (inflow) and outlet (outflow) of the cross-slot cell with flexible Tygon tube (internal diameter 3 mm). The diaphragm-sensing element
utilised for the pressure-drop measurement were available with different pressure
ranges using two different diaphragms, Model-3-34 (0-22 kPa) and Model-3-42
(0-140 kPa), to enable the full working range and provide high accuracy measure-
ments. The voltage output for the pressure transducer, which was digitised using a
Validyne CD223 digital transducer indicator (CD223, Validyne Engineering), was
sampled by an analogue-to-digital converter, ADC, (LabJack U12) at 100 Hz for
60s. The pressure transducer was calibrated at periodic intervals using air against
an MKS Baratron differential pressure transducer (MKS Instruments Inc. USA)
(see Section 4.1.3 for more details).

4.2.4 Flow rate measurements

Two different procedures were adopted to measure the flow rates of the work-
ing fluids flowing through the cross-slot micro-geometry. First of all, flow rate
was measured for each inflow fluid using a high precision metal tube flowmeter
(3750A Ar-Mite Low Flow Armored Flowmeter, BROOKS). This type of metal
tube flowmeter is able to handle a variety of liquids with moderate to high vis-
cosities. These two flowmeters were situated in the pipelines between the pressure
vessels and the cross-slot cell in order to control and measure the flow rate at each
inflow channel of the cross slot. Therefore, the flow rate can be directly quantified
by these flowmeters. According to the manufacturer, the flowmeter measurement
accuracy is 5% of the full scale. The metal tube flowmeter was experimentally cal-
ibrated by comparison to a syringe pump (PHD Ultra Harvard Apparatus), which
has a certified accuracy of 0.35%, in the range 3 - 80 ml/min as depicted in Figure
4.13. This comparison indicated that the flow rate obtained using the metal tube
flowmeter exhibits a good agreement with that obtained using the syringe pump
with an average deviation of approximately 3.5%. Secondly, measurements of flow
rate were also compared with the average flow rate evaluated by measuring the
amount of working fluid discharged and the difference between the two flow rates was approximately 0.5%.

![Experimental Flowrate Data](image)

**Figure 4.13:** Metal tube flowmeter calibration measurements versus Syringe pump measurements.

### 4.2.5 Temperature measurements

The fluid temperature was electronically measured at the inlets and outlets of the cross-slot micro-geometry using sheathed 0.5mm-thick chromel-alumel (K-type) thermocouples (KMQSS, Omega). These thermocouples were positioned at each cylindrical hole of the cross-slot as shown in Figure 4.10. From typical voltage-temperature characteristics of thermocouples, the readings of all these thermocouples were recorded over a period of 60s at a frequency of 100 Hz using the same data acquisition system (LabJack U6) as previously used (see Section 4.1.4). The uncertainty in the temperature measurements with thermocouples is estimated by the manufacturer to be within ±1°C. Also, these thermocouples were experimentally calibrated against a mercury thermometer (±0.1°C) as already discussed in
In the range of temperatures from 0°C to 80°C, these thermocouples utilised have a good linear response.

The temperature distribution patterns in the cross-slot have been measured using a temperature-sensitive fluorescent dye \([3, 90, 91, 184]\). Therefore, the temperature-dependent luminescence intensity of Rhodamine-B dye was conducted using an \textit{in-situ} method for monitoring the temperature distribution. The temperature-sensitive fluid (0.1 g of Rhodamine-B in the one litre of 70% aqueous glycerine solution) was continuously injected through the opposed inflow channels of the cross-slot geometry. Meanwhile, intensity images were captured through the upper glass cover of the cross-slot at certain intervals and the thermocouple readings were instantaneously recorded via a data acquisition system after reaching steady-state conditions. Room-temperature images (isothermal intensity images) of the cross-slot at the visualisation region were captured before each heat transfer experiment. The intensity of images was then normalised with the room-temperature intensity. The normalised intensity data of Rhodamine-B are plotted against fluid temperature in the range from 19°C to 48.5°C as shown in Figure 4.14. Furthermore, the averaged fluid temperature versus luminescent intensity of Rhodamine-B is introduced by capturing at least 10 images during sequential video frames to reduce the random error. The calibration results illustrate that a linear relationship between normalised intensity data and the fluid temperature with an image resolution of 1000 × 1000\ pixels where the normalized intensity decreased linearly with increasing temperature over this temperature range. The intensity values of the treated images are then converted to temperature using the intensity-temperature calibration curve for extracting the temperature profiles in the cross-slot micro-geometry. The normalised temperature contours that are obtained from images captured during the experiment were plotted in a normalised fashion (0 to 1) using image processing by a Matlab programme. It is worth noting that the fluid temperature was recorded in conjunction with the flow visualisation images during all.
experiments in order to monitor the heating of the fluid as a result of illuminating from light sources (LED Arrays). It was found that the LEDs had no effect on the temperature of the working fluid.

![Experimental Data Least Squares Fit](image)

**Figure 4.14:** The calibration data for normalised fluorescence intensity of Rhodamine-B against fluid temperature.

### 4.2.6 Temperature distribution and flow visualisation

The behaviour of fluid flow and temperature distribution in the mixing region and downstream of the cross-slot micro-geometry have been experimentally visualised. The locations of the mixing region and downstream measurement sections (field of view) in the cross-slot micro-geometry are shown in Figure 4.15. The fluid flow was visualised under isothermal conditions, both inflow fluid streams were injected into the cross-slot at room temperature. Whilst the temperature distribution patterns were observed with adiabatic surfaces (thermally insulated materials) of the cross-slot cell and hot and cold fluid streams were injected into the cross-slot through separate opposed inlets.
Rhodamine-B (ACROS Organics), which is soluble in water and has a strong sensitivity of temperature, was chosen as a temperature-sensitive fluorescent dye for visualising the fluid flow and temperature distribution at the mixing region and regions downstream (field of view) of the cross-slot micro-geometry (see Figure 4.15). The temperature-sensitive fluid was prepared by dissolving 0.1 g of Rhodamine-B in the one litre of working fluid. In fact, if the concentration of fluorescent dye is below 0.1 g per 1 l solvent, the temperature gradient in the flow field could be hard to observe in the camera. The technique for the use of temperature-sensitive dye is achieved by converting the luminescent signal of fluorescent dye to temperature through optical arrangement and excitation and the luminescent intensity is then captured using a sensitive CCD camera \[39, 186\].

![Figure 4.15](image)

**Figure 4.15:** The relevant details of the cross-slot micro-geometry test section illustrating locations of inlet ports, outlet ports, mixing region and downstream.

The experimental setup that employed for the purposes of visualising consists of a 1000 × 1000 high-speed digital camera (XS5-M-4 NanoSense MKIII, Dantec Dynamics) equipped with a 6× objective lens (Video Zoom Modules (VZM) 600i Zoom Imaging Lens, Edmund Optics) with a resolution of \(\sim 2\mu m\) per pixel see Figure 4.16 VZM 600i Zoom Imaging Lens enables inspection of a wide range of objects without changing the working distance. Additionally, the parfocal zoom for this lens allows magnifications to be simply changed without refocusing. Meanwhile, long-pass optical
filter (cut-on wavelength 550 nm, FEL0550 Thorlabs) was used to collect the fluorescence emission from the Rhodamine-B. The fluorescent Rhodamine-B dye was excited by two LED Arrays light sources (central wavelength 470 nm, LIU470A Thorlabs), which have been installed to achieve illumination from above as shown in Figure 4.16. Image acquisition and storage were controlled by DynamicStudio (v3.41) software. The field of view within the cross-slot geometry is placed on the stage of view of the digital camera and is continuously illuminated to allow on a qualitative impression of the field flow. For isothermal flows, one injected fluid stream was dyed with Rhodamine-B fluorescent dye and the other was undyed. Therefore, the fluorescent fluid would become illuminated using the LED Arrays light whereas the non-fluorescent fluid would remain non-illuminated in the image. In order to observe the temperature distribution patterns in the non-isothermal cases, both injected fluid streams were dyed using Rhodamine-B fluorescent dye.

![Figure 4.16: Experimental setup of the temperature distribution measurements in the cross-slot micro-geometry using temperature-sensitive fluorescent dye (Rhodamine-B).](image-url)
Images captured during experiments were normalised by reference images taken at room temperature. The intensity values were then converted to temperatures through the calibration curve. The normalised temperature contours were plotted using image processing by a Matlab programme.

### 4.2.7 Experimental procedure

The first step from the experimental procedure of the cross-slot geometry was begun with supplying working fluids to the pressure vessels from the same batch of pre-prepared solution. Controlled measurements of flow rate through the cross-slot micro-geometry were carried out using two identical flowmeters, which supply a constant flow rate with an accuracy of $\pm 3.5\%$. A wide range of volumetric flow rate from 3 to 80 $ml/min$ was covered. The pressure-drop between the inlet and outlet channels of the cross-slot geometry was measured using a pressure transducer corresponding to each separate setting of flow rate. This pressure sensor was connected between the entrance and exit of the cross-slot geometry using plastic tubing with an inner diameter of 3 $mm$. The pressure-drop contribution of the connecting tubes was also estimated to be negligible to the total measured pressure-drop. In addition, the readings of the pressure-drop were recorded at steady-state conditions (the pressure-drop does not change with time) using a computerised data acquisition system. The temperature of hot and cold fluid streams injected into the cross-slot through the opposed inlets, and the outlet fluid temperature, were electronically measured using the conventional voltage-temperature characteristics of the thermocouples. The cold fluid temperature at the cross-slot inlet was approximately $20^\circ C$ whereas the inlet hot fluid temperature was changed depending on the flow rate from approximately $25^\circ C$ to $50^\circ C$ due to heat losses to the ambient prior to the cross-slot test section especially at lower flowrates. Temperature distribution patterns between hot and cold streams
at the thermal mixing region of the cross-slot micro-geometry were also investigated as shown in Figure 4.16. The temperature-sensitive fluid was pumped at the same flow rate into the test section. The experimental measurements of the fluorescence-based temperature were conducted using *in-situ* calibration during each temperature measurement to guarantee identical illumination conditions for both calibration and measurement. In order to carry out the calibration *in-situ*, images were captured *in-situ* using the same experimental arrangement whilst the temperature-sensitive fluid was continuously injected through the opposed inflow channels of the cross-slot geometry. In the experimental measurements, background and room temperature images of the cross-slot geometry were acquired before each experiment. The normalised intensity was then translated to temperature using the experimental calibration curve for correlating the fluorescence intensity to temperature. The temperature distribution profiles were determined for each pixel inside the image as a pixel-by-pixel calibration, considering each pixel as an individual temperature-sensitive sensor. The pixel-by-pixel calibration is capable of eliminating error owing to variations in the intensity response at different locations represented by the digital camera sensor positioned on the top of the cross-slot. The variations in the intensity response are considered one of the main error sources in the fluorescence-based temperature measurements at the micro-scale [89].

### 4.3 Relevant dimensionless parameters

The dynamics of the Newtonian and viscoelastic flow through the micro-scale geometries are usually described by the following dimensionless quantities: Reynolds number, $Re$, which represents the ratio between inertial forces and viscous forces, and Weissenberg number, $Wi$, which characterises the ratio of elastic to viscous
forces. Also, there are several parameters that were chosen and identified to characterise the behaviour of fluid flow and heat transfer using the micro-scale geometries in this study.

Reynolds number often used to distinguish the type of flow regime such as laminar or turbulent \cite{222}. In a square cross-section channel the Reynolds number is defined as:

$$Re = \frac{\rho U_B D_H}{\eta}$$ \hspace{1cm} (4.1)

where $\rho$ and $\eta$ are the density and viscosity of fluid, respectively. $D_H$ is the hydraulic diameter ($D_H = W$, $W$ is the square channel depth) and $U_B$ is the mean fluid velocity. For viscoelastic fluid, the viscosity is a function of the shear rate thus the viscosity value is associated with the characteristic shear-rate. In order to evaluate a characteristic shear-rate ($\dot{\gamma}_{CH} = U_B/W$) the same value of the fluid velocity and the hydraulic diameter of the channel is used. The value of the characteristic shear-rate is then substituted into the Carreau-Yasuda model fit \cite{228} to obtain a characteristic shear viscosity ($\eta_{CH}$) consistent with this shear rate. Therefore, the Reynolds number is redefined as:

$$Re = \frac{\rho U_B W}{\eta_{CH}}$$ \hspace{1cm} (4.2)

A useful parameter for curved planar pipes is the Dean number, $K$, which is a dimensionless number denoting the ratio of the viscous force acting on a fluid flowing in a curved pipe to the centrifugal force and can be defined here as \cite{49}:

$$K = Re \times \sqrt{\frac{W}{Rc}}$$ \hspace{1cm} (4.3)

where $Rc$ is the mean radius of curvature of the serpentine microchannel. In fact, Dean number affects the intensity and shape of secondary flows in curved ducts. From Eq. 4.3, Dean number is the Reynolds number modified by the path
curvature of the curved ducts.

The elastic properties of viscoelastic fluid flows can be described by the Weissenberg number, which is usually used in polymer fluid dynamics involves a ratio of elastic force to viscous force. In a steady shearing flow of a viscoelastic fluid, Weissenberg number is defined as \[20, 120, 135:\]

\[
Wi = \lambda \dot{\gamma}
\]

(4.4)

where, \(\dot{\gamma}\) is the shear rate and \(\lambda\) is the fluid relaxation time. The estimation of the Weissenberg number in the square channel is then taken to be \[76, 77:\]

\[
Wi = \lambda \frac{U_B}{W}
\]

(4.5)

There are different approaches to estimate the relaxation time of viscoelastic fluid by steady-state shear measurement of the first normal-stress difference \[171, 229, 233\], using the measurements of small-amplitude oscillatory-shear (SAOS) in the limit of the angular frequency tending to zero (oscillatory shear mode) \[76, 78, 171\], applying the experimental data of the storage (\(G'\)) and loss (\(G''\)) modulus within a multimode Maxwell model fit and Zimm’s method \[179, 180, 197, 232, 233\] as mentioned in Chapter 3. Also, there are other methods to estimate the relaxation time of viscoelastic fluids such as capillary breakup extensional rheometry (CaBER) \[179, 180, 214\].

Darcy’s friction factor, \(f\), can be defined based on the relation between the wall shear stress and the dynamic pressure, this formula is valid for any fully-developed, incompressible steady flow \[222\], as:

\[
f = \frac{W \Delta P}{l \frac{1}{2} \rho U_B^2}
\]

(4.6)

where \(\Delta P\) is the pressure-drop across the test section and \(l\) is the path-length
of the test section. It is worth mentioning that the pressure-drop from the minor losses is experimentally considered negligible compared with the major losses in the channel. The Darcy’s friction factor-Reynolds number product, \( fRe \), (essentially the pressure-drop normalised by a viscous stress) can be outlined as:

\[
fRe = \frac{W^2}{l} \frac{\Delta P}{\frac{1}{2} U \eta_{CH}}.
\]  

(4.7)

In order to quantify the enhancement of heat transfer in the serpentine microchannel geometry, from the energy conservation equation, the steady-state thermal energy acquired by the fluid flowing through the serpentine microchannel can be expressed as [97, 191]:

\[
Q = \dot{m}C (T_{m,o} - T_{m,i})
\]  

(4.8)

where, \( Q \) represents the net heat transferred to the flowing fluid in the serpentine microchannel, \( \dot{m} \) is the mass flow rate, \( C \) is the specific heat of the fluid, and \( T_{m,i} \) and \( T_{m,o} \) are the mean (bulk) inlet and outlet temperatures, respectively. The mass flow rate in Eq. 4.8 can be obtained by measuring the mass of fluid collected at the exit over a fixed period of time. The average inlet \( (T_{m,i}) \) and outlet \( (T_{m,o}) \) fluid temperature were obtained from time averaged readings of the thermocouples. Also, the specific heat of fluid was estimated at the mean film temperature equal to \((T_{m,i} + T_{m,o})/2 \) [97, 191]. If we assume that at any location along the length of the serpentine microchannel, the walls are at a uniform temperature around the cross-section, then the averaged heat transfer coefficient, \( \bar{h} \), can be determined using Newton’s law of cooling for internal flow under the condition of constant surface temperature [97, 191]:

\[
\bar{h} = \frac{\dot{m}C (T_{m,o} - T_{m,i})}{A_s \Delta T_{\text{lm}}},
\]  

(4.9)

where, \( \bar{h} \) is the averaged convection heat transfer coefficient, \( A_s \) is the heated surface area of the serpentine microchannel \((A_s = P \cdot l, \ P \) is the surface perimeter and
Chapter 4. Experimental arrangement and operational procedure

$l$ is the test section path-length). According to the thermal boundary conditions of the square serpentine microchannel fabrication (the side walls and the bottom wall of the serpentine channel are isothermal and an adiabatic boundary condition on the upper wall), therefore the heated surface area available for convection is equal to $3W \times l$. $\Delta T_{lm}$ is the log-mean temperature difference.

$$ \Delta T_{lm} = \frac{(T_w - T_{m,o}) - (T_w - T_{m,i})}{\ln\left[\frac{(T_w - T_{m,o})}{(T_w - T_{m,i})}\right]} \quad (4.10) $$

where, $T_w$ is the wall temperature of the square serpentine microchannel. The convection heat transfer coefficient value is governed by the geometrical shape of the test section, flow rate or fluid flow velocity and thermophysical properties of the fluid. The experimental data is often correlated in terms of dimensionless groups to facilitate the use in practical applications under special experimental conditions. Thus, the convective heat transfer coefficient is formulated in terms of a dimensionless number as [97, 191]:

$$ Nu = \frac{\bar{h}W}{k}, \quad (4.11) $$

$Nu$ is the mean Nusselt number that is usually utilised in calculations of convective heat transfer. Generally, Nusselt number represents the ratio of convective heat transfer to purely conductive heat transfer between a moving fluid and a solid surface [97]. $k$ is the thermal conductivity of fluid. In internal flow heat transfer, the $Nu$ depends on at least two dimensionless parameters namely, the Reynolds number (which is used to describe the nature and regime of the flow as demonstrated early) and the Prandtl number (which is used to characterise the growth of thermal boundary layer thickness on the heat transfer surface) [97, 191, 221]. According to the thermal boundary conditions of the cross-slot micro-geometry, the heat transfer is determined by mixing between the hot and cold fluid streams.
flowing from the opposing inlets and the surfaces of the cross-slot cell are considered to be adiabatic. Consequently, the Nusselt number cannot be defined as the mean temperature at every plane is a constant. Therefore, the root mean square temperature ($T_{RMS}$) along the outlet channel was adopted as a proxy to evaluate the heat transfer between the two streams in this case. Non-dimensional root mean square temperature is calculated at each cross-sectional plane in the outlet channels as

$$
T_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} (T_i - \overline{T})^2}{n \overline{T}}},
$$

where, $\overline{T} = (T_{in.C} + T_{in.H})/2$ represents the mean temperature and $n$ is the number of cells in the cross-stream plane.

The Prandtl number, $Pr$, which is named after the German physicist Ludwig Prandtl who pioneered the concept of thermal boundary layer in 1904 [40], defined as the ratio of momentum diffusivity to thermal diffusivity and is generally described in the following form [97, 191, 221]:

$$
Pr = \frac{\nu}{\alpha} = \frac{\eta C}{k},
$$

Where $\nu$ is momentum diffusivity (kinematic viscosity) and $\alpha$ is thermal diffusivity. From Eq. (4.13) the Prandtl number is dependent only on the fluid properties and the fluid state because there is no length scale in its definition. Generally, the convection heat transfer rate anywhere along the surface of a body is directly related to the temperature gradient at that location. Therefore, the fluid flow velocity has a strong effect on the temperature profile and the development of the velocity boundary layer relative to the thermal boundary layer has strong influence on the convection heat transfer. Nusselt number in the region of the thermally-developing flow (entrance length of the channel) is usually described as a function
of Graetz number, which is defined as \[97, 191, 221\]:

\[ G_z = \left( \frac{W}{l} \right) RePr, \quad (4.14) \]

### 4.4 The experimental measurement uncertainties

Practical experiments are usually associated with measurement errors from precision, random and systematic errors. For two of these errors, namely precision and systematic errors, it is not possible to reduce them by repeating measurements or by averaging large numbers of readings. There is, however, a common approach to minimise systematic error via calibration of the measurement instruments \[88\]. Accordingly, experimental results are considered incomplete without a determination of the uncertainty of measurement, which identifies the acceptable range of values for the final results within a specified level of confidence. Therefore, for evaluating the uncertainty of measurement, the uncertainty sources in the measurement need to be firstly determined and the magnitude of the uncertainty emerging from each source is then estimated because of an inherent error in the measured values of each parameter.

The main geometric dimensions of the selected test sections, flowrate, pressure and temperature, which represent the independent parameters, are directly measured using various instruments, probes and sensors. The accuracy of these measurement instruments based on manufacturers supplied information, accompanying documentation, and on instruments label of specification may consist of the known sources of errors for example, the resolution, sensitivity, linearity, repeatability, reproducibility, hysteresis and various drifts. The uncertainty in the measurement of these independent parameters is tabulated in Table 4.1.
Typically, the heat transfer enhancement is determined through the dependent parameters (e.g. heat transfer rate or Nusselt number). These dependent parameters are functions of the independent measured variables and the thermophysical properties of the working fluids. Thus, the errors, which express the difference between the measured value and the true value, resulting from the measured independent variables will be implicitly propagated through the calculation procedure into the experimental results according to the parametric relationships involved in the analysis.

A careful analysis of the experimental uncertainty is critical to interpret the experimental data of the dependent parameters. The uncertainty associated with dependent parameters ($R$) based on the measured independent variables ($x_i$) is calculated using root sum square (RSS) method as \[ \varepsilon_R = \left[ \sum \left( \frac{\partial R}{\partial x_i} \varepsilon_i \right)^2 \right]^{0.5} \] (4.15)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty</th>
<th>Measurement range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric dimensions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross-section dimensions ($\mu m$)</td>
<td>±10</td>
<td>0 ~ 1000</td>
</tr>
<tr>
<td>Test-section length (mm)</td>
<td>±0.1</td>
<td>0 ~ 115</td>
</tr>
<tr>
<td>Pressure measurements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model-3-34 pressure transducer (kPa)</td>
<td>±0.25% FS</td>
<td>0 ~ 22</td>
</tr>
<tr>
<td>Model-3-46 pressure transducer (kPa)</td>
<td>±0.25% FS</td>
<td>0 ~ 350</td>
</tr>
<tr>
<td>Temperature measurements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K-type thermocouples ($^\circ C$)</td>
<td>±0.5</td>
<td>0 ~ 80</td>
</tr>
<tr>
<td>Mercury thermometer ($^\circ C$)</td>
<td>±0.1</td>
<td>-10 ~ 100</td>
</tr>
<tr>
<td>Thermophysical properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>±1</td>
<td>0 ~ 2000</td>
</tr>
<tr>
<td>Dynamic viscosity (Pa.s)</td>
<td>±2%</td>
<td>0.01 ~ 0.8</td>
</tr>
<tr>
<td>Specific heat capacity (J/kg.K)</td>
<td>±5%</td>
<td>2400 ~ 2900</td>
</tr>
<tr>
<td>Thermal conductivity (W/m.K)</td>
<td>±10%</td>
<td>0.285 ~ 0.368</td>
</tr>
<tr>
<td>Mass flowrate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volumetric flow rate (ml/min.)</td>
<td>±4%</td>
<td>3 ~ 100</td>
</tr>
<tr>
<td>Weight</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight (g)</td>
<td>±0.002</td>
<td>0.001 ~ 1500</td>
</tr>
</tbody>
</table>
where $\varepsilon_R$ is the uncertainty in the dependent parameter, $R(x_1, x_2, \ldots, x_n)$, and $\varepsilon_i$ is the uncertainty of the independent variable $x_i$. If the dependent parameter, $R(x_1, x_2, \ldots, x_n)$, represents the form of a product of the independent variables, $R = x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}$, the uncertainty equation (4.15) can be rewritten as:

$$\frac{\varepsilon_R}{R} = \left[ \sum \left( \frac{a_i}{x_i} \varepsilon_i \right)^2 \right]^{0.5} \tag{4.16}$$

Whilst, the relative uncertainty of the dependent parameters is normally obtained by dividing the absolute uncertainty by the mean value of the dependent parameters. Applying the same procedure demonstrated in previous publications \[105\, 132\, 201\], the relative uncertainty of Reynolds number, Weissenberg number, friction factor-Reynolds number product ($fRe$) and mean Nusselt number that is associated with the propagated error of the experimental variables can be expressed by Eq. 4.17, 4.18, 4.19 and 4.20, respectively, as follows:

$$\frac{\varepsilon_{Re}}{Re} = \left[ \left( \frac{\varepsilon_\rho}{\rho} \right)^2 + \left( \frac{\varepsilon_{UB}}{UB} \right)^2 + \left( \frac{\varepsilon_W}{W} \right)^2 + \left( \frac{\varepsilon_\eta}{\eta} \right)^2 \right]^{0.5} \tag{4.17}$$

$$\frac{\varepsilon_{Wi}}{Wi} = \left[ \left( \frac{\varepsilon_\lambda}{\lambda} \right)^2 + \left( \frac{\varepsilon_{UB}}{UB} \right)^2 + \left( \frac{\varepsilon_W}{W} \right)^2 \right]^{0.5} \tag{4.18}$$

$$\frac{\varepsilon_{fRe}}{fRe} = \left[ \left( \frac{\varepsilon_{\Delta P}}{\Delta P} \right)^2 + \left( 2 \times \frac{\varepsilon_W}{W} \right)^2 + \left( \frac{\varepsilon_\eta}{\eta} \right)^2 + \left( \frac{\varepsilon_l}{l} \right)^2 + \left( \frac{\varepsilon_{UB}}{UB} \right)^2 \right]^{0.5} \tag{4.19}$$

$$\frac{\varepsilon_{Nu}}{Nu} = \left[ \left( \frac{\varepsilon_\rho}{\rho} \right)^2 + \left( 2 \times \frac{\varepsilon_W}{W} \right)^2 + \left( \frac{\varepsilon_C}{C} \right)^2 + \left( \frac{\varepsilon_l}{l} \right)^2 + \left( \frac{\varepsilon_k}{k} \right)^2 + \left( \frac{\varepsilon_{UB}}{UB} \right)^2 \right]^{0.5} \tag{4.20}$$
where $\varepsilon_{Re}/Re$, $\varepsilon_{Wi}/Wi$, $\varepsilon_{fRe}/fRe$ and $\varepsilon_{Nu}/\overline{Nu}$ are the relative uncertainties in the Reynolds number, $Re$, Weissenberg number, $Wi$, friction factor-Reynolds number product, $fRe$, and mean Nusselt number, $\overline{Nu}$, respectively. $\varepsilon_{\rho}$, $\varepsilon_{\eta}$, $\varepsilon_{C}$ and $\varepsilon_{k}$ are the uncertainties for thermophysical properties, density, $\rho$, dynamic viscosity, $\eta$, specific heat capacity, $C$, and thermal conductivity, $k$, respectively, of working fluids. $\varepsilon_{\Delta P}$ is the pressure-drop, $\Delta P$, uncertainty. $\varepsilon_{W}$ is the square channel depth (= hydraulic diameter, $D_H$), $W$, uncertainty. $\varepsilon_{l}$ is the uncertainty of the micro-scale geometries path-length, $l$. $\varepsilon_{U_B}$ is an uncertainty for the discharging bulk fluid velocity, $U_B$. $\varepsilon_{\lambda}$ is an uncertainty for the relaxation time of viscoelastic fluids. $\varepsilon_{\Delta T_B}$ is an uncertainty for the bulk fluid temperature difference, $\Delta T_B$, between inlet and outlet of the micro-scale test section. $\varepsilon_{\Delta T_{lm}}$ is an uncertainty for the log-mean temperature difference, $\Delta T_{lm}$.

From the uncertainty equation of friction factor-Reynolds number product (Eq. 4.19), the error in the measurement of hydraulic diameter dominates the experimental uncertainty in $fRe$ because the uncertainty in diameter is greater by a factor 2 than the other measured variables \cite{105, 132, 201}. Therefore, in spite of the accurate and calibrated instrument that was employed for measuring the hydraulic diameter of the micro-scale geometries, still there is the minimum uncertainty in $fRe$ as dictated via uncertainty in the hydraulic diameter. Thus, because of the measurement errors of the micro-scale geometries dimensions, there is an uncertainty in the pressure-drop measurements. For small Reynolds number, the accuracy of the flowrate measurement also becomes more significant.

The uncertainty analysis for the mean Nusselt number in Eq. 4.20 reveals that the main parameter contributing effectively to the total uncertainty is the systematic uncertainty of temperature measurement. When the temperature difference, between the inlet and outlet of the test section, is larger this leads to the uncertainty of temperature measurement becoming smaller. In addition, the uncertainty of temperature measurement could be reduced using more sensitive thermocouple
probes which may provide a smaller error. The relative uncertainty associated
with the friction factor-Reynolds number product \((fRe)\) and the mean Nusselt
number under various operating conditions is presented in the discussion chap-
ters. Also, the procedure of uncertainty analyses and sample calculation for some
chosen key parameters is provided in Appendix A.
Chapter 5

Newtonian fluid flow and heat transfer in a serpentine microchannel geometry

Many studies have numerically [45, 136, 182, 183, 205, 206, 230, 231] and experimentally [46, 204] investigated developing and fully-developed flows in curved “wavy” channels and revealed an early transition from laminar to turbulent flow. Also, they observed that heat transfer enhancement was affected greatly by the flow conditions and the geometry of the channel. However, the majority of these previous studies are restricted to purely numerical approaches and to rather low Prandtl numbers. This is attributed to the experimental difficulties due to the restrictions imposed by the small length-scales. Therefore, the thermo-hydrodynamic behaviour of serpentine microchannels requires a more specific and detailed assessment, which is experimentally and numerically addressed in this chapter. Experimental measurements of convective heat transfer and pressure drop are performed for 30/70 and 10/90 per cent by weight mixtures of glycerine/water. The experimental system design and calibration, data acquisition and operational procedures were detailed in Chapter 4. A three-dimensional numerical simulation of
the square cross-section serpentine microchannel has been performed using the commercial CFD software package Fluent to help elucidate the physical mechanisms responsible for the heat transfer enhancement as presented in Section 5.1. The pressure-drop losses together with the heat transfer performance over a wide range of Reynolds number are discussed in Sections 5.2 and 5.3, respectively, to understand the mechanisms of heat transfer enhancement in the square cross-section serpentine microchannel. Moreover, Section 5.4 gives a detailed discussion of numerical simulation results. Section 5.5 addresses the behaviour of chaotic advection in the serpentine microchannel. Finally, the summary of this chapter will be provided in the Section 5.6.

