Robust Sliding-mode Control of Wind Energy Conversion Systems for Optimal Power Extraction via Nonlinear Perturbation Observers

Bo Yang, Tao Yu, Hongchun Shu, Jun Dong, Lin Jiang

Abstract

This paper designs a novel robust sliding-mode control using nonlinear perturbation observers for wind energy conversion systems (WECS), in which a doubly-fed induction generator (DFIG) is employed to achieve an optimal power extraction with an improved fault ride-through (FRT) capability. The strong nonlinearities originated from the aerodynamics of the wind turbine, together with the generator parameter uncertainties and wind speed randomness, are aggregated into a perturbation that is estimated online by a sliding-mode state and perturbation observer (SMSPO). Then, the perturbation estimate is fully compensated by a robust sliding-mode controller so as to provide a considerable robustness against various modelling uncertainties and to achieve a consistent control performance under stochastic wind speed variations. Moreover, the proposed approach has an integrated structure thus only the measurement of rotor speed and reactive power is required, while the classical auxiliary dq-axis current regulation loops can be completely eliminated. Four case studies are carried out which verify that a more optimal wind power extraction and an enhanced FRT capability can be realized in comparison with that of conventional vector control (VC), feedback linearization control (FLC), and sliding-mode control (SMC).

Keyword DFIG, optimal power extraction, FRT, nonlinear perturbation observer, robust sliding-mode control

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1 Introduction

Due to the astonishingly ever-increasing population issue and environmental crisis, both the social and industrial demands of renewable energy keep growing rapidly in the past decade around the globe. As one of the most abundant and mature renewable energy, wind energy conversion systems (WECS) have been paid considerable attention and their proportion in nationwide energy production will rise even faster in future [1]. Nowadays, the most commonly used wind turbine in WECS is based on doubly-fed induction generator (DFIG) because of its noticeable merits: variable speed generation, the reduction of mechanical stresses and acoustic noise, as well as the improvement of the power quality [2].

So far, an enormous variety of studies have been undertaken for DFIG modelling and control, in which vector control (VC) incorporated with proportional-integral (PI) loops is the most popular and widely recognized framework in industry, thanks to its promising features of decoupling control of active/reactive power, simple structure, as well as high reliability [3]. The primary goal of DFIG control system design is to optimally extract the wind power under random wind speed variation, which is usually called maximum power point tracking (MPPT) [4]. Meanwhile, a fault-ride through (FRT) capability is often required so that DFIG can withstand some typical disturbances in power grids [5]. However, one significant drawback of VC is that it cannot maintain a consistent control performance when operation conditions vary as its PI parameters are determined by the one-point linearization, while DFIG is a highly nonlinear system resulted from the fact that it frequently operates under a time-varying and wide operation region by stochastic turbulent wind. Several optimal parameter tuning techniques have been examined to improve the overall control performance of PI control, such as the differential evolutionary algorithm (DE) employed for the performance enhancement of DFIG in the presence of external disturbances [6]. Reference [7] proposed a meta-heuristic algorithm called grouped grey wolf optimizer to achieve MPPT together with an improved FRT capability. In addition, literature [8] adopted particle swarm optimizer (PSO) to enhance the building energy performance. Moreover, a genetic algorithm was developed to minimize the energy consumption of the hybrid energy storage system in electric vehicle [9].

On the other hand, plenty of promising alternatives have been investigated attempting to remedy such inherent flaws of VC. For example, fuzzy-logic was used to deal with onshore wind farm site selection [10]. In reference [11], a feedback linearization control (FLC) was designed for MPPT of DFIG with a thorough modal analysis of generator dynamics, which internal dynamics stability is also proved in the context of Lyapunov criterion. Besides, both the rotor position and speed are calculated based on model reference adaptive system (MRAS) control strategy by [12], such that a fast dynamic response without the requirement of flux estimation can be realized. Furthermore, a robust continuous-time
model predictive direct power control of DFIG was proposed via Taylor series expansion for stator current prediction, which is directly used to compute the required rotor voltage in order to minimize the difference between the actual stator currents and their references over the prediction period [13]. Meanwhile, literature [14] developed an internal model state-feedback approach to control the DFIG currents, which is able to provide robustness to external disturbances automatically and to eliminate the need of disturbance compensation. Additionally, a Lyapunov control theory based controller was devised for rotor speed adjustment without any information about wind data or an available anemometer [15]. A nonlinear robust power controller based on a hybrid of adaptive pole placement and backstepping was presented in [16], which implementation feasibility is validated through field-programmable gate array (FPGA). Moreover, an approximate dynamic programming based optimal and adaptive reactive power control scheme was applied to remarkably improve the transient stability of power systems with wind farms [17].

