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$g-2$ of the muon: status report

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Abstract

A short review is given on the status of the anomalous magnetic moment of the muon, $g-2$. Emphasis is put on recent developments in the different sectors and the accuracy of the Standard Model prediction. We comment on the prospects for further improvements in time for the upcoming $g-2$ experiments and on New Physics explanations of the discrepancy, which has been further consolidated.

1. Introduction

Historically, the anomalous magnetic moments of leptons, $a_\ell = (g-2)_\ell/2$, have been a cornerstone in the development of quantum field theory. They allow for very strong tests of the Standard Model (SM) and are providing increasingly effective constraints of physics beyond. While the electron's anomaly, a_e, currently gives the most accurate determination of the fine structure constant α [1, 2], the muon's anomaly, a_μ, is, due to the much higher mass of the muon, much more sensitive to higher scales. Since the arrival of the accurate measurements of a_μ by E821 at Brookhaven in the late 90's, there exists a significant discrepancy between its Standard Model prediction, $a_\mu^{\rm SM}$, and its experimental value, $a_\mu^{\rm EXP}$, which is completely dominated by the BNL measurements [3]: $a_\mu^{\rm EXP} = 1\ 165\ 920\ 89(63) \times 10^{11}$. This tantalising discrepancy has been objected to much scrutiny, but persists. The situation is depicted in Fig. 1. The current discrepancy amounts to $3-4\,\sigma$, depending on the details of the SM prediction and is, hence, still inconclusive. New experiments are currently under construction at FNAL and at J-PARC, see [5, 6] for status reports. Both experiments aim at improving the experimental uncertainty by about four times compared to the BNL measurement. If the mean value stays the same, this would mean a $5\,\sigma$ discrepancy if the SM prediction is unaltered. However, if the uncertainty of the SM prediction could be halved, the discrepancy would move to a clear 7-8 σ signal for new physics. Alternatively, a change in the mean values would either mean an even stronger signal or stronger constraints on new physics. Clearly, a further improvement in the SM prediction of $g-2$ is highly desirable.

2. The Standard Model prediction of $g-2$

The anomalous magnetic moment receives contributions from all sectors of the SM, and possibly from New Physics (NP): $a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$.

Following a many-year-long effort from Kinoshita and collaborators, the QED contributions, i.e. all contributions which are due to photons and leptons only, are known up to and including five-loop accuracy [7]. The result is $a_\mu^{\rm QED} = 116\ 584\ 718.951\ (0.009)(0.019)(0.007)(.077) \times 10^{-11}$, where the uncertainties come from the lepton masses, the four- and five-loop contributions and the input value for α obtained from measurements using ^{87}Rb atoms [8], respectively. The perturbative series converges very well and the resulting error is negligible compared to the uncertainties of other contributions, see below. However, the four- and five-loop results depend heavily on numerical integrations, and independent

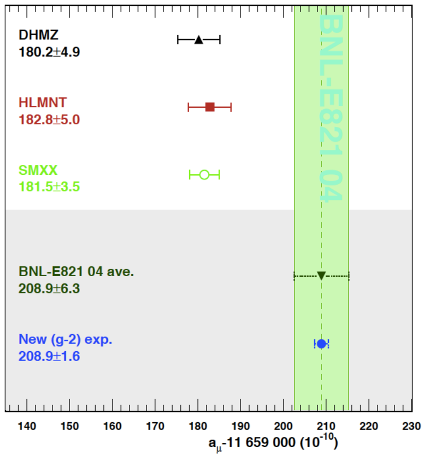

Figure 1: Comparison of recent SM predictions of $g-2$ with the current experimental average. The marker labelled 'SMXX' indicates an anticipated improvement in the SM prediction, while 'New (g-2) exp.' assumes no change in the mean value but a four-fold improvement in the error of the experimental value, as planned for E989 at Fermilab. See [4] for more details. (Figure from [4].)

evaluations of these crucial results are important. In a series of works, this has been achieved for specific classes of four- and five-loop diagrams [9, 10, 11, 12]. All four-loop diagrams with internal leptons have been calculated independently and agree with the results from Kinoshita *et al.*. The not-yet checked 'universal' (purely photonic) four-loop term is small, of the same order as the five-loop contribution and less than a quarter of the $g-2$ discrepancy. Hence, the QED contributions are in very good control.

