**Lifetime deflections of long-span bridges under dynamic and growing traffic load**

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**Abstract:**

Steady traffic growth may pose a safety hazard to in-service bridges, especially long-span bridges subjected to the simultaneous presence of multiple heavy-duty trucks. This study presents a methodology for evaluating the statistical extrapolation of traffic load effects on long-span bridges. As part of the contributions advancing the state of the art, this study addresses several challenging issues, including traffic growth, the resulting dynamic impact and actual traffic patterns. The nonstationarity of the traffic load effects due to traffic growth is considered in a series system compounded by several interval traffic models. The dynamic impacts of traffic loads are simulated by a traffic-bridge coupled vibration system, and its statistical characteristics are captured using the Rice level-crossing model. The actual traffic pattern is simulated by stochastic traffic flows based on the statistics of the weigh-in-motion measurements of a highway bridge. Two numerical examples demonstrate the ability of the interval traffic growth model to capture the nonstationarity of the growing traffic loads. In

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addition, a case study of a long-span suspension bridge shows the effectiveness of implementing the proposed methodology for the statistical extrapolation of the maximum deflection. The numerical results of the case study demonstrate that the degradation of the road roughness conditions leads to more level crossings, but results in a slight increase in the extrapolation of the deflection. However, the traffic growth results in rapid increases in both the extrapolated deflection and the probability of exceedance of the deflection limit.

Keywords:
Long-span bridge; Traffic load; Deflection; Vehicle-bridge interaction; Level-crossing;
Road roughness condition; Traffic growth

Introduction
Due to the intense competition in transportation of goods in the global market, truck overloading has increased and has led to the collapse of numerous in-service highway bridges over the last few decades (Fu et al. 2009; Zhu et al. 2010). A steady growth in traffic volume and vehicle weight may pose safety hazards to in-service bridges (Deng et al. 2016; Wang et al. 2017). Although the live-load model in a design specification has a confidence level that ensures the bridge safety over the expected lifetime, such levels may be underestimated relative to actual traffic loading scenarios involving extremely overloaded trucks. For instance, a case study conducted by Han et al. (2015) showed that 4 of 1319 trucks yielded bridge hogging moments larger than the value estimated by China’s designed live-load model. In addition to the general phenomenon of traffic loading, long-span bridges are flexible and ductile, especially in the event of the simultaneous presence of multiple heavy-duty trucks (Zhou and Chen 2014). To ensure the serviceability of long-span bridges, design specifications have recommended certain deflection limits, such as $L/250$ and $L/300$ for the maximum deflection and the maximum deflection range in the Eurocode 3 (ECS 2005), where
$L$ is the effective length of a bridge. Therefore, the lifetime deflection and the reliability calibration of long-span bridges considering the actual traffic load are worth investigating.

Implementations of site-specific measurements in bridge engineering have been investigated by numerous studies, such as the calibration of design live-load models (Nowak, 1995; Kwon et al. 2011; Enright et al. 2013), bridge fatigue reliability assessment (Lu et al. 2016) and the characteristic traffic load effect extrapolation (Oconnor, 2005; Caprani 2008; OBrien and Engright 2011). Conventional methods for extrapolating traffic load effects are associated with the generalized extreme value (GEV) theory and Rice’s level-crossing theory (OBrien et al. 2015b). Nonstationary traffic load effects on short-span bridges ($L<30$ m) due to traffic growth has been investigated by OBrien et al. (2014). In addition to investigations on the static extrapolation cited above, studies on dynamic extrapolation of the traffic load effects have been conducted. For instance, OBrien et al. (2010) presented an assessment dynamic ratio (ADR) to investigate the influence of vehicle-bridge interaction (VBI) on medium-span bridges. Subsequently, Caprani et al. (2012) utilized the ADR in conjunction with a multivariate GEV theory to investigate the dynamic allowance of a highway bridge under long-term traffic loading. Even though these developments are mostly concentrated on short- to medium-span bridges, a solid foundation has been provided for the extended application to long-span bridges.

The simultaneous presence of multiple trucks increases the complexity of the statistical extrapolation for a long-span bridge based on the actual traffic pattern. The traffic load effect under individual truck loads can be treated following an independent identical distribution (IID), but the load effect induced by the presence of multiple vehicles violates the IID assumption. To solve this problem, Caprani et al. (2008) presented a maximum traffic loading event accounting for the simultaneous presence of multiple trucks to evaluate the maximum traffic load effect. Critical traffic loading scenarios on long-span bridges were presented by
OBrien et al. (2012) utilizing microscale stochastic traffic flows simulated based on weigh-in-motion (WIM) data. Lifetime traffic loading scenarios on medium span bridges were modeled by Enright and OBrien (2013) with the consideration of free-flowing traffic conditions. Several recent studies have emphasized the traffic loading behavior on long-span bridges. For instance, OBrien et al. (2015a) utilized a microscale traffic model to investigate congested traffic loading on long-span bridges with consideration of truck proportions. In this scenario, slow-moving traffic led to greater loading than fully stopped traffic. Caprani et al. (2016) investigated the influence of microscale traffic scenarios on the extrapolated results. Ruan et al. (2016) used site-specific WIM data to estimate the traffic load effect of a multi-pylon cable-stayed bridge and the maximum static friction coefficient of an anti-sliding model for a suspension bridge. The dynamic effects of traffic loading due to deteriorated road surfaces on long-span bridges lead to higher dynamic amplification factors (DAFs) (Chen and Wu 2010). The studies cited above contributed to the understanding of the statistical extrapolation of the traffic load effects on long-span bridges.

