Abstract—The rapid and accurate measurement of the volume of a large cavity is much in demand in practice, especially for a cavity of irregular shape. In this paper, a new method is proposed to measure the volume of a large cavity. This method is based on the measurement of the decay time constant of the cavity. A lossy object with a known averaged absorption cross section is used to aid the measurement. By measuring decay time constants of the cavity with and without the lossy object, the cavity volume can be extracted. It is found that only one antenna is required for the measurement and the whole measurement can be completed simply and rapidly. The method could be applied to both metallic and non-metallic cavities which makes the proposed method a very attractive alternative to existing methods.

Index Terms—Absorption cross section, cavity volume, decay time constant.

I. INTRODUCTION

THE measurement of the volume of a large cavity is required especially in shipping and aircraft industry. In practice, to optimize the capacity of the cargo compartment of a ship or an aircraft, it is necessary to get the knowledge of its volume. An example is shown in Fig. 1. However, the volume of such a cavity is not easy to obtain because of its irregular shape and the complex inner environment. Normally, a 3D (three-dimensional) laser scanner can be employed [2]. The laser scanner first scans the whole profile of the cavity under test and builds its 3D model. Subsequently, the volume of the cavity can be calculated from the 3D model. However, this method is time-consuming and costly. It may not be available for some companies or institutes. And also, before scanning the profile of the cavity, the contents that may block the laser beam in the cavity should be removed, which could be tedious or sometimes impossible in reality. Since such a cavity is normally made of metal, an alternative method based on the statistical theory of electromagnetic waves was proposed in [3]. In this method, the volume of a cavity is extracted by comparing the cavity quality factors in the frequency domain and in the time domain. This method is economical, but it is not efficient enough because many stirring positions are required during the measurement. A more efficient method is in demand for practical purpose.

In this paper, we present a new rapid method to measure the volume of a large cavity. This method is based on the measurement of the cavity decay time constants [4] – [7] with and without a lossy object. A lossy object with a known averaged absorption cross section (ACS) is required for the measurement [8] – [13]. It is found that only one antenna is required in this method. Hence, the hardware requirement is very simple, which makes the method very economical. Also, this method is very efficient because only a few stirring positions are needed to complete the measurement. Consequently, the measurement time can be greatly shortened. Furthermore, by using acoustic waves instead of electromagnetic waves, this method can be generalized and the cavity under test does not have to be metallic.

This paper is organized as follows: Section II presents the

Fig. 1 The cargo compartment of an aircraft and the demand of measuring its volume [1].
II. THEORY

Before studying how to measure the cavity volume using the ACS, it is useful to first introduce how the ACS is measured in an electrically large cavity in electromagnetics. The ACS of a lossy object is defined as the ratio of the power dissipated in the object to the power density of the incident plane waves [8]. The averaged ACS $\langle \sigma_{ACS} \rangle$ of the object under test (OUT) can be characterized using the cavity quality factor $Q$ as [4]

$$\langle \sigma_{ACS} \rangle = \frac{2\pi V}{\lambda} (Q_l^{-1} - Q_u^{-1}),$$

where $V$ is the volume of the cavity, $\lambda$ is the wavelength, $Q_l$ and $Q_u$ are the cavity quality factors with and without the OUT, respectively. The subscripts “$l$” and “$u$” are used to represent the loaded scenario and the unloaded scenario, respectively. $\langle \cdot \rangle$ indicates the average with respect to the incidence angles and polarizations [13].

It has been proved the ACS can be measured in the time domain [4], [13]. In the time domain, $Q_l$ and $Q_u$ can be written as

$$Q_l = \omega \langle \tau_l \rangle \quad \text{and} \quad Q_u = \omega \langle \tau_u \rangle,$$

where $\tau_l$ and $\tau_u$ are the cavity decay time constants with and without the OUT, respectively. $\langle \cdot \rangle$ means the average over all stirring positions [4].