The contributions from the weak interactions are known to two-loop accuracy [13, 14, 15, 16]. With the Higgs mass now known, their value is estimated to be $a_\mu^{\rm EW} = (153.6 \pm 1.0) \times 10^{-11}$ [17]. While the weak contributions are negligible compared to the dominant QED contributions, with the current accuracy of $a_\mu^{\rm EXP}$, one is sensitive to this sector of the SM. Their uncertainty is large compared to the uncertainty of $a_\mu^{\rm QED}$, but small compared to the hadronic uncertainties. As, in addition, estimates of the leading higher-order contributions to $a_\mu^{\rm EW}$ are included in its current value, the electroweak contributions are, similar to the QED contributions, very well under control.

The story is less straightforward for the hadronic contributions. They are divided into hadronic vacuum polarisation (VP) and so-called hadronic light-by-light (HLbL) scattering contributions. Both classes are dominated by contributions from the low mass spectrum of hadronic resonances and can not be calculated within perturbative QCD (pQCD). The VP contributions start at order α^2 with a single HVP insertion in the leading one-loop QED diagram (leading order hadronic VP) and have been estimated, using dispersion relations and experimental cross section data for $e^+e^- \to hadrons$ at leading, next-to leading order (NLO) and recently at next-to-next-to leading order (NNLO). They will be discussed in more detail below, see also [18].

The HLbL contributions enter at α^3 and are, hence, sub-leading compared to the LO HVP contributions. However, so far it has been impossible to estimate them in a model-independent way. Estimates from model calculations are based on meson exchanges, the large N_c limit, chiral perturbation theory and short-distance constraints from the operator product expansion and pQCD. Several groups have made estimates which are, by and large, compatible, and recent estimates mostly used are $a_\mu^{\rm HLbL} = (105 \pm 26) \times 10^{-11}$ (the 'Glasgow consensus', following a conference on $g-2$ in Glasgow) [19] and $a_\mu^{\rm HLbL} = (116 \pm 39) \times 10^{-11}$ [20], see also [21]. Recently, re-evaluations of the axial exchanges in $a_\mu^{\rm HLbL}$ have led to smaller contributions, see [22, 23]. If those estimates were used, then the total HLbL contributions would go down by several units of 10^{-11}, but still within their original uncertainties. In [24], even higher-order HLbL corrections have been estimated to be $a_\mu^{\rm HO\ HLbL} = (3 \pm 2) \times 10^{-11}$ and, therefore, not alter the picture significantly.

To further consolidate the SM prediction of $g-2$, it will be crucial to obtain model independent calculations of $a_\mu^{\rm HLbL}$ and, in fact, prospects to achieve this have improved during the last few years. In recent works different groups have shown ways to constrain either the input into the model calculations, i.e. meson form factors, or the HLbL tensor itself by experimental data, involving dispersion methods or by lattice calculations, see e.g. [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. Recently, a lot of effort has also been invested in developing methods for a direct lattice simulation of $a_\mu^{\rm HLbL}$, see e.g. [36] and references therein. Given all these developments, it is reasonable to assume that the estimate of $a_\mu^{\rm HLbL}$ will be consolidated and a reduction of its all-important error below that of the 'Glasgow consensus' will be possible within a few years.

The HVP contributions, $a_\mu^{\rm HVP}$, currently contribute the biggest uncertainty to $a_\mu^{\rm SM}$. Like $a_\mu^{\rm HLbL}$, they are mostly non-perturbative and, as such, can not be calculated reliably using pQCD. However, unlike in the HLbL case, there are dispersion relations which allow the direct calculation of $a_\mu^{\rm HVP}$ at LO, NLO and NNLO.

At LO,

$$a_\mu^{\rm LO\ HVP} = \frac{m_\mu^2}{12\pi^3}\int_{m_\pi^2}^{\infty}\frac{{\rm d}s}{s}\hat{K}(s)\sigma^0_{\rm had}(s)\,, \qquad (1)$$