In practice, stochastic traffic flow models are an effective and realistic and need to be considered especially for probabilistic applications of long-span bridges during multiple-truck events. Since a bridge lifetime is much longer than the duration of the recorded traffic data, some conventional assumptions in this field might be inappropriate. First, the traffic volume will grow during the lifetime of a bridge, leading to virtual nonstationarity of traffic load effects over the bridge lifetime. This phenomenon apparently violates the stationarity assumption in the GEV theory. Consequently, even though the dynamic effects of traffic loading are not significant for flexible long-span bridges, the deterioration of the road surface will amplify the traffic load effect. These issues increase the complexity of the probabilistic investigation of the traffic load effects. However, to the best of the authors’ knowledge, the influence of some actual traffic load behaviors, such as dynamic effects and traffic growth, on
the statistical extrapolation for long-span bridges remains unclear.

This study aims to present a methodology for the statistical extrapolation of traffic load effects on long-span bridges considering several challenging factors, including traffic growth, dynamic impacts and actual traffic patterns. The actual traffic pattern is simulated via stochastic traffic flows based on the statistics of WIM measurements of a highway bridge. The nonstationary traffic load effects due to traffic growth are considered in a series system composed of several interval traffic models. The traffic dynamic impact is simulated by a traffic-bridge coupled vibration system and its statistical characteristics are captured using Rice level-crossing model. The proposed methodology is demonstrated and verified via two numerical examples and subsequently applied to a case study of a suspension bridge. The effects of road surface deterioration and traffic growth on the statistical extrapolation of the maximum deflection and the probability of exceedance of the deflection limit are investigated.

**Theoretical bases of Rice extrapolation for traffic-bridge interaction**

The traffic dynamic effects on bridges due to traffic-bridge interaction impacts the statistical extrapolation. To investigate the differences between the static and dynamic extrapolations, Rice formula is utilized to count the number of level crossings to capture the difference in the probabilistic models. Herein, the theoretical formulations of the traffic-bridge interaction and Rice formula are introduced.

**Traffic-bridge interaction**

The traffic-bridge interaction comes from the VBI that is conventionally used for DAF estimation. In a VBI system, the vehicle is usually simulated by the degrees of freedom (DOFs) in physical coordinates. For instance, a 3D 2-axle truck model as shown in Figs. 1 can be simulated by 7 DOFs including two rotational ($\theta_{r1}$ and $\theta_{r2}$) and a vertical translational
motions \((Z_{vb})\) of the vehicle rigid body, as well as vertical translational motions \((Z_{vl}^1, Z_{vl}^2, Z_{vl}^3, Z_{vl}^4, Z_{vl}^5)\) of each vehicle wheel, where \(K_{vl}, K_{vr}, C_{vl}, \) and \(C_{vr}\) are the stiffness and the damping terms of the upper and lower suspension system on the left and right wheels, respectively. The equation for the motions in the VBI system can be solved by a modal superposition approach or a step-by-step integration approach in the time domain (Zhang and Xia 2013).

The simultaneous presence of multiple vehicles is unique to long-span bridges compared with short-span bridges. Initially, investigations (Cai and Chen, 2004; Chen and Cai, 2007) suggested that the interaction effects between multiple vehicles on a flexible long-span bridge were insignificant. Subsequently, the up-to-date researches (Zhou and Chen 2015; Chou and Chen 2016) found that interaction effects exist in traffic flows with multiple presences of vehicles in motion. This study utilizes the equivalent dynamic wheel load (EDWL) approach proposed by Chen and Wu (2010) to evaluate the dynamic traffic-flow load effects. The EDWL approach utilizes time-variant forces accounting for the mode shapes and natural frequencies of the bridge to approximate the VBI forces. Eventually, the cumulative EDWL acting on the bridge can be defined as follows:

\[ \{F(t)\}_{eq}^{\text{wheel}} = \sum_{j=1}^{n_v} \left[ \frac{1 - \text{EDWL}_j(t)}{G_j} \right] G_j \cdot \sum_{k=1}^{n_a} \left\{ h_k [x_j(t) + \alpha_k x_j(t) d_j(t)] \right\} \]  

\[ \text{EDWL} = \sum_{k=1}^{n_a} \left( K_{vl} Z_{vl} + C_{vl} \overline{Z}_{vl} \right) \]

where \(\text{EDWL}_j(t)\) is the dynamic load of the \(j\)-th vehicle at time \(t\); \(G_j, x_j, \) and \(d_j\) are the weight, longitudinal location, and transversal location of the center of gravity of the \(j\)th vehicle on the bridge, respectively; \(h_k\) and \(a_k\) are the vertical and torsional mode shapes for the \(k\)th mode of the bridge, respectively; \(n_v\) and \(n_a\) are the number of vehicles on the bridge and the number of axles of the \(j\)-th vehicle, respectively; and \(\overline{Z}_{vl}\) and \(\overline{Z}_{vl}^{ij}\) are the relative vertical displacement
and velocity of the wheel axle with respect to the bridge, respectively. The number of vehicles on the bridge changes with time depending on the stochastic traffic flow, and the road-roughness coefficient (RRC) is considered in the vertical displacement and velocity of the vehicle. The effectiveness of the EDWL approach has been demonstrated by Chen and Wu (2010).

**Rice formula**

With the simulated deterministic load effects, the statistical parameters can be estimated by the tail fittings. The Rice formula (Rice 1945) was chosen in the present study to evaluate both the extreme traffic load effect and the probability of exceedance in the bridge lifetime. The principle of the Rice formula is shown in Figs. 2. In practice, the influence lines of long-span bridges are long enough, and the critical traffic loadings are mostly consistent with intensive vehicle use. Thus, the load effects can be assumed to be a Gaussian random process (Ditlevsen 1994). In addition, interval-based growth in traffic loading evidently satisfies the stationary assumption. Based on the above assumptions, the mean level-crossing rate \( v(x) \) under the condition of a threshold \( l \) and a reference period is expressed as follows (Rice 1945):

\[
v(x) = \frac{\dot{\sigma}}{2\pi\sigma} \exp \left[ -\frac{(x - m)^2}{2\sigma^2} \right] = \frac{1}{R_l}
\]

(2)

where, \( x \) is the traffic load effect, \( m \) and \( \sigma \) are the mean value and standard deviation of the load effect, respectively, and \( \dot{\sigma} \) is the standard deviation of the derivative of the load effect. In general, the level-crossing rate can be expressed by a normalized rate indicated as fitted histograms versus the summation of truncated remaining histograms. The critical step of using the Rice level crossing theory for extrapolation is to determine the optimal starting point, indicated as \( x_{0, opt} \), and the optimal number of class intervals, indicated as \( N_{opt} \) (Beck and Melchers 2004). For these determinations, the conventional approach is to utilize the Kolmogorov-Smirnov (K-S) statistics recommended by Cremona (2001) to check the
confidence level of the predefined starting point and number of class intervals. Eventually, the statistical extrapolation can be evaluated by the derivation of Eq. (2) considering a return period \( R_0 \).