Among all sorts of advanced approaches, sliding-mode control (SMC) is a powerful high-frequency switching control scheme for nonlinear systems with various uncertainties and disturbances, which elegantly features effective disturbance rejection, fast response, and strong robustness [18], thus it is appropriate to tackle the above obstacles. In work [19], the dynamics of a small-capacity wind turbine system connected to the power grid was altered under severe faults of power grids, in which the transient behaviour and the performance limit for FRT are discussed by using two protection circuits of an AC-crowbar and a DC-Chopper. A high-order SMC was applied which owns prominent advantages of great robustness against power grid faults, together with no extra mechanical stress on the wind turbine drive train [20]. In addition, reference [21] wisely chose a sliding surface that allows the wind turbine to operate very closely to the optimal regions, while PSO was used to determine the optimal slope of the sliding surface and the switching component amplitude. Further, an intelligent proportional-integral SMC was proposed for direct power control of variable-speed constant-frequency wind turbine systems and MPPT under several disturbances [22]. Moreover, literature [23] designed a robust fractional-order SMC for MPPT and robustness enhancement of DFIG, in which unknown nonlinear disturbances and parameter uncertainties are estimated via a fractional-order uncertainty estimator while a continuous control strategy is developed to realize a chattering-free manner.

Nevertheless, an essential shortcoming of SMC is its over-conservativeness stemmed from the use of upper bound of uncertainties, while these worst conditions in which the perturbation takes its upper bound does not usually occur. As a consequence, numerous disturbance/perturbation observer based controllers have been examined which aim to provide a more appropriate control performance by real-time compensation of the combinatorial effect of various uncertainties and disturbances, e.g., a high-gain state
and perturbation observer (HGSPO) was adopted to estimate the unmodelled dynamics and parameter uncertainties of multi-machine power systems equipped with flexible alternating current transmission system devices, such that a coordinated adaptive passive control can be realized [24]. Alternatively, a nonlinear observer based adaptive disturbance rejection control (ADRC) was proposed to improve the power tracking of DFIG under abrupt changes in wind speed, which can be applied for any type of optimal active power tracking algorithms [25]. Moreover, reference [26] described a linear ADRC based load frequency control (LFC) to maintain generation-load balance and to realize disturbance rejection of power systems integrated with DFIG. In work [27], sliding-mode based perturbation observer was used to design a nonlinear adaptive controller for power system stability enhancement. On the other hand, disturbance observer based SMC was studied for continuous-time linear systems with mismatched disturbances or uncertainties [28], while the applications of disturbance/perturbation observer based SMC can be referred to the current regulation of voltage source converter based high voltage direct current system [29], LFC of power systems with high wind energy penetration [30], position and velocity profile tracking control for next-generation servo track writing [31], etc. In addition, a derivative-free nonlinear Kalman filter was redesigned as a disturbance observer to estimate additive input disturbances to DFIG, which are finally compensated by a feedback controller that enables the generator’s state variables to track desirable setpoints [32].

This paper proposes a perturbation observer based sliding-mode control (POSMC) of DFIG for optimal power extraction, which novelty and contribution can be summarized as the following four points:

- The combinatorial effect of wind turbine nonlinearities, generator parameter uncertainties, and wind speed randomness is simultaneously estimated online by a sliding-mode state and perturbation observer (SMSPO), which is then fully compensated by a robust sliding-mode controller. Thus no accurate system model is needed. In contrast, other nonlinear approaches need an accurate system model [11] or can merely handle some specific uncertainties, e.g., wind speed uncertainties [15] or parameter uncertainties [16];
- Only the measurement of rotor speed and reactive power is required by POSMC, while various generator variables and parameters are required by references [12,14]. Hence POSMC is relatively easy to be implemented in practice;
- Compared to other SMC schemes [22,23], as the upper bound of perturbation is replaced by its real-time estimate, the inherent over-conservativeness of SMC can be avoided by the proposed method;
- POSMC employs a nonlinear SMSPO to estimate the perturbation, which does not have the malignant effect of peaking phenomenon existed in HGSPO [24], Moreover, its structure is simpler than that of
Figure 1: The configuration of a DFIG connected to the power grid.

Another typical nonlinear observer called ADRC [25].

Four case studies have been undertaken to evaluate the effectiveness of the proposed approaches and compare its control performance against other typical methods, such as VC, FLC, and SMC. The remaining of this paper is organized as follows: Section II is devoted for DFIG modelling while Section III develops the POSMC scheme. In Section IV, the POSMC design of DFIG for optimal power extraction is investigated. Section V provides the simulation results. Lastly, some concluding remarks are summarized in Section VI.