The following results and discussion have formed the basis for recently published paper:


Waleed M. Abed, Richard D. Whalley, David J. C. Dennis and Robert J. Poole, which is included in Appendix B.

5.1 Numerical simulations

Micro-scale analysis is often challenging and expensive with traditional experimental approaches, which are usually used for determining global properties such as, flow velocity, pressure drop and temperature etc. Computational Fluid Dynamics (CFD) lends itself to not only specifying global properties, but in addition local characteristics which can be extracted with relative ease. CFD and experimental methods can complement each other. If a fluid problem is analysed both with CFD and an experimental investigation is ensured and the global properties
are similar, it is a realistic assumption to suggest that the local properties extracted from the CFD are reliable. Therefore, CFD is always best when validated with experimental results where possible.

The application of CFD tools include solving a set of multi-dimensional differential equations developed for flow transport characteristics such as mass flow, momentum, internal energy and potentially turbulence characteristics over a fluid domain of interest. This set of differential equations is complemented with equations for fluid thermophysical properties and auxiliary conditions such as initial and boundary conditions. The main restriction with any CFD technique applied to a complex media is defining a suitable computational geometry for the flow domain in which the fluid motion can be analysed \[8, 217\]. The three-dimensional fluid domain of interest is discretised into a number of cells characterised by nodes, edges and faces leading to the solution of a partial differential equation. This process is referred to as meshing or grid generation. The integral forms of the transport equations are then converted into approximate algebraic equations for each of these cells and the resulting set of nonlinear algebraic system of equations for the entire fluid domain is solved using various semi-explicit schemes.

5.1.1 Simulation formulation

The numerical analysis of the three-dimensional square serpentine microchannel has been conducted using the commercial CFD software package Fluent version 14.5.7 \[95\]. The computational domain involves the inlet portion, the periodic serpentine units (20 half-loops which represent 10 units) and the outlet portion for simulating the geometric configuration of the serpentine microchannel. The flow stream begins from the inlet portion and goes through ten repeating units, which allow an adequate axial distance for flow development and the final unit (10th unit) is then linked to the outlet portion, allowing the flow to exit the channel smoothly
as shown in Figure 5.1 (a). This complete computational model considers for possible flow “non-periodicity” in the serpentine microchannel by comprising several repeating units rather than just one, as well as entry and exit effects. Numerical simulations are performed in the computational domain that has the same geometrical dimensions, such as cross-section, path-length (channel wavelength) and wavy unit number, of the fabricated serpentine microchannel test-section used in the experiments (see Section 4.1). A schematic view of the modelled serpentine microchannel geometry is shown in Figure 5.1. The serpentine microchannel geometry is defined in three-dimensional cartesian coordinates ($x$, $y$ and $z$ indicate the streamwise, wall-normal and spanwise directions, respectively).

**Figure 5.1:** Schematic diagram of the computational domain for the square cross-section serpentine microchannel. $W$ is the square microchannel depth, $L$ is the length of one serpentine unit, $R_c$ is the mean radius of curvature and $A$ is the unit height (amplitude).
5.1.2 Conservation equations and boundary conditions

In the numerical simulation, the computational model was constructed with the following assumptions. The flow is assumed to be incompressible, laminar and steady-state; the fluid properties are assumed constant and temperature independent; negligible viscous dissipation (no heat generation) is also assumed. The conservation equations that need to be solved for governing laminar flow and heat transfer are those of continuity, momentum and energy [221]:

\[ \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (5.1)

\[ \rho (\mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla P + \eta \nabla^2 \mathbf{u} \]  \hspace{1cm} (5.2)

\[ \rho C (\mathbf{u} \cdot \nabla T) = k (\nabla^2 T) \]  \hspace{1cm} (5.3)

where \( \rho \) (density), \( \eta \) (dynamic viscosity), \( k \) (thermal conductivity) and \( C \) (specific heat capacity) are the physical properties of fluid. \( \mathbf{u} \) represents the velocity vector whereas \( P \) and \( T \) represent the static pressure and temperature, respectively.

A fully-developed velocity profile for a straight square channel is imposed at the inlet of the square serpentine microchannel for thermally developing steady laminar flow regime using a finite-volume approach with applied no-slip condition on the microchannel surfaces. The thermal boundary conditions for the numerical investigation are the upper surface is assumed adiabatic whereas the other three surfaces are held at constant temperature in order to simulate the experimental circumstances. The constant temperature of the three surfaces is set at 30 °C whereas the inlet fluid temperature is set at 20 °C.
5.1.3 Model discretisation

The three-dimensional computational domain for the square serpentine microchannel geometry is generated with swept hexahedral volume elements along the microchannel path-length using the commercial pre-processor software ICEM version 14.5.7, which is utilised as the pre-solver software to mesh the geometric domain, discretise grid generation and label boundary definition [96]. ICEM software allows rapid generation of multi-block structured hexahedral volume meshes (complex topologies) and supports a wide variety of solver outputs and post-processing. The cross-sectional area of the microchannel is meshed by a BiGeometric mesh law, which allows a fine structured mesh near the wall and a coarse structured mesh in the middle, with low mesh spacing and a value of 1.05 for the successive grid ratio for resolving the gradients of fluid velocity and temperature near the microchannel surfaces. A uniform mesh law is adopted in the axial flow direction as shown in Figure 5.2.

![Figure 5.2: The geometric mesh (Mesh-2) for the square cross-section serpentine microchannel.](a) The axial direction for a one periodicity unit (b) A cross-sectional view
5.1.4 Solution method

The conservation equations of continuity, momentum and energy for steady laminar fluid flow within a square serpentine microchannel are solved by employing the general-purpose finite-volume based computational fluid dynamics (CFD) software package, Fluent using double precision [95]. The continuity and momentum equations are utilised for calculating velocity vector whereas the energy equation is applied in order to compute temperature distribution and wall heat transfer coefficient. A pressure-based solver is applied using a segregated solver to find a solution to the continuity and momentum equations within a steady-state framework. Pressure staggering option (PRESTO) interpolation scheme was employed for pressure discretisation while, the well-known Semi-Implicit method for Pressure-Linked equations (SIMPLE) algorithm is used to introduce pressure into the continuity equation and flow coupling [163]. The gradient evaluation of scalars was computed by using a Least Squares Cell-Based approach. In addition, the energy equation is activated during the analysis for achieving convective heat transfer in the channel. The equations of momentum and energy are solved with a second-order upwind scheme. Hybrid initialisation method is employed to initialise the fields of velocity and temperature in the computational domain.

An important step in the setup of the computational model is to specify the fluids and their physical properties. The properties that are relevant to the studied problem scope are defined via the User-Defined Materials Database panel using a constant temperature of fluid properties at mean film temperature. All numerical simulations are carried out for convergence criteria of all relative residuals falling to less than $10^{-6}$. The parameters of convection heat transfer and fluid flow such as, velocity, temperature, friction factor and heat transfer coefficient, are computationally obtained across the square serpentine microchannel in the post-processing step.
In order to accurately compare the experimental results with the numerical simulation data, it is necessary to properly specify the heat transfer rate and Nusselt number, $Nu$, particularly when the experimental data are comparatively limited to just the wall temperature and the inlet and outlet fluid temperatures. Thus, in this study, numerical simulations were carried out for verifying experimental analysis and system validation. For numerical simulation under constant wall temperature condition, the computational domain is axially divided into small slices of control volumes and the local peripherally-averaged heat transfer coefficient ($h_x$) at a given axial location ($x$) is evaluated from [97]:

$$h_x = \frac{q''(T_w-T_{m,o})-(T_w-T_{m,i})}{ln\left[\frac{(T_w-T_{m,o})}{(T_w-T_{m,i})}\right]}$$  \hspace{1cm} (5.4)

where the term $q''$ refers to the heat flux. $T_w$ is the microchannel wall temperature and $T_{m,i}$ and $T_{m,o}$ represent the bulk fluid temperature at the inlet and outlet of the control volume, respectively, defined as [97, 191]:

$$T_{m,x} = \frac{\int_{A_c} u_{y,z}T_{y,z}dA_c}{U_B A_c}$$  \hspace{1cm} (5.5)

where $A_c$ is the serpentine channel cross-section area and $U_B$ is the mean (bulk) fluid flow velocity, $u_{y,z}$ is the velocity component parallel to the channel flow direction and $T_{y,z}$ is the local temperature at the point $(y, z)$ of the cross-sectional plane. The local Nusselt number ($Nu_x$) can be expressed as [97, 191]:

$$Nu_x = \frac{h_x W}{k}$$  \hspace{1cm} (5.6)

where $k$ represents the thermal conductivity of the working fluid. The average Nusselt number ($\overline{Nu}$) can be computed from the axially weighted average values
of local Nusselt number ($Nu_x$) along the serpentine microchannel by [97, 191]:

$$\overline{Nu} = \frac{\int_{l} Nu_x dl}{l}$$  \hspace{1cm} (5.7)

The thermal performance of the serpentine microchannel is evaluated by dividing the heat transfer rate by the equivalent value in a straight microchannel, which has similar dimensions (cross-section and path-length) and the thermal boundary conditions of surfaces. Similarly, the relative pressure-drop losses (represented by friction factor) incurred along the microchannel can be assessed with respect to the straight microchannel as [109, 182, 183]:

$$e_f = \frac{f_{\text{Serpentine}}}{f_{\text{Straight}}}$$  \hspace{1cm} (5.8)

$$e_{\overline{Nu}} = \frac{\overline{Nu}_{\text{Serpentine}}}{\overline{Nu}_{\text{Straight}}}$$  \hspace{1cm} (5.9)

where $e_f$ and $e_{\overline{Nu}}$ are the relative pressure-drop and the enhancement of heat transfer, respectively. Moreover, the overall ratio of the enhancement efficiency, $E$, can be predicted by dividing the enhancement of heat transfer to the relative pressure-drop [109, 182, 183]:

$$E = \frac{e_{\overline{Nu}}}{e_f}$$  \hspace{1cm} (5.10)

5.1.5 Mesh quality analysis

Mesh independence studies were performed to ensure that the grid generated for the computational domain is refined to the extent that the resulting solution is no longer significantly affected by the mesh size. Therefore, to reduce the discretisation error and to guarantee the numerical accuracy of the computational results, the grid independence is firstly investigated for independent and stable predictions. The whole domain of a square serpentine microchannel has been meshed with a cross-sectional mesh of $25 \times 25$, $30 \times 30$ and $35 \times 35$ with 176, 220 and 264
intervals in the axial direction for each repeated unit, respectively. The computed values of mean Nusselt number \((Nu)\) and \(fRe\) are tabulated in Table 5.1 for the three different grid sizes. These illustrate that the averaged \(Nu\) and \(fRe\) acquired with a \(30 \times 30\) mesh is within 1.07% and 0.41%, respectively, of that acquired with a \(35 \times 35\) mesh at \(Re = 50\) \((K = 40.8)\) under constant wall temperature conditions and 70/30 per cent concentration of glycerine/water \((Pr = 137)\) as the working fluid. Another procedure has been carried out to verify the grid independence by comparing the temperature profile for the three selected grids \((25 \times 25, 30 \times 30\) and \(35 \times 35)\). The temperature profile, which is perpendicular to the axial flow direction at the centerline of the wall-normal plane, was chosen at the same location for these grids in the serpentine microchannel with \(Re = 50\) \((K = 40.8)\) and \(Pr = 137\) (as depicted in Figure 5.3) \([182, 183]\).

An additional computational validation was achieved by simulating a straight square microchannel, i.e., when the serpentine microchannel amplitude is equal to zero, in order to compare the computational results with previous analytical and numerical data (following the approach by Manglik et al. \([136]\) and Sui et al. \([205]\)). Therefore, a set of numerical calculations for the straight square microchannel, which has the same dimensions (cross-section and path-length) as the square serpentine microchannel, was implemented with \(25 \times 25, 30 \times 30\) and \(40 \times 40\) grid elements in the cross-section and 2600 non-uniform elements size in the axial direction to validate the accuracy of the numerical simulation. The details of the comparison between the present numerical simulation data and Shah and London \([191]\) are provided in Table 5.2 for mesh \((30 \times 30)\) where it can be observed that the deviations are within 0.013% and 0.181% for \(fRe\) and \(Nu\), respectively. Furthermore, an additional comparison with numerical data by Chandrupatla and Sastri (using a finite-difference approximation scheme) \([30]\) for water \((Pr = 7)\) flow in the square straight channel was also conducted. The flow was simulated hydraulically fully-developed and thermally developing under the conditions of constant
wall temperature at \( Re = 327 \) and Graetz number, \( (Gz = \frac{D}{D} Re Pr) = 20 \) was considered, and the agreement shown in Table 5.3 is seen to be fair.

Consequently, a mesh size \( 30 \times 30 \) in the cross-sectional direction and 220 cells in the axial direction for each repeated unit was deemed adequate for the present numerical simulation (\( 2.34 \times 10^6 \) cells in total).

**Table 5.1:** The influences of mesh size on \( Nu \) and \( fRe \) at \( Re = 50 \) (\( K = 40.8 \)) and \( Pr = 137 \). Deviation based on the numerical data for Mesh-3 (\( 35 \times 35 \)).

<table>
<thead>
<tr>
<th>Mesh No.</th>
<th>Mesh size</th>
<th>No. of nodes ( \times 10^6 )</th>
<th>( Nu )</th>
<th>Deviation (%)</th>
<th>( fRe )</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh-1</td>
<td>25 \times 25</td>
<td>1.350</td>
<td>10.0155</td>
<td>3</td>
<td>16.2897</td>
<td>0.96</td>
</tr>
<tr>
<td>Mesh-2</td>
<td>30 \times 30</td>
<td>2.340</td>
<td>10.2244</td>
<td>1</td>
<td>16.1997</td>
<td>0.42</td>
</tr>
<tr>
<td>Mesh-3</td>
<td>35 \times 35</td>
<td>3.724</td>
<td>10.3349</td>
<td>-</td>
<td>16.1325</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 5.2:** Comparison of the present data of computational simulation for mesh \( 30 \times 30 \) with the analytical data of Shah and London \([191]\) for a square straight microchannel.

<table>
<thead>
<tr>
<th></th>
<th>( fRe )</th>
<th>( Nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shah and London ([191])</td>
<td>14.2271</td>
<td>2.976</td>
</tr>
<tr>
<td>Present computational simulation</td>
<td>14.2289</td>
<td>2.9814</td>
</tr>
<tr>
<td>Deviation (%)</td>
<td>0.0128</td>
<td>0.1806</td>
</tr>
</tbody>
</table>

**Table 5.3:** Comparison of the present data of computational simulation with the numerical data of Chandrupatla and Sastri \([30]\) for a square straight channel under constant wall temperature conditions at \( Re = 327 \) and \( Gz = 20 \). Deviation based on the numerical data \([30]\).

<table>
<thead>
<tr>
<th>Mesh size</th>
<th>( Nu )</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh-1, 25 \times 25</td>
<td>4.1877</td>
<td>3.909</td>
</tr>
<tr>
<td>Mesh-2, 30 \times 30</td>
<td>4.1776</td>
<td>3.677</td>
</tr>
<tr>
<td>Mesh-3, 40 \times 40</td>
<td>4.1770</td>
<td>3.663</td>
</tr>
<tr>
<td>Numerical data ([30])</td>
<td>4.0240</td>
<td>-</td>
</tr>
</tbody>
</table>
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5.2 Pressure-drop losses in the serpentine microchannel

The complex flow field expected in the serpentine microchannel leads to relatively higher pressure-drop losses compared to the equivalent straight channel. The experimental procedure to measure pressure drop in the serpentine microchannel geometry was described in Section 4.1.3. A series of experimental measurements were implemented in the square serpentine microchannel for \( Re \ (K) \) varied from 0.7 to 98 (0.6 - 80) using two types of Newtonian fluids, which were prepared by mixing 10 and 30 per cent by weight of distilled water with 90 and 70 per cent glycerine, respectively. The measured values of the density and viscosity for both working fluids across the working temperatures range are provided in Table 5.4.
The experimental and numerical pressure drop findings in the axial flow direction of the serpentine microchannel for two Prandtl numbers are compared to the numerical data for constant curvature single bend from Hwang and Chao [93] as shown in Figure 5.4. The averaged Prandtl numbers for 30/70 and 10/90 per cent by weight of glycerine/water are 137 and 1038, respectively. Figure 5.4 indicates that the present numerical values of \( f \) (where \( f \) is Fanning friction factor, \( f = f_D / 4 \) [222], \( f_D \) is Darcy friction factor defined in Eq. 4.7) as a function of Dean number \( (K) \) collapse with the predicted values of Ref. [93] and the experimental results of the \( f \) exhibit an excellent agreement with the numerical data.

Typically, in a curved microchannel, a secondary flow is created in the form of a symmetric pair of counter-rotating vortices called Dean’s vortices [49]. These two symmetric vortices are induced by the centrifugal body force that is imposed on the axial flow direction and play a vital role in mixing the flow. In contrast, most techniques of fluid mixing, especially for flows in micro-scale channels, are dependent on the classical approach of Aref of fluid mixing called “chaotic advection” [10, 11]. The chaotic advection is based on the increase in interfacial area of fluid that passes through the curved or serpentine passages. Fluid elements are stretched and folded to increase the interfacial area of the flowing streams. In chaotic advection the exponential growth in stretching of fluid interfaces lead to the significant enhancement of diffusive fluid mixing. The experimental \( f \) results at \( Pr = 1038 \) are slightly increased from 14.2 to 14.6 (\( f = 14.227 \), for fully-developed laminar flow regime in a straight square cross-section ducts [191, 222]) over a narrow range of \( K \) from 0.6 to 4.7 (due to a high fluid viscosity) therefore it is expected that the fluid mixing occurring in this case is as a result of the rotational movement of fluid elements due to the identical pair of vortices along the serpentine flow direction. In other words the effect of chaotic advection on the fluid mixing is negligible in this case. The experimental measurements of \( f \) at \( Pr = 137 \) are significantly
increased from 14.2 to 18.7 over a range of $K$ from 6 to 80. Therefore the fluid flow structure is significantly changed with the appearance of Dean’s vortices at the outer wall of the serpentine microchannel [10, 182, 205, 206, 230]. The deviations of $fRe$ between the experimental measurements and simulated data are 0.9% and 0.8% with $Pr= 1038$ and 137, respectively. The averaged values of uncertainties for all experimental values of the friction factor-Reynolds number product ($fRe$) can be determined by a similar procedure as depicted in Eq. 4.19 (see Section 4.4). The $fRe$ uncertainties ranged from 2.7% to 4.4% for $Pr= 1038$ and from 3.2% to 4.7% for $Pr= 137$.

Figure 5.5 shows velocity magnitude contours with velocity vectors perpendicular to the main flow direction at different values of $K$ (0.53, 42.6 and 106.5) for $Pr= 137$. The increases in $K$ are related to changes in the secondary flow shape, where the centers of the counter-rotating vortices moving towards the outer wall due to the fluid elements near the microchannel centerline having larger velocity than fluid near the walls of the microchannel as is shown by Figure 5.5. Thus, they would tend to flow outward around the curve of the microchannel.

<table>
<thead>
<tr>
<th>Temperature ($^\circ$C)</th>
<th>30% water/ 70% glycerine</th>
<th>10% water/ 90% glycerine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta$ (Pa.s)</td>
<td>$\rho$ (kg/m$^3$)</td>
</tr>
<tr>
<td>20</td>
<td>0.0226</td>
<td>1184.6</td>
</tr>
<tr>
<td>22</td>
<td>0.0203</td>
<td>1183.7</td>
</tr>
<tr>
<td>24</td>
<td>0.0186</td>
<td>1183.2</td>
</tr>
<tr>
<td>26</td>
<td>0.0167</td>
<td>1181.8</td>
</tr>
<tr>
<td>28</td>
<td>0.0154</td>
<td>1181.3</td>
</tr>
</tbody>
</table>

Table 5.4: The measured values of density and viscosity for the working fluids across the working temperature range.
5.3 Nusselt number in the serpentine microchannel

A series of experiments were implemented for Reynolds numbers ranging from 0.7 to 98 ($K = 0.6 \text{ - } 80$) with three-dimensional numerical simulations to investigate convective heat transfer within the serpentine microchannel geometry. As mentioned in Chapter 4, the average wall temperature was extrapolated from the measured wall temperatures at axial locations along the side walls in both sides at a distance of $1\text{mm}$ from the wall of the serpentine channel as shown in Figure 4.4 (b). The side walls and the bottom of the channel were made of copper ($k_{Cu} = 385 \text{ W/mK}$), therefore they are maintained at essentially isothermal boundary conditions whereas the insulating properties of the PVC ($k_{PVC} = 0.25 \text{ W/mK}$) ensured...
an adiabatic boundary condition on the upper wall. The large thermal conductivity of the copper ensured that, although the “wall” temperature was actually measured 1mm away, the error associated with this assumption was very small. Additional temperature measurements at 8mm from the wall suggested that the “true” wall temperature (determined by extrapolation) agreed with the assumed value to within 0.01 °C (see Table 5.5 for more details). Additionally the inlet and outlet temperatures of the fluid flowing through the serpentine channel were measured in the upstream reservoir and the downstream reservoir. The whole facility was placed in a water bath continuously-stirred and maintained at a temperature
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of 30 °C so that the measured wall temperature remained constant to 30 ±0.2 °C in reality. Typical inlet temperatures were 22 ±0.5 °C whilst the outlet temperature varied depending on conditions (different Reynolds numbers) between 24 and 30 °C. It is worth noting that the temperature measurement uncertainty is relatively large for a small temperature variation between the inlet and outlet of the serpentine microchannel. The system was allowed to reach steady-state before measuring and sampling the flow rate and temperatures for each test.

The experimental measurements of averaged Nusselt number within the steady laminar range are compared in Figure 5.6 to the present numerical data for Prandtl numbers of 137 and 1083. The averaged values of $Nu$ appear to match best with the computational predictions under thermally-developing flow along the serpentine microchannel. The experimental results of the averaged $Nu$ show good agreement with the present numerical data where the deviations of the mean $Nu$ between the experimental results and computational simulation data are 10.7% and 8.6% with $Pr = 1038$ and 137, respectively (i.e. well within the uncertainty of the experiments). The effect of secondary flow does not appear to modify $Nu$ at the lower values of Dean number. As a result, the fluid flow behaves essentially as flow in a straight microchannel even within the serpentine microchannel because of the dominance of the viscous forces, which suppress the formation of secondary flow. However, with rising $K$, the helical swirl flow tends to dominate the flow (inertial forces more significant than viscous forces) thus it is consistent with the augmentation of heat transfer with increasing values of $K$ [205, 206, 230]. From Figure 5.6 it is clear that there is a remarkable effect of $Pr$ on the convective heat transfer within the serpentine microchannel because the thermal boundary layer thickness declines with increasing $Pr$ [109, 231]. In the inset of Figure 5.6 it is illustrated that the averaged $Nu$ values versus ($Pr \times K^2$) essentially collapse [93]. The error in calculating the averaged $Nu$ comes from the energy balance equation parameters as mentioned in Eq. 4.20. The percentage error that comes
from the measured temperatures of the walls and fluid is approximately 90% of the total error of the averaged $Nu$, whilst the mass flow rate and the serpentine microchannel dimensions contribute 8% and 2%, respectively, to the total error of the averaged $Nu$. The averaged $Nu$ uncertainties ranged from 21% to 22.6% for $Pr= 1038$ and from 25.3% to 30.3% for $Pr= 137$. In fact, the agreement between experiment and simulation would suggest these are extremely conservative estimates for the experimental uncertainty and, in reality, the uncertainty in the thermocouple values (and ultimately $Nu$) is much lower than these estimates.

The results of the heat transfer enhancement ($e_{Nu}$) and the relative pressure-drop losses ($e_f$) from the numerical simulation are presented in Figure 5.7 versus the variations of $K$ from 0.53 to 106.5 for $Pr= 137$ and from 0.05 to 14 for $Pr= 1038$. From Figure 5.7 it can be observed that the serpentine microchannel is able to enhance the performance of heat transfer relative to the straight microchannel over the entire range of $K$. Simultaneously, the relative pressure-drop losses rise with increasing $K$. However, this increase is much smaller over the whole range of $K$ than the heat transfer enhancement. The maximum percentage increase of relative pressure-drop losses for both Prandtl numbers, 1038 and 137, is up to 2% and 39%, respectively, whereas the enhancement of heat transfer for $Pr= 1038$ and 137 increases by 56% and 158%, respectively.

In order to investigate the heat transfer performance improvement of the serpentine microchannel, the results of the enhancement efficiency ratio ($E$), which is the ratio between the heat transfer enhancement and the relative losses of pressure drop, from the computational simulation are presented in Figure 5.8 versus the variations of $K$ for $Pr= 1038$ and $Pr= 137$. The ratio of enhancement efficiency increases for both Prandtl numbers, 1038 and 137, up to 53% and 85%, respectively.
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Table 5.5: The measurements of the wall temperature of serpentine channel at two different positions of wall.

<table>
<thead>
<tr>
<th>Water bath temperature</th>
<th>Average temperature at a 1mm</th>
<th>Average temperature at a 8mm</th>
<th>Extrapolation internal surface temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>°C</td>
<td>°C</td>
<td>°C</td>
</tr>
<tr>
<td>17.5</td>
<td>17.41</td>
<td>17.43</td>
<td>17.407</td>
</tr>
<tr>
<td>29.5</td>
<td>29.34</td>
<td>29.42</td>
<td>29.330</td>
</tr>
<tr>
<td>39.2</td>
<td>38.95</td>
<td>39.05</td>
<td>38.937</td>
</tr>
</tbody>
</table>

Figure 5.6: Comparison between the measured values of averaged Nusselt number ($\overline{Nu}$) and the averaged Nusselt number obtained from numerical simulation versus the variation of Dean number. Inset: log-log scale for the averaged $\overline{Nu}$ versus $(Pr \times K^2)$. 

Numerical data, $Pr = 137$
Experimental results
Numerical data, $Pr = 1038$
Experimental results
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**Figure 5.7:** The numerical data of the heat transfer enhancement and the relative losses of pressure-drop versus the variation of Dean number. Inset: log-log scale.

**Figure 5.8:** The numerical data of the enhancement efficiency ratio versus the variation of Dean number. Inset: log-log scale.
5.4 Flow and heat transfer patterns in the serpentine microchannel

As a result of the existence of secondary flows in the flow fields of the serpentine microchannel geometry, the heat transfer performance of the serpentine microchannel is much higher than that of the equivalent straight microchannel with the same cross-section, as also evidenced by previous numerical and experimental investigations [136, 183, 205, 206, 230, 231]. The computational simulation data in terms of the pressure-drop losses ($fRe$) and $Nu$ shows a good agreement with the experimental results. Therefore, this validation provides confidence that the other available data from the numerical simulation can be used to enable an understanding of the convective heat transfer and fluid flow behaviours within the serpentine microchannel.

Although the simulations were performed over a large range of $K$, the normalised velocity magnitude along the axial direction of the serpentine microchannel with $Pr=137$ are presented in Figure 5.9 for just the minimum and maximum values of $K$. In Figure 5.9 at $K=0.53$, the fluid flow along the axial direction of the serpentine microchannel is largely invariant where the velocity profile is almost constant through the entire flow path-length because of the dominance of viscous forces (there are no secondary flows). Increasing $K$ to 106.5 the inertial forces begin to dominate, the fluid mixing increases noticeably and encourages the convective heat transfer in the serpentine microchannel. Additionally, the velocity contours and vectors are shown on the enlarged sections of the second unit of serpentine microchannel for both values of $K$ in Figure 5.9. These sections show the fluid flow behaviour in the serpentine microchannel and how the flow can enhance heat transfer with increasing $K$. The formation of Dean’s vortices in the cross-section and their spatial development along the axial flow direction encourage fluid mixing.
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Figure 5.10 illustrates the contours of non-dimensional velocity magnitude \((u/U_B)\), secondary flow vectors and non-dimensional temperature \(((T-T_B)/(T_w-T_B))\) with \(Pr= 137\) in the first unit of the serpentine microchannel cross-section at different axial locations \((X_1 = 0L, X_2 = 0.5L\) and \(X_3 = L)\) and at an equivalent location \((X_2)\) for a straight microchannel at \(Re= 0.65\) \((K = 0.53)\). As shown in Figure 5.10 (a) the flow behaviour in the cross-sectional plane of the serpentine microchannel is approximately similar to that in the straight microchannel due to the low value of \(Re (K)\) and the dominance of the viscous forces. The symmetric pair of vortices (Dean’s vortices) is formed in the cross-sectional plane of the serpentine microchannel (see Figure 5.10 (b)). The appearance of these symmetrical vortices boosts rotational motion of fluid elements along the serpentine microchannel compared to the straight microchannel. However, the fluid mixing is almost insignificant and \(Nu\) is essentially unaffected. The non-dimensional temperature patterns on the locations \((X_1 = 0L, X_2 = 0.5L\) and \(X_3 = L)\) of the first unit of cross-sectional planes and at equivalent location \((X_2)\) of the straight microchannel are exhibited in Figure 5.10 (c). These figures of the temperature contours show the similarity in behaviour with inhomogeneous temperature distribution in both microchannels. Consequently, the convective heat transfer in the serpentine microchannel is limited. In contrast, once the \(Re (K)\) increases to 130.4 \((106.5)\), the fluid flow pattern in the serpentine microchannel is qualitatively altered. The intensity of symmetric vortices in the cross-sectional plane of the serpentine microchannel increases due to the increasing variance between the gradient of the radial fluid pressure and the centrifugal body forces as shown in Figure 5.11 (a) and (b). Furthermore, at the locations \(X_1\) and \(X_3\), the upper vortex rotates in the clockwise direction and the lower vortex rotates in the counter-clockwise direction whereas the two vortices at the \(X_2\) location reverse their rotation direction \[136, 183\]. The center of the single-vortex-pair for all locations tends to shift towards the outer wall of the serpentine microchannel because of the centrifugal body forces (the transfer of momentum) as shown in Figure 5.11 (a) and (b).
The alternating shape of the serpentine microchannel coupled with the increasing value of \( K \) can lead to “chaotic advection” in the flow \([10, 182, 205, 206, 230]\). Both the chaotic advection and the secondary flow vortices encourage the mixing of fluid elements in the cross-sectional and axial directions of the serpentine microchannel compared to the straight microchannel. The mixing ensures a more homogenous temperature distribution, which in turn leads to an enhanced convective heat transfer within the serpentine microchannel compared to the convective heat transfer in the straight microchannel at the same value of \( Re \) as depicted in Figure 5.11 (c).