Note that the extrapolated value \( x \) based on the level-crossing rate can also be defined as the value exceeded with a probability in a reference period. Therefore, the cumulative distribution function (CDF) of the maximum load effects can be written as follows:

\[
F_{\text{max}}(x, T) = 1 - a = 1 - \left[ 1 - \exp\left( -\frac{T}{R_0} \right) \right] = \exp\left\{ -(Tv_x \exp\left[ -\frac{(x - m)^2}{2\sigma^2} \right]) \right\}
\]  

(3)

where, \( T \) is a general time period related to a reference period \( T_{\text{ref}} \), i.e., a 100-year lifetime of a bridge in the present study, and \( a \) is a probability of exceedance, i.e., approximately 10% between \( T_{\text{ref}} = 100 \) years and \( R_0 = 1000 \) years. It is clear that the CDF can be estimated easily given the level-crossing function as shown in Eq. (3) for a stationary process. It is worth noting that these equations can only be used for stationary traffic loads without considering traffic growth. The growing traffic case is shown in next section.

**Methodology for evaluating maximum load effects considering interval traffic growth**

Traffic growth leads to a nonstationary density of vehicles on the bridge, which directly affects the maximum traffic load effects. An interval traffic growth model is utilized in the present study to divide the lifetime traffic loads into several intervals, each of which can be assumed to be non-growing and stationary. An improved Rice formula for combining the interval level-crossing models is presented in a detailed framework.

**Interval traffic growth model**

In general, the traffic volume grows continually over a given period. Such steady growth is usually defined as an annual growth rate (AGR) that is compounded between two years. Since
the traffic density over the bridge lifetime is nonstationary, the continual growth model is inappropriate for use in the load effect extrapolation. However, the traffic volume can be assumed to be stationary over short periods, such as one or two years. This short period is defined as an interval in this study.

An example of the interval traffic growth is shown in Figs. 3, where the curve is the volume of average daily truck traffic (ADTT) representing the assumed traffic growth and the histograms are the volumes of the ADTT in the 10-year interval. It is obvious that the traffic volume is constant during a given interval period rather than growing with the curve. The advantage of the interval growth model is that Rice’s formula can be applied to each interval. The shortcomings of the interval growth model are that the result is only an estimate and that its accuracy mostly depends on the number of intervals. Obviously, an increase in the intervals improves the accuracy of the result but will also lead to additional computational effects.

**Improved Rice extrapolation account for interval traffic growth**

As mentioned before, the Rice formula, as shown in Eqs. (4) and (5), cannot be directly used for traffic load effects under growing traffic load conditions because the traffic density is nonstationary over the bridge lifetime. However, this formula is effective for individual time intervals, as shown in Fig. 3(a). The question then becomes how to combine these individual interval probability models to extrapolate the maximum value over a specified return period. On the basis of related research conducted by Caprani et al. (2008) and Zhou et al. (2016), this study utilizes a series system as shown in Fig. 3(b), to combine the intervals. Therefore, the CDF of the maximum lifetime value can be estimated by multiplying each interval CDF, which can be evaluated via Eq. (3), by $T$, which is equal to the interval period. Assuming that the $T_{ref}$ can be divided into $N_{int}$ intervals, the lifetime CDF can be estimated using the interval CDF products:
where, $F_{\text{max}}^{G}(x,T)$ is the maximum traffic load effect of a bridge in a reference period $T$, considering traffic growth, $N_{\text{int}}$ is the number of intervals, $T_{\text{int}}=T/N_{\text{int}}$ is the interval period, $F_{\text{max},i}(x,T)$ is the CDF of the maximum traffic load effects in the $i$th interval, and $v_{0,i}$, $m_{i}$ and $\sigma_{i}$ are the mean crossing rate, the mean value and the standard deviation of the load effects in the $i$th interval, respectively. Thus, the maximum load effects in a return period can be estimated based on the system CDF as shown in Eq. (4) and the general form of the CDF as shown in Eq. (3), written as

$$R_{i} = \frac{-T_{\text{ref}}}{\ln F_{\text{max}}^{G}(x,T_{\text{ref}})}$$

where, each maximum load effect corresponds to a return period.

Herein, the critical steps of combining the interval traffic load effects for the extrapolation are as follows: (a) estimating the CDF of the traffic load effects in each interval based on Eq. (3); (b) combining the interval CDFs in a series system to formulate the actual CDF via Eq. (4); and (c) computing the return periods with respect to the maximum load effects via Eq. (5). The maximum load effect can also be interpolated from Gumbel probability paper.

In addition to the maximum extrapolation, the Rice formula can also estimate the probability of the maximum load effect exceeding a predefined limit over a reference period in terms of the probability of exceedance. A formulation of the probability of exceedance is written as follows:

$$p(z,t) \approx 1 - F_{\text{max}}(z,T_{\text{ref}})$$

where, $z$ is a predefined threshold for the traffic load effect, and $F_{\text{max}}(z,T_{\text{ref}})$ denotes the CDF of the maximum lifetime load effect at the limit $z$. This equation provides an approach for the
quantification of the probability of traffic load effects exceeding a limit.