Substituting (2) to (1), we obtain

$$\langle \sigma_{ACS} \rangle = \frac{V}{c} \langle (\tau_l)^{-1} - (\tau_u)^{-1} \rangle,$$

where $c$ is the speed of light in free space. The cavity decay time constants $\tau_l$ and $\tau_u$ can be extracted from the time-domain power response of the cavity. Since the static power in the cavity decays exponentially in the time domain, $\tau_l$ and $\tau_u$ can then be obtained from the slope of $\ln(\text{power})$. The detailed procedure of extracting the cavity decay time constant $\tau$ from the $S$-parameters can be found in [13].

It can be clearly seen from (3), the cavity volume $V$ is required to calculate the ACS. (3) can be transformed to the form

$$V = c \cdot \frac{\langle \sigma_{ACS} \rangle}{\langle (\tau_l)^{-1} - (\tau_u)^{-1} \rangle}.$$

We can see that if the ACS of the OUT is known or pre-calibrated, (4) provides a method to obtain the cavity volume. The quantities that we need to measure is $\tau_l$ and $\tau_u$. In this method, only one antenna is required since $\tau$ can be obtained from $S_{11}$. Furthermore, because $\tau$ is very robust [13], only a few stirring positions are needed to extract it accurately. Thus, this method could be very efficient.

III. MEASUREMENT

To verify the proposed method, measurements were conducted in the frequency range of 2.8-4.2 GHz in the RC at the University of Liverpool. The volume of the RC is $3.6 \text{ m} \times 4.0 \text{ m} \times 5.8 \text{ m} = 83.52 \text{ m}^3$. Two mechanical stirrers are installed in the RC. One is mounted in the corner and the other is set close to the ceiling. A double-ridged waveguide horn antenna
(SATIMO SH 2000) was used in the measurement. The antenna was mounted on a turn-table platform to introduce source-stir positions [14]. It was connected to port 1 of the VNA via a cable running through the bulkhead of the RC. A piece of RF absorber was selected as the OUT which was placed on the support (a carton box). In the measurement, the turn-table platform was rotated stepwise to 18 different positions (10° for each step). At each source-stir position, the VNA swept the S-parameters over the full frequency span. It should be noted that the antenna should direct away from the OUT to avoid line-of-sight illumination (to provide a random environment). Therefore, a directional antenna was used and the turn-table platform was rotated 180° instead of 360°. During the measurement, the two stirrers of the RC were not used because in practice it may not have a stirring system in a cavity. A general measurement setup is shown in Fig. 2(a). The measurement setups without and with the OUT are shown in Fig. 2(b) and Fig. 2(c), respectively.

The measurement was conducted with the following 5 steps.

Step 1: Calibrate the VNA including the cables.

**Step 2:** Place the antenna, the turn-table platform and the support inside the RC, excluding the OUT.

**Step 3:** Connect the antenna to the cable connected to VNA port 1 and record the S-parameters ($S_{11}$) at each source-stir position.

**Step 4:** Keep the measurement setup unchanged, place the OUT on the support and repeat Step 3.

**Step 5:** Extract the cavity decay time constants with and without the OUT and calculate the volume of the RC using (4).

It is worth mentioning that $\tau$ can be measured in the time domain directly or in the frequency domain. Here, the frequency-domain measurement is adopted because it normally gives a larger dynamic range than the time-domain measurement. The time-domain response is obtained from the inverse Fourier transform of the measured frequency-domain response.

In our measurement, 10,001 points were sampled in the frequency range of 2.8-4.2 GHz. $S_{11}$ in this frequency span at each source-stir position was collected. The inverse fast Fourier transform (IFFT) was then applied to $S_{11}$. Since $\tau$ is frequency dependent, thus we used an elliptic band-pass filter of order 10 with 200-MHz bandwidth to filter $S_{11}$ [13], as shown in Fig. 3(a). Then the IFFT was applied to the filtered $S_{11}$. Subsequently, the least-square fit was applied to $\ln(|\text{IFFT}(S_{11})|^2)$ and its least-square fit.
\[ \ln(|\text{IFFT}(S_{11})|^2) \] to extract the slope \( k \), and the cavity decay time constant was obtained as \( \tau = -1/k \). To avoid the influence of the early-time response \([6]\) and the noise floor, only part of the signal (where the power in the RC reaches steady state) was used for the least-square fit, as shown in Fig. 3(b). By sweeping the center frequency of the filter, \( \tau \) with different center frequencies was obtained as shown in Fig. 4. The thin curves are the measured \( \tau \) for different source-stir positions and the thick curves are the averaged \( \tau \) for all source-stir positions. The averaged \( \tau \) was used to calculate the cavity volume. The robustness of \( \tau \) can be observed in Fig. 4.