Figure 5.9: The normalised velocity magnitude in the central plane of the axial direction of the serpentine microchannel at two different Dean numbers \((K)\) for \( Pr = 137 \).
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Figure 5.10: The distribution of non-dimensional velocity magnitude and temperature with $Re \ (K) = 0.65 \ (0.53)$ in the first unit of the serpentine microchannel cross-section at different axial locations for $Pr = 137$: (a) non-dimensional velocity magnitude contours, (b) secondary flow vectors, and (c) non-dimensional temperature fields.
The distribution of non-dimensional velocity magnitude and temperature with $Re (K) = 130.4$ ($106.3$) in the first unit of the serpentine microchannel cross-section at different axial locations for $Pr = 137$: (a) non-dimensional velocity magnitude contours, (b) secondary flow vectors, and (c) non-dimensional temperature fields.
5.5 Chaotic advection in the serpentine microchannel

Chaotic advection has been utilised as a technique for stirring fluids in micro-devices [11]. Chaotic advection can be thought of as an intermediate state that lies between laminar and turbulent advection [10] and is one of the most efficient approaches to promote fluid mixing in curved or wavy microchannels. In previous numerical simulations, the existence of chaotic advection was found in “zigzag” channels in the steady, laminar flow regime [230]. The simulations revealed that the chaotic advection occurred at a Reynolds number higher than 200 for the same zigzag geometry. To further study the characteristics of chaotic advection, according to Sui et al. [204–206] and Zheng et al. [230], the fluid mixing resulting from chaotic advection can be distinguished by tracking passive massless tracers to construct the particle trajectories in the flow field by adopting a Lagrangian particle tracking technique. The instantaneous positions of each tracer particle can be determined via time integration using the following expression:

\[
x(t) = \int_0^t u(x(t')) \, dt'
\]

(5.11)

where \( x \) is the position vector, \( u \) is the velocity vector and \( t \) is the time. This time integration is directly performed using a post-processing routine in the commercial software (Fluent) by applying a standard fourth-order Runge-Kutta scheme at the required velocity values. The Eulerian velocity field is used for estimating the instantaneous velocity of the tracer particles by adopting spatial interpolation. Second-order accuracy is achieved using a Taylor series expansion and least-squares-based schemes for analysing velocity flow fields for the trajectories of the massless tracer particles. The constant integration time (time-step) utilised to advect the massless particles is \( \Delta t = 0.002 \, s \) or \( \nu \Delta t / W^2 = 0.04 \), which is optimised
via a series of computational trials. This time-step size is considered adequate to obtain independent Poincarè sections [175]. In this investigation, Poincarè sections are produced by tracking trajectories of 1800 tracers through 10 periodic units of the serpentine microchannel. Tracer particles are evenly distributed along a straight line of the cross-sectional plane in the inlet section; see Figure 5.12(a). At a very low value of $Re= 0.65$ ($K= 0.53$), the behaviour of the fully-developed flow in the serpentine microchannel can be approximately considered as the flow of the straight microchannel as depicted in the Poincarè section in Figure 5.12(b). In Figure 5.12(c) a moderate stretching and folding of the tracer particles lines can be noticed for an increased $Re= 52.2$ ($K= 42.6$). It is apparent that although the resulting arrangement of the particles is complex, it is still a recognisable pattern which is symmetric in the plane of symmetry of the secondary flow. When the $Re$ ($K$) rises to 130.4 (106.5) there is no longer a recognizable pattern in the resulting positions of the tracer particles, as shown in Figure 5.12(d). The tracer particles have almost filled the whole cross-section plane of the microchannel. This chaotic distribution of the tracer particles is a typical signature of chaotic advection, which significantly boosts fluid mixing with a concomitant enhancement in convective heat transfer. It is notable from the colours in Figure 5.12(d) that flow does not cross the mid-plane of the channel and all particles remain in the same half of the channel. This is due to the presence of the secondary flow vortices, which are symmetric in this plane.
5.6 Summary

Quantitative experimental measurements of convective heat transfer and pressure drop were obtained using mixtures of glycerine/water as working fluids within a square cross-section serpentine microchannel \((R_i/W = 1)\). The averaged Prandtl numbers for 30/70 and 10/90 per cent by weight of glycerine/water are 137 and 1038, respectively. Steady-state, fully developed laminar flow and thermally-developing conditions were investigated with the upper wall insulated and other
side walls held at constant temperature over a range of Dean number from 6 to 80 for $Pr= 137$ and from 0.6 to 4.7 for $Pr= 1038$. Complementary three-dimensional numerical simulations were also conducted using the commercial Computational Fluid Dynamics (CFD) software package Fluent for the same conditions over a range of Dean numbers from 0.07 to 106.5.

The experimental results of the pressure-drop losses (in terms of $fRe$) exhibit a good agreement with the present numerical data. The results show that the growth of Dean’s vortices promotes fluid mixing in the serpentine microchannel and leads to an enhancement of the convection heat transfer. Therefore, the serpentine microchannel is able to enhance the performance of heat transfer relative to the straight microchannel over the entire range of Dean number ($K = 0.56 - 80$). Meanwhile, at these values of Prandtl number the relative pressure-drop losses increase with the increasing Dean number. However, they are significantly smaller over the whole range of Dean number than the enhancement of heat transfer.

The alternating shape of the serpentine microchannel coupled with the increasing value of Dean number can lead to “chaotic advection” in the flow [10, 11, 182, 201, 206, 230]. Both the chaotic advection and the secondary flow vortices encourage the mixing of fluid elements in the cross-sectional and axial direction of the serpentine microchannel compared to the straight microchannel. The mixing ensures a more homogenous temperature distribution, which in turn leads to an enhanced convective heat transfer within the serpentine microchannel compared to the convective heat transfer in the straight microchannel at the same value of $Re$.

At very low value of $Re= 0.65$ ($K = 0.53$), the behaviour of the fully-developed flow in the serpentine microchannel can be approximately considered as the flow of the straight microchannel. A moderate stretching and folding of the tracer particles lines can be noticed for an increased $Re= 52.2$ ($K = 42.6$). It is apparent that although the resulting arrangement of the particles is complex, it is still a
recognisable pattern which is symmetric in the plane of symmetry of the secondary flow. When the $Re (K)$ rises to 130.4 (106.5) there is no longer a recognisable pattern in the resulting positions of the tracer particles.
Chapter 6

Viscoelastic fluid flow and heat transfer in a serpentine microchannel geometry

The key point in this chapter is to consider the use of polymers added to Newtonian liquids to induce viscoelastic instabilities as a means of enhancing convective heat transfer in a serpentine microchannel flow domain. The experimental results of a comprehensive investigation into the flow of a series of viscoelastic fluids (several concentrations of polyacrylamide, PAA) and two types of Newtonian fluids (solvents) through the square cross-section serpentine microchannel are presented. The rheological characterisation for all viscoelastic solutions used is documented in Chapter 3. Details of the experimental set-up and test section for the serpentine microchannel geometry and the measurement instruments have been described in Chapter 4.

This chapter starts with a detailed explanation for the flow behaviour of viscoelastic fluids at low Reynolds number in the serpentine microchannel in Section 6.1. The investigation of the pressure drop of selected viscoelastic solutions is reported
in Section 6.2 with the aim of confirming the effect of elastic turbulence on the flow in the absence of inertial effects ($Re < 1$). The results of Newtonian fluids are also presented for comparative purposes. In Section 6.3 the results are reported for the convective heat transfer in the serpentine microchannel using elastic turbulence as well as studying the influences of both viscoelasticity and shear-thinning viscosity individually on the convective heat transfer and fluid flow in Section 6.4.

The following results and discussion have formed the basis for recently published papers:


Waleed M. Abed, Richard D. Whalley, David J. C. Dennis and Robert J. Poole which are included in Appendix B.

### 6.1 The flow behaviour of viscoelastic fluids at low Reynolds number

Microfluidic devices have usually been associated with negligible inertial effects (no inertial fluid instabilities) because of the predominantly laminar flow, i.e. the small scale of dimensions ($\leq 1000\mu m$), for these micro-devices leading to small Reynolds numbers [51]. Consequently, the process of mixing fluid in the micro-geometries relies on molecular diffusion, which is usually extremely slow compared with convection. Mixing at the micro-scale is difficult to achieve (as molecular diffusion is dominant) and thus the enhancement of convective heat transfer is problematic.
under these conditions. Various approaches have been investigated for promoting mixing in micro-devices, involving using bubbles \[69\], electro-hydrodynamic instability \[210\], flow focusing \[115\], and bas-relief patterning \[202\]. However, these methods are generally considered inefficient in the process of mixing fluid under conditions of very low flowrates, i.e. very low Reynolds numbers. One of the proposed approaches to enhance heat transfer in these conduction-dominated regimes is to use viscoelastic fluids, which are prepared by adding a small amount of high molecular-weight polymer to a Newtonian solvent, in order to introduce non-linear effects and promote the appearance of instabilities even at low \(Re\). 

Viscoelastic fluids can exhibit flow instabilities \[20\], when polymer coils stretch and bend along streamlines which creates hoop-stress. Adding small concentrations of high-molecular-weight polymer to Newtonian fluids moderately increases viscosity (two-fold or more than the solvent viscosity) and provides elasticity, allowing micro-flows to undergo viscoelastic instability. Viscoelastic fluid flows in such geometries have then been seen to exhibit “turbulent-like” characteristics such as chaotic and randomly fluctuating fluid motion across a broad range of spatial and temporal scales and have led this phenomena to be called “elastic turbulence” \[20, 76, 78, 127, 187, 208, 232, 233\]. In fact, the concept of elastic instabilities appeared as a phenomenon for viscoelastic fluid flows in the 1990s \[20, 120, 122, 138, 162, 192\]. Larson et al. \[122\] reported theoretical and experimental results that show that purely-elastic instabilities can arise in viscoelastic fluid flows by a coupling of the first normal-stress difference and streamline curvature. This seminal work was followed by a series of experimental investigations that used different geometries containing curved streamlines to study elastic instabilities in low \(Re\) fluid flows. This included swirling flow between parallel disks \[76, 78, 187\], in serpentine or wavy channels \[26, 77, 78, 127, 208\] and in concentric cylinder devices \[18, 78\]. A common feature of the above-mentioned geometries is the existence of curved streamlines in the base flow with a sufficient velocity.
gradient across the streamlines. As a result, the elastic stresses grow nonlinearly with shear-rate and can dramatically affect the flow behavior even at low $Re$ \[20, 120\]. Elastic instabilities and resulting non-linear interactions between elastic stresses generated within the flowing high-molecular-weight polymer solutions and the streamline curvature are “purely-elastic” in nature, driven by the elastic (normal) stresses developed in the flow and occur at Reynolds numbers far removed from the usual turbulence observed for Newtonian fluids which is, of course, inertial in nature. Steinberg and co-workers \[26, 76–78\] reported that elastic instabilities of highly-elastic fluids can be exploited to augment fluid mixing in serpentine or curved channels. Their experimental results revealed that elastic stresses can lead to efficient mixing in curved channels (see Figure 6.1 (a)) at low $Re$ \[77\]. For Newtonian fluid at low Reynolds number ($Re = 0.16$), the flow remains stable with a sharp and clear interface between fluorescent dyed and undyed fluid as shown in Figure 6.1 (b). In contrast, the flow of viscoelastic solutions in the same curved channel shows elastic instability which leads to irregular flow and mixing in the absence of inertia as shown in Figure 6.1 (c - e). They also observed that viscoelastic fluid flows with negligible inertia ($Re < 1$), that have been stretched along curved streamlines, can undergo a series of flow transitions from viscometric laminar flow, to clearly chaotic flow, and eventually to fully-developed “elastic turbulence”.

Several investigations motivated by the seminal work of Groisman and Steinberg (see, for example, \[26, 165, 208\]) have found that viscoelastic instabilities can be excited in micro-geometries. Efficient mixing (up to orders of magnitude more rapid versus the pure solvent \[77\]) can be achieved; however, it is relative to a highly viscous solvent and in the presence of strong shear. The effects of viscoelastic fluids in micro-scale geometries become significant even for dilute solutions because of the small flow time scales and the high shear rates achievable. If the velocity gradients are sufficiently large, then polymer molecules that are “coiled”
at equilibrium will begin to stretch. The molecular stretching is often distinguished by the Weissenberg number (\(Wi\), as defined in Eq. 4.5). Availability of these features in the flow simultaneously makes the \(Re\) small and the Weissenberg number large, which quantifies non-linear elastic effects. From earlier publications [26, 76, 78, 127, 187, 208, 232, 233], purely-elastic instabilities have been observed at low \(Wi\) (order 1) in a range of flows and, at high \(Wi\) (order 10), elastic turbulence observed as a result of the combination of elastic stresses and streamline curvature. Elastic turbulence can arise in the serpentine channel flow at typical Weissenberg numbers as shown in Table 6.1.

To summarise: the combination between elastic stresses generated within the flowing high-molecular-weight polymer solutions and the streamline curvature in the base flow leads to large qualitative change in flows and large instabilities, these instabilities progress to “elastic turbulence” and large orders of magnitude gains in mixing rate were observed in elastic turbulent flows [77]. It is worth mentioning the intermediate “transition” region between the steady and stable flow and turbulence flow in the case of viscoelastic flow is very short and quickly developed to the elastic turbulence compared to the hydrodynamic turbulence flow in the case of Newtonian flow [148]. According to previous studies, the first direct experimental evidence of convective heat transfer enhancement using elastic turbulence within the square serpentine microchannel will be presented in this chapter.

**Table 6.1:** Experimental values of typical Weissenberg number for onset of a purely-elastic instability and elastic turbulence in the serpentine channel flow.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Weissenberg number range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Onset of elastic turbulence</td>
</tr>
<tr>
<td>Groisman and Steinberg [78]</td>
<td>3.2</td>
</tr>
<tr>
<td>Burghela et al. [26]</td>
<td>1.4-3.5</td>
</tr>
<tr>
<td>Feng-Chen Li et al. [127]</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Figure 6.1: Efficient mixing in curved channel at low Reynolds number using dilute polymer solutions (after [77]). (a) Schematic drawing of the square cross-section serpentine channel, \( d = 3 \text{ mm}, R_1 = 3 \text{ mm}, R_2 = 6 \text{ mm}, N = 30 \). (b) Snapshot of flow behavior of Newtonian fluid (65% sucrose and 1% NaCl in water) at \( N = 15 \) and \( Re = 0.16 \) where there is no mixing, while for viscoelastic fluid (80 ppm PAA in Newtonian fluid) with \( Re = 0.16 \) and \( Wi = 6.7 \) at (c) \( N = 4 \), (d) \( N = 15 \) (compare to (b)), and (e) \( N = 27 \) unstable flow and improved mixing is observed. Bright regions represent fluid with fluorescent dye.

6.2 Pressure-drop results and discussion

Details of the measurements of pressure-drop for the square serpentine microchannel are described in Section 4.1.3. The experimental measurements of steady-state pressure drop, \( \Delta P \), for all working fluids (both shear-thinning and Boger solutions) acquired along the serpentine channel between an upstream reservoir and a downstream reservoir for a range of flow rates between 0.2 \text{ ml/min.} and 24.8 \text{ ml/min.}
are presented in Figure 6.2. The pressure-drops for the Newtonian fluids (W/GLY and W/SUC) exhibit a linear increase across the whole range of flow rates. The experimental results of pressure drop for viscoelastic solutions (both shear-thinning, PAA-W/GLY, and Boger, PAA-W/SUC, solutions) in Figure 6.2 show that, above the elastic instability threshold, the flow in the serpentine microchannel exhibits a significant increase in the flow resistance. Therefore, this is accompanied by a noticeable increase in the pressure losses [77, 78, 180].

![Figure 6.2: Pressure-drop measurements versus flow rate for Newtonian (W/GLY and W/SUC) and viscoelastic (PAA-W/GLY and PAA-W/SUC) solutions.](image)

Figure 6.3 shows the Darcy friction factor, $f$ (as defined in Eq. 4.6), versus $Re$ (as defined in Eq. 4.2 where the characteristic shear-viscosity, $\eta_{CH}$, was obtained at a characteristic shear rate, $\dot{\gamma}_{CH}$, from the Carreau-Yasuda model fit [228] to the steady-shear viscosity measurements at the mean film temperature equal to $(T_{m,i} + T_{m,o})/2$, where $T_{m,i}$ and $T_{m,o}$ denote the mean (bulk) fluid temperature
at the inlet and outlet reservoirs, respectively) for all working fluids. The experimental measurements of friction factor were measured when the upper wall of the serpentine channel was insulated and the side walls were maintained at constant temperature. Figure 6.3 indicates that the values of friction factor for Newtonian fluids (W/GLY and W/SUC) collapse with the theoretical (Darcy) equation \( f = 57/Re \), for straight square cross-section ducts \([191, 221, 222]\) for fully-developed isothermal laminar flow \((Re = 0.2 - 5.7)\) where the friction factor values decline linearly with \(Re\) on a log-log plot. Although secondary flow may be generated due to the combination of large axial normal stresses with streamline curvature (e.g., as in the classical paper of Dean \([49]\)), we find here that the Newtonian fluid flow behaves essentially as flow in a straight channel even within the serpentine channel owing to the dominance of the viscous forces, which suppress the formation of secondary flow (Dean’s vortices \([49]\)), see Section 5.2 for more details. The maximum deviation between the measured values of friction factor and the well-known Darcy’s formula is within \(\pm 7.6\%\). Therefore, experiments with Newtonian fluids emphasised the validity of the present experimental techniques and measuring system.

The friction factor values for viscoelastic solutions, which were selected to understand elastic turbulence effects on heat transfer in the serpentine channel, are also shown in Figure 6.3. The measured values of friction factor for these viscoelastic solutions demonstrate a significant increase compared with the Newtonian solution over the same range of \(Re\). In the viscoelastic fluid flow (even with very low \(Re\)) a secondary flow may develop due to the combination of a first normal-stress difference and streamline curvature \([172]\). However, the large increases in the non-dimensional pressure drop for the viscoelastic solutions are probably attributable to the appearance of elastic turbulence above a certain flowrate (see e.g. data in Table 6.1).
The friction factor-Reynolds number product - essentially the pressure-drop normalised by a viscous stress - is shown in Figure 6.4 as a function of Weissenberg number, \( Wi \) (as defined in Eq. 4.5 where the longest relaxation time (\( \lambda_o \)) obtained from the small-amplitude oscillatory-shear data was evaluated at the mean film temperature). The values of \( fRe_{(N-\text{Newt.})} \), normalised by \( fRe_{(\text{Newt.})} \) for laminar fully-developed Newtonian fluid flow in a straight square duct (\( fRe_{(\text{Newt.})} = 57 \)) \cite{191,221,222}, rise rapidly initially with increasing \( Wi \) before levelling off. From previous publications \cite{26, 77, 78, 127, 172, 208, 232}, it is known that, creeping flows of viscoelastic solutions (\( Re \to 0 \)) through a serpentine channel develop firstly at low \( Wi \) to a steady secondary flow \cite{172} before the onset of a purely-elastic instability, which leads eventually to oscillatory time-dependent flow at \( Wi \) of order one for constant-viscosity fluids (\( Wi \sim 3.2 \) \cite{78}, 1.4 - 3.5 \cite{26} see Table 6.1). Beyond this first purely-elastic instability, the flow becomes increasingly complex and then develops to elastic turbulence, which is observed beyond
$Wi > 6.7 \ [78], 10 \ [26], 7.5 - 15 \ [127]$. Figure 6.4 shows that for $Wi < 5$ the pressure drop is similar to that for a Newtonian fluid. With increasing Weissenberg number ($5 < Wi < 25$) the purely-elastic instability develops and leads to an increase in the normalised pressure-drop. The increase in the normalised pressure-drop values for all viscoelastic solutions beyond $Wi \approx 25$ are much greater than the Newtonian limit, suggesting that the complexity of the elastic instabilities increase with increasing $Wi$. The highest non-dimensional values of pressure-drop in terms of normalised $fRe$ increase approximately from 1.86 for 50-W/GLY to 4.82 for 200-W/GLY for shear-thinning solutions meanwhile for Boger solutions they range from 1.48 for 80-W/SUC to 4.67 for 500-W/SUC. Therefore, from the experimental results of pressure drop for viscoelastic solutions in Figure 6.4 one can observe that there is a proportional relation between the pressure losses and the concentration of polymer.

**Figure 6.4:** Normalised friction factor-Reynolds number product, $fRe$, versus Weissenberg number, $Wi$, for PAA-W/GLY and PAA-W/SUC solutions. (a) $Wi < 5$, Viscometric laminar flow, the pressure-drop is essentially the same as the Newtonian values. (b) $5 < Wi < 25$, Viscoelastic instabilities, purely-elastic instabilities lead to an increase in the pressure-drop. (c) $Wi > 25$, Elastic turbulence, significant increases in normalised pressure drop.
6.3 Heat transfer results and discussion

An experimental investigation of the heat transfer characteristics in the square cross-section serpentine microchannel using two different groups of viscoelastic working fluids (shear-thinning and Boger solutions) was performed. While, the details of the measurements of temperature are described in Section 4.1.4. The thermal boundary conditions of the serpentine microchannel were formed as follows: the copper side walls and the bottom wall of the serpentine channel are considered to be isothermal whereas the insulating properties of the PVC ensured an adiabatic boundary condition on the upper wall of the channel. The cold working fluid, which was pumped through the serpentine microchannel, is heated by the hot walls of the serpentine channel that is placed in a water bath at 30°C.

Measurements of the temperature difference between outlet and inlet ($T_{m,o} - T_{m,i}$) for viscoelastic solutions (shear-thinning and Boger solutions) versus $Wi$ are shown in Figure 6.5. The temperature of fluid in the inlet and outlet represent, in fact, the average fluid temperature in the reservoirs of the upstream and downstream of the serpentine microchannel. As can be seen in Figure 6.5, the temperature difference decreases progressively with increasing $Wi$ as a consequence of the reduced residence time in the microchannel (higher $Wi$ correspond to higher flowrates and hence lower residence times). The effect of elastic turbulence on enhancing the heat transfer can most readily be seen for the higher concentration solutions (e.g. 200- W/GLY and 500- W/SUC) where the temperature increase remains approximately constant (approximately 8°C) despite this reduced residence time.

The enhancement of heat transfer by elastic turbulence can be quantified via the estimation of mean Nusselt number, $\overline{Nu}$ (as defined in Eq. 4.11). The mean $Nu$ values for Newtonian solutions (W/GLY and W/SUC) and viscoelastic solutions (shear-thinning and Boger solutions) against Graetz number, $Gz$ (as defined in Eq. 4.14 where the averaged values of Prandtl number were obtained at the mean film
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temperature for all working solutions), are shown in Figure 6.6. The averaged $Nu$
for Newtonian fluid flow collapses to the numerical predictions [30] for a thermally-
developing laminar flow through a straight square duct under the condition of
constant wall temperature. The curvature of the serpentine microchannel has
apparently little impact for Newtonian fluid flows at such low $Gz$ (up to 14.6). On
the other hand, there is a significant increase of the $\overline{Nu}$ for all viscoelastic solutions.
It is expected that this enhancement of heat transfer is due to elastic turbulence
[26, 77, 78, 127, 208], which evolves in the flow and has previous been seen to
enhance mixing of a passive scalar [76, 77]. Elastic turbulence generated in the flow
of viscoelastic solutions is shown to augment the convective heat transfer in the
serpentine microchannel by approximately 200% for 50-W/GLY and 80-W/SUC
and reaches up to 380% for 200-W/GLY and 500-W/SUC under creeping-flow
conditions in comparison to that achieved by the equivalent Newtonian fluid flow
at identical Graetz number.

![Figure 6.5: Temperature difference measurements between outlet and inlet, $T_{m,o} - T_{m,i}$, versus Weissenberg number, $Wi$, for viscoelastic solutions (shear-thinning and Boger solutions).](image-url)
Chapter 6. Viscoelastic flow and heat transfer in a serpentine microchannel

Figure 6.6: Mean Nusselt number, $\overline{Nu}$, versus Graetz number, $Gz$, for PAA-W/GLY and PAA-W/SUC solutions. Solid line is the numerical solution for a thermally-developing laminar flow through a straight square duct under the condition of constant wall temperature [30].

The $\overline{Nu}$ data, normalised by the equivalent $\overline{Nu}$ for a Newtonian fluid, is presented against $Wi$ in Figure 6.7. According to the development of viscoelastic fluid flow through the serpentine microchannel with the absence of inertial effects [26, 78, 127] previously described early in Section 6.2 (see Figure 6.4 for more details), Figure 6.7 shows that the behaviour of viscoelastic flow is essentially Newtonian at very low $Wi$ ($< 25$), and the flow here can be considered to be quasi viscometric as the secondary flow does not appear to affect $\overline{Nu}$ much. Beyond a critical Weissenberg number, $Wi \approx 25$ [26, 77, 78, 127], the flow undergoes a purely-elastic instability. With increasing $Wi$, the viscoelastic flow evolves to fully-developed elastic turbulence [26, 76, 78, 127, 187, 208, 232, 233]. Thus, we posit that, the increase in convective heat transfer here is due to the influence of such elastic turbulence, which is created by the non-linear interaction between
elastic stresses generated within the polymeric solutions and the streamline curvature of the serpentine geometry \[26, 76, 78, 127, 187, 208, 232, 233\]. From Figure 6.7, there is an excellent overlap of the data for all dilute viscoelastic solutions showing an enhancement in convective heat transfer with increasing \(Wi\). However, the normalised values of \(\overline{Nu}\) for the semi-dilute viscoelastic solution (500-W/SUC) increase more sharply against \(Wi\) than the dilute solutions. Shear-thinning influences may be significant because of the large shear rates encountered in this micro-scale flow, as might thermal development effects. Therefore, Figure 6.8 illustrates the relation between the normalised \(\overline{Nu}\) and modified Weissenberg number, \(Wi^*\), which we define as:

\[
Wi^* = \left(\frac{W}{l}\right) PrWi, \tag{6.1}
\]

The modified Weissenberg number, which represents the ratio of elastic stress to thermal diffusion stress, combines geometric dimensions (depth, \(W\), and path-length, \(l\), of the square serpentine channel), Prandtl number \((Pr)\) and Weissenberg number \((Wi)\) to describe the convective heat transfer under thermally-developing conditions. It is worthwhile to note that the momentum diffusivity for all viscoelastic solutions is much greater than the thermal diffusivity, such that \(Pr >> 1\). The experimental results in Figure 6.8 show that Boger solutions can enhance convective heat transfer over a \(Wi^*\) range of approximately between 250 and 750. While, the shear-thinning solutions need a wide range of \(Wi^*\) to enhance convective heat transfer from around 500 to 3250. Essentially, the results are indicating that the mean Nusselt number is a function of both \(Wi^*\) and the degree of shear-thinning of the fluids. Generally, the findings in Figure 6.8 are consistent for each viscoelastic solution group depending on the rheological characteristics and thermal properties. Overall the collapse against the classical \(Wi\) (Figure 6.7) is better, especially for dilute solutions.
Executing a standard error analysis, the averaged values of uncertainties for friction factor-Reynolds number product \((fRe)\) and Nusselt number \((\overline{Nu})\) were conducted by following the same approach as shown previously in Eq. 4.19 and Eq. 4.20, respectively (see Section 4.4 for more details). The minimum and maximum uncertainty values of \(fRe\) and \(\overline{Nu}\) are provided in Table 6.2 for all working fluids. The greatest uncertainty in determining \(\overline{Nu}\) came from the measured temperatures of the walls and fluid at the inlet and outlet serpentine channel (approximately 90% of the total error of the averaged \(\overline{Nu}\)), whilst the flow rate and the serpentine channel dimensions contribute about 10% to the total error of the \(\overline{Nu}\). The agreement between the \(\overline{Nu}\) from the present experiments for Newtonian fluids and numerical data [30] (see Figure 6.6) suggested that these are conservative estimates for the experimental uncertainty and, in reality, the uncertainty in the thermocouple values (and ultimately the \(\overline{Nu}\)) is lower than these values in Table 6.2 suggest.

Finally we confirmed that any contributions from natural convection in our set-up are expected to be negligible, with estimates of typical Grashof numbers (i.e. the ratio of buoyancy and viscosity forces) being \(10^{-3}\). Thus, buoyancy forces are essentially negligible for all of the results shown here.