A computational framework for extrapolating the maximum load effect of long-span bridges

Based on the interval traffic growth model and the improved Rice formula, a computational framework is presented for evaluating the maximum load effects of long-span bridges using WIM measurements. The flowchart of the computational framework is shown in Fig. 4. The entire procedure outlined in the flowchart is mainly composed of three categories: traffic load simulation, the load effect computation, and the probabilistic extrapolation. The procedures associated with the categories are described below.

The first module depicted in Fig. 4 is the traffic load simulation based on recorded WIM data. With available WIM data, filtering procedures should be conducted to exclude invalid data and to select effective data that contribute to the maximum traffic load effect. In the present study, lightweight cars were removed from the recorded data because most of the critical loading scenarios usually involve dense traffic flows with a high proportion of heavy trucks. The statistical parameters of the traffic flow can be divided into two groups (O’Connor and O’Brien, 2005): (a) those modeling the individual vehicle feature at small scales (i.e., the vehicle configuration, the gross vehicle weight, the vehicle spacing and the driving speed); and (b) those modeling the traffic feature at large scales (i.e., proportions of vehicle types and traffic volume). The most critical parameter is the vehicle spacing, defined as the space between two vehicles in the same driving lane. The vehicle spacing is a unique and critical factor for the traffic flow simulation on long-span bridges and is usually measured by the headway, which is time variant depending on the traffic density (OBrien and Enright 2012). The extreme traffic load effects are mostly induced by dense traffic flows with small vehicle spacings. Therefore, the PDF of the vehicle spacing in dense traffic flows is an important factor impacting the maximum traffic loading on a long-span bridge.
In general, traffic loads on a bridge can be simulated with a mathematical model in the time domain or in the space domain. A stochastic traffic flow is one of these mathematical models composed of individual vehicles formulated with statistical parameters. However, there is a problem with utilizing numerous daily traffic flows to conduct probabilistic and dynamic analyses because the step-by-step integration of the VBI system solution is extremely time consuming. Note that the purpose of using the daily traffic flow load model is to probabilistically model extreme traffic load effects. The maximum value is affected by the upper tail of the load effects produced by critical traffic loading scenarios. Therefore, critical traffic loading scenarios identified in a daily stochastic traffic flow can be utilized for the daily maxima simulation. The principle of identifying the critical traffic loading scenario is related to the static influence line analysis for determining the maximum load effects in the daily stochastic traffic flow. Based on this assumption, a step-by-step search strategy is adopted to identify the critical loading scenario. These procedures are as follows: (1) generating the stochastic traffic flows via MCS and the statistics of the WIM data; (2) specifying the effective range of the loading scenario on the bridge according to the bridge length; (3) moving the predefined range forward along the simulated daily stochastic traffic flows to calculate the static traffic load effect; (4) identifying the maximum load effect and the corresponding loading scenario; and (5) repeating steps (3) and (4) for the remaining daily traffic flows.

The second module is the traffic load effect computation. Two special factors are considered in this module: the VBI and the traffic growth. For the VBI, the dynamic vehicle load of the bridge under the critical loading scenario can be evaluated using the EDWL approach with the consideration of the RRC. The result is a time history that is then used for counting the number of level crossings. For the traffic growth, the interval traffic growth model is used to simulate traffic growth with an assumed AGR. The daily maxima in each
interval are estimated using the static influence lines.

The third module is the probabilistic extrapolation. The first step is to count the number of level crossings based on the estimated traffic load history or the interval daily maxima. The level-crossing rate as shown in Fig. 2(b), can be fitted to the histograms of the number of crossings. Then, the maximum traffic load effect over a return period can be extrapolated based on the Rice formula in Eq. (3) or the interval model in Eq. (4). Finally, the probability of exceedance of the predefined limit can be evaluated from the CDF of the maximum value.

Obviously, there are some key points in the proposed method. Firstly, a higher number of traffic intervals will lead to a more accurate extrapolation but will also lead to more computations. Additionally, because the actual traffic volume in each traffic interval will grow instead of being constant as assumed in the proposed approach, the extrapolated value will be slightly underestimated. In addition, the consideration of lightweight cars should be further developed.

**Verification examples**

Two numerical examples, including an individual GVW extrapolation and a deflection extrapolation of an idealized long-span bridge, are presented to verify the effectiveness of the interval traffic growth model.

**Individual GVW extrapolation**

A numerical example from OBrien et al. (2014) is presented here to verify the effectiveness of the interval traffic growth model. In this example, the block length is 1 day, the total lifetime is 25,000 days, the truck weights follow a normal distribution of $W \sim N(50,5)$ in tons, and the initial traffic volume is 1000 trucks per day. The objective of this numerical study is to extrapolate the maximum individual GVW over a bridge lifetime considering the daily traffic volume growth rate of 0.016% (an average AGR of 4.1%). OBrien et al. (2014) used a day-
by-day growth model to simulate the traffic growth, but the present study adopted a 10-year
time interval. The GEV fitting utilized the entire 100-year daily maxima, while the Rice
fitting utilized the 10-year 30% upper daily maxima. The following discussion focuses on the
accuracy of the extrapolation, as the computational efficiency is the same.

Fig. 5(a) shows the annual crossing rates of the 1st, 5th and 10th interval periods fitted
to histograms simulated by the daily maxima over 10 years. The mean value of the crossing
rate apparently moves to the right hand side, and the mean level-crossing rate shows a slight
decrease. Fig. 5(b) plots the maxima and fittings on Gumbel probability paper, where the
100-year daily maxima are shown as point symbols, the GEV fitting are shown as dash lines,
and the Rice fittings are shown as solid lines. For the case of non-growing traffic, both the
GEV and Rice extrapolations are in agreement with the value (77.50 t) provided by Obrien et
al. (2014). However, for the case of growing traffic, the Rice extrapolation more accurately
fits the reference value (80.60 t). Note that the exact value (78.842 t) for the case of non-
growing traffic computed by the normal distribution of GVW to the power of 1000 is larger
than the extrapolated value. This phenomenon can be explained by the fact that the
convergence of a normal distribution to a Gumbel distribution is extremely slow and both the
Rice and GEV extrapolations are underestimated.