2 DFIG Modelling

A schematic diagram of DFIG connected to a power grid is illustrated in Fig. 1, in which the wind turbine is connected to an induction generator through a mechanical shaft system, while the stator is directly connected to the power grid and the rotor is fed through a back-to-back converter [7].
2.1 Wind turbine

The aerodynamics of wind turbine can be generically characterized by the power coefficient $C_p(\lambda, \beta)$, which is a function of both tip-speed-ratio $\lambda$ and blade pitch angle $\beta$, in which $\lambda$ is defined by

$$\lambda = \frac{\omega_m R}{v_{\text{wind}}}$$

where $R$ is the blade radius, $\omega_m$ is the wind turbine rotational speed and $v_{\text{wind}}$ is the wind speed. Based on the wind turbine characteristics, a generic equation employed to model $C_p(\lambda, \beta)$ can be written as [33]

$$C_p(\lambda, \beta) = c_1 \left( \frac{c_2}{\lambda_i} - c_3 \beta - c_4 \right) e^{-\frac{c_3}{\lambda_i}} + c_6 \lambda$$

with

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^2 + 1}$$

The coefficients $c_1$ to $c_6$ are chosen as $c_1=0.5176$, $c_2=116$, $c_3=0.4$, $c_4=5$, $c_5=21$ and $c_6=0.0068$ [34].

Particularly, Fig. 2 demonstrates the power coefficient curve $C_p(\lambda, \beta)$ against tip-speed-ratio $\lambda$ and blade pitch angle $\beta$. Note that this paper adopts a simple wind turbine which blade pitch angle $\beta$ is a constant as a simplification of wind turbine modelling [35].

The mechanical power that wind turbine can extract from the wind is calculated by

$$P_m = \frac{1}{2} \rho \pi R^2 C_p(\lambda, \beta) v_{\text{wind}}^3$$

where $\rho$ is the air density. In the MPPT, the wind turbine always operates under the sub-rated wind speed, in which the aim of controller is to track the optimal active power curve which is obtained by
connecting each maximum power point at various wind speed. Under such circumstance, the pitch angle control system is deactivated thus \( \beta \equiv 0 \) [36]. When the wind speed is beyond the rated value, then the control objective will be changed to control the pitch angle, in which the value of pitch angle \( \beta \) will be a variable and tuned in the real-time [37].

2.2 Doubly-fed induction generator

The generator dynamics are described as follows [7,11,33]:

\[
\frac{di_{qs}}{dt} = \frac{\omega_b}{L_s} \left( - R_1 i_{qs} + \omega_b L_s' i_{ds} + \frac{\omega_b}{\omega_s} e_{qs}' - \frac{1}{T_i \omega_b} e_{ds}' - v_{qs} + \frac{L_m}{L_{tr}} v_{qr} \right) \tag{5}
\]

\[
\frac{di_{ds}}{dt} = \frac{\omega_b}{L_s} \left( - \omega_b L_s' i_{qs} - R_1 i_{ds} + \frac{1}{T_i \omega_b} e_{qs}' + \frac{\omega_b}{\omega_s} e_{ds}' - v_{ds} + \frac{L_m}{L_{tr}} v_{dr} \right) \tag{6}
\]

\[
\frac{de_{qs}'}{dt} = \omega_b \omega_s \left[ R_2 i_{ds} - \frac{1}{T_i \omega_b} e_{qs}' + \left( 1 - \frac{\omega_b}{\omega_s} \right) e_{ds}' - \frac{L_m}{L_{rr}} v_{dr} \right] \tag{7}
\]

\[
\frac{de_{ds}'}{dt} = \omega_b \omega_s \left[ - R_2 i_{qs} - \left( 1 - \frac{\omega_b}{\omega_s} \right) e_{qs}' - \frac{1}{T_i \omega_b} e_{ds}' + \frac{L_m}{L_{rr}} v_{qr} \right] \tag{8}
\]

where \( \omega_b \) is the electrical base speed and \( \omega_s \) is the synchronous angular speed; \( e_{ds}' \) and \( e_{qs}' \) are equivalent d-axis and q-axis (dq-) internal voltages; \( i_{ds} \) and \( i_{qs} \) are dq- stator currents; \( v_{ds} \) and \( v_{qs} \) are dq- stator terminal voltages; \( v_{dr} \) and \( v_{qr} \) are dq- rotor voltages, respectively. The remained parameters are covered in Appendix.

The active power \( P_e \) produced by the generator can be calculated by

\[
P_e = e_{qs}' i_{qs} + e_{ds}' i_{ds} \tag{9}
\]

Here, the q-axis is aligned with stator voltage while the d-axis leads the q-axis. Thus, one can directly obtain that \( v_{ds} \equiv 0 \) and \( v_{qs} \) equals to the magnitude of the terminal voltage. Finally, the reactive power \( Q_s \) is given by

\[
Q_s = v_{qs} i_{ds} - v_{ds} i_{qs} = v_{qr} i_{ds} \tag{10}
\]
2.3 Shaft system

The shaft system is simply modelled as a single lumped-mass system with a lumped inertia constant denoted as $H_m$, calculated by [34].

$$H_m = H_t + H_g$$  \hspace{1cm} (11)

where $H_t$ and $H_g$ are the inertia constants of the wind turbine and the generator, respectively.