where $\hat{K}(s) \sim O(1)$ is a well-known kernel function and $\sigma^0_{\rm had}$ is the total hadronic cross section which starts at the one-pion threshold. The superscript 0 indicates that the bare cross section must be used, i.e. $\sigma_{\rm had}$ undressed w.r.t. VP (running α) effects, as those are part of the higher order VP contributions. However, note that QED final state radiation effects must be included as part of the hadronic cross section, as they can not be added separately at the required level of precision. At low energies, $\sigma_{\rm had}$ is measured individually in many different hadronic channels, whose contributions have to be added. At higher energies, from above about $1.8 - 2$ GeV, such a channel summation becomes impossible and one relies on measurements of the inclusive hadronic cross section and/or estimates based on pQCD. Many experiments have contributed to this programme over recent decades, with many more current and future measurements having been discussed at this conference. Several groups have compiled the total hadronic cross section, evaluated $a_\mu^{\rm HVP}$ based on Eq. 1 (and corresponding dispersion integrals at NLO) and combined their results with the other SM contributions, see e.g. [21] for a detailed review. Two examples for $a_\mu^{\rm SM}$ compilations are depicted in Fig. 1, labelled DHMZ [37] and HLMNT [38]. Since then, many more data sets have become available, either obtained through the method of radiative return which has become feasible with the high luminosity at modern colliders, or via direct energy scan, as discussed in detail at this conference [39, 40, 41, 42, 43, 44]. For recent updates of hadronic cross section compilations and determinations of $a_\mu^{\rm SM}$ which are based on a model independent direct data compilation and integration, see [22, 18].[1] In the following, we will briefly report on progress of our own compilation.

Our previous estimate, HLMNT11 [38], employed a non-linear χ^2-minimisation which included fitted renormalisation factors as nuisance parameters representing the energy independent systematic uncertainties, see also [46, 47]. We have checked that, contrary to some warnings in the literature, this method (in our case) does not lead to any significant bias in the channel compilations used in our previous determinations of $a_\mu^{\rm HVP}$.

However, recent precise data, especially radiative return data in the $\pi^+\pi^-$ and K^+K^- channels, have energy-dependent uncertainties and non-trivial bin-to-bin correlations from both statistics and systematics, therefore necessitating the use of full covariance matrices in fits. To achieve this, we use a method advocated in [48], employing an iterated fit which has been shown to not incur a bias. Our new procedure does not include nuisance parameters, but makes full use of all energy dependent systematic errors in the fit, thus allowing for an increased fit flexibility.

Being not only the largest contribution to $a_\mu^{\rm HVP}$, but also the largest contribution to its uncertainty, the $\pi^+\pi^-$ channel requires a great deal of scrutiny. The most recent data in this channel are from KLOE(12) [49] and BESIII(15) [50] and became available only after our last major evaluation [38]. Both data sets were obtained via radiative return and were released with full covariance matrices. In our revised fit procedure, their inclusion, in addition to all data from the older radiative return and direct scan measurements, leads to a significantly improved result.

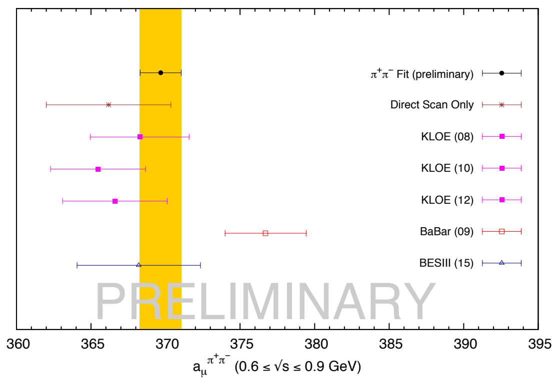

Figure 2: Comparison of mean values of individual radiative return data sets and the combination of the direct scan measurements contributing to the $\pi^+\pi^-$ channel, against the fit of all contributing data. All references are cited in [38], except for KLOE(12) [49] and BESIII(15) [50], which are more recent than our previous analysis.

In Fig. 2, estimates for $a_\mu^{\pi^+\pi^-}$ in the range from $0.6 - 0.9$ GeV from individual (groups of) data sets are compared with the result from our full fit which achieves a combination of all data with a global $\chi^2_{\rm min}/{\rm d.o.f.} \sim 1.3$ and a significantly improved error. Clearly, the new KLOE12 data confirm their earlier measurements and are, for the integral, in very good agreement with the new BESIII and the older direct scan data, while us-