**Table 6.2:** Minimum and maximum uncertainty values of \(fRe\) and \(\overline{Nu}\).

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>(fRe) uncertainty</th>
<th>(\overline{Nu}) uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min. (%)</td>
<td>Max. (%)</td>
</tr>
<tr>
<td>W/GLY</td>
<td>2.73</td>
<td>6.51</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>2.88</td>
<td>7.21</td>
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<tr>
<td>100-W/GLY</td>
<td>3.42</td>
<td>10.42</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>4.04</td>
<td>16.70</td>
</tr>
<tr>
<td>W/SUC</td>
<td>2.77</td>
<td>4.26</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>2.96</td>
<td>5.67</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>3.25</td>
<td>14.70</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>4.55</td>
<td>16.37</td>
</tr>
</tbody>
</table>
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**Figure 6.7:** Normalised Mean Nusselt number, $\overline{Nu}$, versus Weissenberg number, $Wi$, for PAA-W/GLY and PAA-W/SUC solutions. (a) $Wi < 5$, Viscometric laminar flow, the normalised $\overline{Nu}$ is essentially the same as the Newtonian values. (b) $5 < Wi < 25$, Viscoclastic instabilities, purely-elastic instabilities lead to an increase in the normalised $\overline{Nu}$. (c) $Wi > 25$, Elastic turbulence flow, significant increases in normalised $\overline{Nu}$.

**Figure 6.8:** Normalised Mean Nusselt number, $\overline{Nu}$, versus modified Weissenberg number, $Wi^*$, for PAA-W/GLY and PAA-W/SUC solutions.
6.4 The influence of shear-thinning on the characteristics of convective heat transfer and fluid flow

Purely-elastic flow instability in the microfluidic serpentine channel was experimentally investigated using two groups of viscoelastic fluids, namely shear-thinning solutions and approximately constant-viscosity Boger solutions, with different PAA concentrations as detailed in Chapter 3. In order to assess the role of fluid elasticity compared to flow inertia, at least four independent runs for each viscoelastic solution were performed with additional experiments for the Newtonian solvent (with no elastic contribution) at comparable flow rates (0.2 - 24.8 ml/min) which verified that the flow remained stable.

Differences between the constant-viscosity and shear-thinning solutions both in the pressure-drop data (Figure 6.4) and in the heat transfer data (Figures 6.7 and 6.8) may be due to a number of different reasons. Firstly we have used the longest relaxation time, determined from small-amplitude oscillatory-shear data in the limit of low frequency, at the mean film temperature to determine the Weissenberg number. As the data in Figures 3.17, 3.18 and 3.19 show, however, the relaxation time exhibits frequency/rate dependence and a characteristic relaxation time - determined at the mean wall shear rate for example - might offer better collapse of the data. One method to determine such a rate-dependent relaxation time in the non-linear regime is to use first normal-stress difference data as shown in Figures 3.20 and 3.21. Unfortunately reploting the data using this $N_1$ data did not prove successful, most probably as a consequence of the large uncertainty associated with measurements of $N_1$ for such low concentration fluids. A secondary reason for the differences may be due to the estimate of the viscous stress within the flow which we estimated as a characteristic shear rate ($U_B/W$)
multiplied by a characteristic viscosity determined at the same shear rate (at the mean film temperature). The use of a simple characteristic shear rate is clearly a simplification as the shear-thinning fluids will have flatter velocity profiles in the laminar regime and hence higher wall shear rates and the mean film temperature is only a first order correction. We note also that the relaxation time exhibited different temperature and shear-rate dependence for each of the two fluid types (e.g. constant-viscosity or shear-thinning). Additionally, a conventional “dimensional analysis” approach to the problem would suggest extra dependency with an extra dimensionless group for the shear-thinning fluids (e.g. the $n$ parameter of the Carreau-Yasuda model provided in Table 3.4). Even the onset of the first purely-elastic instability has recently been shown to be affected by shear-thinning, such that data collapse cannot solely be achieved based on a Weissenberg number alone once shear-thinning becomes significant [27]. Given all of the above issues, the reasonable collapse found against one single dimensionless group, either $Wi$ or $Wi^*$, for the six different fluids is perhaps not unreasonable.

### 6.5 Summary

Convective heat transfer and pressure-drop measurements were quantitatively measured using two groups of viscoelastic fluids, namely shear-thinning solutions and approximately constant-viscosity Boger solutions. The thermal boundary conditions were such that the upper wall of the serpentine channel was insulated (adiabatic) and the side walls were maintained at constant temperature. The normalised values of non-dimensional pressure drop in terms of $fRe$ - essentially the pressure-drop normalised by a viscous stress - for the highly-elastic viscoelastic solutions (both shear-thinning and Boger solutions) increase monotonically with increasing $Wi$ and were significantly higher than the Newtonian limit which we
attribute to the appearance of so-called “elastic turbulence” at high $Wi$. The elastic turbulence is generated by the non-linear interaction between elastic normal stresses created within the flowing high-molecular-weight polymer solutions and the streamline curvature of the serpentine channel. The elastic turbulence created in the flow of these viscoelastic solutions is able to boost the heat transfer by approximately 200% for low polymer concentrations (50-W/GLY and 80-W/SUC) and reaches up to an increase of 380% for high polymer concentrations (200-W/GLY and 500-W/SUC) whilst keeping the Graetz number sufficiently low such that inertia is essentially negligible. A modified Weissenberg number, defined as the ratio of elastic stress to thermal diffusion stress, is able to approximately collapse the normalised data of mean Nusselt number for each viscoelastic solution group. Elastic turbulence is therefore a method which can be employed to enhance convective heat transfer and to boost micro-mixing technologies in practical micro-scale applications.
Chapter 7

Newtonian fluid flow and heat transfer in a cross-slot micro-geometry

The possibility of utilising a steady symmetry-breaking instability in Newtonian fluids in the cross-slot micro-geometries to promote convective heat transfer at low or moderate values of Reynolds numbers is discussed in this chapter. Detailed temperature distributions are measured by monitoring the fluorescent intensity using microscopy techniques with a temperature-sensitive fluorescent dye (Rhodamine-B). In the experiment, the surfaces of the cross-slot micro-geometry are considered to be adiabatic and the hot and cold fluid streams are injected into the cross-slot cell through the opposed inlets. In addition, a three-dimensional numerical simulation of the cross-slot geometry has been executed using the commercial CFD software package Fluent to clarify the effect of the purely-inertial flow instability on the heat transfer in the cross-slot micro-geometry. As far as the influence of the instability of Newtonian fluids (symmetry-breaking bifurcations) in the cross-slot flow on heat transfer is concerned, no work has been previously reported to characterise this effect on convective heat transfer.
Experimental measurements of convective heat transfer and pressure drop are performed for a 30%/70% (w/w) mixture of glycerine/water. The experimental system design and calibration, data acquisition and operational procedures were detailed in Chapter 4. The detail of a three-dimensional numerical simulation of the cross-slot micro-geometry using the commercial CFD software package Fluent is presented in Section 7.1. While, the comparison between the numerical and experimental results is explained in Section 7.2. The discussion of temperature distribution and flow visualisation outlined in detail in Section 7.3. In addition, the transition behaviour between symmetric flow and asymmetric flow in the square cross-slot geometry is described in Section 7.4. Heat transfer enhancement in the cross-slot micro-geometry using purely-inertial flow instability is explained in Section 7.5.

The following results and discussion have formed the basis for recently submitted paper:


Waleed M. Abed, Allysson F. Domingues, David J. C. Dennis and Robert J. Poole.

7.1 Numerical simulations

The numerical simulations have been performed in our research group by Domingues [54] using the finite-volume method within ANSYS workbench (Fluent) version 14.5.7 [95]. As experimental results will be compared to these simulations, a brief outline is provided here. The three-dimensional computational domain of the cross-slot micro-geometry consists of two bisecting square channels with opposing inlets and opposing outlets as shown in Figure 7.1 (a), resulting in a flow field with a stagnation point at the centre of symmetry (when the flow is symmetric, the flow
velocity is zero at the stagnation point) [84]. The geometry and parameters in the numerical simulations are essentially the same as those used for the experimental investigation. The length of each inlet and outlet channel was defined as 60 times its width $W$ ($60W = 30\text{mm}$). This length assures that the fluid velocity no longer depends upon the axial distance and the flow is hydrodynamically fully-developed [57] for all Reynolds numbers studied.

**Figure 7.1:** (a) Schematic diagram of the cross-slot showing the relevant details: the two opposing inlets, two outlets and the stagnation point and (b, c and d) the group 1 computational domain, (b) Mesh-1, (c) Mesh-2 and (d) Mesh-3, of the cross-slot geometry.
In the numerical simulations, the fluid flow was assumed incompressible, laminar and steady-state; the fluid properties were assumed constant and temperature independent. No-slip boundary condition at the side walls was also applied. Boundary conditions are specified in the following way: a uniform velocity profile is imposed at the inlet sections and outflow boundary conditions are used for the outlets. Thermal boundary conditions were defined for the heat transfer problem as: there is no heat flux through the walls (adiabatic walls), therefore, the heat transfer is determined by mixing between the hot and cold fluid streams flowing from the opposing inlets in order to simulate the experimental investigation. The fluid temperatures of the hot and cold streams at the inlets of the cross-slot cell are 45 °C and 20 °C, respectively, in order to approximate the experimental investigation.

A modified SIMPLE algorithm developed by Patankar and Spalding [163] was employed for solving the equations in discretised form (pressure-velocity coupling scheme). The equations of momentum and energy are solved with a second-order upwind scheme. The fluid properties that are relevant to the studied problem scope are defined at constant temperature (mean film temperature). All numerical simulations are performed for convergence criteria of all relevant residuals falling to less than $10^{-10}$.

7.1.1 Computational meshes and bifurcation parameter

A consistent mesh refinement was conducted to estimate the numerical accuracy of the simulations [65]. A preliminary series of simulations was carried out with two groups of three different uniform hexahedral (all quad) computational meshes (as in Table 7.1). In the first group the number of cells in the cross-section was varied $(25 \times 25$, $50 \times 50$ and $75 \times 75)$ for fixed number of biased cells (growth factor 100:1 towards the centre) along the inlet and outlet channels as shown in
Figure 7.1 (b, c and d). The second group explores the effect of the variation of the number of cells along the channels (25, 50 and 100) given a fixed number of cells at the cross-section (50 × 50).

**Table 7.1:** Grid independence study for estimating the numerical accuracy using different mesh characteristics for the cross-slot by comparing the analytical solution for the normalised maximum flow velocity \(u_{\text{max}}/U_B = 2.0979\) \(^{221}\) in a square channel in fully-developed flow with the numerical results and also evaluating the maximum value for the transverse velocity and the normalized root mean square temperature at outlet plane (depicted in Figure 7.2). \(N_x\) and \(N_y\) represents the number of divisions in \(x\) and \(y\) directions at the cross section. \(N_c\) represents the number of division along the inlet and outlet channels and \(Gr\) represents the growth rate (bias factor) from the cells along the channels. Error represents the percentage error between the analytical solution of \(u_{\text{max}}/U_B\) and the present numerical simulation.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>(N_x)</th>
<th>(N_y)</th>
<th>(N_c)</th>
<th>(Gr)</th>
<th>(u_{\text{max}}/U_B)</th>
<th>Error (%)</th>
<th>(w_{\text{max}}/U_B)</th>
<th>(T_{RMS}/T)</th>
<th>Cells (\times 10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mesh-1</td>
<td>25</td>
<td>25</td>
<td>100</td>
<td>100</td>
<td>2.0783</td>
<td>0.943</td>
<td>2.1101</td>
<td>0.2122</td>
<td>0.3321</td>
</tr>
<tr>
<td>Mesh-2</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>2.0938</td>
<td>0.196</td>
<td>2.1892</td>
<td>0.2242</td>
<td>1.2004</td>
</tr>
<tr>
<td>Mesh-3</td>
<td>75</td>
<td>75</td>
<td>100</td>
<td>100</td>
<td>2.0942</td>
<td>0.177</td>
<td>2.2008</td>
<td>0.2273</td>
<td>2.8635</td>
</tr>
<tr>
<td><strong>Group 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mesh-4</td>
<td>50</td>
<td>50</td>
<td>25</td>
<td>100</td>
<td>2.0931</td>
<td>0.229</td>
<td>2.1895</td>
<td>0.2243</td>
<td>0.5904</td>
</tr>
<tr>
<td>Mesh-5</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>2.0932</td>
<td>0.225</td>
<td>2.1894</td>
<td>0.2242</td>
<td>0.7554</td>
</tr>
<tr>
<td>Mesh-2</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>2.0938</td>
<td>0.196</td>
<td>2.1892</td>
<td>0.2242</td>
<td>1.2004</td>
</tr>
</tbody>
</table>

The first chosen criterion to determine the accuracy is the maximum streamwise velocity of the fully-developed part of the inlet channel with the analytical solution for a square duct \(u_{\text{max}}/U_B = 2.0979\) \(^{221}\). The variation of this quantity with increasing mesh refinement for a typical simulation \((Re = 100)\) is shown in Figure 7.2 (a). First, it can be noted that the variation of \(u_{\text{max}}/U_B\) between the meshes is less than 1% when varying the number of cells in the cross section (group 1). The error found for meshes varying the number of cells along the channel (group 2) can be regarded as negligibly small. Previous numerical investigations were carried out by Refs. \(^{85, 173}\) for Newtonian fluids flowing through cross-slot micro-geometries and a symmetry-breaking phenomenon at the stagnation point was observed beyond a critical \(Re\). A steady and symmetric flow for low Reynolds number is replaced by a steady and asymmetric flow beyond a critical
value of \( Re \). A steady symmetric flow at the \( y - z \) outlet plane of the cross-slot geometry (as shown in Figure 7.3 (a)) represents a zero velocity contribution of \( w/U_B \) along the central \( y - z \) line, consequently the diffusive flow mixing regime is dominant (\( w \) represents the spanwise velocity component in the \( z \) direction). Otherwise, a non-zero contribution of \( w/U_B \) indicates flow across the \( y - z \) line, i.e. a breaking of symmetry. The bifurcation parameter is represented by the maximum transverse velocity \( (w_{max}/U_B) \) along the \( y - z \) line at the outlet channel for different Reynolds numbers [85]. In Figure 7.2 (b) the variation of this quantity \( (w_{max}/U_B) \) with varying cell number can be seen and it was found that the error was less than 1% when increasing the number of cells from 1.2 million to 2.86 million.

Another procedure has been carried out to verify the mesh independence by examining the normalised root-mean-square of the temperature, \( T_{RMS}/\overline{T} \) (see Eq. 4.12), at the \( y - z \) outlet plane of the cross-slot geometry with varying the number of cells as shown in Figure 7.2 (c). It was found that the percentage of error is equal to 1.36% when increasing the number of cells from 1.2 million to 2.86 million. Therefore, based on this analysis of the grid dependency, all simulations were carried out using mesh Mesh-2 (1.2 million cells).
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Figure 7.2: (a) Effect of mesh refinement on the maximum normalised velocity ($u_{\text{max}}/U_B$) at the inlet channel in fully-developed region. (b) Evaluation of the transverse maximum velocity ($w_{\text{max}}/U_B$) at the inlet plane to the outlet channel when increasing the number of cells. (c) Evaluation of the normalized root mean square temperature ($T_{\text{RMS}}/\bar{T}$) at the inlet plane to the outlet channel when increasing the number of cells. $Re = 100$ and $Pr = 100$. 

---

The diagrams illustrate the impact of mesh refinement on the maximum normalised velocity and transverse maximum velocity, as well as the normalized root mean square temperature, over varying numbers of cells. Each mesh is represented with distinct markers, highlighting the analytical solutions and refined outcomes as the number of cells increases. The Reynolds number ($Re$) and Prandtl number ($Pr$) are specified as constraints for these simulations.
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(a) Enlarged schematic diagram of the cross-slot geometry

(b) Increasing $Re$

(c) Decreasing $Re$

**Figure 7.3:** (a) Enlarged schematic diagram of the cross-slot micro-geometry depicting the numerical spanned region by $-W/2 \leq y \leq W/2$, $-W/2 \leq z \leq W/2$ in the plane at $x=W/2$, (b, c) presenting transverse velocity profiles along the centreline ($z=0$) with increasing and decreasing Reynolds number, respectively.
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### 7.2 Experimental bulk flow measurements and comparison with numerical results

The Newtonian working fluid used for the experimental measurements was a mixture of glycerine and distilled water with a concentration of 70% glycerine (by weight). The density and viscosity of the mixed solution were measured at the mean fluid temperature (the average fluid temperature between hot and cold inflow streams). The measured values of the density and viscosity are given in Table 7.2. Additionally, in order to measure the temperature distribution in the cross-slot micro-geometry, Rhodamine-B was chosen as a temperature-sensitive fluorescent dye for preparing the temperature-sensitive fluid (0.1 g of Rhodamine-B in one litre of working fluid) [90, 91, 184].

An applied pressure produced from both pressure vessels has been used to drive the working fluid into the cross-slot cell at constant and equal flow rates as mentioned in the experimental chapter in Section 4.2. Steady-state pressure-drop across an inlet/outlet channel pair of the cross-slot geometry was experimentally measured using a differential pressure transducer of high accuracy. Minor pressure losses (the inlet and outlet manifolds and corresponding sudden contraction and expansion) associated with the measured pressure drop are estimated using the traditional relationships used in micro-scale [105, 201] where the percentage of these losses to the major pressure drops ranges from just 0.11% at low flow rate to 2.3% at high flow rate. Thus, the pressure-drop across an individual inlet/outlet pair was approximately equal to that in the pressure-drop in the channel.

The experimental measurements and numerical data of friction factor-Reynolds number product (defined in Eq. 4.7) versus Reynolds number are shown in Figure 7.4. As depicted in Figure 7.4, the present numerical values of $fRe$ as a function of Reynolds number collapse perfectly with the theoretical (Darcy) analysis ($fRe = \ldots$)
57, for straight square cross-section ducts and fully-developed laminar flow \[191\]).
Meanwhile, the experimental results of the \( fRe \) show a good agreement with the theoretical analysis and the present numerical data with a range of Reynolds number from 3 to 80. The maximum deviation between the measured values of \( fRe \) and the Darcy’s equation is within 2.6\%. As illustrated in Figure 7.4, the flow within the cross-slot geometry behaves essentially as flow in a straight channel and any additional pressure loss due to the pressure of the cross, even after the onset of the instability (\( Re > 40 \)) is small and within our measurement uncertainty. For very low Reynolds numbers, there is a small amount of scatter in the measured pressure-drop data which can be attributed to the large relative uncertainties at low flow rates and small pressure drops. The uncertainty of \( fRe \), which is associated with the estimated error of each term in Eq. 4.7, is carefully estimated by following the same manner described in Section 4.4. The minimum and maximum uncertainty values of \( fRe \) are 5.5\% and 8.7\%, respectively. Dimensional inaccuracies may play a significant role in the determination of the \( fRe \) uncertainty (approximately 65\% of the total error of the \( fRe \)) as indicated by Eq. 4.7 \[105, 201\]. The accuracy of the flow rate also becomes more important for small Reynolds numbers.

As previously illustrated, the thermal boundary conditions for the cross-slot cell are that the walls are assumed to be adiabatic (thermally insulated materials) whereas the hot and cold fluid streams are injected into the cross-slot through the opposed inlets. Fundamentally, assuming no heat losses, the heat gained by the cold fluid stream must be equal to the heat rejected by the hot fluid stream. Therefore, the mean temperatures between the inlets and exits should, under these ideal conditions, be equal. The averaged fluid temperatures were measured at the entrances \( (T_{in.C} \) and \( T_{in.H} \)) and exits \( (T_{out}) \) in order to quantify any potential systematic deviation from the ideal conditions. Figure 7.5 shows the non-dimensional mean temperature \( (\theta = \frac{T_{in.C}}{T_{in.H} - T_{in.C}}) \) against flowrate (Reynolds numbers). The
average deviation of non-dimensional mean temperature from the predicted value (0.5) is 6%. The measured values of dimensionless temperature exhibit some slight scatter in the region of symmetric flow (Reynolds number \( \lesssim 40 \) [107, 150]) due to the uncertainty in the temperature measurements at low Reynolds numbers. The average value of dimensionless temperature is typically \( \approx 0.47 \), i.e. an error of 6.6% due to heat losses, except for a few points at very low Reynolds numbers where heat losses are more significant due to longer residence times.

**Table 7.2:** The measured values of density and viscosity for the working fluid (30% water/70% glycerine) across the working temperature range.

<table>
<thead>
<tr>
<th>Temperature ( ^\circ C )</th>
<th>30% water/70% glycerine</th>
<th>( \eta ) (Pa.s)</th>
<th>( \rho ) (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0226</td>
<td>1184.6</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.0186</td>
<td>1183.2</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.0154</td>
<td>1181.3</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.0131</td>
<td>1181.0</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.0116</td>
<td>1179.4</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.0097</td>
<td>1176.2</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>0.0085</td>
<td>1173.5</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 7.4:** Friction factor-Reynolds number product, \( fRe \), versus Reynolds number.

**Figure 7.5:** Friction factor-Reynolds number product, \( fRe \), versus Reynolds number.
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\[
\text{Re} = \frac{(T_{\text{out}} - T_{\text{in,C}})}{(T_{\text{in,H}} - T_{\text{in,C}})}
\]

---

**Figure 7.5:** Normalised mean temperature versus Reynolds numbers.

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### 7.3 Discussion of temperature distribution and flow visualisation

Both the experimental measurements and the numerical simulations provide a quantification of the temperature distribution and hence an understanding of the convective heat transfer in the cross-slot geometry. Independent isothermal experiments were conducted for each experiment to show how a passive scalar was mixed by the flow. The experiments also provide a qualitative picture of the mixing of the two streams (dyed and undyed), which will closely reflect the temperature distribution if it is primarily determined by the fluid mixing. Figure 7.6 (a, d and g) shows images which represent dye visualisation for isothermal flow (both inlets at room temperature $21 \pm 0.5 \, ^\circ C$), whilst the experimental and numerical temperature distribution patterns are shown in Figure 7.6 (b, e and h) and 7.6.
(c, f and i) respectively. These distributions are shown for the center of the cross-slot and also for regions 20W downstream of the stagnation point. The field of view (FOV) is 2.13 mm × 2.13 mm. The numerical temperature distribution in the cross-slot was computed through a half depth (W/2), i.e., top-down integral, of the channel and then compared with the experimental visualisation of the temperature-sensitive dye (top-down view). The isothermal images for Re = 36.6 in Figure 7.6 (a, d and g) illustrate a distinct sharp symmetric boundary between the dyed and undyed inflow streams at the stagnation point and this separation is also observable along the outflow channels, although the boundary between the two streams is not as sharp, indicating there is a very small amount of diffusive mixing. There is certainly no significant mixing between streams at low Reynolds numbers and the flow appears steady, laminar and symmetric. As such, in terms of the heat transfer between the two streams, only conduction/diffusive occurs at the interface. This is clearly shown in the experimental temperature patterns in Figure 7.6 (b, e and h), which shows two streams, one hot and the other cold, separated by a very narrow region where conduction occurs. Although some small variations are observed (particularly in the hot stream) they are primarily associated with the inaccuracy of the imaging technique rather than an indication of any significant heat transfer occurring. The experimental picture is reinforced by the equivalent numerical simulations in Figure 7.6 (c, f and i), which show two sharply separated streams at very different temperature and an extremely narrow region between the two streams where conduction occurs.

Figures 7.7 and 7.8 same as Figure 7.6 but for Reynolds numbers equal to 46.3 and 80.6, respectively. When the Reynolds number increases beyond a certain critical value (≈ 40) [107, 150], it is known that a symmetry-breaking bifurcation occurs at the stagnation point because an axially-oriented spiral vortex is created along the outflow channels. This is apparent in the images in Figures 7.7 (a, d and g) and 7.8 (a, d and g) as this spiral vortex leads to promote the mixing process in
the cross-slot. The two streams are now much less distinct after they have met and the dye through the outlet arms is much more uniform than it was for $Re = 36.6$.

Convective heat transfer arises between the hot and cold fluids as presented in Figures 7.7 and 7.8 (b, e, h, c, f and i). The three-dimensional behaviour of the bifurcated flow enhances the heat transfer between the hot and cold fluids by convection along the outflow channels. For $Re = 46.3$ the experimental temperature distribution still shows some signs of two distinct hot and cold streams, but they are clearly much less distinct than at $Re = 36.6$. This is in reasonably good agreement with the numerical simulations, although the two streams appear more distinctly in the simulations, it is observable that some convective heat transfer has occurred. Where the two streams meet it appears that the cold stream is “dominating” over the hot stream. In reality, in this case, the hot stream has been swept underneath the cold stream and therefore, due to the visualisation technique, it is seen less prominently. The experiments at $Re = 80.6$ in Figure 7.8 (b, e and h) show an increasingly even temperature distribution, whereas, although the simulations in Figure 7.8 (c, f and i) still show two streams, greater mixing is noticeable. In particular a very similar temperature distribution pattern is observed in the experiments and simulations for $Re = 80.6$ where the two streams meet. It is clear that the single central vortex is dominating the flow and entwining the two streams.

Overall, comparison of the asymmetric ($Re > 40$) to the symmetric regime ($Re < 40$) shows that the mixing quality between inflow streams is clearly improved leading to a more even temperature distribution along the outflow channels. This mixing is a direct consequence of the vortex created by the instability. Concerning the numerical simulation (Figure 7.8 (c, f and i)), a single central vortex is observed in agreement with the experiments for a Reynolds number equal to 80.6 \[85\].
To quantitatively assess the enhancement of heat transfer in the outflow channels of the cross-slot, histograms of normalised temperature are constructed for the symmetric ($Re = 36.6$) and asymmetric ($Re = 80.6$) flow regimes using the experimental images and numerical data corresponding to the FOV $20W$ downstream of the stagnation point. Figure 7.9 (a) shows the probability density function (PDFs) of this normalised temperature distribution ($\theta$). As illustrated from Figure 7.9 (a), two peaks are observed in both the experimental and numerical data around $\theta = 0$ (which corresponds to the cold inlet stream temperature) and $\theta = 1$ (which corresponds to the hot inlet stream temperature) for the symmetric flow regime ($Re = 36.6$). This is because there is a clear separation (no mixing) between the hot and cold streams along the outflow channels. There is a quantitative difference between the experiments and simulations in that the peaks in the experimental data are not as distinct. This difference can be attributed to the accuracy of the captured images using the temperature-sensitive fluorescent dye with the digital camera and heat losses. Qualitatively, however, the temperature distribution from experimental and numerical data shows a reasonable agreement.

In the asymmetric flow regime ($Re = 80.6$), the distribution of the normalised temperature from the experiment resembles a Gaussian distribution with its peak at approximately $\theta = 0.6$ as shown in Figure 7.9 (b). This indicates that because of the convective mixing between the hot and cold streams the temperature distribution has been significantly altered towards a more uniform distribution (which would be $\theta = 0.5$ by definition). In fact, there are virtually no values corresponding to either $\theta = 0$ or $\theta = 1$, showing that the extremes of temperature have been eliminated, which is in stark contrast to the symmetric flow regime. The normalised temperature from the numerical simulations shows an approximately flat distribution except at the extremes of temperature, which occur much less frequently, as depicted in in Figure 7.9 (b). Although the simulations show that the temperature is not as uniform as seen in the experiments it is still apparent
from this flat distribution that the spiral vortex has had a significant influence and the distribution is much more uniform than in the symmetric case ($Re = 36.6$). The difference between the experiments and the simulations could be attributed to the imaging method.
Figure 7.6: Symmetric flow behaviour and temperature distribution patterns in the cross-slot micro-geometry, (a, d and g) Grayscale images illustrating the development of isothermal flow structure at the location of cross-slot region and 20W downstream of the stagnation point for Reynolds number = 36.6, (b, e and h) Experimental visualisation of the temperature-sensitive dye and (c, f and i) Numerical temperature distribution at the location of cross-slot region and 20W downstream of the stagnation point.
Figure 7.7: Symmetric flow behaviour and temperature distribution patterns in the cross-slot micro-geometry, (a, d and g) Grayscale images illustrating the development of isothermal flow structure at the location of cross-slot region and downstream of the stagnation point for Reynolds number = 46.3, (b, e and h) Experimental visualisation of the temperature-sensitive dye and (c, f and i) Numerical temperature distribution at the location of cross-slot region and downstream of the stagnation point.
Figure 7.8: Symmetric flow behaviour and temperature distribution patterns in the cross-slot micro-geometry, (a, d and g) Grayscale images illustrating the development of isothermal flow structure at the location of cross-slot region and 20W downstream of the stagnation point for Reynolds number = 80.6, (b, e and h) Experimental visualisation of the temperature-sensitive dye and (c, f and i) Numerical temperature distribution at the location of cross-slot region and 20W downstream of the stagnation point.

\[ \theta = \frac{T - T_{m,C}}{T_{m,H} - T_{m,C}} \]
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Figure 7.9: Experimental and numerical histograms of probability density function (PDF) for the normalised temperature distribution in the 20W downstream of the stagnation point for the cross-slot geometry.
7.4 The transition behaviour between symmetric flow and asymmetric flow

This section focuses on the description of the flow behaviour within the cross-slot micro-geometry. The aim is to understand the topological changes in the flow close to the transition region. Several independent computational simulations have been carried out with increasing Reynolds number and decreasing Reynolds number. The numerical simulations were performed by firstly simulating at very high $Re$, well beyond critical conditions, and then by reducing the Reynolds number in small steps near to the critical condition to evaluate the transitional point from symmetric to asymmetric flow, which is represented by a non-zero value for the normalised transverse velocity $w$ along the centreline ($y = W/2 - y = -W/2$) at the $y-z$ outlet plane (see Figure 7.3 for more details). The variation of initial conditions on the simulations by increasing or decreasing the Reynolds number beyond the critical Reynolds number shows clearly a hysteretic behaviour as has been observed previously [85, 107, 108].