The following inferences can be obtained from the numerical results: (a) the
nonstationarity of the extreme distribution of the GVWs demonstrated by the level-crossing
curves moving to the left side at higher intervals because the higher traffic volume increases
the daily maxima and the number of higher-level crossings; (b) both the GEV and Rice
fittings fit the daily maxima fairly well for the non-growing traffic condition since the GVW
is stationary over the reference period; (c) the GEV fitting deviates from the tail of the daily
maxima due to the nonstationarity of the traffic volume during the bridge lifetime; and (d) the
Rice fitting is close to the tail, as the nonstationarity of the growing traffic has been captured
by the proposed method utilizing an interval traffic growth model. This example demonstrates the effectiveness of the proposed interval model for extrapolating a maximum individual GVW considering traffic volume growth.

**Deflection extrapolation of an idealized long-span bridge**

Since the objective is to apply the proposed method to long-span bridges, a second numerical example is presented to extrapolate the deflection of an idealized long-span bridge associated with the first example. This bridge and the traffic pattern are shown in Fig. 6, in which the vehicle spacing follows a normal distribution of $S_i \sim N(100, 10)$ in meters, and the other parameters are the same as those in the first example. The objective of this example is to extrapolate the maximum deflection of the bridge over a 1000-year return period. The AGR is supposed to be between 0 and 1%.

The interval period is 2,500 days (effective working days in 10 years), and the lifetime includes 10 intervals. Step-by-step simulations based on the influence lines of the bridge were conducted to evaluate the daily maxima. Three typical annual crossing rates are shown in Fig. 7(a). The crossing rate apparently moves to the right side and has a strong nonstationarity. The numerical results and a return period line are plotted in Fig. 7(b). It is observed from the daily maxima that (a) the simulated daily maxima for the case of the no-growth model form a nearly straight line when plotted on the Gumbel probability paper; and (b) the daily maxima for the case of growing traffic model move to the left and form a curve when plotted on Gumbel probability paper. In addition, it is observed from the fittings that (a) for the case of non-growing traffic, the GEV and Rice fittings to the full data or the interval data are both well suited for extrapolating; (b) for the case of growing traffic, the GEV and Rice interval models yield better extrapolations than those of the GEV and Rice full data models; and (c) even though the Rice full data model has a relatively better extrapolation due to the optimal starting point, the interval growth model provides a much better extrapolation.
These phenomena can be explained by the following inferences. First, when considering traffic growth, the probability density of the traffic load effects is time-variant and will move to the higher-value side. The time-variant probability density violates the IID assumption, resulting in worse fitting via the general GEV distribution or Rice formula. Second, the Rice interval fitting is better for providing the extrapolation from the starting point of the final interval (105 mm in this example). Finally, the interval models provide better fittings for the higher-value data rather than the lower-value data. This pattern may be due to an insufficient number of intervals, as shown in Fig. 3(b) to capture a higher probability of exceedance.

**Case study**

A suspension bridge and its WIM data are utilized to demonstrate the effectiveness of the proposed computational framework. The dynamic characteristics and traffic growth are considered in order to investigate their influences on the probabilistic extrapolation.

**Weigh-in-motion measurements of a suspension bridge**

Nanxi Yangtze River Bridge is a long-span highway suspension bridge in Sichuan, China. A pavement WIM system composed of scales or pressure sensors embedded into the road pavement was installed on the bridge. More details of the bridge and the WIM measurements can be found in Liu et al. (2015) and Lu et al. (2016). A filtering process was conducted to identify and to remove invalid records, where the vehicles with the GVW less than 3 t were removed. An overview of the filtered WIM data is summarized in Table 1. The maximum overload rate was about 200% over the GVW limit (55 t) in China for a 6-axle truck (MOCAT 2004). A dense traffic flow composed of such extremely overloaded trucks is literally a hazard to the safety of long-span bridges.

Based on the WIM data and the assumption of stochastic traffic flow, the trucks were classified into 6 categories indicated as V1 ~ V6, where V1 denotes 2-axle light trucks and
V2 ~ V6 denote 2- to 6-axle trucks. The hourly traffic volume per lane was analyzed as shown in Fig. 8(a). The hourly traffic volumes of the busy traffic period between 9:00 and 19:00 were utilized for statistical analysis of the truck spacing. The probability density of the truck spacing is accurately fitted by a lognormal distribution function (LNDF) as shown in Fig. 8(b).

It is worth mentioning that the vehicle spacing in terms of traffic density is a significant factor impacting the traffic load effects on medium- to long-span bridges. This study ignores the effect of lightweight cars on the bridge deflection computation by removing lightweight cars from the database. Therefore, the truck spacing in this study denotes the gap between two following trucks in a traffic lane, which is different from the vehicle spacing, which is defined as the vehicle gap between vehicles in the actual traffic stream. The effective truck spacing between two trucks (rather than the actual vehicle spacing) was utilized in the present study. The truck spacing makes sense in the context of probabilistic domain because the PDF of the truck spacing was fitted to the actual WIM data. Although the truck spacing does not represent an actual traffic state, the PDF has captured the statistical characteristics of the trucks in the actual traffic flows.