The electromechanical dynamics is then written by

$$\frac{d\omega_m}{dt} = \frac{1}{2H_m} (T_m - T_e - D\omega_m)$$  \hspace{1cm} (12)

where $\omega_m$ is the rotational speed of the lumped-mass system which equals to the generator rotor speed $\omega_r$ when both of them are given in per unit (p.u.); $D$ represents the damping of the lumped system; and $T_m$ denotes the mechanical torque given as $T_m = P_m/w_m$, respectively.

3 Perturbation Observer based Sliding-mode Control

Consider an uncertain nonlinear system which has the following canonical form

$$\begin{align*}
\dot{x} &= Ax + B(a(x) + b(x)u + d(t)) \\
y &= x_1
\end{align*}$$  \hspace{1cm} (13)

where $x = [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$ is the state variable vector; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and system output, respectively; $a(x) : \mathbb{R}^n \mapsto \mathbb{R}$ and $b(x) : \mathbb{R}^n \mapsto \mathbb{R}$ are unknown smooth functions; and $d(t) : \mathbb{R}^+ \mapsto \mathbb{R}$ represents a time-varying external disturbance. The $n \times n$ matrix $A$ and $n \times 1$ matrix $B$ are of the canonical form as follows

$$A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}_{n \times n}, \quad B = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}_{n \times 1}$$  \hspace{1cm} (14)
The perturbation of system (13) is defined as [24]

$$\Psi(x, u, t) = a(x) + (b(x) - b_0)u + d(t)$$  \hspace{1cm} (15)$$

where \(b_0\) is the constant control gain.

From the original system (13), the last state \(x_n\) can be rewritten in the presence of perturbation (15), which yields

$$\dot{x}_n = a(x) + (b(x) - b_0)u + d(t) + b_0u = \Psi(x, u, t) + b_0u$$  \hspace{1cm} (16)$$

Define an extended state \(x_{n+1} = \Psi(x, u, t)\). Then, system (13) can be directly extended into

$$\begin{align*}
\dot{y} &= x_1 \\
\dot{x}_1 &= x_2 \\
\vdots \\
\dot{x}_n &= x_{n+1} + b_0u \\
\dot{x}_{n+1} &= \dot{\Psi}(\cdot)
\end{align*}$$  \hspace{1cm} (17)$$

The new state vector becomes \(x_e = [x_1, x_2, \cdots, x_n, x_{n+1}]^T\), and the following three assumptions are made

- A.1 \(b_0\) is chosen to satisfy: \(|b(x)/b_0 - 1| \leq \theta < 1\), where \(\theta\) is a positive constant.

- A.2 The functions \(\Psi(x, u, t) : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^+ \mapsto \mathbb{R}\) and \(\dot{\Psi}(x, u, t) : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^+ \mapsto \mathbb{R}\) are bounded over the domain of interest: \(|\Psi(x, u, t)| \leq \gamma_1\), \(|\dot{\Psi}(x, u, t)| \leq \gamma_2\) with \(\Psi(0, 0, 0) = 0\) and \(\dot{\Psi}(0, 0, 0) = 0\), where \(\gamma_1\) and \(\gamma_2\) are positive constants.

- A.3 The desired trajectory \(y_d\) and its up to \(n\)-th-order derivative are all continuous and bounded.

Here, Assumptions A.1 and A.2 ensure the closed-loop system stability with perturbation estimation, while assumption A.3 guarantees POSMC can drive the system state \(x\) to track a desired trajectory \(x_d = [y_d, y_d^{(1)}, \cdots, y_d^{(n-1)}]^T\).

Throughout this paper, \(\tilde{x} = x - \hat{x}\) refers to the estimation error of \(x\) whereas \(\hat{x}\) represents the estimate of \(x\). In the consideration of the worst case, e.g., \(y = x_1\) is the only measurable state, an \((n+1)\)-th-order SMSPO for the extended system (17) is designed to simultaneously estimate the system states and
perturbation, shown as follows

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + \alpha_1 \dot{x}_1 + k_1 \text{sat}(\dot{x}_1, \epsilon_o) \\
\vdots & \\
\dot{x}_n &= \dot{\Psi}(\cdot) + \alpha_n \dot{x}_1 + k_n \text{sat}(\dot{x}_1, \epsilon_o) + b_0 u \\
\dot{\Psi}(\cdot) &= \alpha_{n+1} \dot{x}_1 + k_{n+1} \text{sat}(\dot{x}_1, \epsilon_o)
\end{align*}
\]  

(18)

where \( \alpha_i, i = 1, 2, \cdots, n + 1 \), are the Luenberger observer constants which are chosen to place the poles of

\[s^{n+1} + \alpha_1 s^n + \alpha_2 s^{n-1} + \cdots + \alpha_{n+1} = (s + \lambda_n)^{n+1} = 0\]

being in the open left-half complex plane at \(-\lambda_n\), with

\[\alpha_i = C_{n+1}^i \lambda^i_n, \quad i = 1, 2, \cdots, n + 1.\]  

(19)

where \( C_{n+1}^i = \frac{(n+1)!}{i!(n+1-i)!} \).