As we can see, under the unloaded scenario (without OUT), the variation between the \( \tau \) for one stirring position and the averaged \( \tau \) is within about \( \pm 10\% \) and under the loaded scenario (with OUT), it is within about \( \pm 5\% \). This is because \( \tau \) is determined by the diffuse loss of the RC which is not sensitive to the source-stir positions \([13]\).

To measure the volume of the RC, the ACS of the OUT was first calibrated in the frequency range of interest, as shown in Fig. 5. It can be seen that the ACS of the OUT is \( 0.1078 \text{ m}^2 \) in the frequency band of 3.0-4.0 GHz. And also, the ACS seems frequency independent. The reason is, when the OUT is electrically large, the ACS only determined by its surface area which does not depend on the frequency \([8]\). Actually, this provides a faster method to obtain the ACS of an RF absorber and consequently, a faster and simpler method to measure the cavity volume with ACS. This point will be discussed later.
In practice, a cavity is hardly vacant or well shielded. It may be loaded with cargoes or have apertures (such as windows or ventilation openings). To verify the validity of the proposed method in a practical environment, we measured the volume of the RC in three different scenarios: the well-shielded scenario, the open-door scenario, and the cargo-loaded scenario, as shown in Fig. 6 (a), (b) and (c), respectively. In the following part, the measurement results under these three different scenarios using the proposed method are detailed.

In the well-shielded scenario, the door of the RC is closed. This scenario is corresponding to a cavity with a high $Q$ factor. The measurement result is shown in Fig. 7 (a). The reference value (the real value of the RC volume, 83.52 m$^3$ here) and the average value (the averaged value of the RC volume in the frequency span of interest) are also plotted. It can be seen that the measured value is close to the reference value. The maximum difference is about 6.8% at 3.403 GHz. The difference between the average value and the reference value is only about 1.2%, which is very small. In practice, there may be some apertures on a cavity, such as windows, ventilation openings or open doors. This scenario is corresponding to a cavity with a relatively low $Q$ factor. To emulate a cavity with apertures, the front door of the RC is open and the whole measurement using the proposed method is repeated. The results are shown in Fig. 7(b). As can be seen, the maximum difference between the measured value and the reference value is approximately 5.8% at 3.423 GHz. The difference between the average value and the reference value is only approximately 0.26%, which is negligible. Sometimes a cavity is loaded with cargoes, such as the compartment of an aircraft or the refrigerated warehouse of a supermarket. When a cavity is loaded with cargoes, its $Q$ factor will decrease. To emulate a cavity loaded with cargoes, two pieces of RF absorbers were placed at the corners of the RC to decrease its $Q$ factor. The measurement result is depicted in Fig. 7 (c). As we can see, the maximum difference between the measured value and the reference value is around 5% at 3.182 GHz. The difference between the average value and the reference value is around 0.5%, which can be neglected.

It should be noted that in practice we may not have a chance to calibrate the ACS of the OUT. Fortunately, it has been proved that, for an electrically large RF absorber of convex shape, its ACS and surface area $S$ satisfy $\text{ACS} = S/4$ [8]. That is, the ACS of an RF absorber is a quarter of its surface area. By utilizing this theory, the proposed method can be further simplified and we do not need to calibrate the ACS of the OUT.