**Extreme deflection considering dynamic traffic loads**

As the first task, the commercial program ANSYS was utilized to construct the bridge finite element (FE) model shown in Fig. 9. In the FE model, the stiffening steel box girders and the concrete pylons were modeled using beam elements, and the hangers and the cables were modeled using link elements. Additionally, the bridge tower elements and the cable elements were separated and the top joints were connected by a set of coupled degrees of freedom. This study takes into consideration the pre-tension forces in the main cable and hangers as well as the self-weight of the bridge, which are essential parameters in the mechanical analysis of a suspension bridge. The pre-tension forces and dead loads were considered in the FE model in
the first time step, followed by modal and static analysis. The value of the pre-tension forces and the secondary dead loads were initially determined by the design values and were subsequently updated to bring the structural modal characteristics in agreement with the measured data. It is worth noting that the structural stiffness accounting for the pre-tension forces and the dead load was considered but that the load effect due to the pre-tension forces and the dead load was excluded because the present study concentrated only on the traffic load effect. It is acknowledged that long-span bridges are geometrically nonlinear, especially under heavy traffic loads. The present study, however, is limited to a linear analysis as a first step implementing the improved stochastic analysis. The advancement of the improved stochastic analysis can be extended to the nonlinear case but the computational efficiency should be further developed.

Table 2 summarizes the first 5 fundamental mode characteristics of the FE model. In total, 50 mode frequencies and mode shapes were used in the mode superposition. The vehicle physical properties in this case study were adopted from Yin et al. (2011). The RRC in this study was defined based on the International Organization for Standardization (1995). The RRCs for classifications of “Good”, “Average”, and “Poor” are $32 \times 10^{-6}$, $128 \times 10^{-6}$, and $512 \times 10^{-6}$, respectively. The RRC coefficients were simulated via inverse Fourier transformation approach in the time domain.

As the second task, the dynamic traffic load effects at the critical points of the bridge were simulated. Initially, conditions involving a 2-axle truck with a GVW of 10 t, a driving speed $\nu$ of 20 m/s, and an RRC of “Good” were considered to determine the static and dynamic deflections of the bridge girders. The static analysis was conducted by considering a moving concentrated force $F$ of 100 kN, and the dynamic analysis was conducted via the EDWL approach considering vehicle-bridge interaction. Fig. 10 shows the mean vertical deflections of the three potential points versus the truck loading position on the bridge. It is
observed that the most critical point is the L/4 (quarter-span) point. The maximum deflections due to static and dynamic loads are 0.029 m and 0.032 m, respectively. Therefore, the subsequent investigation focuses on the quarter-point of the bridge.

Subsequently, the critical traffic loading scenarios were identified from the simulated daily stochastic traffic flows. The procedures are as follows: firstly, the vehicle type was randomly sampled according to the proportion of each vehicle type; secondly, the vehicle weight and the driving lane of the vehicle was determined according to their probability densities; thirdly, the vehicle spacing of the following vehicle was determined; finally, the above procedures were repeated to generate a traffic flow model. The stochastic traffic flows were first put into the static influence line to conduct a static analysis. The static deflection was analyzed to identify the critical traffic loading scenario that was then utilized for dynamic analysis. Only a critical loading scenario in each 10 h stochastic traffic flow is necessary for the extreme traffic load effect analysis because the dynamic analysis of a 10 h daily traffic load is time consuming. An illustration of the process for generating the critical loading scenario on a 4-lane bidirectional bridge is shown in Figs. 11, where traffic lanes 1 and 2 are in the same direction, and traffic lanes of 3 and 4 are in the opposite direction. A daily maximum deflection was identified from the static deflection histories in Fig. 11(a), and the corresponding critical traffic loading scenario was extracted from the daily stochastic traffic load as shown in Fig. 11(b). Obviously, the critical loading scenario is a small part (0.1%) of the daily traffic flow. As a result, the time-consuming computation due to the step-by-step integration can be greatly reduced by utilizing the critical traffic loading scenario rather than the entire stochastic daily traffic flows. Therefore, the static analysis can identify the critical loading scenarios for dynamic analysis, thereby improving the efficiency of the dynamic computation of numerous traffic flows.
In total, 1,000 days of traffic flows were simulated via Monte-Carlo simulation and the probability density models of the current WIM traffic data. These critical traffic streams are free flowing with a constant velocity \( v = 20 \text{ m/s} \). Each time-variant concentrated force was computed via the EDWL approach in modal superposition, and was then put into ANSYS to conduct the traffic load effect analysis. Fig 12(a) shows two traffic loading scenarios for better understanding the dynamic and static deflection histories shown in Fig. 12(b). A reference line in Fig. 12(b) counts the number of level crossings at the deflection level \( a = -0.667 \text{ m} \), where terms \( N_{\text{static}} \) and \( N_{\text{dynamic}} \) are the number of crossings for the static and dynamic histories, respectively. Note that the deflection level was utilized to count the number of crossings, whereas the deflection threshold was utilized to evaluate the probability of exceedance. It is observed, from Fig. 12(b), that the dynamic histories fluctuate around the static histories. This makes the numbers of level crossings different. The number of dynamic crossings is always higher than that of static crossings. This numerical result shows the advantage of the Rice formula in capturing the dynamic effects.

Based on the Rice formula, the down-crossing histograms and estimated crossing rates of the dynamic and static histories were estimated, as shown in Fig. 13, where \( x_0 \) is the optimal starting point estimated by the K-S test. It is observed that the dynamic effect has a higher crossing rate, while the mean value and the standard derivation have negligible differences. With the fitted level-crossing models, extrapolations of the maximum deflection were estimated. Fig. 14 shows the results of the static and dynamic extrapolations for a 1000-year return period.

As shown in Fig. 14, the maximum deflections over a return period of 1000 years are 1.445 m, 1.458 m, 1.464 m and 1.475 m for the static results for “Good”, “Average” and “Poor” roughness conditions, respectively. It is observed that the lifetime dynamic ratio, i.e., the ratio between dynamic and static extrapolations, for the poor road roughness condition is
Therefore, although poorer RRCs lead to a larger number of level crossings, its influence on the maximum traffic load effect extrapolation of a long-span bridge appears to be negligible.