[1] In an alternative approach, Benayoun *et al.* [45] fit the hadronic cross section data with a model based on Hidden Local Symmetry which provides additional constraints and correlations between different channels. Their preferred estimate of the resulting value for $a_\mu^{\rm HVP}$ is slightly lower and has a smaller error compared to the previously discussed evaluations, leading to a $g-2$ discrepancy above $4\,\sigma$.

ing exclusively BaBar data would lead to a much larger value of $a_\mu^{\pi^+\pi^-}$. However, there is some tension in the spectral functions. This is visible in Fig. 3, which

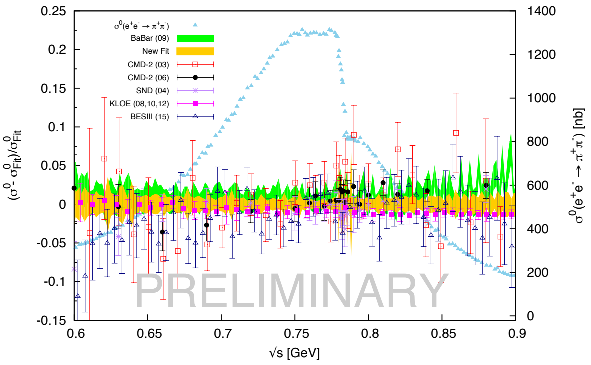

Figure 3: Normalised difference as a function of energy between the leading measurements and the fit of all data in the ρ-ω interference region of the $\pi^+\pi^-$ channel. The (light blue) triangles display the fitted bare cross section $e^+e^- \to \pi^+\pi^-$ in nb.

shows normalised differences between individual sets and our full fit. When compared to the corresponding figure in [38], there is clear improvement of the combination in the important peak and ρ-ω interference regions. However, especially at lower and higher energies, the spectral functions of the different experiments somewhat disagree with each other and the BaBar measurement is, in general, considerably higher than the full $\pi^+\pi^-$ combination. Nevertheless, the inclusion of the additional data sets and the improved algorithm lead to a much consolidated combination. New, high precision $\pi^+\pi^-$ data sets are also expected from CMD-3 [42] and BaBar [18] in the near future, so the picture here should only improve.

The improvements in available hadronic cross section data, along with previously unmeasured data in multi-pion and multi-kaon final states, have allowed for hugely positive progress in our estimate of $a_\mu^{\rm HVP}$. An example of this is the K^+K^- channel, depicted in Fig. 4. Here new radiative return data from BaBar [51] and direct scan data from SND [52], along with better treatment of the previously sizeable and too conservative additional uncertainty due to the treatment of radiative corrections, has resulted in a much better fit and greatly reduced error.

Above 1.8 – 2 GeV, $a_\mu^{\rm HVP}$ is determined through either estimates from pQCD, or the combination of inclusive hadronic R-ratio measurements. Recently, new precise measurements of R were completed with the KEDR detector in Novosibirsk [53, 54], which agree well with

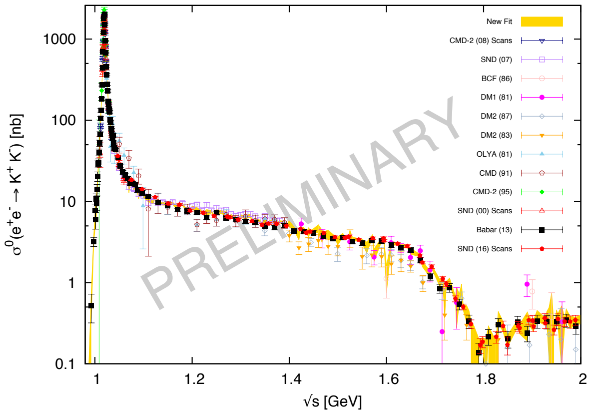

Figure 4: Bare cross section $e^+e^- \to K^+K^-$ from threshold to 2 GeV showing all contributing data sets and the combined fit of all data (yellow band). All references are cited in [38], except for BaBar(13) [51] and SND(16) [52], which are more recent than our previous analysis.

pQCD. The lowest few data points can be seen in Fig. 5, which highlights the agreement between the upper end of the sum of exclusive hadronic final states and the fit of inclusive R data in our analysis, and compares both to the prediction from pQCD.