Figure 7.10 presents the maximum values of the normalised transverse velocity ($w_{\text{max}}/U_B$) along the centreline ($y = W/2 - y = -W/2$) at the $y-z$ inlet plane of the outlet channel of the cross-slot for different Reynolds numbers. The critical Reynolds number, $Re_c$, represented by a non-zero value for $w_{\text{max}}/U_B$, differs for each case depending on the initial conditions ($Re_c = 39$ when decreasing and $Re_c = 53$ when increasing), in broad agreement with previously reported values by Haward et al. [85]. Therefore, these two solution branches exist between $Re = 39$ and $Re = 53$ clearly demonstrating that the system is hysteretic. Also depicted in Figure 7.10 is a comparison of the temperature contours (discussed already in detail in section 7.3) which show that an increase of Reynolds number beyond the critical value leads to a more uniform temperature distribution due to convective mixing and heat transfer. The distributions shown for $Re = 46.3$ are
for the decreasing $Re$ branch as the instability has clearly occurred. While, the solution branch for increasing $Re$ that is stable up to $Re = 53$ cannot physically be achieved in the laboratory experiments, presumably because the disturbances inherent in the experiment (e.g. from fluctuations in the inlet velocity or geometric disturbances) are too large for the flow to remain stable/symmetric.

Symmetry-breaking bifurcations occurring in the flow of cross-slot beyond the critical Reynolds numbers ($Re_c$) can transition the flow regime from diffusive to convective mixing. This analysis of the transition from the symmetric flow to the asymmetric flow assists to understand better how the flow regimes in the cross-slot geometry can affect on the fluid mixing and thereby enhancing convective heat transfer.

### 7.5 Heat transfer enhancement in the cross-slot micro-geometry

In order to quantify the improvement of heat transfer in the asymmetric and convective flow regime compared with the symmetric flow regime, the root mean square temperature ($T_{RMS}$) analysis [113] was employed to evaluate the heat transfer between the two different streams (hot and cold) within the outlet channels of the cross-slot micro-geometry with adiabatic walls. This approach is necessary because the Nusselt number cannot be calculated under this thermal condition (as the mean temperature at any cross-sectional plane downstream of the cross is constant from energy conservation). As previously illustrated in Figure 7.10, the flow becomes asymmetric above a critical Reynolds number ($\approx 40$) in the square cross-section cross-slot geometry. Thus, in order to obtain symmetric cases beyond the critical $Re$ a quarter cross-slot was simulated and symmetry was imposed as
Figure 7.10: Numerical results showing the purely-inertial instability and a hysteretic behaviour for a laminar Newtonian fluid flow in the cross-slot micro-geometry. The figure shows the maximum transverse velocity ($w_{\text{max}}/U_B$) along the centerline ($z = 0$) with increasing and decreasing Reynolds number, respectively. The critical Reynolds numbers ($Re_c$) are 53 when increasing and 39 when decreasing. The insets show, (a, d, g) dye visualisation illustrating the development of flow structures for Newtonian fluid flow in the cross-slot micro-geometry, (b, e, h) experimental and (c, f, i) numerical temperature distribution at the location of cross-slot region and 20W downstream of the stagnation point.
the boundary conditions to avoid the instability, therefore ensuring the flow is always symmetric, even at $Re > 40$. It is important to note that poor heat transfer between the two streams is characterised by high values of $T_{RMS}$, as enhanced heat transfer will work to make the distribution more uniform (i.e., uniform temperature distribution) and therefore reduce the $T_{RMS}$.

Figure 7.11 shows the numerical results for $T_{RMS}$ over a range of Reynolds number from 30 to 100 with different Prandtl number simulations (1, 10, 100) in the outlet channels of a cross-slot geometry and also in a quarter-geometry (symmetry-imposed). The $T_{RMS}$ versus an inverted Graetz number, which is defined in Section 4.3 ($Gz^{-1} = x/(WRePr)$), was used to show the behaviour along the length of the outlet channels under different flow conditions. In the case of Reynolds number lower than critical value ($Re < 40$), Figure 7.11 illustrates that the normalised values of $T_{RMS}$ for both cases (symmetry-imposed and cross-slot) collapse entirely along the outlet channel in the same pattern even with varying Prandtl number (1 - 100) due to the poor mixing and diffusion-dominant heat transfer. This is completely consistent with the experimental and numerical temperature contours in Figure 7.6 which shows the high segregation between the hot and cold fluid streams. In contrast, for Reynolds numbers greater than the critical value ($Re > 40$), Figure 7.11 shows that the normalised values of $T_{RMS}$ for the cross-slot geometry have effectively enhanced heat transfer between the hot and cold streams (as presented previously in Figures 7.7 and 7.8) for all Prandtl numbers. In addition, the normalised $T_{RMS}$ in Figure 7.11 exhibits different mixing patterns depending on Prandtl number where, a fluid with $Pr >> 1$ would have much higher momentum diffusivity than thermal diffusivity [97, 191].

The $T_{RMS}$ ratio between symmetric (symmetry-imposed) and asymmetric (cross-slot) cases ($\frac{(T_{RMS})_{Sym}}{(T_{RMS})_{Asym}}$) was adopted to evaluate the enhancement of heat transfer as shown in Figure 7.12 it is important to note that the values of $T_{RMS}$ for symmetric flow regime are considered higher than the $T_{RMS}$ for
asymmetric flow regime by definition (see Eq. 4.12). The enhancement of heat transfer exhibits a strong dependency on Prandtl number as depicted in Figure 7.12 (a, b and c). For numerical simulations at $Pr = 1$ (momentum diffusivity equal to thermal diffusivity), the heat transfer enhancement increases monotonically from approximately 1.6 to 3.7 with increasing Reynolds number from 40 to 100, respectively, as shown in Figure 7.12 (a) due to the effect of the instability. However, for high Prandtl number (10 and 100), the enhancement of heat transfer reduces significantly and deviates from its monotonic trend with $Re \geq 70$ ($Re = 70$ and $Re = 100$, see Figure 7.12 (b) and (c)) due to the slower rate of molecular thermal diffusion causing a slight drop in the efficiency of the heat transfer along the outlet channel. In fact, for $Pr = 10$ and $Pr = 100$, a $Re$ of 50 gives superior heat transfer performance far downstream of the cross centre than $Re = 70$ and $Re = 100$. Closer to the cross centre this is not the case and the monotonic trend with $Re$ can still be observed. Therefore, this unusual behaviour at $Re = 70$ and 100 is may be also due a hydrodynamic development length effect (from the centre of the cross) related to the decay of the axially-oriented spiral vortex formed by the instability. To make this more clear, the bifurcation parameter $w_{max}/U_B$ versus $Gz^{-1}$ for $Pr = 10$ has been plotted in Figure 7.13 and comparing with the normalised $T_{RMS}$ values along the outlet channels for the same Prandtl number (see Figure 7.11 (b)). It is clear that the vortex intensity drops rapidly at a certain $Gz^{-1}$ indicating a transition from a convective to a conduction dominated regime. For the higher Reynolds numbers with the unusual behaviour ($Re = 70$ and 100) this occurs at a lower $Gz^{-1}$. This indicates that the point at which the vortex is largely eliminated has a non-trivial relationship to $Re$, which the scaling with $Gz^{-1}$ does not fully resolve. However, this would only be of primary concern for very long outlet channel lengths. After the vortex has been eliminated conduction dominates and it is seen that the $T_{RMS}$ curves start to converge and the heat transfer enhancement plateaus and then starts to drop ($Pr = 100$ has not reached this point, but it would be expected at higher $Gz^{-1}$). In the extreme case where
the length of the channel tends towards infinity the flow would approach uniformity such that $T_{RMS}$ would approach zero and the enhancement measure would be unity (i.e. no enhancement). However, the advantage of the instability is that it can provide a much higher level of heat transfer in a relatively short channel.

The inertial instability in the cross-slot micro-geometry with adiabatic walls has shown a significant capacity to enhance the heat transfer for different Prandtl number. However, it can be noted that the variation in the behaviour with varying $Pr$ shows some complexity.

![Figure 7.11: Normalised $T_{RMS}$ calculations as function of inverted Graetz number for different Prandtl numbers (1, 10 and 100 respectively). Dashed lines represents symmetric cases, which were numerically imposed, and solid lines represents asymmetric cases for different values of $Re$.](image-url)
Figure 7.12: Heat transfer enhancement: $T_{RMS}$ ratio calculated between the symmetric and asymmetric cases for values of $T_{RMS}$ and $Re$ shown in Figure 7.11 (a, b and c) and different Prandtl number (1, 10 and 100 respectively).

Figure 7.13: The bifurcation parameter (the normalised transverse maximum velocity, $w_{max}/U_B$) versus inverted Graetz number over the range of Reynolds numbers (30 - 100) for Prandtl number = 10.
7.6 Summary

The behaviour of fluid flow and heat transfer in the cross-slot geometry were investigated through experimental measurements and supporting numerical simulations in this chapter. Experimentally the walls were approximately adiabatic and the hot and cold fluid streams were injected into the cross-slot cell through opposed inlets over a range of Reynolds number from 3 to 80.6. Temperature distribution patterns have been successfully obtained in the cross-slot micro-geometry using a temperature-sensitive fluorescent dye (Rhodamine-B). A numerical model of the cross-slot micro-geometry was created to clarify the effect of the purely-inertial flow instability on the heat transfer involved in the current experimental design.

It is seen that when the Reynolds number increases above a certain critical value ($Re \geq 40$) [85, 107, 150], there is the formation of an axially-oriented spiral vortex. This spiral vortex acts to enhance the mixing of the fluid along the outlet channels and comparisons with equivalent results for the symmetric regime show that the heat transfer performance is clearly improved.

Numerical simulations with boundary conditions matching the experiments agree well with the experimental results, particularly with respect to the critical $Re$ of the instability, although there are some quantitative differences in the details of the captured temperature distributions, which can be attributed to the limitations of the experimental imaging technique. A hysteretic behaviour was observed in the simulations, which was absent from the experiments, and showed the existence of a solution branch on which the instability was delayed until $Re = 53$ (similar effects were observed in the isothermal study of Haward et al [85]). These simulations suggest that the enhancement of heat transfer increases with increasing $Re$, although for long outlet channels (in which the vortex has decayed) the picture is less clear and the data for $Re > 70$ exhibits a complex behaviour. However, the
asymmetric flow field beyond the instability always exhibits an enhancement of the heat transfer.
Chapter 8

Viscoelastic fluid flow and heat transfer in a cross-slot geometry

A great deal of recent literature, both experimental [13, 56, 79, 84, 164] and numerical [1, 6, 19, 42, 43, 158, 169, 170], has focused on viscoelastic fluid flow through a cross-slot geometry over a wide range of Weissenberg/Deborah numbers and aspect ratios using a large variety of viscoelastic fluids and constitutive models. The main intention in this chapter is to conduct a preliminary investigation on the extensional flow instabilities (symmetry-breaking bifurcation) that arise in the elongational or extensional flow of polymeric viscoelastic fluids in a cross-slot geometry with the absence of inertial effects as a means of boosting thermal mixing (temperature distribution) between two inflow streams at different temperatures and thereby enhancing convective heat transfer.

Details of the experimental set-up and test section for the cross-slot micro-geometry and the measurement instruments have been described in the experimental chapter (Chapter 4). The thermal boundary conditions for the cross-slot cell are that the surfaces are assumed to be adiabatic whereas the hot and cold fluid streams are injected into the cross-slot through the opposed inlets. Rhodamine-B has been
employed as a temperature-sensitive fluorescent dye to measure the temperature distribution in the cross-slot micro-geometry.

A comprehensive set of experimental measurements of the flow of a dilute viscoelastic fluid (100 ppm PAA in a 70%-glycerine-based solution, 100-W/70GLY) through the cross-slot micro-geometry is presented in this chapter. The rheological properties of the tested viscoelastic fluid have been characterised in the rheology chapter (Chapter 3). The experimental measurements include pressure drop and fluid temperature measurements as well as flow visualisation and temperature distribution over a wide range of Weissenberg numbers.

This chapter is organised as follows: Section 8.1 discusses experimentally pressure losses associated with the steady motion of viscoelastic fluid in the cross-slot micro-geometry. While, in Section 8.2 measurements of mean temperature in the cross-slot geometry are discussed and presented. Finally, the characterisation of temperature distribution and flow patterns are presented and discussed in Section 8.3.

8.1 Discussion of pressure drop measurements

Details of the measurements of pressure drop for the cross-slot geometry are previously described in the experimental chapter in Section 4.2.3. The pressure-drop measurements across the cross-slot geometry were obtained with flow into both opposing inlets and out of both opposing outlets (see Figure 4.11 for more details) using an accurate differential pressure transducer with two different diaphragms in order to cover the whole range of applied pressures. The pressure drop associated with flows of the Newtonian fluid (W/70GLY) and the viscoelastic fluid (100-W/70GLY) through the cross-slot micro-geometry as a function of flow rate is shown in Figure 8.1. The pressure drop of the Newtonian fluid (W/70GLY)
through the cross-slot micro-geometry was measured to serve as a baseline for comparison with the viscoelastic fluid and to evaluate the performance of the experimental system.

![Graph](image)

Figure 8.1: Pressure drop measured in the cross-slot micro-geometry versus flow rate for the Newtonian fluid (W/70GLY) and the viscoelastic fluid (100-W/70GLY).

First note that a comparison between the pressure drop measured for the Newtonian fluid (W/70GLY) and the viscoelastic fluid (100-W/70GLY) shown in Figure 8.1 confirms that the steady-state pressure drop for the W/70GLY increases linearly over the entire range of flow rates as expected. For the 100-W/70GLY, the pressure drop is seen to increase linearly at low flow rates (up to approximately 0.5 ml/min) whilst, with further increasing of the flow rate, the pressure drop begins to deviate from the linear viscous fluid behaviour due to the onset of strong elastic effects, which is accompanied by a noticeable increase in the pressure losses [77, 78, 180]. The pressure-drop measurements for the viscoelastic fluid flows are greater than those for the Newtonian fluid flows within the tested range of flow rates. At low flow rate, this increase in the pressure drop is as a direct consequence
of the existence of polymer molecules (PAA) making the fluid more viscous than the comparator fluid [16, 20].

The Darcy friction factor, \( f = 2\Delta PW/\rho lU_B^2 \) where \( U_B \) is a bulk flow velocity, \( W \) is the depth of the square cross-section cross-slot geometry and \( l \) is the total length of the inflow and outflow channels of the cross-slot geometry and \( \rho \) is the fluid density, has been employed to express the dimensionless pressure drop in the cross-slot geometry. The friction factor-Reynolds number product, \( fRe \) (described in Eq. 4.7), is shown in Figure 8.2 as a function of Reynolds number for both Newtonian and viscoelastic fluids. Figure 8.2 indicates that the values of \( fRe \) for the Newtonian fluid (W/70GLY) agree very well with the theoretical (Darcy) equation \( (fRe = 57, \text{for straight square cross-section ducts}) \) for fully-developed laminar flow (\( Re = 0.5 - 15.2 \)). The maximum deviation between the measured values of \( fRe \) and the well-known Darcy’s formula is within ±2.2%.

This behaviour of the dimensionless pressure-drop for the Newtonian fluid flows within the cross-slot illustrated in Figure 8.2 confirms that inertial contributions (inertial instabilities) to the fully-developed pressure drop are negligible for this tested range of Reynolds numbers (see Chapter 7 where flow remains symmetric for \( Re < 40 \)). Therefore, the experiments with Newtonian fluids emphasise the validity of the present experimental techniques and measuring system.

In contrast, the \( fRe \) values for the viscoelastic fluid (100-W/70GLY), which was selected to understand the effects of elastic instabilities on the behaviour of heat transfer in the cross-slot geometry, is also shown in Figure 8.2. The measured values of \( fRe \) for this viscoelastic solution demonstrate a significant increase compared with the Newtonian solution (W/70GLY) over the same range of \( Re \). In the viscoelastic fluid flow (even with very low \( Re \)) the combination of extensional flow and the nonlinear properties of viscoelastic solution features within the cross-slot micro-geometry give rise to large pressure drops [13, 79, 83, 84, 164, 198]. This extensional flow between the two inflow streams at the stagnation point is probably
responsible for elastic instabilities and thus generating a large increase in the pressure drop above a certain flowrate (Weissenberg number) [23, 37, 152, 156, 180].

The normalised friction factor-Reynolds number product - essentially the pressure-drop normalised by a viscous stress (by definition, Reynolds number represents the ratio between the momentum flux and viscous shear stress [221, 222]) - is shown in Figure 8.3 as a function of Weissenberg number ($5.8 \leq Wi \leq 136$). The values of $fRe_{(N-Newt.)}$, normalised by $fRe_{(Newt.)}$ for laminar fully-developed Newtonian fluid flow in a straight square duct ($fRe = 57$) [191, 222], measured experimentally through the cross-slot geometry increase monotonically from the Newtonian limit at low $Wi$. Above a critical Weissenberg number ($Wi_c$), a rapid increase in the normalised values of $fRe_{(N-Newt.)}$ is observed before levelling off due to the onset of elastic instabilities. As the Weissenberg number is increased still further, the normalised values of $fRe_{(N-Newt.)}$ appear to saturate, plateauing at a value of $(fRe_{(N-Newt.)}/fRe_{(Newt.)}) \approx 2.4$. This ultimate saturation in the
normalised values of $fRe_{(N-Newt.)}$ observed in the current experiments is probably attributable to a sustained elastic turbulent state \cite{47,74}.

![Figure 8.3](image)

**Figure 8.3:** Normalised friction factor-Reynolds number product, $fRe$, versus Weissenberg number, $Wi$, for the viscoelastic fluid (100-W/70GLY) in the cross-slot micro-geometry.

From the previous publications \cite{1,6,13,19,42,43,56,79,84,158,164,169,170}, it is known that, the planar elongational flow field produced by the cross-slot device develops firstly at low $Wi$ from a stable symmetric (Newtonian-like) flow to a stable asymmetric (symmetry-breaking bifurcation) flow and eventually to an unsteady asymmetric (time-dependent) flow and the direction of asymmetry flips between two outflow channels with time. Figure 8.3 exhibits that for $Wi < 25$ the non-dimensional pressure drop increases monotonically from the Newtonian limit. With further increasing $Wi$ ($Wi > 25$), the normalised values of $fRe_{(N-Newt.)}$ rise significantly as a result of the development of purely-elastic instabilities from a stable asymmetric (symmetry-breaking bifurcation) state to an unsteady asymmetric (time-dependent) regime.
8.2 Discussion of mean temperature measurements

As mentioned previously in the experimental chapter, the cross-slot cell was configured from thermally-insulated materials to ensure no (or minimum) heat loss to the surroundings. The mean fluid temperature at the inlets and outlets of the cross-slot geometry was electronically measured using K-type thermocouples, which were placed at each cylindrical hole of the cross-slot cell as shown in Figures 4.10 and 4.11.

The variation of measured fluid temperatures at the inflow and outflow locations of the cross-slot geometry against Weissenberg numbers is presented in Figure 8.4 for the viscoelastic fluid (100-W/70GLY). The mean cold fluid temperature, $T_{in,C}$, at the inlet of the cross-slot is approximately constant at the room temperature (20 ± 0.5°C) with varying Weissenberg numbers ($Wi = 5.8 - 136$) as depicted in Figure 8.4. While, the mean temperature of hot fluid, $T_{in,H}$, at the inlet of the cross-slot varies from 25.8 °C to 48.2 °C with increasing Weissenberg number due to the heat loss through the transport pipelines (flexible Tygon tube, outer diameter 6mm) depending on the fluid residence time. The averaged fluid temperatures at the exits, $T_{out}$, fall between the hot and cold fluid temperatures at the entrances as predicted from an energy balance (see Figure 8.4).

The energy balance between the hot and cold fluid streams is assessed assuming no or negligible heat losses to the surroundings to validate the current thermal boundary conditions. Therefore, the heat gained by the cold fluid stream should be equal to the heat rejected by the hot fluid stream (assuming equal flow rates in each inlet channel). According to the energy balance between the hot and cold flowing streams in the cross-slot geometry, the non-dimensional mean temperature
is then given by,

\[ \theta = \frac{T_{\text{out}} - T_{\text{in,C}}}{T_{\text{in,H}} - T_{\text{in,C}}} \quad (8.1) \]

From Eq. 8.1 if the mean fluid temperature at the exits (\(T_{\text{out}}\)) equals to the average between the hot (\(T_{\text{in,H}}\)) and cold (\(T_{\text{in,C}}\)) fluid temperatures at the entrances, \(\theta\) will equal to 0.5 and there is no heat losses to the ambient as assumed. Otherwise, the values of \(\theta\) will deviate from the expected value (0.5). Figure 8.5 illustrates the non-dimensional mean temperature (\(\theta\)) versus Weissenberg number. The average deviation of non-dimensional mean temperature from the predicted value (0.5) is a 6.7\%. The average value being \(\theta \approx 0.47\), which is slightly less than 0.5 due to heat losses.

**Figure 8.4:** The variation of measured mean fluid temperatures at the inlets and outlets of the cross-slot cell against Weissenberg numbers for the viscoelastic fluid (100-W/70GLY). The solid lines represent the best fitting lines for experimental data.
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8.3 Observation and characterisation of temperature distribution and flow patterns

Flow visualisation was utilised to observe and characterise the behaviour of flow and temperature distribution in the cross-slot region and 20W downstream of the cross with increasing Weissenberg number using the technique described in Section 4.2.6.

First of all, a set of isothermal experiments was conducted to understand the flow behaviour in the cross-slot using viscoelastic fluids. Fluorescent dye (Rhodamine-B) at a concentration 0.1 g in one litre of working fluid was added for visualisation. The coloured solution of Rhodamine-B was forced into one inlet of the cross-slot.
cell, whilst undyed solution was forced into the other inlet. In this case, both inflow fluid streams were set at room temperature. Secondly, the dyed fluid was forced from one pressure vessel at room temperature and from the other pressure vessel with a heating element to generate two inflow streams at different temperatures as described in the experimental chapter in Section 4.2.1.

The flow behaviour of the Newtonian fluid has been detailed in Chapter 7. To briefly recap, the snapshots a, d and g in Figure 7.6 for Newtonian fluid (W/70GLY) at $Re = 36.6$ reveal a sharp interface at the stagnation point and $20W$ downstream of the cross-slot between the coloured (bright) and uncoloured (dark) fluid streams. The flow remains steady, laminar and symmetric and there is no significant mixing at this range of Reynolds number except a very small amount of diffusive mixing. Consequently, this is indicated by two sharply separated temperature distributions between the hot and cold flowing streams as depicted experimentally in images b, e and h and numerically in images c, f and i in Figure 7.6. Therefore, within this tested range of Reynolds number ($Re < 36.6$), an inertial instability is not expected to occur in the cross-slot geometry. For all experiments performed with the viscoelastic fluid the Reynolds number was significantly less than 36.6, making it possible to neglect inertial influences.

As is well-known, the flow behaviour of viscoelastic fluids is entirely different (more complex) than that found for Newtonian fluids in micro-scale geometries. Viscoelastic fluid (100-W/70GLY) has been employed to investigate the flow behaviour and temperature distribution in the cross-slot micro-geometry with the aim of understanding the effect of elastic instabilities on the temperature distribution in this micro-device. The large viscosity ($\eta_o = 0.75 \text{ Pa.s}$ at a shear rate $\dot{\gamma} = 0.01 \text{ 1/s}$) and relatively long relaxation times ($\lambda_o = 0.675 \text{ s}$) for the 100-W/70GLY fluid eliminate inertial influences while permitting flows to be studied at high Weissenberg numbers. In addition, variations in the rheological properties of the viscoelastic fluid (100-W/70GLY) can occur due to thermal mixing between
the hot and cold streams. Thus, time-temperature superposition with a shift factor was employed to adjust the shear rate, viscosity and relaxation time to the desired temperature (see more details in the rheology chapter in Section 3.6).

In Figure 8.6, the grayscale images represent isothermal dye visualisation for the flow of viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point at low Weissenberg number, \( Wi = 5.8 (Re = 0.06) \). At such extremely low Reynolds number (\( Re << 1 \)), the impact of inertia is basically negligible, and hence the instabilities in the flow of viscoelastic fluid only occur as a result of elastic effects. Three different instants of dye visualisation images d, e and f in Figure 8.6 of the isothermal flow patterns at the centre of cross-slot show that the interface between the coloured (bright) and uncoloured (dark) viscoelastic fluid streams is slightly deformed (symmetry-breaking) and becomes wavy, although the two streams are still distinct. In addition, the flow patterns are still essentially steady-state and do not alter significantly with time although there is some weak time dependency as can be seen in d, e and f in Figure 8.6. This viscoelastic flow behaviour is totally consistent with previous experimental [13, 80, 82, 84, 198] and numerical [42, 43, 169, 170] studies. Images a, b, c, g, h and i in Figure 8.6 display the flow field at the regions 20W downstream of the stagnation point over three different instants. These images reveal a sharp interface between the coloured and uncoloured viscoelastic fluid streams except for an extremely small amount of diffusive mixing.
Figure 8.6: The behaviour of isothermal flow (dye visualisation patterns) of the viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point for $Wi = 5.8$ ($Re = 0.06$).
(a) Instant-1  (b) Instant-2  (c) Instant-3
(Three different instants at 20W downstream of the stagnation point)

(d) Instant-4  (e) Instant-5  (f) Instant-6
(elucidative temperature distribution patterns at different instants of the oscillation cycle at
the centre of the cross-slot)

(g) Instant-7  (h) Instant-8  (i) Instant-9
(Three different instants at 20W downstream of the stagnation point)

$$\theta = \frac{T - T_{m,C}}{T_{m,H} - T_{m,C}}$$

**Figure 8.7:** Experimental visualisation of the temperature-sensitive dye of the viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point for $Wi = 5.8$ ($Re = 0.06$).
In this case ($Wi = 5.8$), the elastic instability (symmetry-breaking bifurcation) between inflow streams is inadequate to strongly mix the viscoelastic fluid flowing through the cross-slot micro-geometry. Images in Figure 8.7 denote the temperature distribution in the cross-slot region and $20W$ downstream of the stagnation point. Because of the insufficient mixing occurring between the inflow streams at $Wi = 5.8$, the temperature distribution images in Figure 8.7 show two distinctly separated streams at very different temperatures in the centre of the cross-slot and, even at the $20W$ distance downstream away from the stagnation point, an extremely narrow region between the two streams where conduction occurs. We believe the small differences at the top parts of images g, h and i in Figure 8.7 are due to experimental artefacts.

When Weissenberg number is increased to $Wi = 48.6$ ($Re = 1.02$), the flow field for the 100-W/70GLY fluid becomes more clearly time-dependent. This is captured through a series of isothermal grayscale images captured at different times depicting the different flow patterns as shown in Figure 8.8. The grayscale images d, e and f in Figure 8.8 at the centre of the cross-slot illustrate that the interface between the coloured and uncoloured viscoelastic fluid streams oscillates and it is not as sharp as the stable asymmetric flow case at $Wi = 5.8$. The extensional flow instabilities lead to stretching and folding of viscoelastic fluid elements and the asymmetric fluid direction flips between the two outflow channels with time [13, 80, 82, 84, 169, 170, 198]. The time-dependent elastic instability in the cross-slot micro-geometry causes distorted regions of coloured and uncoloured fluid along the outflow channels with time (the coloured fluid no longer divides equally between the two outflow streams) as shown in the regions $20W$ downstream of the stagnation point in Figure 8.8. Thus, this complex structure of viscoelastic fluid flow arising in the cross-slot geometry can promote mixing.

The instantaneous experimental temperature distribution of the 100-W/70GLY fluid at the centre of the cross-slot is presented in Figure 8.9 for $Wi = 48.6$. The
hot and cold inflow streams overlap and mix together as a result of the time-dependent purely-elastic instability. Although the experimental temperature distribution still exhibits some signs of two distinct hot and cold streams, they are obviously much less distinguishable than at $Wi = 5.8$. The instantaneous temperature distribution at the regions $20W$ downstream of the stagnation point indicates that some convective heat transfer occurs between the hot and cold streams. It is observable that this thermal mixing between the hot and cold streams is attributed to the unsteady flow. As the Weissenberg number is further increased to $Wi = 118.5$ ($Re = 7.26$), the effect of unsteady elastic instability on the quality of fluid mixing in the cross-slot is significantly increased as shown in the grayscale images in Figure 8.10. The inflow streams of viscoelastic fluid that met at the stagnation point in the centre of the cross-slot are most strongly stretched because the strain rate is highest. This improvement in the fluid mixing leads to enhancement in the convective heat transfer between the hot and cold fluid streams as illustrated in Figure 8.11. The three different instants of the temperature patterns at the regions $20W$ downstream of the stagnation point shown in Figure 8.11 reveal that the temperature distribution has been significantly modified towards a more uniform temperature distribution between the hot and cold streams.
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Figure 8.8: The behaviour of isothermal flow (dye visualisation patterns) of the viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point for $Wi = 48.6$ ($Re = 1.02$).
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Figure 8.9: Experimental visualisation of the temperature-sensitive dye of the viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point for $Wi = 48.6$ ($Re = 1.02$).
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Figure 8.10: The behaviour of isothermal flow (dye visualisation patterns) of the viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point for $Wi = 118.5$ ($Re = 7.26$).
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Figure 8.11: Experimental visualisation of the temperature-sensitive dye of the viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also for the regions 20W downstream of the stagnation point for $Wi = 118.5$ ($Re = 7.26$).
Thermal mixing between two inflow streams in the outflow channels of the cross-slot is quantitatively assessed via histograms of normalised temperature for the experimental images at 20W downstream of the stagnation point. The histograms are constructed for two flow regimes: weakly stable asymmetric (symmetry-breaking bifurcation) flow regime ($Wi = 5.8$, $Re = 0.06$) and unsteady asymmetric (time-dependent) flow regime ($Wi = 118.5$, $Re = 7.26$). The probability density function (PDFs) is evaluated by taking the average of the normalised temperature distribution ($\theta$) for the three different instants with the camera frame rate 10 FPS (shown in Figures 8.7 and 8.11) at 20W downstream of the stagnation point for both viscoelastic flow regimes as illustrated in Figure 8.12.