**Lifetime maximum deflection assessment considering interval traffic growth**

The European commission predicts a sustainable annual growth ratio of truck traffic volume between 1.5% and 2% (European Commission 2008). Therefore, in the present study, the linear AGR of the traffic volume was set to 0, 1%, 2%, and 3%, and the traffic featured free-flowing conditions. The 100-year lifetime of the bridge was divided into 10 intervals in which the traffic volume was stationary and non-growing. In total, 1000 days of daily maxima for each interval were utilized to estimate the level-crossing model as shown in Fig. 15. It is observed that the level-crossing rate is constant for the non-growing traffic model, while the level-crossing curves apparently move to the left with traffic growth, with a higher traffic growth rate leading to a larger shift. The traffic growth for long-span bridges not only results in a large traffic volume but also leads to a higher traffic density on the bridge. Increases in both of these parameters lead to a rapid growth of the extrapolation of the maximum deflection. This phenomenon is in agreement with the numerical results presented in the verification examples.

Based on the estimated level-crossing models, the extrapolation of maximum deflection over a 1,000-year return period was estimated based on Eq. (6). Fig. 16 plots the extrapolations in the bridge lifetime accounting for the traffic growth. It is obvious that the traffic growth leads to a rapid growth of the maximum deflection. As a result, an AGR of 3% increases to the extrapolation of the lifetime maximum deflection by 18%.

The probability of exceedance of a deflection limit is a serviceability criterion for a bridge under traffic loading. The exceedance criterion was defined as the maximum deflection of the quarter-point of the bridge girder crossing the deflection limit specified in native/local
specifications. Therefore, it is important to define a threshold deflection for the prototype bridge under traffic loading. As mentioned in the introduction, different design codes have different limits, such as \( L/350 \) and \( L/800 \) for long-span bridges and simply supported bridges, respectively (AASHTO, 2015). In the present study, the deflection limit was set to \( L/400 \) according to China’s code (MOCAT 2007).

The probability assessment of the maximum deflections is shown in Figs. 17. Fig. 17(a) shows estimated CDFs of the maximum deflections in the lifetime estimated and plotted on Gamble probability paper, and Fig. 17(b) shows the probabilities of exceedance of the deflection limit over the 100-year period. The \( x \)-axis values at the cross point in Fig. 17(a) are similar to the values shown in Fig. 16. Therefore, the CDF and the extrapolations are in agreement. The probabilities of exceedance for an \( x \)-axis value of 2.05 m in Fig. 17(b) are \( 1.1 \times 10^{-11}, 2.0 \times 10^{-9}, 4.9 \times 10^{-8} \) and \( 2.7 \times 10^{-7} \) for traffic growth ratios of 0, 1%, 2% and 3%, respectively.

It is inferred that traffic growth has a significant influence on the probability of exceedance. This influence is greater for a lower limit, but weaker for a higher limit. In addition, an increase in the traffic growth rate leads to a higher rate of increase in the probability of exceedance. This phenomenon can be explained by the fact that traffic growth not only leads to a larger daily maxima of an individual truck weight but also increases the traffic density on the bridge. Therefore, a reasonable traffic growth model is critical for evaluating the maximum traffic load effect over the lifetimes of a bridge.

Conclusions

This study presents a methodology for statistical extrapolation of traffic load effects over the lifetime of long-span bridges. Advancements have been made in several challenging areas related to realistically addressing vehicle-bridge interactions, actual traffic patterns and traffic growth. The actual traffic pattern was modeled via stochastic traffic flows simulated based on
weigh-in-motion measurements. The continuously growing traffic loads were considered as a series system composed of interval traffic loads. The nonstationarity of the growing traffic load was captured in this model. The methodology was verified by two numerical examples and was subsequently applied to the lifetime maximum deflection extrapolation of a suspension bridge. The following conclusions have been drawn from the numerical studies.

(1) Traffic volume growth leads to a time-variant level-crossing rate, which violates the IID in the GEV extrapolation theory. Therefore, the conventional model deviates from tail of the maxima plotted on Gumbel probability paper and provides poor extrapolations. However, the interval traffic growth model in series has the ability to capture the nonstationality of growing traffic load effects. The interval fitting of the GEV or Rice extrapolations represents the tail fairly well and provides relatively accurate extrapolations.

(2) Not only does traffic growth results in a higher daily maximum GVW, but it also increases the traffic density on the bridge, which is the main reason leading to the significant increase of in lifetime traffic load effect.

(3) The vehicle-bridge coupled vibration under worse road roughness condition leads to more level crossings of the bridge deflection, but it does not significantly impact the mean value and the standard deviation of the level-crossing rate. As a result, the lifetime dynamic assessment ratio is less than 2.1%.

(4) For the site-specific traffic condition of the suspension bridge, the annual traffic growth rate of 3% leads to an 18% increase in the extrapolated maximum deflection for within a 1000-year return period.

(5) The traffic growth has a significant influence on the probability of exceedance of the deflection limit. Such influence is more significant for a lower threshold deflection level, but it is minor for a higher limit. In addition, a higher traffic growth rate leads to a rapid increase in the probability of exceedance.
Although the proposed methodology was applied to the lifetime maximum deflection extrapolation of a suspension bridge, it can also be extended to extrapolate of the maximum bending moment, the maximum cable force, and the longitudinal displacement of other long-span bridges. The findings from the presented study provide a basis for extension in the following direction. Firstly, as an alternative approach of MCS, cellular automaton and Markov chain sampling can be utilized to simulate the microscale behaviour of vehicles, such as changes in the vehicle spacing on the bridge. Secondly, improvements can be made through focusing on congested traffic conditions rather than on free-flow traffic conditions and adjustments in the model since congested traffic conditions were found to be more critical to the maximum deflection. Thirdly, the approximation of the traffic load by removing the highly proportioned lightweight cars from the WIM database was found to be potentially critical since it may result in the distortion of the vehicle spacing in the simulated stochastic traffic flow. Finally, nonlinear stochastic analysis should be considered in the proposed approach, but the computational efficiency associated with the demanding nonlinear calculations is a key problem.