In addition, positive constants \( k_i \) are the sliding surface constants, in which

\[k_1 \geq |\dot{x}_2|_{\text{max}}\]  

(20)

where the ratio \( k_i/k_1 (i = 2, 3, \cdots, n+1) \) are chosen to put the poles of \( p^n + (k_2/k_1)p^{n-1} + \cdots + (k_{n}/k_1)p + (k_{n+1}/k_1) = (p + \lambda_k)^n = 0 \) to be in the open left-half complex plane at \(-\lambda_k\), yields

\[\frac{k_{i+1}}{k_1} = C_n^i \lambda^i_k, \quad i = 1, 2, \cdots, n.\]  

(21)

where \( C_n^i = \frac{n!}{i!(n-i)!} \).

Moreover, \( \text{sat}(\dot{x}_1, \epsilon_o) \) function is employed to replace conventional \( \text{sgn}(\dot{x}_1) \) function, such that the malignant effect of chattering in SMSPO resulted from discontinuity can be reduced, which is defined as

\[\text{sat}(\dot{x}_1, \epsilon_o) = \dot{x}_1/|\dot{x}_1| \text{ when } |\dot{x}_1| > \epsilon_o \text{ and } \text{sat}(\dot{x}_1, \epsilon_o) = \dot{x}_1/\epsilon_o \text{ when } |\dot{x}_1| \leq \epsilon_o.\]

In addition, \( \epsilon_o \) denotes the observer thickness layer boundary.

Define an estimated sliding surface as

\[\hat{S}(x, t) = \sum_{i=1}^{n} \rho_i (\dot{x}_i - y_d^{(i-1)})\]  

(22)

where the estimated sliding surface gains \( \rho_i = C_{n-i}^{n-1} \lambda_{c}^{n-i}, \quad i = 1, \cdots, n \), place all poles of the estimated sliding surface at \(-\lambda_c\), where \( \lambda_c > 0.\)
The POSMC for system (13) is designed as

$$
\alpha = \frac{1}{b_0} \left[ y_d^{(n)} - \sum_{i=1}^{n-1} \rho_i (\hat{\gamma}_{i+1} - y_d^{(i)}) - \zeta \hat{S} - \varphi \operatorname{sat}(\hat{S}, \epsilon_c) - \hat{\Psi}(\cdot) \right] 
$$

(23)

where \( \zeta \) and \( \varphi \) are control gains which are chosen to fulfill the attractiveness of the estimated sliding surface \( \hat{S} \). In addition, \( \epsilon_c \) is the controller thickness layer boundary.

### 4 POSMC Design of DFIG for Optimal Power Extraction

This paper aims to apply POSMC on the rotor-side converter (RSC) of DFIG for an MPPT, while the dynamics of the grid-side converter (GSC) is ignored. The maximum power point (MPP) is defined as an operating point of the wind turbine at which maximum mechanical power can be extracted from the wind turbine [11].

Choose the tracking error \( e = [e_1 \ e_2]^T \) of rotor speed \( \omega_r \) and stator reactive power \( Q_s \) as the outputs, yields

$$
\begin{align*}
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix}
= & \begin{bmatrix}
f_1 - \omega^*_r \\
f_2 - Q^*_s
\end{bmatrix} + B 
\begin{bmatrix}
v_{dr} \\
v_{qr}
\end{bmatrix}
\end{align*}
$$

(24)

where

$$
\begin{align*}
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
= & \frac{\dot{\omega}_r}{2H_m} - \frac{1}{2H_m} \left\{ w_b \left[ (1 - \frac{\omega_s}{\omega_c}) (e'^q_{ds} i_{qs} - e'^q_{qs} i_{ds}) - \frac{1}{\omega_s \omega_c} (e'^q_{qs} i_{qs} + e'^q_{ds} i_{ds}) \right] + \frac{\omega_m \omega_r}{\omega_c} \left[ e'^q_{qs} + e'^q_{ds} \right] \\
+ \omega_s L_s e'^q_{qs} i_{ds} - e'^q_{ds} i_{qs} - R_1 (e'^q_{qs} i_{qs} + e'^q_{ds} i_{ds}) - e'^q_{qs} v_{qs} + e'^q_{ds} v_{ds}
\right\}
\end{align*}
$$

(25)

and

$$
B = \begin{bmatrix}
-\frac{\omega_m L_d}{2H_m L_m i_{rr}} \left( e'^q_{qs} - i_{qs} \right) & -\frac{\omega_m L_d}{2H_m L_m i_{rr}} \left( e'^q_{qs} + i_{ds} \right) \\
-\frac{\omega_m L_d}{L_m i_{rr} v_{qs}} & -\frac{\omega_m L_d}{L_m i_{rr} v_{ds}}
\end{bmatrix}
$$

(26)

(27)

(28)
where \( B \) is the control gain matrix. As \( \det(B) = -\frac{\omega_r^2L_2^2v_{\text{ms}}}{2M_2v_{\text{ms}}^2} (\frac{\epsilon_d^*}{\omega_r} + i_d) \neq 0 \), it is invertible and the transformed system is linearizable over the whole operation range.