As shown from Figure 8.12 (a), the PDFs shows peaks around $\theta = 0$ (which corresponds to the cold inlet stream temperature) and $\theta = 1$ (which corresponds to the hot inlet stream temperature) for the stable asymmetric (symmetry-breaking bifurcation) flow regime ($Wi = 5.8$). This is because of the insufficient mixing between the hot and cold streams along the outflow channels as depicted in Figure 8.7. While, in the unsteady asymmetric (time-dependent) flow regime ($Wi = 118.5$), the distribution of the normalised temperature resembles a Gaussian distribution with its peak at $\theta = 0.5$ as shown in Figure 8.12 (b). This indicates that because of the convective mixing between the hot and cold streams the temperature distribution has been significantly altered towards a more uniform distribution as shown earlier in Figure 8.11.

In summary, the unsteady asymmetric (time-dependent) instability observed for viscoelastic flow in the cross-slot micro-geometry can enhance convective heat transfer.
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(a) Stable asymmetric (symmetry-breaking bifurcation) flow regime ($Wi = 5.8$, $Re = 0.06$)

\[ \theta = \frac{(T - T_{in,C})}{(T_{in,H} - T_{in,C})} \]

(b) Unsteady asymmetric (time-dependent) flow regime ($Wi = 118.5$, $Re = 7.26$)

\[ \theta = \frac{(T - T_{in,C})}{(T_{in,H} - T_{in,C})} \]

**Figure 8.12:** Histograms of probability density function (PDF) for the normalised temperature distribution in the 20W downstream of the stagnation point for the cross-slot geometry.
8.4 Summary

A preliminary investigation of the viscoelastic fluid flow behaviour and temperature distribution in the square cross-section cross-slot micro-geometry was studied experimentally in this chapter. The walls of the cross-slot geometry were approximately adiabatic and the hot and cold viscoelastic fluid streams were injected into the cross-slot cell through opposed inlets over a range of Weissenberg number from 5.8 to 136. A temperature-sensitive fluorescent dye (Rhodamine-B) has been used to measure and observe temperature distribution patterns in the cross-slot micro-geometry. The evolution of flow structures and temperature distribution were observed and characterised in the region of cross-slot and also 20W downstream of the stagnation point with increasing Weissenberg number. For all viscoelastic fluid experiments performed, the Reynolds number was significantly less than the critical Reynolds number ($\approx 40$ [107, 150] see Chapter 7 for more details) for inertial instability in the cross-slot geometry. Therefore, it is possible to neglect significant inertial influences.

The experimental measurements of pressure drop associated with flows of the viscoelastic fluid (100-W/70GLY) through the cross-slot micro-geometry were shown that, for $Wi < 25$, the non-dimensional pressure drop increases monotonically from the Newtonian limit whereas, with further increasing $Wi$ ($Wi > 25$), the normalised values of pressure drop rise significantly as a result of the development of extensional elastic instabilities from a weakly stable asymmetric (symmetry-breaking bifurcation) state to an unsteady asymmetric (time-dependent) regime.

In the case of weakly stable asymmetric flow instability ($Wi = 5.8$), the insufficient mixing occurring between the inflow viscoelastic streams in the cross-slot geometry, the field of temperature distribution shows two distinctly separated streams at very different temperature in the centre of the cross-slot and, even at the 20W distance downstream away from the stagnation point, an extremely narrow region
between the two streams where conduction occurs. While, the complex structure of viscoelastic fluid flow arisen in the cross-slot geometry in the case of unsteady elastic instability (time-dependent elastic instability) is seen to promote fluid mixing. Therefore, the temperature distribution has been significantly changed towards a more uniform temperature distribution between the hot and cold streams at the regions $20W$ downstream of the stagnation point.
Chapter 9

Conclusions and future recommendations

This thesis has presented a systematic experimental investigation of the hydrodynamics and heat transfer characteristics of Newtonian and viscoelastic flows in two micro-scale geometries (a serpentine microchannel and a cross-slot micro-geometry). Two different regimes of instabilities, i.e. purely-inertial and purely-elastic, have been employed to enhance heat transfer in these micro-systems. Although summaries have been included in each chapter of the results (Chapter 5, Chapter 6, Chapter 7 and Chapter 8) in this thesis, in this final chapter, the key findings from each chapter will be collated to provide an overall summary. This chapter is organised as follows: the conclusions drawn from this work are initially separated into four subsections in Section 9.1 and recommendations for future research in the field are provided in Section 9.2.
9.1 Conclusions

9.1.1 Serpentine microchannel geometry

Newtonian fluid flow and heat transfer

A comprehensive experimental investigation of the fluid flow and heat transfer characteristics was carried out using mixtures of glycerine/water as working fluids within a square cross-section serpentine microchannel of unity curvature ratio \((R_i/W = 1)\) as detailed in Chapter 5 of this thesis. The averaged Prandtl numbers (based on the mean film temperature) for 30/70 and 10/90 per cent by weight of glycerine/water are 137 and 1038, respectively. Steady-state, fully developed laminar flow at inlet and thermally-developing conditions were investigated with the upper wall insulated and other side walls held at constant temperature over a range of Dean number from 6 to 80 for \(Pr = 137\) and from 0.6 to 4.7 for \(Pr = 1038\). A complete model of the serpentine microchannel comprising 10 repeating units together with the inlet and outlet sections was developed to simulate steady-state flow behaviour and convective heat transfer characteristics. The conservation equations were solved using the commercial Computational Fluid Dynamics (CFD) software package Fluent with the necessary boundary conditions to simulate the experimental set-up.

The following key conclusions for the Newtonian fluid flow and heat transfer in the serpentine microchannel geometry have been determined from Chapter 5:

1. The experimental and numerical findings of dimensionless pressure drop, \(fRe\), of the serpentine microchannel for two Prandtl numbers exhibit an excellent agreement where the deviations of \(fRe\) between the experimental measurements and simulated data are 0.9% and 0.8% with \(Pr = 1038\) and 137, respectively. Also, the present numerical values of \(fRe\) collapse with
the predicted values of the numerical data for a constant curvature single bend from Hwang and Chao \cite{93}.

2. The experimental results of the averaged Nusselt number show good agreement with the present numerical data where the deviations of the mean Nusselt number between the experimental results and numerical data are 10.7\% and 8.6\% with $Pr = 1038$ and 137, respectively (i.e. well within the uncertainty of the experiments).

3. At low Dean number $K = 0.53$, the fluid flow along the axial direction of the serpentine microchannel is largely invariant where the velocity profile is almost constant through the entire flow path-length because of the dominance of viscous forces, which suppress the formation of secondary flow. With increasing Dean number to $K = 106.5$, the inertial forces begin to dominate and the formation of Dean’s vortices (a pair of counter-rotating vortices) in the cross-section and their spatial development along the axial flow direction encourage fluid mixing and thereby enhancing convective heat transfer in the serpentine microchannel.

4. The alternating shape of the serpentine microchannel coupled with the increasing value of Dean number can lead to “chaotic advection” in the flow. Both the chaotic advection and the secondary flow vortices encourage the mixing of fluid elements in the cross-sectional and axial directions of the serpentine microchannel compared to the straight microchannel. The mixing ensures a more homogenous temperature distribution, which in turn leads to an enhanced convective heat transfer within the serpentine microchannel compared to the convective heat transfer in a straight microchannel at the same value of Reynolds number.
5. As a result of the existence of secondary flows in the flow fields of the serpentine microchannel geometry, the heat transfer performance of the serpentine microchannel is higher than that of the equivalent straight microchannel with the same cross-section over the entire range of Reynolds numbers (Dean numbers). Simultaneously, the relative pressure-drop losses in the serpentine microchannel rise with increasing Dean number. However, this increase is much smaller over the whole range of Dean number than the heat transfer enhancement. The maximum percentage increase of relative pressure-drop losses for both Prandtl numbers, 1038 and 137, is up to 2% and 39%, respectively, whereas the enhancement of heat transfer for $Pr = 1038$ and 137 increases by 56% and 158%, respectively.

**Viscoelastic fluid flow and heat transfer**

The characteristics of convective heat transfer and fluid flow within a square cross-section serpentine microchannel with the upper wall insulated and other side walls held at constant temperature were experimentally studied for two groups of polymeric viscoelastic fluids, shear-thinning solutions (PAA-W/GLY) and approximately constant-viscosity Boger solutions (PAA-W/SUC). The elastic turbulence was created by the non-linear interaction between elastic stresses generated within the flowing high-molecular-weight polymer solutions and the streamline curvature of the serpentine geometry. In order to confirm elastic turbulence in this serpentine microchannel, pressure drop across the serpentine channel was measured.

Based on the experimental results discussed in Chapter 6, the following main conclusions can be summarised:

1. Above the elastic instability threshold, the flow of viscoelastic solutions (both shear-thinning and Boger solutions) in the serpentine microchannel exhibits
a significant increase in the flow resistance. This flow resistance is accompanied by a noticeable increase in the pressure-drop losses. Therefore, the friction factor values for these viscoelastic solutions demonstrate a significant increase compared with the Newtonian solution over the same range of Reynolds number. The large increases in the non-dimensional pressure drop for the viscoelastic solutions are probably attributable to the appearance of elastic turbulence above a certain flowrate where a secondary flow may develop due to the combination of an elastic normal-stress difference and streamline curvature [77, 78, 172, 208, 232].

2. The values of the product of friction factor-Reynolds number for the viscoelastic solutions ($fRe_{(N-Neut.)}$) normalised by friction factor-Reynolds number product for laminar fully-developed Newtonian fluid flow ($fRe_{(Neut.)}$) in a straight square duct versus Weissenberg number indicated that, for $Wi < 5$, the normalised non-dimensional pressure drop is similar to that for a Newtonian fluid (the values lie on the Newtonian limit) and with further increasing Weissenberg number ($5 < Wi < 25$) the purely-elastic instability develops and leads to an increase in the normalised pressure-drop. The increase in the normalised non-dimensional pressure-drop values for all viscoelastic solutions beyond $Wi \approx 25$ are much greater than the Newtonian limit, suggesting that the complexity of the elastic instabilities increase with increasing $Wi$.

3. The experimental values of non-dimensional pressure drop revealed that there is a proportional relation between the pressure losses and the concentration of polymer. The highest non-dimensional values of pressure drop in terms of normalised $fRe$ increase approximately from 1.86 for 50-W/GLY to 4.82 for 200-W/GLY for shear-thinning solutions meanwhile for Boger solutions they range from 1.48 for 80-W/SUC to 4.67 for 500-W/SUC.
4. Elastic turbulence generated in the flow of viscoelastic solutions is shown to enhance the convective heat transfer in the serpentine microchannel by approximately 200% for 50-W/GLY and 80-W/SUC and reaches up to 380% for 200-W/GLY and 500-W/SUC under creeping-flow conditions in comparison to that achieved by the equivalent Newtonian fluid flow at identical Graetz number.

5. The experimental results of normalised averaged Nusselt number for the viscoelastic solutions against Weissenberg number showed that there is an excellent overlap of the data for all dilute viscoelastic solutions showing an enhancement in convective heat transfer with increasing Weissenberg number. However, the normalised values of averaged Nusselt number for the semi-dilute viscoelastic solution (500-W/SUC) increase more sharply against Weissenberg number than the dilute solutions.

9.1.2 Cross-slot micro-geometry

Newtonian fluid flow and heat transfer

The key conclusions from a steady symmetry-breaking instability in Newtonian fluids in the cross-slot micro-geometry to promote convective heat transfer at low or moderate values of Reynolds numbers determined from Chapter 7 can be summarised as following:

1. The experimental results of the dimensionless pressure drop ($fRe$) show a good agreement with the theoretical (Darcy) analysis ($fRe = 57$, for straight square cross-section ducts and fully-developed laminar flow [191]) and the present numerical data with a range of Reynolds number from 3 to 80. The maximum deviation between the measured values of $fRe$ and the Darcy’s equation is within 2.6%. 
2. The flow within the cross-slot geometry behaves essentially as flow in a straight channel and any additional pressure loss due to the pressure of the cross, even after the onset of the instability ($Re > 40$), is small and within our measurement uncertainty.

3. For low Reynolds numbers ($Re < 40$), there is certainly no significant mixing between the dyed and undyed inflow streams at the stagnation point and also along the outflow channels and the flow appears steady, laminar and symmetric. In this case, the experimental and numerical temperature patterns observed that only conduction/diffusive occurs at the interface between the two streams along the outflow channels (very narrow region where conduction occurs).

4. A symmetry-breaking bifurcation occurs at the stagnation point when the Reynolds number increases beyond a certain critical value ($\approx 40$) and an axially-oriented spiral vortex is created along the outflow channels. Therefore, this spiral vortex promotes the mixing process in the cross-slot. This three-dimensional behaviour of the bifurcated flow enhances the heat transfer between the hot and cold fluids by convection along the outflow channels.

5. The probability density function (PDFs) of the normalised temperature distribution ($\theta$) revealed that two peaks are observed in both the experimental and numerical data around $\theta = 0$ (which corresponds to the cold inlet stream temperature) and $\theta = 1$ (which corresponds to the hot inlet stream temperature) for the symmetric flow regime ($Re = 36.6$) due to a clear separation (no mixing) between the hot and cold streams along the outflow channels.

6. In the asymmetric flow regime ($Re = 80.6$), the distribution of the normalised temperature from the experiment resembles a Gaussian distribution with its peak at approximately $\theta = 0.5$ because of the convective mixing between the
hot and cold streams and the temperature distribution has been significantly altered towards a more uniform distribution.

7. The critical Reynolds number \((Re_c)\) associated with the steady symmetry-breaking instability in Newtonian fluids in the cross-slot micro-geometries differs for each case depending on the initial conditions \((Re_c = 39\) when decreasing and \(Re_c = 53\) when increasing), in broad agreement with previously reported values by Haward et al. [85]. Therefore, these two solution branches exist between \(Re = 39\) and \(Re = 53\) clearly demonstrating that the system is hysteretic.

8. The normalised values of the root mean square temperature \((T_{RMS})\) for both cases (symmetry-imposed and cross-slot) collapse entirely along the outlet channel in the same pattern even with varying Prandtl number (1 - 100) in the case of Reynolds number lower than critical value \((Re < 40)\) due to the poor mixing and diffusion-dominant heat transfer. For Reynolds numbers greater than the critical value \((Re > 40)\), the normalised values of the \(T_{RMS}\) show effectively enhanced heat transfer between the hot and cold streams for all Prandtl numbers.

9. The heat transfer enhancement based on \(((T_{RMS})_{Sym.}/(T_{RMS})_{Asym.})\) increases monotonically from approximately 1.6 to 3.7 with increasing Reynolds number from 40 to 100, respectively, at \(Pr = 1\) due to the effect of the instability.

**Viscoelastic fluid flow and heat transfer**

From a preliminary investigation of the viscoelastic fluid flow behaviour and temperature distribution in the square cross-section cross-slot micro-geometry studied in Chapter 8, the major conclusions of the experimental results are summarised by the following points:
1. The measured values of friction factor-Reynolds number product, $fRe$, for the viscoelastic solution (100-W/70GLY) demonstrate a significant increase compared with the Newtonian solution (W/70GLY) over the same tested range of Reynolds numbers because the combination of extensional flow and the nonlinear properties of viscoelastic solution features within the cross-slot micro-geometry give rise to large pressure drops.

2. The normalised values of friction factor-Reynolds number product, $fRe(\text{N} - \text{Newt.})$, for the viscoelastic solution (100-W/70GLY) measured experimentally through the cross-slot geometry increase monotonically from the Newtonian limit at low Weissenberg number ($Wi < 25$) and, above a critical Weissenberg number ($Wi > 25$), a rapid increase in the normalised values of $fRe(\text{N} - \text{Newt.})$ is observed before levelling off.

3. The isothermal dye visualisation for the flow of viscoelastic fluid (100-W/70GLY) at the centre of the cross-slot and also 20W downstream of the stagnation point at low Weissenberg number, $Wi = 5.8$, showed that the first weakly unstable elastic instability (symmetry-breaking bifurcation) between inflow streams is inadequate to strongly mix the viscoelastic fluid flowing through the cross-slot micro-geometry. Therefore, the temperature distribution shows two distinctly separated streams at very different temperatures in the centre of the cross-slot, even at the 20W distance downstream away from the stagnation point. As the Weissenberg number is further increased to $Wi = 118.5$ ($Re = 7.26$), the effect of strong unsteadiness on the quality of fluid mixing in the cross-slot is significantly increased. Thus, the instantaneous experimental temperature distribution of the 100-W/70GLY fluid has been significantly modified towards a more uniform the temperature distribution between the hot and cold streams.

4. The probability density function (PDFs) of the average of the normalised
temperature distribution ($\theta$) at 20W downstream of the stagnation point shows peaks around $\theta = 0$ and $\theta = 1$ for the weakly stable asymmetric (symmetry-breaking bifurcation) flow regime ($Wi = 5.8$) due to the insufficient mixing between the hot and cold streams along the outflow channels. While, in the unsteady asymmetric (strongly time-dependent) flow regime ($Wi = 118.5$), the distribution of the normalised temperature resembles a Gaussian distribution with its peak at $\theta = 0.5$ because of the convective mixing between the hot and cold streams.

To summarise, micro-geometries or “microfluidic systems” are extensively used in many applications, such as bioengineering devices, microelectronic devices, cooling systems of computer chips, and mini or micro-heat exchangers, rather than their traditional large-scale counterparts. Therefore, the experimental results with supporting numerical calculations for Newtonian flow obtained through this thesis show that both inertial instability and elastic instability can be exploited to enhance heat transfer in the micro-scale geometries, a serpentine microchannel and a cross-slot geometry. If the flow in the micro-geometry can reach to moderate values of Reynolds number, for example beyond 40 in the cross-slot, using the inertial instability is the most effective way to promote heat transfer as it causes only a slight increase in the pressure drop. In the case of length scales of micro-geometry on the order of tens of microns that makes the inertial effect extremely small, and the flow is dominant by viscous forces, elastic instability is the right way to enhance heat transfer but the relative increase in the pressure drop is more major. Thus, the precise choice of which mechanism to use will be geometry and fluid dependent.
9.2 Suggestions for further work

Although this thesis has provided a significant contribution to the enhancement of convective heat transfer in the micro-scale geometries using two different flow regimes of instabilities (purely-inertial and purely-elastic), there are still some areas that require further investigation. Suggestions for future work are given below.

1. Application of temperature-sensitive fluid for measuring the local temperature or temperature distribution field in a serpentine microchannel would be crucial for the next step to obtain the local heat transfer rate using Newtonian and viscoelastic fluids and thus a greater understanding of the detailed thermal performance of serpentine microchannel.

2. The polymer chosen to study the effect of elastic instabilities on the convective heat transfer within micro-scale geometries in this thesis was polyacrylamide, which is high viscoelasticity and also shear-thinning. It is worthwhile to investigate the type effect of polymer, for example polyethylene-oxide (PEO), on the heat transfer enhancement in these micro-scale geometries. In addition, the effect of using worm-like surfactants instead of polymers would also be interesting.

3. One of the major aims of the present study was to provide a detailed experimental database for the enhancement of heat transfer through micro-scale geometries (a serpentine microchannel and a cross-slot micro-geometry) using Newtonian and viscoelastic fluids. The thermal boundary conditions that were applied to study the effect of purely-inertial and purely-elastic instabilities on the convective heat transfer through the micro-scale geometries in this thesis were somewhat limited. It would be interesting to study the
effect of different thermal boundary conditions on the heat transfer by applying for example constant heat flux on the walls instead of constant wall temperature.

4. The preliminary experiments undertaken to investigate the enhancement of convective heat transfer in the cross-slot micro-geometry using viscoelastic fluid may be extended to study more deeply the effect of different polymer types and concentrations on the heat transfer enhancement.

5. Alternative micro-scale geometry, which can be employed to create purely-inertial and purely-elastic instabilities due to the presence of a stagnation point, is so-called mixing-separating cell, and also called counter-current shearing flows. The mixing-separating geometry, which is the same composition of the cross-slot geometry, consists of two parallel channels with opposite inlets interacting through central gap in the separating internal wall as shown in Figure 9.1. Further numerical and experimental research could be conducted within a mixing-separating cell in order to characterise firstly the flow instabilities, both purely-inertial and purely-elastic, that leads to a bifurcation of steady flows. Moreover, numerical and experimental investigations may be performed to study the effect of the purely-inertial and purely-elastic instabilities on the heat transfer in the mixing-separating geometry.
Figure 9.1: Schematic diagram of the mixing-and-separating flow geometry (after [2]).
References


References


[107] V. N. Kalashnikov and M. G. Tsiklauri, *Ordered three-dimensional structures resulting from instability of two-dimensional flow in crossed channels*, Fluid


Appendix A

Uncertainty analysis procedure

A.1 The uncertainties in the independent parameters

Surface area:

The uncertainty of the surface area for a serpentine microchannel is calculated as:

\[ A_s = 4Wl \]  

(A.1)

Partial derivation with respect to \( W \) and \( l \) gives,

\[ \frac{\partial A_s}{\partial W} = 4l \quad \text{and} \quad \frac{\partial A_s}{\partial l} = 4W \] ; Substituting these partial derivations in Eq. 4.15 gives,

\[ \varepsilon_{A_s} = \left[ \left( \frac{\partial A_s}{\partial W} \varepsilon_W \right)^2 + \left( \frac{\partial A_s}{\partial l} \varepsilon_l \right)^2 \right]^{0.5} = \left[ (4l\varepsilon_W)^2 + (4W\varepsilon_l)^2 \right]^{0.5} \]  

(A.2)

In relative form,

\[ \frac{\varepsilon_{A_s}}{A_s} = \left[ \left( \frac{\varepsilon_W}{W} \right)^2 + \left( \frac{\varepsilon_l}{l} \right)^2 \right]^{0.5} \]  

(A.3)
Appendix A. Uncertainty analysis procedure

Putting the values of $W = 1.075 \text{ mm}$, $l = 114.25 \text{ mm}$, $A_s = 4Wl = 491.275 \text{ mm}^2$, $\varepsilon_W = \pm 0.01 \text{ mm}$, $\varepsilon_I = \pm 0.1 \text{ mm}$ in Eqs. A.2 and A.3, the absolute and relative uncertainties in surface area are estimated to be $(\varepsilon_{A_s}) = \pm 4.6 \text{ mm}$ and $(\varepsilon_{A_s}/A_s) = \pm 0.934\%$, respectively. Other parameters are similarly calculated.

Manual mass flow rate measurements:

The flow rate for Newtonian and viscoelastic fluids within the serpentine microchannel and the cross-slot micro-geometry was manually measured using bucket-weigh-stopwatch method. A digital stopwatch and a digital weighing scale were used in this measurement. The uncertainty in mass flow rate measurement using bucket-weigh-stopwatch method was obtained as follows,

$$\dot{m} = \frac{m}{t} \tag{A.4}$$

Getting partial derivatives, $\frac{\partial \dot{m}}{\partial m} = t^{-1}$ and $\frac{\partial \dot{m}}{\partial t} = mt^{-2}$, and then substituting these partial derivations in Eq. A.4 to give,

$$\varepsilon_{\dot{m}} = \left[ \left( \frac{\partial \dot{m}}{\partial m} \varepsilon_m \right)^2 + \left( \frac{\partial \dot{m}}{\partial t} \varepsilon_t \right)^2 \right]^{0.5} \tag{A.5}$$

In relative form, the absolute uncertainty in in mass flow rate (Eq. A.5) takes the following form,

$$\frac{\varepsilon_{\dot{m}}}{\dot{m}} = \left[ \left( \frac{\varepsilon_m}{\dot{m}} \right)^2 + \left( \frac{\varepsilon_t}{t} \right)^2 \right]^{0.5} \tag{A.6}$$

Mean film temperature:

By partial derivation of mean film temperature ($(T_{m,i} + T_{m,o})/2$), the following can be deduced,

$$T_f = \frac{T_{m,i} + T_{m,o}}{2} \tag{A.7}$$
\[ \frac{\partial T_f}{\partial T_{m,i}} = \frac{1}{2} \text{ and } \frac{\partial T_f}{\partial T_{m,o}} = \frac{1}{2}; \text{ the absolute uncertainty is,} \]

\[ \varepsilon_{T_f} = \left[ \left( \frac{\partial T_f}{\partial T_{m,i}} \varepsilon_{T_{m,i}} \right)^2 + \left( \frac{\partial T_f}{\partial T_{m,o}} \varepsilon_{T_{m,o}} \right)^2 \right]^{0.5} \]  

(A.8)

The relative uncertainty is,

\[ \frac{\varepsilon_{T_f}}{T_f} = \left[ \left( \frac{\varepsilon_{T_{m,i}}}{T_f} \right)^2 + \left( \frac{\varepsilon_{T_{m,o}}}{T_f} \right)^2 \right]^{0.5} \]  

(A.9)

**Bulk fluid temperature difference**

By partial derivation of bulk fluid temperature difference (\( \Delta T_B \)), the following are obtained,

\[ \Delta T_B = T_{m,o} - T_{m,i} \]  

(A.10)

\[ \frac{\partial \Delta T_B}{\partial T_{m,o}} = 1 \text{ and } \frac{\partial \Delta T_B}{\partial T_{m,i}} = -1; \text{ the absolute uncertainty is,} \]

\[ \varepsilon_{\Delta T_B} = \left[ \left( \frac{\partial \Delta T_B}{\partial T_{m,o}} \varepsilon_{T_{m,o}} \right)^2 + \left( \frac{\partial \Delta T_B}{\partial T_{m,i}} \varepsilon_{T_{m,i}} \right)^2 \right]^{0.5} \]  

(A.11)

The relative uncertainty is,

\[ \frac{\varepsilon_{\Delta T_B}}{\Delta T_B} = \left[ \left( \frac{T_{m,o}}{\Delta T_B} \right)^2 + \left( \frac{T_{m,i}}{\Delta T_B} \right)^2 \right]^{0.5} \]  

(A.12)

**Log-mean temperature difference**

The log-mean temperature difference (\( \Delta T_{lm} \)) in the current study is defined by Eq. 4.10 From the partial derivatives the uncertainty is calculated as follows:

\[ \frac{\partial \Delta T_{lm}}{\partial (T_w - T_{m,o})} = \frac{ln[T_w - T_{m,o}]}{(T_w - T_{m,i})} - \left( \frac{T_w - T_{m,i}}{T_w - T_{m,o}} \right) \left( ln[T_w - T_{m,o}] \right)^2 \]
and

\[ \frac{\partial \Delta T_{lm}}{\partial (T_w - T_{m,i})} = -\frac{\ln[(T_w - T_{m,o})/(T_w - T_{m,i})]}{(\ln[(T_w - T_{m,o})/(T_w - T_{m,i})])^2} \]

The absolute uncertainty,

\[ \varepsilon_{\Delta T_{lm}} = \left[ (\frac{\partial \Delta T_{lm}}{\partial (T_w - T_{m,o})}\varepsilon_{(T_w - T_{m,o})})^2 + (\frac{\partial \Delta T_{lm}}{\partial (T_w - T_{m,i})}\varepsilon_{(T_w - T_{m,i})})^2 \right]^{0.5} \] (A.13)

**A.2 The uncertainties in the dependent parameters**

**Reynolds number:**

The Reynolds number is previously defined in Chapter 6 in Eq. 4.1, \( \frac{\partial Re}{\partial \rho} = \frac{U_B W}{\eta}, \frac{\partial Re}{\partial W} = \frac{\rho U_B}{\eta} \), \( \frac{\partial Re}{\partial U_B} = \frac{\rho W}{\eta} \), and \( \frac{\partial Re}{\partial \eta} = -\frac{\rho U_B W}{\eta} \); therefore, the absolute uncertainty is,

\[ \varepsilon_{Re} = \left[ \left( \frac{\partial Re}{\partial \rho} \varepsilon_{\rho} \right)^2 + \left( \frac{\partial Re}{\partial W} \varepsilon_{W} \right)^2 + \left( \frac{\partial Re}{\partial U_B} \varepsilon_{U_B} \right)^2 + \left( \frac{\partial Re}{\partial \eta} \varepsilon_{\eta} \right)^2 \right]^{0.5} \] (A.14)

\[ \varepsilon_{Re} = \left[ \left( \frac{U_B W}{\eta} \varepsilon_{\rho} \right)^2 + \left( \frac{\rho U_B}{\eta} \varepsilon_{W} \right)^2 + \left( \frac{\rho W}{\eta} \varepsilon_{U_B} \right)^2 + \left( -\frac{\rho U_B W}{\eta} \varepsilon_{\eta} \right)^2 \right]^{0.5} \] (A.15)

Relative uncertainty,

\[ \frac{\varepsilon_{Re}}{Re} = \left[ \left( \frac{\varepsilon_{\rho}}{\rho} \right)^2 + \left( \frac{\varepsilon_{W}}{W} \right)^2 + \left( \frac{\varepsilon_{U_B}}{U_B} \right)^2 + \left( \frac{\varepsilon_{\eta}}{\eta} \right)^2 \right]^{0.5} \] (A.16)

Similarly, the absolute and relative uncertainties of Weissenberg number (see Eq. 4.5) are expressed as follows:

**The absolute uncertainty,**

\[ \varepsilon_{W_i} = \left[ \left( \frac{U_B}{W} \varepsilon_{\lambda} \right)^2 + \left( \frac{\lambda}{W} \varepsilon_{U_B} \right)^2 + \left( \frac{\lambda U_B}{W^2} \varepsilon_{W} \right)^2 \right]^{0.5} \] (A.17)
The relative uncertainty,

\[ \frac{\varepsilon_{W_i}}{W_i} = \left[ \left( \varepsilon_{\lambda} \frac{\lambda}{X} \right)^2 + \left( \frac{\varepsilon_{U_B}}{U_B} \right)^2 + \left( \frac{\varepsilon_{W}}{W} \right)^2 \right]^{0.5} \]  

(A.18)

**Darcy friction factor:**

The absolute and relative uncertainties of Darcy friction factor defined in Eq. 4.6 are described as follows:

The absolute uncertainty,

\[ \varepsilon_f = \left[ \left( \frac{2W}{U_B^2} \varepsilon_{\Delta P} \right)^2 + \left( \frac{2\Delta P}{U_B^2} \varepsilon_W \right)^2 + \left( \frac{-2W\Delta P}{U_B^2} \varepsilon_l \right)^2 + \left( \frac{-2W\Delta P}{U_B^2} \varepsilon_\rho \right)^2 + \left( \frac{-2W\Delta P}{U_B^2} \varepsilon_{U_B} \right)^2 \right]^{0.5} \]  

(A.19)

The relative uncertainty,

\[ \frac{\varepsilon_f}{f} = \left[ \left( \frac{\varepsilon_{\Delta P}}{\Delta P} \right)^2 + \left( \frac{\varepsilon_W}{W} \right)^2 + \left( \frac{\varepsilon_l}{l} \right)^2 + \left( \frac{\varepsilon_\rho}{\rho} \right)^2 + \left( \frac{-2\varepsilon_{U_B}}{U_B} \right)^2 \right]^{0.5} \]  

(A.20)

**Friction factor-Reynolds number product**

Friction factor-Reynolds number product is defined by Eq. 4.7 from where by partial derivatives the absolute uncertainty is calculated as follows:

\[ \frac{\partial fRe}{\partial f} = Re \text{ and } \frac{\partial fRe}{\partial Re} = f \]

\[ \varepsilon_{fRe} = \left[ \left( \frac{\partial fRe}{\partial f} \varepsilon_f \right)^2 + \left( \frac{\partial fRe}{\partial Re} \varepsilon_{Re} \right)^2 \right]^{0.5} = \left[ \left( \frac{\varepsilon_f}{f} \right)^2 + \left( \frac{\varepsilon_{Re}}{Re} \right)^2 \right]^{0.5} \]  

(A.21)

The relative uncertainty,

\[ \frac{\varepsilon_{fRe}}{fRe} = \left[ \left( \frac{\varepsilon_{\Delta P}}{\Delta P} \right)^2 + \left( 2 \times \frac{\varepsilon_W}{W} \right)^2 + \left( \frac{\varepsilon_\eta}{\eta} \right)^2 + \left( \frac{\varepsilon_l}{l} \right)^2 + \left( \frac{\varepsilon_{U_B}}{U_B} \right)^2 \right]^{0.5} \]  

(A.22)

**Heat transfer rates**
Heat transfer rate is given in Eq. 4.8 from where by partial derivatives the uncertainty is calculated as follows: $\frac{\partial Q}{\partial \dot{m}} = C \Delta T_B$, $\frac{\partial Q}{\partial C} = \dot{m} \Delta T_B$ and $\frac{\partial Q}{\partial \Delta T_B} = \dot{m}$. 