Acknowledgements
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References
ANSYS [Computer software]. ANSYS, Canonsburg, PA


Tables Captions

Table 1. Overview of the filtered WIM measurements

Table 2. The first five order mode frequencies of the suspension bridge
Table 1. Overview of the filtered WIM measurements

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Figures Captions

Figs. 1. Physical models of a 2-axle truck: (a) elevation view; (b) side view

Figs. 2 Basic principles of Rice’s formula: (a) level crossings; (b) fitting to the crossings

Figs. 3. An example of an interval traffic growth model: (a) interval ADTTs; (b) a series system model

Fig. 4. Flowchart of the proposed computational framework for the lifetime maximum traffic load effect extrapolation

Figs. 5. Analytical results of the first example: (a) annual crossing rates; (b) daily maxima and fittings on Gumbel paper

Fig. 6. An idealized long-span bridge and crossing vehicles

Figs. 7. Analytical results of the second example: (a) annual crossing rates; (b) daily maxima and fittings plotted on Gumbel paper

Figs. 8. Statistics of the WIM measurements: (a) hourly traffic volume; (b) truck spacing of the busy traffic flow

Fig. 9. Finite-element model and dimensions of the suspension bridge

Fig. 10. Deflection histories of critical points of the girder under a 2-axle truck load

Figs. 11. An example of identifying the critical loading scenario: (a) a daily deflection history; (b) a critical loading scenario

Fig. 12. An example to show the difference between the numbers of crossings of static and dynamic histories

Fig. 13. Histograms and fittings of the numbers of crossings

Fig. 14. Extrapolations of the maximum deflections considering the RRC

Fig. 15. Time-variant level-crossing rates accounting for traffic growth

Fig. 16. Extrapolation of the lifetime maximum deflection accounting for traffic growth

Figs. 17. Probabilistic assessment of the bridge deflection under growing traffic loads: (a)
CDFs plotted on Gamble paper; (b) probability of exceedance
Figure 2

(a) Up crossing
Level 1
Down crossing
Level 2

(b) Optimal starting point
Rice's fitting
Interval length
Figure 3

(a) Actual traffic growth

A 10-year interval

(b) $P_1 = F_{max_1}(x, T_{int})$  $P_2 = F_{max_2}(x, T_{int})$  $P_n = F_{max_n}(x, T_{int})$
Traffic load simulation:

- WIM measurements
  - Filtering
  - Statistics of traffic flows
    - Stochastic traffic simulation
      - Daily critical traffic-loading scenarios

Load effect computation:

- Considering VBI
  - Bridge mode shapes and frequencies
    - RRCs
    - EDWLs
      - Dynamic load effect histories
- Or
  - Considering traffic growth
    - Bridge static influence lines
      - AGR
      - Interval growth
      - Daily maxima in each interval

Probabilistic extrapolation:

- Counting the number of crossings
  - Fitting the level-crossing rates to the histograms of the number of level-crossings
    - Return period
      - Extrapolation of the maximum traffic load effects
        - Threshold value
          - Lifetime probability of exceedance of the limit
Figure 5

(a) Annual number of crossings

(b) 1000-year return period

-OBrien's value

-GEV (full)

-Rice (interval)

-Log[-log(F_{max})]

-GVW (t)

-Daily maxia (no-growth)

-Daily maxia (AGR=4.1%)
Deflection: $y = G_i \sin \left( \frac{x \pi}{L} \right)$

$G_i \sim \mathcal{N}(50, 5) \text{ t}$

$G_{\text{VW}} = 50 \text{ t}$

$x = L/2 = 50 \text{ m}$

$S_i \sim \mathcal{N}(100, 10) \text{ m}$

$50 \text{ mm}$
Figure 8

(a) Number of trucks per hour over time, with dense traffic highlighted.

(b) Probability density of truck spacing, showing a lognormal distribution with $X \sim LN(6.21, 1.40)$.
Figure 9

- WIM system
- $L = 820 \text{ m}$
- $89 \text{ m}$
- $145 \text{ m}$
Figure 10

Vehicle location on the bridge (m) vs. Vertical deflection (m)

- Static
- Dynamic

Points of interest:
- L/4 point
- 3L/8 point
- L/2 point

Click here to download Figure Fig. 10.pdf
Figure 11

(a) Time (h) vs. Deflection (m)

(b) Location (m) vs. Traffic lane vs. GVW (t)
Critical point

Scenario 1

Scenario 2

(a)

(b)

\[ N_{Satic}^1 = 1 \]
\[ N_{Dynamic}^1 = 2 \]
\[ N_{Satic}^2 = 0 \]
\[ N_{Dynamic}^2 = 2 \]

\[ v = 20 \text{ m/s} \]

RRC = Good

Sample 1

Sample 2

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(a) Diagram showing the critical point for Scenario 1 and Scenario 2.

(b) Graph showing time vs. deflection with static and dynamic lines, indicating a good RRC.

Figure 12
Deflection level (m)
Number of crossings

Figure 13

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Static fitting
Dynamic fitting

Click here to download Figure Fig. 13.pdf
Figure 14

The figure shows the relationship between the maximum deflection (in meters) and the return period (in years) for different levels of condition: Static, Good, Average, and Poor. The deflection values increase as the return period increases, with the Static condition showing the highest deflection for all return periods.

Click here to download Figure Fig. 14.pdf
Figure 15
Figure 16: Extrapolated deflection (m) vs. service period (years) for different AGR rates.

- **AGR = 0**
- **AGR = 1%**
- **AGR = 2%**
- **AGR = 3%**
Figure 17

(a) Graph illustrating the relationship between the deflection level and the probability of exceedance for different AGR values. The horizontal line at -log[-log(-F_{max}(x,100))] represents the return period of 1000 years.

(b) Graph showing the logarithmic scale of the probability of exceedance for various AGR values. The vertical line at L/400 = 2.05 m indicates the calculated deflection level.