Recently, there has also been a lot of effort and progress in lattice calculations aiming at a precise prediction of $a_\mu^{\rm HVP}$ from first principles, see [55] for a discussion at this conference, and references therein. While the current simulations are not yet competitive with the data-driven dispersive approach discussed above, the recent improvements are impressive. The main future challenges, required for a meaningful comparison at the precision level, are the inclusion of QED corrections, the evaluation of the so-called disconnected contributions and, last but not least, the reliable estimate of the systematic uncertainties.

Hadronic VP contributions can also be calculated at higher order, employing dispersion integrals similar to (1) and the same compilations of hadronic cross section data as used at LO. At next-to-leading order (NLO), they are negative and result in a reduction of the leading order result by more than 1.4%, see [47, 38, 37, 18, 22]. Their uncertainty is not relevant for the error of $a_\mu^{\rm SM}$. Recently, the next-to-next-to leading order (NNLO) hadronic VP contributions were calculated to be $a_\mu^{\rm NNLO\ HVP} = 1.24 \times 10^{-10}$ [56]. This is somewhat bigger than expected and slightly shifts the total HVP contributions upwards.

Taking all this into account, where do we currently stand when summing all SM contributions and com-

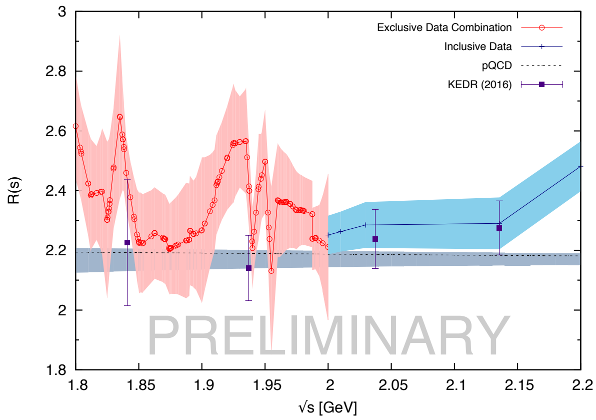

Figure 5: Energy region around 2 GeV where the fit of inclusive hadronic R-ratio measurements replaces the combined sum of exclusive hadronic final states. The recent KEDR measurement [53] is individually marked and included in the inclusive data fit represented by the band. The estimate from pQCD (including mass corrections and a small contribution due to the inclusive QED correction) is included for comparison as a dashed line with an error band which is dominated by the variation of the renormalisation scale.

paring $a_\mu^{\rm SM}$ with $a_\mu^{\rm EXP}$? The precise value and error of $a_\mu^{\rm SM}$ depends, of course, on the details chosen for the hadronic contributions, both HLbL and VP. Nevertheless, with the recent and current developments, not only has the status of $g-2$ as displayed in Fig. 1 been consolidated, but the discrepancy has become slightly more significant. The progress of our own compilation leads to preliminary estimates that result in a slight lowering of the mean value for $a_\mu^{\rm HVP}$, with a significantly reduced uncertainty which, in turn, increases the discrepancy. This is in line with other updated evaluations using direct data integration, which quote $3.6\,\sigma$ [18] and $3.8\,\sigma$ [22] (see also footnote 1).

3. Physics beyond the Standard Model

The now longstanding $g-2$ discrepancy has inspired many attempts to formulate New Physics (NP) models (which solve $g-2$ and other puzzles) and to use $g-2$ to constrain such scenarios. Supersymmetric extensions of the SM remain a possible, good candidate for TeV scale NP [57, 58, 59]. While, given the direct exclusion limits from LHC and other searches, highly constrained models like the CMSSM are looking increasingly disfavoured as explanations of g–2, nature could have chosen large mass splittings with lighter sleptons and, simultaneously, large SUSY masses in the hadronic sector. Even an extended Higgs sector alone would already be sufficient to solve the discrepancy [60]. In this scenario, as well as for studies of supersymmetry, estimates based on one-loop calculations may lead to misleading results and higher-order analyses are required. Tools for this are becoming available, see e.g. [61].

A lot of other scenarios have been studied, which also aim at solving deviations from the SM in other sectors. Examples include TeV scale leptoquarks, where one new scalar could explain $g-2$ and anomalies seen by BaBar and the LHC in the flavour sector [62]. However, with no direct signal for high-scale NP, possible extensions of the SM at low energies have recently attracted increased attention. Examples include a light extra gauge boson (Z') with non-standard couplings to evade existing constraints from light leptons [63, 64, 65] or exchange particles for a fifth force ('dark photons'), see e.g. [66]. Similarly, axion-like particles could lead to enhanced VP and LbL contributions and solve the g–2 discrepancy [67].