Absolute uncertainty,

$$\varepsilon_Q = [(C \Delta T_B \varepsilon_{\dot{m}})^2 + (\dot{m} \Delta T_B \varepsilon_C)^2 + (\dot{m} C \varepsilon_{\Delta T_B})^2]^{0.5} = [(\frac{\varepsilon_{\dot{m}}}{\dot{m}})^2 + (\frac{\varepsilon_C}{C})^2 + (\frac{\varepsilon_{\Delta T_B}}{\Delta T_B})^2]^{0.5}$$ (A.23)

Relative uncertainty,

$$\frac{\varepsilon_Q}{Q} = [(\frac{\varepsilon_{\dot{m}}}{\dot{m}})^2 + (\frac{\varepsilon_C}{C})^2 + (\frac{\varepsilon_{\Delta T_B}}{\Delta T_B})^2]^{0.5}$$ (A.24)

**Nusselt number**

heat transfer coefficient is deduced from Newton’s law of cooling ($h = Q/A_s \Delta T_{lm}$).

From there the uncertainty is calculated as follows:

$$\varepsilon_h = [(\frac{\varepsilon_Q}{A_s \Delta T_{lm}})^2 + (\frac{\varepsilon_{A_s} Q}{A_s \Delta T_{lm}^2})^2 + (\frac{\varepsilon_{\Delta T_{lm}} Q}{A_s \Delta T_{lm}^2})^2]^{0.5}$$ (A.25)

Relative uncertainty,

$$\frac{\varepsilon_h}{h} = [(\frac{\varepsilon_Q}{Q})^2 + (\frac{\varepsilon_{A_s}}{A_s})^2 + (\frac{\varepsilon_{\Delta T_{lm}}}{\Delta T_{lm}})^2]^{0.5}$$ (A.26)

Therefore, the relative uncertainty in Nusselt number (see Eq. 4.11) is evaluated as:

$$\frac{\varepsilon_{Nu}}{Nu} = \left[\left(\frac{\varepsilon_{\rho}}{\rho}\right)^2 + \left(2 \times \frac{\varepsilon_W}{W}\right)^2 + \left(\frac{\varepsilon_C}{C}\right)^2 + \left(\frac{\varepsilon_l}{l}\right)^2 + \left(\frac{\varepsilon_k}{k}\right)^2 + \left(\frac{\varepsilon_{UB}}{UB}\right)^2 + \left(\frac{\varepsilon_{\Delta T_B}}{\Delta T_B}\right)^2 + \left(\frac{\varepsilon_{\Delta T_{lm}}}{\Delta T_{lm}}\right)^2\right]^{0.5}$$ (A.27)
Appendix B

Published papers
Letter

Enhancing heat transfer at the micro-scale using elastic turbulence

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Abstract

Small concentrations of a high-molecular-weight polymer have been used to create so-called “elastic turbulence” in a micro-scale serpentine channel geometry. It is known that the interaction of large elastic stresses created by the shearing motion within the fluid flow with streamline curvature of the serpentine geometry leads initially to a purely-elastic instability and then the generation of elastic turbulence. We show that this elastic turbulence enhances the heat transfer at the micro-scale in this geometry by up to 300% under creeping flow conditions in comparison to that achieved by the equivalent Newtonian fluid flow.

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Keywords:
Elastic turbulence
Viscoelasticity
Serpentine channel
Micro-mixing
Heat transfer

In so-called “creeping flow”, i.e., flows for which the Reynolds number (Re) remains small (Re < 1), Newtonian fluids remain laminar and steady. Consequently efficient mixing and heat transfer to the fluid are problematic for very viscous systems or liquid flows at small scales (e.g., microfluidics) as they are essentially diffusion/conduction dominated. One method to circumnavigate these problems is to make the fluid non-linear by the addition of small amounts of high molecular-weight polymer. The resulting viscoelastic solution enables fluid flows at arbitrarily small values of Re to exhibit “turbulent-like” characteristics such as randomly fluctuating fluid motion excited across a broad range of temporal and spatial scales [1–6]. Steinberg and co-workers [1–4] showed that highly-elastic viscoelastic fluids can undergo a series of flow transitions from viscometric laminar flow, to periodic flow, to apparently chaotic flow, and then to fully developed elastic turbulence (ET) in conditions of negligible inertia (Re < 1) and this has been shown in a range of flows: swirling flow between parallel disks [1,4,5], in serpentine or wavy channels [2–4,6] and in concentric cylinder devices [4,7]. The instabilities and resulting non-linear interactions are “purely-elastic” in nature – driven by the elastic (normal) stresses developed in flow – and occur at Reynolds numbers far removed from the usual turbulence observed for Newtonian fluids which is inertial in nature (critical Re on the order of 1000 for internal flows). Although the original work of Groisman and Steinberg [1] has elicited a significant degree of interest (and the passive-scalar mixing effectiveness of the regime has been mentioned repeatedly [1,3–7]) outside of the quantitative studies on passive scalar mixing [8,9], little work has yet been carried out to assess this effectiveness in other typical “mixing” scenarios. An exception to this is the study of Poole et al. [10] where ET was used to create oil in polymer solution emulsions in a swirling flow between parallel disks arrangement (similar to that used in Ref. [1]) where for a Newtonian oil and continuous phase, at identical conditions, no emulsification occurred at all. Flows containing streamline curvature are ideal for encouraging elastic instabilities and elastic turbulence as it is generally accepted that purely-elastic instabilities arise as a consequence of both elastic normal stresses and streamline curvature [11]—although some analytical work [12] and experimental evidence [13] are beginning to show that even parallel shear flows may exhibit ET providing the initial perturbation is sufficiently strong.

The growth of “microfluidic” research, and the major fundamental interest and applications of such flows [14], has revealed previously unobserved instabilities and flow phenomena that occur solely due to viscoelasticity. In fact some of the key publications on ET [4,6,13] have used such micro-geometries to access the required parameter space (low inertia, high elastic stresses). The small scale nature of such flows leads directly to the viscoelastic behavior observed: the small length scale simultaneously makes the Reynolds number (Re ≡ ρUD/µ) small and the Deborah (De ≡ λU/D) or Weissenberg (Wi ≡ λU/D) numbers, which characterize the degree of elasticity in the flow, large (where ρ

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The entire facility was housed in a Techne TE-10A water bath continuously-stirred and maintained at a temperature of 30°C (leading to typical fluid temperature increases of 4°C–8°C). The copper bottom and copper side walls guaranteed isothermal boundary conditions and the insulating properties of the PVC ensured an adiabatic boundary condition on the upper wall. The surface temperature of the serpentine channel was monitored by four K-type thermocouples each embedded 1 mm from the channel side walls. The enhancement of heat transfer generated by the complex fluid flowing through the serpentine channel was quantified by measuring the temperature difference between the two reservoirs (before and after the serpentine channel) with K-type thermocouples. The K-type thermocouples had a constant uncertainty of ±1°C and were calibrated against a mercury thermometer of certified accuracy (±0.1°C).

The pressure drop along the channel was measured by a Valdyne DP15–26 differential pressure transducer. The pressure transducer estimated the streamwise pressure gradient (∆P), from which the friction factor (f = |ΔP/0.5ρU²|)/(D/L), where U is the average velocity, D is the hydraulic diameter, and L is the path-length equal to 111.25 mm in our set-up) could be determined, by measuring the difference in pressure across two pressure taps installed on the upper wall of each reservoir. The pressure transducer used two different diaphragms to capture the full working range: one had a working range of 0.2 bar whilst the other had a range of 2 bar, both are said to be accurate to ±0.25% full scale, and both diaphragms were periodically calibrated against an MKS Baratron differential pressure transducer (1000 torr fsd).

Fluid was pumped through the serpentine channel by a regulated pressure vessel. The fluid was discharged into a beaker and weighed by a Denver TP-1502 precision balance allowing a measurement of the mass flow rate (uncertainty ±0.03 mg). The working fluids were solutions of a high-molecular-weight molecular-weight polymer, wereach use of high shear rates, viscous solvents, and an extremely high-shear-rate viscosity, η∞ is the infinite-shear-rate viscosity, λCY is a constant which characterizes the...
onset of shear-thinning, \( n \) is a power-law index, and \( a \) is a fitting parameter. We use this shear-rate dependent viscosity to define our Reynolds number \( Re = \rho UD_h/n_1 \) and Prandtl number \( (Pr = \sigma n_{th}/k_t) \) where \( c \) is the heat capacity and \( k_t \) is the thermal conductivity of the fluid) where the viscosity \( \eta_{th} \) is determined at a characteristic shear rate corresponding to \( \dot{\gamma} = \gamma_s/D_h \) and at a mean film temperature \( (T_h + T_i)/2 \) where \( T_h \) and \( T_i \) are the mean fluid temperature at the outlet and inlet reservoirs, respectively. The specific heat and thermal conductivity were assumed to be that of the solvent following several other studies [19,20]. The shear-rate dependent polymer relaxation time \( (\lambda) \) has been estimated by small-amplitude-oscillatory-shear (SAOS) measurements \( (\lambda = G'/(G'' \omega)) \). The longest relaxation times \( (\lambda_{max}) \) have been obtained from the SAOS measurements, by estimating the relaxation times in the limit of the angular velocity tending to zero (see Table 1). The slight concentration dependence of the relaxation time suggests that the highest concentration solution may just be semi-dilute in agreement with the critical overlap concentration estimate from the intrinsic viscosity. It was also possible to measure the first normal-stress difference for both solutions, although the values are very close to the resolution of the instrument: estimating a relaxation time from stress relaxation measurements, e.g., following Ref. [21], were not attempted. The important fluid properties for the fluids used in this study are listed in Table 1 at both 20°C and 26°C (the latter being a typical mean film temperature).

To quantify the enhancement of heat transfer by elastic turbulence we calculate the Nusselt number \( (Nu) \) defined as

\[
Nu = \frac{\dot{m}C_h(T_h - T_i)}{k_tA_s \Delta T_{lm}},
\]

where \( \dot{m} \) is the mass flow rate, \( A_s \) is the surface area of the channel, and \( \Delta T_{lm} = (T_w - T_h) - (T_w - T_i) / (T_w - T_h) / (T_w - T_i) \) is the log-mean temperature difference with \( T_w \) being the mean wall temperature.

Figure 3(a) shows the changes in the Nusselt number with increasing Graetz number \( (Gz = Dh/L \cdot Re \cdot Pr) \). The Newtonian fluid flow collapses to the numerical predictions for a thermally developing laminar flow through a straight square duct [24] suggesting the curvature of the serpentine channel has little influence for Newtonian fluids at such low Graetz numbers. Beyond a certain flowrate, the addition of viscoelasticity enhances the heat transfer causing an increase in the Nusselt number. From previous experimental and numerical work for isothermal flows in our group [15–17] and elsewhere [2–4,6], we know that creeping \( (Re \to 0) \) viscoelastic fluid flows through such serpentine channels firstly at low \( Wi \) number develop a steady secondary flow [15] before the onset of a purely-elastic instability leads to oscillatory time-dependent flow at Weissenberg numbers of order one (~0.6 [16], 1.4–3.5 [3], 3.2 [2,4]). Beyond this first linear instability the flow
becomes increasingly complex and, as previously discussed, developed elastic turbulence is observed beyond \( Wi > 7–15 \) [2–4,6]. Hence for the parameter space of our investigation we are at a sufficiently high \( Wi \) number to be in a fully elastic turbulence regime and therefore we believe the increase in heat transfer we observe here is due to elastic turbulence, which has been created by the non-linear interaction between elastic stresses generated within the polymer solutions and the streamline curvature of the serpentine geometry [2–4]. This scenario is illustrated in the pressure-drop data shown in Fig. 3(b) where we can see that at very low flow-rates (\( Wi < 5 \)) the pressure-drop and Nusselt number (Fig. 3(c)) are, to within the experimental uncertainties, essentially the same as the Newtonian values. Thus, perhaps surprisingly, the stationary secondary-flow driven by the interaction of the first normal-stress difference and channel curvature shown numerically in isothermal flow by Poole et al. [15] does not appear to modify the heat transfer significantly in contrast to the secondary-flows driven by the second normal-stress difference for much more concentrated polymer solutions observed in straight ducts [20]. Beyond this Weissenberg number the purely-elastic instability leads to an increase in the pressure-drop but the Nusselt number is only marginally affected (\( 5 < Wi < 25 \)). Beyond \( Wi = 25 \), where the pressure drop data plateaus, significant increases in normalized Nusselt number are observed (Fig. 3(c)). At the highest flow-rates achievable this leads to a maximum \( 30\% \) increase compared to the equivalent Newtonian value (e.g., identical Graetz number).

In this experimental investigation we have shown that it is possible to enhance the heat transfer by up to \( 30\% \) in micro-scale geometries at low Graetz number using elastic turbulence. At the same flow-rates for equivalent Newtonian fluids, e.g., either the solvent or identical Graetz number, the Nusselt number remains within \( 10\% \) of the conduction limit. The elastic turbulence has been created by the non-linear interaction between elastic stresses generated within the flowing high-molecular-weight polymer solutions and the streamline curvature of the serpentine geometry. Outside of its fundamental scientific interest, the use of elastic turbulence to enhance the heat transfer could have impacts for micro-mixing technologies and in the design of lab-on-a-chip devices.

Acknowledgments

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References


Table 1

<table>
<thead>
<tr>
<th>Properties 20°C (26°C)</th>
<th>Solution</th>
<th>Newtonian</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_s ) (Pa·s)</td>
<td>0.253 (0.215)</td>
<td>0.369 (0.297)</td>
</tr>
<tr>
<td>( n_\infty ) (Pa·s)</td>
<td>0.186 (0.168)</td>
<td>0.261 (0.196)</td>
</tr>
<tr>
<td>( \chi_\infty )</td>
<td>0.564 (0.509)</td>
<td>0.501 (0.624)</td>
</tr>
<tr>
<td>( n )</td>
<td>0.364 (0.246)</td>
<td>0.434 (0.272)</td>
</tr>
<tr>
<td>( a )</td>
<td>0.450 (0.451)</td>
<td>0.644 (0.476)</td>
</tr>
<tr>
<td>( k )</td>
<td>1.123 (0.623)</td>
<td>1.714 (1.459)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.186 (0.168)</td>
<td>0.261 (0.196)</td>
</tr>
</tbody>
</table>

\[ k/(W m^{-1} K^{-1}) \] [22] | 0.405 (0.411) | 0.405 (0.411) |

\[ c/(kg K^{-1}) \] [23] | 2655 (2685) | 2655 (2685) | 2655 (2685)

\[ \rho/(kg m^{-3}) \] | 1317–1328 | 1317–1328 | 1317–1328

\[ Pr \] | 1006–1241 | 1123–1642 | 570.6–632

\[ 1006–1241 \ 1123–1642 \ 570.6–632 \]
Experimental investigation of the impact of elastic turbulence on heat transfer in a serpentine channel

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1. Introduction

The practical applications of micro-scale systems, such as bio-engineering devices, micro-electronic devices, cooling systems of computer chips and mini or micro-scale heat exchangers have recently received a great deal of attention due to the development of fabrication technologies for these systems. In the micro-scale systems (with “small” dimensions approximately less than 1000 microns), however, flow of Newtonian fluids at very low Reynolds numbers, \( Re = \rho UD/\eta < 1 \) where \( \rho \) and \( \eta \) are fluid density and dynamic viscosity, respectively, \( U \) is a characteristic velocity scale and \( D \) is a characteristic length scale is inherently laminar and steady [1]. As a consequence, mixing at the micro-scale is difficult to achieve (as molecular diffusion is dominant) and thus the enhancement of convective heat transfer is problematic under these conditions. One of the proposed approaches to enhance heat transfer in these conduction-dominated regimes is to use viscoelastic fluids, which are prepared by adding a small amount of high molecular-weight polymer to a Newtonian solvent, in order to introduce non-linear effects and promote the appearance of instabilities even at low \( Re \). Viscoelastic fluid flows in such geometries have then been seen to exhibit “turbulent-like” characteristics such as chaotic and randomly fluctuating fluid motion across a broad range of spatial and temporal scales and have led this to be called “elastic turbulence” [2–10].

The concept of elastic instability appeared as a well-known phenomenon for viscoelastic fluid flows in the 1990s. Larson et al. [11] reported theoretical and experimental results that showed that purely-elastic instabilities can arise in viscoelastic fluid flows by a coupling of the first normal-stress difference and streamline curvature. This seminal work was followed by a series of experimental investigations that used different geometries containing curved streamlines to study elastic instabilities in low \( Re \) fluid flows. This included swirling flow between parallel disks [2–4], in serpentine or wavy channels [3,5–8] and in concentric cylinder devices [3,9]. Elastic instabilities and resulting non-linear interactions between elastic stresses generated within the flowing high-molecular-weight polymer solutions and the streamline curvature are “purely-elastic” in nature, driven by the elastic (normal) stresses developed in the flow and occur at Reynolds numbers far removed from the usual turbulence observed for Newtonian fluids which is, of course, inertial in nature. Steinberg and co-workers...
Table 1

<table>
<thead>
<tr>
<th>Authors</th>
<th>Weissenberg number range</th>
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<tbody>
<tr>
<td></td>
<td>Onset of elastic turbulence</td>
</tr>
<tr>
<td>Groisman and Steinberg, 2004</td>
<td>3.2</td>
</tr>
<tr>
<td>Burghela et al., 2004</td>
<td>1.4–3.5</td>
</tr>
<tr>
<td>Li et al., 2010</td>
<td>7.5</td>
</tr>
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</table>

[2,3,5,6] reported that elastic instabilities of highly-elastic fluids can be exploited to augment fluid mixing in serpentine or curved channels. Typically, viscoelastic fluid flows with negligible inertia (Re < 1) that have been stretched along curved streamlines can undergo a series of flow transitions from viscometric laminar flow, to clearly chaotic flow, and eventually to fully developed “elastic turbulence”. The experimental study by Poole et al. [12] used elastic turbulence in a swirling flow between two parallel disks to create oil-in-polymer-solution emulsions. Their results showed that for Newtonian oil and continuous phase no emulsification occurred at all. While in the case of the viscoelastic oil solutions, at identical conditions, a good emulsion was observed due to the mixing effects of elastic turbulence.

Prior to this understanding of elastic turbulence, Hartnett and co-workers conducted a set of experimental [13–15] and numerical [16] investigations to study the fundamental characteristics of fully-developed laminar convective heat transfer under different combinations of thermal boundary conditions using various types of viscoelastic fluids in straight ducts. The results indicated that viscoelastic solutions show higher convective heat transfer as compared to Newtonian fluids flow under identical conditions (Re = 314–1974, Pr = 42.9–79). These increases are attributed to secondary flows, arising from the second normal-stress differences imposed on the surfaces boundaries, which occur in viscoelastic fluids in laminar flow through rectangular or square cross-section ducts. When the concentration of polymer increases this leads to an increase in the strength of the secondary flow and a decrease in the thermal development entrance length. Further numerical studies [17–20] investigated the impact of such secondary flows on the convective heat transfer of the viscoelastic fluids. The results of these numerical studies were consistent with the original experimental results of Hartnett and co-workers [13–16].

The effects of viscoelastic fluids in micro-scale geometries become significant even for dilute solutions because of the small flow time scales and the high shear rates achievable. Examples of these features in the flow simultaneously makes the Re small and the Weissenberg number large, Wi (=λU/D, where λ is the relaxation time of the viscoelastic fluid), which quantifies non-linear elastic effects. From earlier publications [2–9], purely-elastic instabilities have been observed at low Wi (order 1) in a range of flows and, at high Wi (order 10), elastic turbulence observed as a result of the combination of elastic stresses and streamline curvature. Elastic turbulence can arise in the serpentine channel flow at typical Wi numbers as shown in Table 1. The possibility of using elastic turbulence to enhance convective heat transfer in a square serpentine microchannel was quantitatively studied by our research group in a short letter [21] utilising a sucrose-based Boger fluid. These initial experimental findings show that elastic turbulence is able to enhance convective heat transfer in the serpentine channel geometry by up to 300% compared to the equivalent Newtonian fluid flow. Very recently Traore et al. [22] studied the role of elastic turbulence in an axisymmetric swirling (von Karman) flow with a cooled lower stationary wall and compared the temperature fluctuations with those of a low diffusivity tracer. PIV measurements for both isothermal and non-isothermal flows revealed no significant effect of the heat transfer process on the flow topology. The results confirmed our own recent results [21], that elastic turbulence can enhance the heat transfer efficiency above the conductive limit although no Nusselt numbers could be determined in the set-up used by Traore et al. [22]. In the current paper, we give a more complete account of our experiments on the convective heat transfer in the square serpentine microchannel using elastic turbulence as well as studying the influences of both viscoelasticity and shear-thinning viscosity individually on the convective heat transfer by adjusting the solvent viscosity, polymer concentration and imposed shear rate.

2. Experimental setup

The experimental rig that has been used to investigate the behaviour of the flow and convection heat transfer of viscoelastic fluids is shown in Fig. 1. It consists of a copper serpentine channel which is mounted on a plastic frame (PVC) that includes reservoirs at either end of the channel that contains pressure tappings. The fluid is pumped through the serpentine channel using a pressure vessel where the amount of driven liquid flows between an upstream reservoir and a downstream reservoir and then to a collection container where it can be weighed for measuring the flow rate. This pressure vessel is connected to a compressed air supply, which is used to control the flow rate. A Denver Instrument TP-1502, which has an uncertainty ±0.03 g at stable load, was used to measure the flow rate by weighing the amount of collected fluid with time.

The square cross-section serpentine channel, which was comprised of 20 half-loops with inner and outer radii of R1 = 1 mm and R2 = 2 mm, respectively, was configured in a (20 mm × 84 mm) piece of copper as shown in Fig. 2. The channel had a square cross-section W = 1.075 ± 0.01 mm (measured with a Nikon EPIMPHOT TME inverted microscope 100× magnification setting, 1230 pixels = 1 mm) and was smoothly joined on either side by straight channel sections with a total length of 77 mm; see Fig. 2. The upper channel wall was fabricated from 12 mm-thick PVC. Therefore, the copper side and bottom walls of the serpentine channel were considered to be isothermal whereas the upper wall was adiabatic.

The measurements of pressure drop along the channel are obtained via two pressure taps, which are placed on the upper wall of an upstream reservoir and a downstream reservoir, using a wet-wet Validyne DP15–26 differential pressure transducer. The pressure transducer utilised two different diaphragms to cover the full working range of 0 < ΔP < 200 kPa. The voltage output of each of the diaphragms was periodically calibrated for the differential pressure range of 0 < ΔP < 20 kPa and the other range of 0 < ΔP < 200 kPa using an MKS Baratron differential pressure transducer and both are accurate to ±0.25% full scale. The pressure-drop reading was electronically sampled by an analogue to digital converter at 100 Hz for 60 s after reaching steady-state conditions (approximately 30 min per flow rate). Measurement of the pressure in the reservoirs meant that issues related to hole-pressure error were minimised but resulted in any fluctuation information being severely damped: thus our pressure-drop data is restricted to mean values.
K-type thermocouples were positioned in the upstream reservoir and the downstream reservoir for measuring the temperature of the fluid flowing before and after the serpentine channel. Further, four K-type thermocouples were embedded at axial locations along the side walls in both sides at a distance of 1 mm from the wall of the serpentine channel to monitor the wall temperature of the serpentine channel as shown in Fig. 2(b). The side walls and the bottom of the channel were made of copper (\(k_{Cu} = 385 \text{ W/mK}\)), therefore they are maintained at essentially isothermal boundary conditions whereas the insulating properties of the PVC (\(k_{PVC} = 0.25 \text{ W/mK}\)) ensured an adiabatic boundary condition on the upper wall. The large thermal conductivity of the copper ensured that, although the “wall” temperature was actually measured 1 mm away, the error associated with this assumption was very small. Additional temperature measurements at 8 mm from the wall suggested that the “true” wall temperature (determined by extrapolation) agreed with the assumed value to within 0.01 °C (see Table 2 for more details). The readings of all these thermocouples were electronically recorded over a period of 60 s at a frequency of 100 Hz once steady-state conditions had been obtained. Measuring the temperature in the reservoirs removed any issues regarding physically disturbing the flow in the serpentine channel but, like the pressure-drop data, restricted the data collection to mean values. The thermocouples, which had an accuracy of approximately \(\pm 1^\circ\text{C}\), were calibrated against a mercury thermometer of certified accuracy (\(\pm 0.1^\circ\text{C}\)) in the range 0–80 °C. The whole facility was placed in a Techne TE-10A water bath continuously-stirred and maintained at a temperature of 30 °C to achieve constant temperature boundary conditions to the copper walls (in reality the measured wall temperature remained constant to \(30 \pm 0.2^\circ\text{C}\)). Typical inlet temperatures were \(22 \pm 0.5^\circ\text{C}\) whilst the outlet temperature varied depending on conditions between 24 and 30 °C.

Executing a standard error analysis, the averaged values of uncertainties for friction factor-Reynolds number product, \(fRe\), and Nusselt number, \(Nu\), were conducted by following the same approach as adopted in Ref. [23]. The minimum and maximum uncertainty values of \(fRe\) and \(Nu\) are provided in Table 3 for all working fluids. The greatest uncertainty in determining \(Nu\) came from the measured temperatures of the walls and fluid at the inlet and outlet serpentine channel (approximately 90% of the total error of the averaged \(Nu\)), whilst the flow rate and the serpentine channel dimensions contribute about 10% to the total error of the averaged \(Nu\). As will be shown, the agreement between the averaged \(Nu\) from the present experiments for Newtonian fluids and numerical data [23,24] (see Fig. 12) will suggest that these are conservative estimates for the experimental uncertainty and, in reality, the uncertainty in the thermocouple values (and ultimately the averaged \(Nu\)) is lower than these values in Table 3 suggest.

Finally we confirmed that any contributions from natural convection in our set-up are expected to be negligible, with estimates of typical Grashof numbers (i.e. the ratio of buoyancy to viscous forces) being \(10^{-3}\). Thus, buoyancy forces are essentially negligible for all of the results shown here.

3. Working fluids preparation and rheological characteristics

A high-molecular-weight polyacrylamide (PAA) (\(M_w \sim 1.8 \times 10^5\) g/mole, Polysciences) was dissolved at different concentrations in two types of Newtonian solvents to produce two groups of viscoelastic fluids [25]. The first type of Newtonian solvent, which was a mixture of 10% distilled water and 90% glycerine (1.26 relative density, ReAgent Chemical Services) by weight (hereafter W/GLY mixture), was used to prepare shear-thinning solutions. The second Newtonian solvent, which was an aqueous solution of 65% sucrose (Biochemical grade, 190–192 °C melting point, Sucrose ACROS Organics) and 1% sodium chloride (NaCl) in distilled water all by weight (hereafter W/SUC solution), was employed to prepare approximately constant-viscosity elastic solutions, usually called Boger fluids [26]. Therefore, the effects of shear-thinning viscosity are investigated by dissolving small amounts of PAA in a W/GLY solvent whilst approximately constant-viscosity elastic liquids are studied by adding small amounts of PAA to a W/SUC solvent.

Three shear-thinning solutions containing 50 ppm, 100 ppm and 200 ppm concentration by weight of the PAA dissolved in the aqueous glycerine mixture were prepared. These shear-thinning solutions are henceforth referred to as 50-W/GLY, 100-W/GLY and 200-W/GLY, respectively. The Boger solutions were produced by dissolving 80 ppm, 120 ppm and 500 ppm concentration by weight of the same polymer in the aqueous sucrose solution, are hereafter referred to as 80-W/SUC, 120-W/SUC and 500-W/SUC respectively.
Fig. 2. Schematic diagram of the serpentine channel.