The time derivative of \( T_m \) in Eq. (26) is calculated by

\[
\dot{T_m} = \frac{\partial T_m}{\partial \omega_r} \frac{d\omega_r}{dt} + \frac{\partial T_m}{\partial v_{\text{wind}}} \frac{dv_{\text{wind}}}{dt}
\]

(29)

where

\[
\frac{\partial T_m}{\partial \omega_r} = \frac{1}{2} \rho A v_{\text{wind}}^3 \left\{ c_1 e^{-c_5 \left( \frac{v_{\text{wind}}}{R \omega_r} - 0.035 \right)} \left[ \frac{c_2 c_3 v_{\text{wind}}^2}{R^2 \omega_r^3} - \frac{(2c_2 + 0.035c_2c_5 + c_4c_5)v_{\text{wind}}}{R \omega_r^3} + \frac{0.035c_2 + c_4}{\omega_r^2} \right] \right\}
\]

(30)

\[
\frac{\partial T_m}{\partial v_{\text{wind}}} = \frac{1}{2} \rho A v_{\text{wind}}^2 \left\{ c_1 e^{-c_5 \left( \frac{v_{\text{wind}}}{R \omega_r} - 0.035 \right)} \left[ -\frac{c_2 c_3 v_{\text{wind}}^2}{R^2 \omega_r^3} + \frac{(4c_2 + 0.035c_2c_5 + c_4c_5)v_{\text{wind}}}{R \omega_r^3} - \frac{0.105c_2 + 3c_4}{\omega_r^2} \right] \right\}
\]

(31)

Assume all the nonlinearities are unknown, define the perturbations \( \Psi_1(\cdot) \) and \( \Psi_2(\cdot) \) for system (25) as

\[
\begin{bmatrix}
\Psi_1(\cdot) \\
\Psi_2(\cdot)
\end{bmatrix} =
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix} + (B - B_0)
\begin{bmatrix}
v_{\text{dr}} \\
v_{\text{qr}}
\end{bmatrix}
\]

(32)

where the constant control gain \( B_0 \) is given by

\[
B_0 =
\begin{bmatrix}
b_{11} & 0 \\
0 & b_{22}
\end{bmatrix}
\]

(33)

Then system (25) can be rewritten as

\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} =
\begin{bmatrix}
\Psi_1(\cdot) \\
\Psi_2(\cdot)
\end{bmatrix} + B_0
\begin{bmatrix}
v_{\text{dr}} \\
v_{\text{qr}}
\end{bmatrix} -
\begin{bmatrix}
\tilde{\omega}_r^* \\
Q_s^*
\end{bmatrix}
\]

(34)

Define \( z_{11} = \omega_r \) and \( z_{12} = \dot{\omega}_r \), a third-order SMSPO is adopted to estimate \( \Psi_1(\cdot) \) as

\[
\begin{bmatrix}
\dot{z}_{11} = \dot{z}_{12} + \alpha_{11} \tilde{\omega}_r + k_{11} \text{sat}(\tilde{\omega}_r, \epsilon_o) \\
\dot{z}_{12} = \dot{\Psi}_1(\cdot) + \alpha_{12} \tilde{\omega}_r + k_{12} \text{sat}(\tilde{\omega}_r, \epsilon_o) + b_{11} v_{\text{dr}} \\
\dot{\Psi}_1(\cdot) = \alpha_{13} \tilde{\omega}_r + k_{13} \text{sat}(\tilde{\omega}_r, \epsilon_o)
\end{bmatrix}
\]

(35)
where observer gains $k_{11}$, $k_{12}$, $k_{13}$, $\alpha_{11}$, $\alpha_{12}$, and $\alpha_{13}$, are all positive constants.

Define $z_{21} = Q_s$, a second-order sliding-mode perturbation observer (SMPO) is employed to estimate $\Psi_2(\cdot)$ as

$$
\begin{aligned}
\dot{z}_{21} &= \dot{\Psi}_2(\cdot) + \alpha_{21} \dot{Q}_s + k_{21} \text{sat}(\dot{Q}_s, \epsilon_o) + b_{22} v_{qr} \\
\dot{\Psi}_2(\cdot) &= \alpha_{22} \dot{Q}_s + k_{22} \text{sat}(\dot{Q}_s, \epsilon_o)
\end{aligned}
$$

(36)

where observer gains $k_{21}$, $k_{22}$, $\alpha_{21}$, and $\alpha_{22}$, are all positive constants.