To validate this idea, another piece of RF absorber was selected as the OUT. The size of the base of the absorber was 0.5 m × 0.5 m × 0.06 m. The pyramids of the absorber were fully covered with aluminum foil (because the surface area of the pyramids was not easy to measure) and only the base was subject to absorption, as can be seen from Fig. 8. Thus, the surface area absorbing the electromagnetic waves is 0.37 m$^2$ and its theoretical ACS is 0.0925 m$^2$. The measured and the theoretical ACS are compared in Fig. 9. As can be seen, the average value of the measured ACS is about 0.0825 m$^2$ in the frequency range of 3.0-4.0 GHz which is 0.01 m$^2$ smaller (10.8% smaller) than the theoretical value. The reason is, in reality, the absorber is not an ideal “black body”, i.e., it cannot absorb all the electromagnetic waves that hit on its surface because of the scattering, the diffraction, and the reflection. Consequently, the measured ACS is smaller than the theoretical value. It is worth mentioning that for ease of surface area calculation, an absorber of regular shape (such as rectangular parallelepiped or spherical shape) is preferred. Again, the measurement was conducted by...
following the aforementioned procedure and three different scenarios (well-shielded scenario, open-door scenario, and cargo-loaded scenario) were studied. The theoretical ACS value is used to calculate the volume of the RC. The results are shown in Fig. 10. We can see that the measured RC volume under the three different scenarios is close to the reference value. The maximum differences under the well-shielded scenario, the open-door scenario, and the cargo-loaded scenario are about 20.8%, 19.4%, and 19.5%, respectively. From (4), we can obtain

\[ \frac{dV}{V} = \frac{d\langle \sigma_{ACS} \rangle}{\langle \sigma_{ACS} \rangle} \]  

(5)

That is, the measurement error margin of the cavity volume is determined by that of the ACS. From Fig. 9, we know that

\[ \frac{d\langle \sigma_{ACS} \rangle}{\langle \sigma_{ACS} \rangle} = \frac{0.0925 - 0.0825}{0.0825} \times 100\% \approx 12\% \]  

(6)

Therefore, the difference of cavity volume between the measured average value and the reference value should be around 12%. As shown in Fig. 10, the differences between the average value and the reference value under the well-shielded scenario, the open-door scenario, and the cargo-loaded scenario are about 11.5%, 11.6%, and 9.7%, respectively, which agree well with the theoretical prediction in (6).

The measurement results are summarized in Table I. A comparison between this proposed method and the method proposed in [3] is made. It can be seen that the proposed one-antenna method using source stirring and calibrated ACS can finish the measurement with minimum errors in the shortest time. The proposed one-antenna method using source stirring and theoretical ACS has the merit of short measurement time as well, but its measurement errors are relatively bigger. For practical purposed, the proposed one-antenna method using source stirring and calibrated ACS is most recommended.

It should be pointed out that the method of using electromagnetic waves is only valid for conducting cavities and the OUT should absorb radio waves. If the cavity is made of non-conducting materials such as concrete or bricks, acoustic waves should be used to detect the volume of the cavity. In acoustics, we have [15]

\[ \langle \sigma_\alpha \rangle = \frac{24 \langle \ln 10 \rangle V}{c_0} \left( T_{60,1}^{-1} - T_{60,u}^{-1} \right) \]  

(7)

where \( \langle \cdot \rangle \) represents the average over different microphone positions. \( \langle \sigma_\alpha \rangle \) is the averaged ACS of the OUT (acoustic wave absorbers), \( V \) is the volume of the cavity and \( c_0 \) is the sound speed in the air. \( T_{60,1} \) and \( T_{60,u} \) are the reverberation time (the decay time for a 60 dB sound pressure level decrease) of the cavity with and without the OUT inside, respectively. (7) can be converted to

\[ V = \frac{c_0}{24 \langle \ln 10 \rangle} \cdot \frac{\langle \sigma_\alpha \rangle}{T_{60,1}^{-1} - T_{60,u}^{-1}} \]  

(8)

\( T_{60,1} \) and \( T_{60,u} \) can be extracted from the power delay profiles of the cavity with and without the OUT inside. Thus, the feasibility of the proposed method is generalized. It is not only limited to conducting cavities.