Table 2
The measurements of the wall temperature of the serpentine channel at two different positions within the wall.

<table>
<thead>
<tr>
<th>Water bath temperature (°C)</th>
<th>Average temperature at 1 mm from wall (°C)</th>
<th>Average temperature at 8 mm from wall (°C)</th>
<th>Extrapolated internal surface temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.5</td>
<td>17.41</td>
<td>17.43</td>
<td>17.407</td>
</tr>
<tr>
<td>29.5</td>
<td>29.34</td>
<td>29.42</td>
<td>29.330</td>
</tr>
<tr>
<td>39.2</td>
<td>38.95</td>
<td>39.05</td>
<td>38.937</td>
</tr>
</tbody>
</table>

Table 3
Minimum and maximum uncertainty values of $f_{Re}$ and $Nu$.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>$f_{Re}$ uncertainty Min. (%)</th>
<th>$f_{Re}$ uncertainty Max. (%)</th>
<th>$Nu$ uncertainty Min. (%)</th>
<th>$Nu$ uncertainty Max. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/GLY</td>
<td>2.73</td>
<td>6.51</td>
<td>8.01</td>
<td>9.78</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>2.88</td>
<td>7.21</td>
<td>8.37</td>
<td>10.84</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>3.42</td>
<td>10.42</td>
<td>11.65</td>
<td>13.14</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>4.04</td>
<td>16.70</td>
<td>12.98</td>
<td>16.78</td>
</tr>
<tr>
<td>W/SUC</td>
<td>2.77</td>
<td>4.26</td>
<td>8.17</td>
<td>9.97</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>2.96</td>
<td>5.67</td>
<td>8.96</td>
<td>11.05</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>3.25</td>
<td>14.70</td>
<td>12.23</td>
<td>13.79</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>4.55</td>
<td>16.37</td>
<td>15.01</td>
<td>19.29</td>
</tr>
</tbody>
</table>

Table 4 provides an overview of the composition of the studied solutions.

All rheological measurements for Newtonian (solvents) and viscoelastic solutions were conducted using a TA Instruments AR1000N controlled-stress rheometer with an acrylic cone-and-plate geometry (60 mm diameter, 2° cone angle) with an uncertainty in viscosity of ±2% [27].

The shear viscosity, $\eta_s$, for all viscoelastic solutions against shear rate, $\dot{\gamma}$, is illustrated in Fig. 3 at 20°C. The Carreau–Yasuda model [28] was adopted to fit the experimental viscosity data:

$$\eta_{CY} = \eta_\infty + \frac{(\eta_0 - \eta_\infty)}{[1 + (\dot{\gamma}_{CY})^a]^n/a}$$  (1)
Table 4
An overview of the working fluids.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>PAA concentration (w/w)</th>
<th>Solvent (X+H₂O)</th>
<th>NaCl</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/GLY</td>
<td>–</td>
<td>Glycerine 90</td>
<td>–</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>50</td>
<td>Glycerine 90</td>
<td>–</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>100</td>
<td>Glycerine 90</td>
<td>–</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>200</td>
<td>Glycerine 90</td>
<td>–</td>
</tr>
<tr>
<td>W/SUC</td>
<td>–</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>80</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>120</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>500</td>
<td>Sucrose 65</td>
<td>1</td>
</tr>
</tbody>
</table>

where, \( \eta_\infty \) are zero-shear-rate viscosity and infinite-shear-rate viscosity, respectively, \( \lambda_c \) is a constant that represents the inverse shear rate at the onset of shear-thinning, \( a \) is a fitting parameter introduced by Yasuda et al. [28] and \( n \) is a power law index. All fitting parameters for the Carreau–Yasuda model, which are tabulated in Table 5, have been determined using the least-squares-fitting method outlined in Escudier et al. [27]. Fig. 3 shows that both viscoelastic solutions exhibit shear-rate dependent viscosity. However, the PAA-W/GLY solutions possess significantly greater shear-thinning effects than the PAA-W/SUC solutions.

The critical overlap concentration, \( c^* \), which corresponds to the approximate concentration when the polymer coils in solution begin to overlap with each other [29,30], has been determined by plotting in log–log form the zero-shear viscosity, \( \eta_\infty \), versus a wide range of PAA concentrations as shown in Fig. 4. Therefore, the viscoelastic solutions can be characterised as dilute solutions \( (c < c^*) \) or semi-dilute solutions \( (c > c^*) \) depending on the value of \( c^* \). As can be seen from Fig. 4, the critical overlap concentration for the shear-thinning solutions over a range of fourteen concentrations from 15 ppm to 3000 ppm (w/w) is approximately 325 ppm whilst for the Boger solutions over a range of eleven concentrations from 25 ppm to 1500 ppm (w/w), it is around 450 ppm at 20 °C. Concentrations of working fluids for both shear-thinning and Boger solutions can be regarded as dilute solutions with the exception of 500-W/SUC, which can be considered as a semi-dilute solution \( (c/c^* < 1) \). The non-dimensional concentration, \( c/c^* \), for all working fluids are provided in Table 5.

Small amplitude oscillatory shear stress (SAOS) tests were used to measure the storage modulus, \( G' \), which measures the energy of elastic storage and the state of the structured materials, and the loss modulus, \( G'' \), which represents viscous dissipation or loss of energy. Therefore, for Newtonian fluids (or any inelastic fluids) the relaxation time, \( \lambda \), is equal to zero and this leads to \( G' = 0 \) and \( G'' = \mu_0 \omega \), where \( \omega \) is the angular velocity (rad/s). A frequency sweep is carried out within the linear viscoelastic region (such that the oscillation frequency is changed while the oscillatory shear stress is kept constant at a low value such that the results are independent of the precise stress value applied). Fig. 5 shows the frequency/shear-rate dependent polymer relaxation time \( \lambda \) from the measurements of SAOS for all working solutions at 20 °C. The values of the longest relaxation time, \( \lambda_{long} \), which are estimated in the limit of the angular velocity approaching to zero, are also summarised in Table 5 for all working fluids. Moreover, an additional procedure has also been carried out to estimate
the mode-averaged longest relaxation time by applying the experimental data of the storage and loss modulus within a multimode Maxwell model fit [25,31] i.e. $\lambda_{\text{Maxwell}}\equiv \sum_{i}^{n} \lambda_{i} / \sum_{i}^{n} \eta_{i}$ where $\lambda_{i}$ are the relaxation modes and $\eta_{i}$ the viscous modes (typically three modes were required for a satisfactory fit).

The estimated values of the longest relaxation time for the selected viscoelastic solutions are listed in Table 5 and it can be seen that they are very similar to $\lambda_{0}$. The relaxation time of a dilute polymeric solution can also be evaluated according to Zimm’s theory as [32]:

$$\lambda_{Zimm} = F \frac{[n]M_{w} \eta_{i}}{N_{A} k_{B} T}$$  \hspace{1cm} (2)

where $N_{A}$ is the Avogadro’s constant, $k_{B}$ is the Boltzmann’s constant, $T$ is the absolute temperature, $M_{w}$ is the molecular weight of polymer, $\eta_{i}$ is the solvent viscosity and $[n]$ the intrinsic viscosity which can be estimated as $[n] \sim 1/\zeta$ [33] to be 3077 and 2222 mll/g for PAA-W/Gly and PAA-W/Suc, respectively, using the experimentally-determined values of critical overlap concentration. The prefactor $F$ is defined by the Riemann Zeta relationship [33,34] to be $F=\frac{\zeta_{0}}{\sum_{i=1}^{n} \frac{1}{\zeta_{i}}}$ = 0.5313, where $\zeta$ is the solvent quality parameter which can be obtained for a good solvent from the exponent in the Mark-Houwink correlation $[n] = KM^{(3\nu -1)}$ to yield $(3\nu - 1) = 0.8 \Rightarrow \zeta = 0.6$ where, $K$ is equal to $49.0 \times 10^{-3}$ with intrinsic viscosity in units of (ml/g) and $(3\nu -1)$ is equal to 0.8 in water [35]. Therefore, the estimated relaxation times for PAA-W/Gly and PAA-W/Suc are 2.5 s and 1.4 s, respectively. The theoretical Zimm relaxation times obtained can be compared to those determined from the SAOS measured in the cone-and-plate rotational rheometer (see Table 5), showing good agreement.

Fig. 6 shows experimental results of the first normal-stress difference, $N_{f} (= \pi \sigma_{xx} - \pi \sigma_{yy})$ for polymeric solutions at the concentrations used for the convective heat transfer experiments. The data of the first normal-stress difference was fitted with a power-law (the solid lines in Fig. 6) between shear rates $\dot{\gamma} > 10 \text{ (1/s)}$ because of the limited experimental resolution (below the sensitivity of the rheometer) and the shear rates $\dot{\gamma} < 200 \text{ (1/s)}$, above which the shear flow became unstable owing to viscoelastic instabilities. Data were obtained with 20 s of equilibration time at each value of shear rate and over several trials (at least four runs for each sample). The temperature was kept constant at 20°C by using a Peltier plate to set the temperature of the solution sample to within $\pm 0.1$ °C. The first normal-stress difference can be estimated from the total normal force for a cone-and-plate geometry using [36],

$$N_{f} = \frac{2F}{\pi R^{2}}$$  \hspace{1cm} (3)

where $F$ is the total normal force (N) and $R$ is the cone radius (m). The experimental values of normal force for all viscoelastic solutions may be slightly lower than the true values because of inertial effects, which is known as the ‘negative normal stress effect’ [31] and was corrected by adding the inertial contributions using [36],

$$\Delta F = \frac{3\pi \rho \omega^{2} R^{4}}{40}$$  \hspace{1cm} (4)

where $\Delta F$ is the difference in the normal force owing to inertia ($N$), $\rho$ is the density of the sample and $\omega$ is the angular velocity (rad/s). A density meter (Anton Paar DMA 35 N) with a quoted precision of 0.001 g/cm³ was utilised for evaluating the density of the working fluids.

Differential scanning calorimetry (DSC) – Model V24.11 DSC Q2000, TA Instruments, USA – was used to measure the specific heat capacity by the modulated method to obtain a measurement of heat capacity for all selected viscoelastic solutions. The DSC was fully computer-controlled with rapid energy compensation and equipped with automatic data analysis software to calculate heat capacity from the heat flow data. Accuracy and reproducibility of heat capacity measurements on the DSC were first validated with the standard samples of sapphire, which are run under the same conditions that are subsequently used for all samples. The inset of Fig. 7 shows the experimental results of a sapphire sample (sample mass = 22.6 mg and heating rate 2°C/min.) in the range from 742.1 to 874.2 K together with standard sapphire data [37] for temperatures from 10 to 83.3°C. Comparison with standard sapphire data illustrates that the accuracy of the experimental sapphire measurements was within 0.5%. Fig. 7 displays the experimental specific heat capacity results for both PAA-W/Gly and PAA-W/Suc solutions in the range of temperatures between 10 and 83.3°C. The data representation has been plotted as an average from at least three runs for each sample. The experimental

Table 5

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>$\eta_{1}$ (Pa·s)</th>
<th>$\eta_{c}$ (Pa·s)</th>
<th>$\lambda_{cP}$ (s)</th>
<th>$\alpha$ (Dimensionless)</th>
<th>$\pi$ (Dimensionless)</th>
<th>$\lambda_{s}$ (s)</th>
<th>$\lambda_{\text{Maxwell}}$ (s)</th>
<th>$\lambda_{Zimm}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/Gly</td>
<td>0.208</td>
<td>1.145</td>
<td>0.220</td>
<td>19.517</td>
<td>1.372</td>
<td>0.501</td>
<td>0.154</td>
<td>1.25</td>
</tr>
<tr>
<td>50-W/Gly</td>
<td>1.145</td>
<td>0.220</td>
<td>22.122</td>
<td>1.763</td>
<td>0.595</td>
<td>0.307</td>
<td>2.11</td>
<td>2.24</td>
</tr>
<tr>
<td>100-W/Gly</td>
<td>3.487</td>
<td>0.304</td>
<td>25.399</td>
<td>1.205</td>
<td>0.647</td>
<td>0.615</td>
<td>5.64</td>
<td>5.49</td>
</tr>
<tr>
<td>W/Suc</td>
<td>0.160</td>
<td>0.186</td>
<td>0.564</td>
<td>0.364</td>
<td>0.421</td>
<td>0.178</td>
<td>1.12</td>
<td>0.87</td>
</tr>
<tr>
<td>80-W/Suc</td>
<td>0.253</td>
<td>0.261</td>
<td>0.490</td>
<td>0.434</td>
<td>0.645</td>
<td>0.267</td>
<td>1.71</td>
<td>1.63</td>
</tr>
<tr>
<td>120-W/Suc</td>
<td>0.369</td>
<td>0.348</td>
<td>1.238</td>
<td>0.274</td>
<td>0.490</td>
<td>1.111</td>
<td>4.03</td>
<td>3.87</td>
</tr>
<tr>
<td>500-W/Suc</td>
<td>0.769</td>
<td>0.348</td>
<td>0.490</td>
<td>0.274</td>
<td>0.490</td>
<td>1.111</td>
<td>4.03</td>
<td>3.87</td>
</tr>
</tbody>
</table>

Fig. 6. First normal-stress difference against shear rate for PAA-W/Gly and PAA-W/Suc solutions. Solid lines represent a power-law fit to the normal-stress data.

[36], [25,31]
results indicate that the specific heat capacities increase gradually with increasing temperature for all viscoelastic solutions. However, the specific heat capacities of PAA-W/SUC solutions are slightly greater than the values of PAA-W/GLY solutions over the entire temperature range. The data in Fig. 7 shows that there is a slight effect from the different polymer concentrations in the chosen viscoelastic solutions on the values of specific heat capacity but no clear trends are apparent and the differences are within the repeatability of the technique for identical fluids. Thus we conclude, for the fluids used here, the addition of polymers to a Newtonian solvent does not alter its specific heat capacity in agreement with previous studies in the literature [38].

The thermal conductivity of the samples W/GLY, 200-W/GLY, W/SUC and 500-W/SUC has been measured using the Fox-50 device, which is a commercial instrument manufactured by Laser-Corp Thermal Conductivity Instrument. Measurements of the thermal conductivity are taken from the thermal contact resistance via the guarded heat flow meter technique. Pyrex was selected to calibrate the Fox-50 device because it is close to the expected value of the thermal conductivities. The experiments were conducted with a temperature difference between the hot and cold plates at 10 °C at two different mean temperatures (10 °C and 40 °C). The average values of the thermal conductivity (at least three runs for each sample) for the selected samples were then determined. The results suggest that the addition of small amounts of polymers has a negligible effect on the values of the thermal conductivities as any small variation in the results is within the uncertainty and repeatability of the measurement (±10%). Therefore, the current results are consistent with the results of Lee et al. [39], who demonstrated that the addition of polymer up to 10,000 ppm (w/w) to Newtonian solvents does not change values of thermal conductivity for these resulting solutions. Therefore, it is possible to use the thermal conductivity values of aqueous glycerine solutions [40] for all PAA-W/GLY solutions and sucrose solution values [41,42] for PAA-W/SUC solutions (see data in Table 6). We note here, that although the negligible effect of dilute polymers on the specific heat capacity and thermal conductivity is in good agreement with the prevalent view in the literature [38], these results are in marked constant to the recent results of Traore et al. [22] where significant differences in the thermal diffusivity are reported for similar solutions to one of those used here (W/SUC).

Finally to estimate the activation energy for each of the two solvents we measured the viscosity across the temperature range used in the experiments (20–30 °C, 293–303 K) and fitted an exponential Arrhenius-type fit to the data. Doing so gives an estimation of the activation energy of 1710 kJ/kg for the glycerol solvent and 1300 kJ/kg for the sucrose solvent indicating a stronger temperature-dependence for the glycerol-based fluids.

### 4. Results and discussion

The experimental measurements of steady-state pressure drop, ΔP, for all working fluids acquired along the serpentine channel between an upstream reservoir and a downstream reservoir for a range of flow rates between 0.2 ml/min and 24.8 ml/min are presented in Fig. 8. The pressure-drops for the Newtonian fluids exhibit a linear increase across the whole range of flow rates. Fig. 9 shows the Darcy friction factor, \( f = 2 \Delta P W / (\rho L \mu L^2) \), where \( \mu \) is a bulk flow viscosity, \( W \) and \( L \) are the depth and path-length of the square serpentine channel, respectively, versus \( Re = \rho U_b W / \eta_{CH} \), where \( \eta_{CH} \) is the characteristic shear viscosity, which was obtained at a characteristic shear rate, \( \dot{\gamma}_{CH} = U_b W / \mu \), using the Carreau-Yasuda model fit [28] to the steady-shear viscosity measurements at the mean film temperature equal to \( T_m = (T_{ni} + T_{no}) / 2 \), where \( T_{ni} \) and \( T_{no} \) denote the mean (bulk) fluid temperature at the inlet and outlet reservoirs, respectively, for all working fluids. The experimental measurements of friction factor were measured when the upper wall of the serpentine channel was insulated and the side walls were maintained at constant temperature. Fig. 9 indicates that the values of friction factor for Newtonian fluids (W/GLY and W/SUC) collapse with the theoretical (Darcy) equation \( f = \mu / 2 \rho L \eta_{CH} \) for fully-developed isothermal laminar flow \( (Re = 0.2–5.7) \) where the friction factor values decline linearly with \( Re \) on a log-log plot. Although secondary flow may be generated due to the combination of large axial normal stresses with streamline curvature (e.g., as in the classical paper of Dean [44]), we find here that the Newtonian fluid flow behaves essentially as flow in a straight

### Table 6

Thermal properties for all working fluids.

<table>
<thead>
<tr>
<th>Working solutions</th>
<th>( \rho ) (kg/m³)</th>
<th>( C ) (kJ/kg K)</th>
<th>( K ) (W/mK)</th>
<th>( Pr ) (Dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W/GLY</td>
<td>1233</td>
<td>2477</td>
<td>0.305</td>
<td>0.3015 [40]</td>
</tr>
<tr>
<td>50-W/GLY</td>
<td>1234</td>
<td>2492</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>100-W/GLY</td>
<td>1236</td>
<td>2496</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>200-W/GLY</td>
<td>1237</td>
<td>2454</td>
<td>–</td>
<td>0.285</td>
</tr>
<tr>
<td>W/SUC</td>
<td>1311</td>
<td>2606</td>
<td>0.368</td>
<td>0.402 [41]</td>
</tr>
<tr>
<td>80-W/SUC</td>
<td>1315</td>
<td>2561</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>120-W/SUC</td>
<td>1315</td>
<td>2538</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>500-W/SUC</td>
<td>1318</td>
<td>2611</td>
<td>0.365</td>
<td>–</td>
</tr>
</tbody>
</table>
channel even within the serpentine channel owing to the dominance of the viscous forces, which suppress the formation of secondary flow [23] (Dean’s vortices [44]). The maximum deviation between the measured values of friction factor and the well-known Darcy’s formula is within ±7.6%. Therefore, experiments with Newtonian fluids emphasised the validity of the present experimental techniques and measuring system. In contrast, the friction factor values for viscoelastic solutions, which were selected to understand elastic turbulence effects on heat transfer in the serpentine channel, are also shown in Fig. 9. The measured values of friction factor for these viscoelastic solutions demonstrate a significant increase compared with the Newtonian solution over the same range of Re. In the viscoelastic fluid flow (even with very low Re) a secondary flow may develop due to the combination of a first normal-stress difference and streamline curvature [45]. However, the large increases in the non-dimensional pressure drop for the viscoelastic solutions are probably attributable to the appearance of “elastic turbulence” above a certain flowrate (see e.g. data in Table 1).

The friction factor–Reynolds number product – essentially the pressure-drop normalised by a viscous stress – is shown in Fig. 10 as a function of Wi = λoγWi, where λo is the longest relaxation time, which was evaluated at the mean film temperature. The values of fRe, normalised by fRe for laminar fully-developed Newtonian fluid flow in a straight square duct (Re = 57), rise rapidly initially with increasing Wi before levelling off. From previous publications [3, 6–8, 45, 46], it is known that, creeping flows of viscoelastic solutions (Re → 0) through a serpentine channel develop firstly at low Wi to a steady secondary flow [45] before the onset of a purely-elastic instability, which leads eventually to oscillatory time-dependent flow at Wi of order one for constant-viscosity fluids (Wi ~ 3.2 [3, 1.4–3.5] [6]). Beyond this first purely-elastic instability, the flow becomes increasingly complex and then develops to elastic turbulence, which is observed beyond Wi > 6.7 [3, 10] [6], 7.5–15 [7]. Fig. 10 shows that for Wi < 5 the pressure drop is similar to that for a Newtonian fluid. With increasing Weissenberg number (5 < Wi < 25) the purely-elastic instability develops and leads to an increase in the normalised pressure-drop for all viscoelastic solutions beyond Wi ≈ 25 are much greater than the Newtonian limit, suggesting that the complexity of the elastic instabilities increase with increasing Wi [21]. The highest non-dimensional values of pressure-drop in terms of normalised fRe increase approximately from 1.86 for 50-W/GLY to 4.67 for 200-W/GLY for shear-thinning solutions meanwhile for Boger solutions they range from 1.48 for 80-W/SUC to 4.82 for 500-W/SUC.

Measurements of the temperature difference between outlet and inlet (Tm,o - Tm,i) for viscoelastic solutions (shear-thinning and Boger solutions) versus Wi are shown in Fig. 11. The temperature of fluid in the inlet and outlet represent, in fact, the average fluid temperature in the reservoirs of the upstream and downstream of the serpentine channel. As can be seen in Fig. 11, the temperature difference decreases progressively with increasing Wi as a consequence of the reduced residence time in the channel. The effect of elastic turbulence on enhancing the heat transfer can most readily be seen for the higher concentration solutions (e.g. 200-W/GLY and 500-W/SUC) where the temperature increase remains approximately constant (approximately 8 °C) despite this reduced residence time.
The enhancement of heat transfer by elastic turbulence can be more fully quantified via the estimation of mean Nusselt number, $\overline{Nu}$, which represents the ratio of convective heat transfer to purely conductive heat transfer between a moving fluid and a solid surface [43], defined as,

$$\overline{Nu} = \frac{\dot{m}CW}{KA} \frac{(T_{m,o} - T_{m,i})}{\Delta T_{lm}}$$

(5)

where $\dot{m}$ denotes the mass flow rate, $k$ and $C$ represent the thermal conductivity and specific heat capacity of the fluid, respectively. $A_t$ is the heated surface area of the square serpentine channel ($=458 \text{ mm}^2$). $\Delta T_{lm}$ is the log-mean temperature difference and is defined as [43]:

$$\Delta T_{lm} = \left[ (T_w - T_{m,o}) - (T_w - T_{m,i}) \right]$$

(6)

where $T_w$ is the channel wall temperature.

The mean $Nu$ values for Newtonian solutions (W/GLY and W/SUC) and viscoelastic solutions (shear-thinning and Boger solutions) against Graetz number, $Gz (=W/LRePr$, where $Pr (=Ck/C_s/k)$ is Prandtl number where the averaged values of Prandtl number at the mean film temperature for all working solutions are listed in Table 6), are shown in Fig. 12. The averaged $Nu$ for Newtonian fluid flow collapses to the numerical predictions [24] for a thermally-developing laminar flow through a straight square duct under the condition of constant wall temperature. The curvature of the serpentine channel has apparently little impact for Newtonian fluid flows at such low $Gz$ (up to 14.6). On the other hand, there is a significant increase of the $\overline{Nu}$ for all viscoelastic solutions. It is expected that this enhancement of heat transfer is due to elastic turbulence [3,5–8], which apparently evolves in the flow and results in enhanced mixing [2,5]. Elastic turbulence generated in the flow of viscoelastic solutions is shown to augment the convective heat transfer in the serpentine microchannel by approximately 200% for 50-W/GLY and 80-W/SUC and reaches up to 380% for 200-W/GLY and 500-W/SUC under creeping-flow conditions in comparison to that achieved by the equivalent Newtonian fluid flow at identical Graetz number.

The $\overline{Nu}$ data, normalised by the equivalent $\overline{Nu}$ for a Newtonian fluid in a square duct, is presented against $Wi$ in Fig. 13. Fig. 13 shows that the behaviour of viscoelastic flow is essentially Newtonian at very low $Wi$ (<25), and the flow here can be considered to be quasi-viscous as the secondary flow does not appear to effect $Nu$ much. Beyond a critical Weissenberg number, $Wi_c \approx 25$ [3,6,7], the flow undergoes a purely-elastic instability. With increasing $Wi$, the viscoelastic flow evolves to fully-developed "elastic turbulence" [3–8]. Thus, we posit that, the increase in convective heat transfer here is due to the influence of such elastic turbulence, which is created by the non-linear interaction between elastic stresses generated within the polymeric solutions and the streamline curvature of the serpentine geometry [3–8]. From Fig. 13, there is an excellent overlap of the data for all dilute viscoelastic solutions showing an enhancement in convective heat transfer with increasing $Wi$. However, the normalised values of $\overline{Nu}$ for the semi-dilute viscoelastic solution (500-W/SUC) increase more sharply against $Wi$ than the dilute solutions. Shear-thinning influences may be significant because of the large shear rates encountered in this
micro-scale flow as might thermal development effects. Therefore, Fig. 14 illustrates the relation between the normalised $N_{\mu}$ and modified Weissenberg number, $Wi^*$, which we define as:

$$Wi^* = \frac{W}{L} Pr Wi.$$  \hspace{1cm} (7)

The modified Weissenberg number, which represents the ratio of elastic stress to thermal diffusion stress, combines geometric dimensions (depth, W, and path-length, L, of the square serpentine channel), Prandtl number ($Pr$) and Weissenberg number ($Wi$) to describe the convective heat transfer under thermally-developing conditions. It is worthwhile to note that the momentum diffusivity for all viscoelastic solutions is much greater than the thermal diffusivity, such that $Pr > 1$. The experimental results in Fig. 14 show that Boger solutions can enhance convective heat transfer over a $Wi^*$ range of approximately between 250 and 750. While, the shear-thinning solutions need a wide range of $Wi^*$ to enhance convective heat transfer from around 500 to 3250. Essentially, the results are indicating that the mean Nusselt number is a function of both $Wi^*$ and the degree of shear-thinning of the fluids. Generally, the findings in Fig. 14 are consistent for each viscoelastic solution group depending on the rheological characteristics and thermal properties. Overall the collapse against the classical $Wi$ (Fig. 13) is better, especially for dilute solutions.

Differences between the constant viscosity and shear-thinning solutions both in the pressure-drop data (Fig. 10) and in the heat transfer data (Figs. 13 and 14) may be due to a number of different reasons. Firstly we have used the longest relaxation time, determined from small amplitude oscillatory shear data in the limit of low frequency, at the mean film temperature to determine the Weissenberg number. As the data in Fig. 5 shows, however, the relaxation time exhibits frequency/rate dependence and a characteristic relaxation time – determined at the mean wall shear rate for example – might offer better collapse of the data. One method to determine such a rate-dependent relaxation time in the non-linear regime is to use first normal-stress difference data as shown in Fig. 6. Unfortunately our attempts to replot the data using this data did not prove successful, most probably as a consequence of the large uncertainty associated with measurements of $N_1$ for such low concentration fluids. A secondary reason for the differences may be due to the estimate of the viscous stress within the flow which we estimated as a characteristic shear rate ($U/W$) multiplied by a characteristic viscosity determined at the same shear rate (at the mean film temperature). The use of a simple characteristic shear rate is clearly a simplification as the shear-thinning fluids will have flatter velocity profiles in the laminar regime – and hence higher wall shear rates – and the mean film temperature is only a first order correction. We note also that the relaxation time exhibited different temperature and shear-rate dependence for each of the two fluid types (e.g. constant-viscosity or shear thinning). The activation energy estimates for the two solvents also suggest that the glycerine-based (shear-thinning) solutions are likely to exhibit greater temperature dependence adding further complication to any attempt to collapse the data onto a single curve. Additionally, a conventional “dimensional analysis” approach to the problem would suggest extra dependency with an additional dimensionless group for the shear-thinning fluids (e.g. the $n$ parameter of the Carreau-Yasuda model provided in Table 5). Even the onset of the first purely-elastic instability has recently been shown to be affected by shear-thinning, such that data collapse cannot solely be achieved based on a Weissenberg number alone once shear-thinning becomes significant [47]. Given all of the above issues, the reasonable collapse found against one single dimensionless group, either $Wi$ or $Wi^*$, for the six different fluids is perhaps not unreasonable.

5. Conclusion

Convective heat transfer and pressure-drop measurements were quantitatively measured using two groups of viscoelastic fluids, namely shear-thinning solutions and approximately constant-viscosity Boger solutions. The thermal boundary conditions were such that the upper wall of the serpentine channel was insulated (adiabatic) and the side walls were maintained at constant temperature. The normalised values of non-dimensional pressure drop in terms of $Re$ – essentially the pressure-drop normalised by a viscous stress – for the highly-elastic viscoelastic solutions (both shear-thinning and Boger solutions) increase monotonically with increasing $Wi$ and were significantly higher than the Newtonian limit which we attribute to the appearance of so-called “elastic turbulence” at high $Wi$. The elastic turbulence is generated by the non-linear interaction between elastic normal stresses created within the flowing high-molecular-weight polymer solutions and the streamline curvature of the serpentine channel. The elastic turbulence created in the flow of these viscoelastic solutions is able to boost the heat transfer by approximately 200% for low polymer concentrations (50-W/GLY and 80-W/SUC) and reaches up to an increase of 380% for high polymer concentrations (200-W/GLY and 500-W/SUC) whilst keeping the Graetz number sufficiently low such that inertia is essentially negligible. A modified Weissenberg number, defined as the ratio of elastic stress to thermal diffusion stress, is able to approximately collapse the normalised data of mean Nusselt number for each viscoelastic solution group. We suggest that elastic turbulence is therefore a method which can be employed to enhance convective heat transfer and to boost micro-mixing technologies in practical micro-scale applications.

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