The estimated sliding surface of system (25) is chosen by

$$
\begin{bmatrix}
\hat{S}_1 \\
\hat{S}_2
\end{bmatrix} =
\begin{bmatrix}
\rho_1 \hat{z}_{11} - \omega_r^* + \rho_2 \hat{z}_{12} - \dot{\omega}_r^* \\
\hat{z}_{21} - Q_s^*
\end{bmatrix}
$$

(37)

where $\rho_1$ and $\rho_2$ are the positive sliding surface gains. The attractiveness of the estimated sliding surface (37) ensures rotor speed $\omega_r$ and reactive power $Q_s$ can effectively track to their reference.

The POSMC of system (25) is designed as

$$
\begin{bmatrix}
v_{dr} \\
v_{qr}
\end{bmatrix} = B_0^{-1}
\begin{bmatrix}
\dot{\omega}_r^* - \rho_1 \hat{z}_{12} - \dot{\omega}_r^* - \zeta_1 \hat{S}_1 - \varphi_1 \text{sat}(\hat{S}_1, \epsilon_c) - \dot{\Psi}_1(\cdot) \\
\dot{Q}_s^* - \zeta_2 \hat{S}_2 - \varphi_2 \text{sat}(\hat{S}_2, \epsilon_c) - \dot{\Psi}_2(\cdot)
\end{bmatrix}
$$

(38)

where positive control gains $\zeta_1$, $\zeta_2$, $\varphi_1$, and $\varphi_2$ are chosen to guarantee the convergence of system (25).

During the most severe disturbance, both the rotor speed and reactive power may reduce from their initial value to around zero within a short period of time $\Delta$. Thus the boundary values of the state and perturbation estimates can be calculated by $|\hat{z}_{11}| \leq |\omega_r^*|$, $|\hat{z}_{12}| \leq |\omega_r^*|/\Delta$, and $|\dot{\Psi}_1(\cdot)| \leq |\omega_r^*|/\Delta^2$, $|\hat{z}_{21}| \leq |Q_s^*|$, and $|\dot{\Psi}_2(\cdot)| \leq |Q_s^*|/\Delta$, respectively. Note that the selection of $B_0$ (33) fully decouples system (25) into two single-input single-output (SISO) systems (34). As a consequence, control inputs $v_{dr}$ and $v_{qr}$ can independently regulate rotor speed $\omega_r$ and reactive power $Q_s$.

To this end, the overall POSMC structure of DFIG is illustrated by Fig. 3, in which only the measurement of rotor speed $\omega_r$ and reactive power $Q_s$ at the RSC side is required. Moreover, one can readily find from Fig. 3 that POSMC has an integrated structure which does not need any auxiliary dq-axis current regulation loops that usually required by VC [3]. At last, the obtained control inputs (38) are modulated by the sinusoidal pulse width modulation (SPWM) technique [38].
Figure 3: The overall POSMC structure of DFIG.
5 Case Studies

The proposed POSMC has been applied to achieve an MPPT of a DFIG connected to the power grid, which control performance is compared to that of conventional VC [3], FLC [11], SMC [20], under four cases, i.e., step change of wind speed, random wind speed variation, FRT capability, and system robustness against parameter uncertainties. Since the control inputs might exceed the admissible capacity of RSC at some operation point, their values must be limited. Here, $v_{dr}$ and $v_{qr}$ are scaled proportionally as: if $v_r = \sqrt{v_{dr}^2 + v_{qr}^2} > v_{r,max}$, then set $v_{dr,lim} = v_{dr}v_{r,max}/v_r$ and $v_{qr,lim} = v_{qr}v_{r,max}/v_r$ [11], respectively. Besides, the controller parameters are tabulated in Table 1. The simulation is executed on Matlab/Simulink 7.10 using a personal computer with an IntelR CoreTmI7 CPU at 2.2 GHz and 4 GB of RAM.

<table>
<thead>
<tr>
<th>Table 1: POSMC parameters for the DFIG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>rotor controller gains</strong></td>
</tr>
<tr>
<td>$b_{11} = -2500$</td>
</tr>
<tr>
<td>$\rho_1 = 750$</td>
</tr>
<tr>
<td>$\rho_2 = 1$</td>
</tr>
<tr>
<td>$\zeta_1 = 50$</td>
</tr>
<tr>
<td>$\phi_1 = 40$</td>
</tr>
<tr>
<td>$\epsilon_c = 0.2$</td>
</tr>
<tr>
<td><strong>rotor observer gains</strong></td>
</tr>
<tr>
<td>$\alpha_{11} = 30$</td>
</tr>
<tr>
<td>$\alpha_{12} = 300$</td>
</tr>
<tr>
<td>$\alpha_{13} = 1000$</td>
</tr>
<tr>
<td>$\Delta = 0.01$</td>
</tr>
<tr>
<td>$k_{11} = 20$</td>
</tr>
<tr>
<td>$k_{12} = 600$</td>
</tr>
<tr>
<td>$k_{13} = 6000$</td>
</tr>
<tr>
<td><strong>reactive power controller gains</strong></td>
</tr>
<tr>
<td>$b_{22} = 6000$</td>
</tr>
<tr>
<td>$\zeta_2 = 10$</td>
</tr>
<tr>
<td>$\phi_2 = 10$</td>
</tr>
<tr>
<td>$\epsilon_c = 0.2$</td>
</tr>
<tr>
<td><strong>reactive power observer gains</strong></td>
</tr>
<tr>
<td>$\alpha_{21} = 40$</td>
</tr>
<tr>
<td>$\alpha_{22} = 400$</td>
</tr>
<tr>
<td>$k_{21} = 15$</td>
</tr>
<tr>
<td>$k_{22} = 600$</td>
</tr>
</tbody>
</table>