### III. COMPARISON OF MEASURED RC VOLUME

<table>
<thead>
<tr>
<th>Measurement methods</th>
<th>Scenarios</th>
<th>Maximum relative error</th>
<th>Mean relative error</th>
<th>Standard deviation</th>
<th>Measurement time</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-antenna method in [3] (mechanical stirring)</td>
<td>Well-shielded</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Approx. 2 hrs</td>
</tr>
<tr>
<td></td>
<td>Open-door (90°)</td>
<td>17.3%</td>
<td>7.3%</td>
<td>2.84</td>
<td>Approx. 2 hrs</td>
</tr>
<tr>
<td></td>
<td>Cargo-loaded</td>
<td>10.2%</td>
<td>1.7%</td>
<td>2.19</td>
<td>Approx. 2 hrs</td>
</tr>
<tr>
<td>One-antenna method in [3] (source stirring)</td>
<td>Well-shielded</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Approx. 40 mins</td>
</tr>
<tr>
<td></td>
<td>Open-door</td>
<td>26.9%</td>
<td>2.6%</td>
<td>6.05</td>
<td>Approx. 40 mins</td>
</tr>
<tr>
<td></td>
<td>Cargo-loaded</td>
<td>13.8%</td>
<td>1.8%</td>
<td>3.67</td>
<td>Approx. 40 mins</td>
</tr>
<tr>
<td>Proposed one-antenna method (source stirring &amp; calibrated ACS)</td>
<td>Well-shielded</td>
<td>6.8%</td>
<td>1.2%</td>
<td>2.15</td>
<td>Approx. 10 mins</td>
</tr>
<tr>
<td></td>
<td>Open-door (90°)</td>
<td>5.8%</td>
<td>0.2%</td>
<td>2.59</td>
<td>Approx. 10 mins</td>
</tr>
<tr>
<td></td>
<td>Cargo-loaded</td>
<td>5.0%</td>
<td>0.5%</td>
<td>2.06</td>
<td>Approx. 10 mins</td>
</tr>
<tr>
<td>Proposed one-antenna method (source stirring &amp; theoretical ACS)</td>
<td>Well-shielded</td>
<td>20.8%</td>
<td>11.5%</td>
<td>3.44</td>
<td>Approx. 10 mins</td>
</tr>
<tr>
<td></td>
<td>Open-door (90°)</td>
<td>19.4%</td>
<td>11.6%</td>
<td>3.95</td>
<td>Approx. 10 mins</td>
</tr>
<tr>
<td></td>
<td>Cargo-loaded</td>
<td>19.5%</td>
<td>9.7%</td>
<td>3.47</td>
<td>Approx. 10 mins</td>
</tr>
</tbody>
</table>

Source stirring & calibrated ACS means using source-stir technique and the calibrated ACS value; source stirring & theoretical ACS means using source-stir technique and the theoretical ACS value; 90° means the door is open with 90 degrees.

### IV. CONCLUSION AND DISCUSSIONS

In this paper, a rapid and accurate measurement method has been developed to measure the volume of a large cavity. An RF absorber with a known averaged ACS is selected as an OUT to aid the measurement. Using this method, the cavity volume can be obtained by measuring its decay time constants with and without the OUT. The proposed method has been validated with both theory and measurement studies. It is found that the measurement can be completed rapidly with a simple
measurement setup using this method, which makes it an ideal way of measuring the cavity volume. Furthermore, by using acoustic waves, the proposed method can be generalized and the cavity under test does not have to be conducting.

The preconditions of the proposed method should be pointed out. First, the environment inside the cavity under test should be reverberant, i.e., the $Q$ factor of the cavity should not be too low. If its $Q$ is very low, i.e., the cavity is very lossy, it will be very difficult to realize a statistically uniform field inside the cavity. Second, the loss of the OUT should not be too small. As can be seen from (4), the calculation of the cavity volume requires the difference of the cavity decay time constants with and without the OUT. If the OUT loss is too small compared with the cavity loss, it may not be possible for the cavity to perceive the difference of the loss (i.e., the difference of the $Q$ factors with and without OUT), which will result in an inaccurate measurement. Third, during the measurement, the line-of-sight illumination of the antenna to the OUT should be avoided. Or the inaccurate measurement of $r_1$ could occur because most of the power from the antenna will be captured and absorbed by the OUT before being reverberated by the cavity. Last but not least, the proposed method is valid for a single cavity while not for coupled cavities because for coupled cavities, the loss mechanism is different [16] – [18] and (1) is no longer hold.

All in all, the paper has presented an efficient and accurate way of measuring the cavity volume with low hardware requirement. A possible extension of this work would be the on-site measurement practice.

REFERENCES