4. Future prospects and conclusions

The anomalous magnetic moment of the muon remains in the focus of attention. The construction of the new experiments at FNAL and at J-PARC is progressing well, and first results from E989, with statistics comparable to that of the BNL measurement, are expected already in 2018. For the theoretical calculation of g–2, all sectors of the SM prediction have been further scrutinised. For QED, independent calculations have confirmed all numerically leading parts of the 4-loop and some 5-loop contributions, and the errors are very small. With the Higgs mass known, the error of the weak contributions has also been reduced. For the hadronic corrections, whose errors dominate the error of $a_\mu^{\rm SM}$, important progress has been made. In the case of the hadronic LbL contributions, a proof of concept already exists for lattice calculations, whereas new dispersive approaches, together with experimental data for meson form factors, will allow to further constrain the model calculations or to replace them, at least in-part, by data-driven evaluations. For the hadronic VP corrections, a substantial error reduction is possible already with currently available data sets. Anticipated data from several experiments will allow to improve the situation even further. For the most important $\pi^+\pi^-$ channel, new analyses ar under way by CMD-3 and BaBar. Both these experiments, together with SND, will also contribute to sub-leading and multi-hadron channels, which will make the iso-spin relations previously used to estimate contributions of missing channels unnecessary. In addition, new scan measurements for the inclusive cross section at higher energies are expected from BESIII. This rich experimental programme works hand-in-hand with further improved analysis techniques, as well as calculations for radiative corrections and data combination. Consequently, the $g-2$ discrepancy has been consolidated at a level of $3.6\,\sigma$ or above, fuelling further theoretical work to explain the discrepancy by NP or to constrain popular BSM models.

We would like to thank the organisers of tau2016 for a very productive and enjoyable workshop.

References

[1] D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. **100** (2008) 120801.
[2] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. D **91** (2015) no.3, 033006
[3] G.W. Bennett *et al.* [Muon g-2 Coll.], Phys. Rev. D **73** (2006) 072003.
[4] T. Blum, A. Denig, I. Logashenko, E. de Rafael, B. Lee Roberts, T. Teubner and G. Venanzoni, arXiv:1311.2198 [hep-ph].
[5] J. Mott, in these proceedings.
[6] Y. Sata, in these proceedings.
[7] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109** (2012) 111808.
[8] R. Bouchendira, P. Clade, S. Guellati-Khelifa, F. Nez and F. Biraben, Phys. Rev. Lett. **106** (2011) 080801.
[9] P.A. Baikov, A. Maier and P. Marquard, Nucl. Phys. B **877** (2013) 647.
[10] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, Nucl. Phys. B **879** (2014) 1.
[11] A. Kurz, T. Liu, P. Marquard, A.V. Smirnov, V.A. Smirnov and M. Steinhauser, Phys. Rev. D **92** (2015) no.7, 073019.
[12] A. Kurz, T. Liu, P. Marquard, A. Smirnov, V. Smirnov and M. Steinhauser, Phys. Rev. D **93** (2016) no.5, 053017.
[13] A. Czarnecki, B. Krause and W.J. Marciano, Phys. Rev. D **52** (1995) R2619.
[14] A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. **76** (1996) 3267.
[15] S. Peris, M. Perrottet and E. de Rafael, Phys. Lett. B **355** (1995) 523.
[16] A. Czarnecki, W.J. Marciano and A. Vainshtein, Phys. Rev. D **67** (2003) 073006; Erratum: Phys. Rev. D **73** (2006) 119901.
[17] C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, Phys. Rev. **D88** (2013) 053005.
[18] M. Davier, in these proceedings.
[19] J. Prades, E. de Rafael and A. Vainshtein, Adv. Ser. Direct. High Energy Phys. **20** (2009) 303.
[20] A. Nyffeler, Phys. Rev. D **79** (2009) 073012.
[21] F. Jegerlehner and A. Nyffeler, Phys. Rept. **477** (2009) 1.
[22] F. Jegerlehner, EPJ Web Conf. **118** (2016) 01016.
[23] V. Pauk and M. Vanderhaeghen, Eur. Phys. J. C **74** (2014) no.8, 3008.
[24] G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera and P. Stoffer, Phys. Lett. B **735** (2014) 90.
[25] V. Pauk and M. Vanderhaeghen, Phys. Rev. D **90** (2014) no.11, 113012.
[26] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP **1409** (2014) 091.
[27] G. Colangelo, M. Hoferichter, B. Kubis, M. Procura and P. Stoffer, Phys. Lett. B **738** (2014) 6.
[28] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP **1509** (2015) 074.
[29] M. Procura, G. Colangelo, M. Hoferichter and P. Stoffer, EPJ Web Conf. **118** (2016) 01030.
[30] A. Nyffeler, Phys. Rev. D **94** (2016) no.5, 053006.
[31] A. Gérardin, H.B. Meyer and A. Nyffeler, Phys. Rev. D **94** (2016) no.7, 074507.
[32] G. Antoine, H.B. Meyer and A. Nyffeler, arXiv:1611.02190 [hep-lat].
[33] F.-G. Cao, in these proceedings.
[34] Yaqian Wang, in these proceedings.
[35] F. Redmer, in these proceedings.
[36] T. Izubuchi, in these proceedings.
[37] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **71** (2011) 1515; Erratum: Eur. Phys. J. C **72** (2012) 1874.
[38] K. Hagiwara, R. Liao, A.D. Martin, D. Nomura and T. Teubner, J. Phys. G **38** (2011) 085003.
[39] K. Griessinger, in these proceedings.
[40] W. Gradl, in these proceedings.
[41] M. Achasov, in these proceedings.
[42] S. Eidelman, in these proceedings.
[43] H. Hu, in these proceedings.
[44] C. Shen, in these proceedings.
[45] M. Benayoun, P. David, L. DelBuono and F. Jegerlehner, Eur. Phys. J. C **75** (2015) no.12, 613.
[46] K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, Phys. Lett. B **557** (2003) 69.
[47] K. Hagiwara, A.D. Martin, D. Nomura and T. Teubner, Phys. Rev. D **69** (2004) 093003.
[48] R.D. Ball *et al.* [NNPDF Coll.], JHEP **1005** (2010) 075.
[49] D. Babusci *et al.* [KLOE Coll.], Phys. Lett. B **720** (2013) 336.
[50] M. Ablikim *et al.* [BESIII Coll.], Phys. Lett. B **753** (2016) 629.
[51] J.P. Lees *et al.* [BaBar Coll.], Phys. Rev. D **88** (2013) no.3, 032013.
[52] M.N. Achasov *et al.* [SND Coll.], Phys. Rev. D **94** (2016) no.11, 112006.
[53] V.V. Anashin *et al.*, arXiv:1610.02827 [hep-ex].
[54] V.V. Anashin *et al.*, Phys. Lett. B **753** (2016) 533.
[55] B. Chakraborty, in these proceedings.
[56] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, Phys. Lett. B **734** (2014) 144.
[57] D. Stöckinger, J. Phys. G **34** (2007) R45.
[58] J.P. Miller, E. de Rafael, B.L. Roberts and D. Stöckinger, Ann. Rev. Nucl. Part. Sci. **62** (2012) 237.
[59] M. Bach, D. Stöckinger, H. Stöckinger-Kim and J.h. Park, EPJ Web Conf. **118** (2016) 01034.
[60] A. Cherchiglia, P. Kneschke, D. Stöckinger and H. Stöckinger-Kim, arXiv:1607.06292 [hep-ph].
[61] P. Athron *et al.*, Eur. Phys. J. C **76** (2016) no.2, 62.
[62] M. Bauer and M. Neubert, Phys. Rev. Lett. **116** (2016) no.14, 141802.
[63] J.P. Lees *et al.* [BaBar Coll.], Phys. Rev. D **94** (2016) no.1, 011102.
[64] W. Altmannshofer, C. Y. Chen, P. S. Bhupal Dev and A. Soni, Phys. Lett. B **762** (2016) 389.
[65] A. Lusiani, in these proceedings.
[66] J.L. Feng, B. Fornal, I. Galon, S. Gardner, J. Smolinsky, T.M.P. Tait and P. Tanedo, Phys. Rev. Lett. **117** (2016) no.7, 071803.
[67] W. J. Marciano, A. Masiero, P. Paradisi and M. Passera, arXiv:1607.01022 [hep-ph].