5.1 Step change of wind speed

A series of four consecutive step changes of wind speed $v_{wind} = 8-12$ m/s are tested, in which a 1 m/s wind speed increase is added during each step change to briefly mimic a gust. The MPPT performance of all controllers is compared in Fig. 4. It shows that POSMC can extract the maximal wind energy with less oscillations, meanwhile it can also regulate the active power and reactive power more rapidly and smoothly compared to that of other algorithms.

5.2 Random variation of wind speed

A stochastic wind speed variation is tested to examine the control performance of the proposed approach, which starts from 8 m/s and gradually reaches to 12 m/s, as demonstrated by Fig. 5. The system responses are provided in Fig. 6, from which it can be clearly observed that POSMC is able to achieve the least oscillations of rotor speed error and reactive power thanks to the online perturbation compensation. Additionally, its power coefficient is the closest to the optimum thus the wind energy can be optimally
Figure 4: MPPT performance to a series of step change of wind speed from 8 m/s to 12 m/s.
extracted under random wind speed variations.

5.3 FRT performance

With the rapidly ever-growing integration of WECS into the main power grid, it often requires that
WECS can realize FRT when the power grid voltage is temporarily reduced due to a fault or a sudden
load change occurred in the power grid, or can even address the generator to stay operational and not
disconnect from the power grid during and after the voltage drop [39,40]. A 625 ms voltage dip staring at
t=1 s from nominal value to 0.3 p.u. and restores to 0.9 p.u. is applied [41], while the system responses
are presented by Fig. 7. One can definitely find that POSMC is able to effectively suppress the power
oscillations and maintain the largest wind power extraction during FRT, while VC requires the longest
time to restore the system from such harmful contingencies.

Lastly, the estimation performance of perturbation observers during the FRT has also been carefully
monitored, as shown in Fig. 8. It gives that the perturbations can be rapidly estimated in around 250
ms while the relative high-frequency oscillations emerged in the initial phase is due to the discontinuity
of power grid voltage and sliding-mode mechanism caused in perturbation observer loop.

5.4 System robustness with parameter uncertainties

A series of plant-model mismatches of stator resistance \( R_s \) and mutual inductance \( L_m \) with \( \pm 20\% \) un-
certainties are undertaken to evaluate the robustness of POSMC, in which a 0.25 p.u. voltage drop at
power grid is tested while the peak value of total active power \( |P_e| \) is recorded for a clear comparison. It
presents from Fig. 9 that the variation of \( |P_e| \) obtained by POSMC is the smallest among all approach-
es, i.e., around 2.3% variation of \( |P_e| \) to the stator resistance \( R_s \) and 1.4% variation to that of mutual
inductance \( L_m \), respectively. This is because of its elegant merits of the full perturbation compensation
Figure 6: MPPT performance to a random variation of wind speed from 8 m/s to 12 m/s.
Figure 7: System responses under FRT (a 625 ms voltage dip starting at $t=1$ s from nominal value to 0.3 p.u. and restores to 0.9 p.u.).
and sliding-mode mechanism, such that the greatest robustness can be provided. Obviously, FLC has
the largest variation against parameter uncertainties as it requires an accurate system model, i.e., around
19.7% variation of $|P_e|$ to the stator resistance $R_s$ and 22.5% variation to that of mutual inductance $L_{m}$,
respectively.

Table 2: IAE indices (in p.u.) of different control schemes calculated in different cases

<table>
<thead>
<tr>
<th>Method</th>
<th>Case</th>
<th>Step change of wind speed</th>
<th>Random variation of wind speed</th>
<th>Fault-ride through</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IAE$_{Q_1}$</td>
<td>IAE$<em>{V</em>{dc1}}$</td>
<td>IAE$_{Q_1}$</td>
<td>IAE$<em>{V</em>{dc1}}$</td>
</tr>
<tr>
<td>VC</td>
<td>2.18E-02</td>
<td>4.29E-03</td>
<td>4.77E-03</td>
<td>2.36E-03</td>
</tr>
<tr>
<td>FLC</td>
<td>1.43E-02</td>
<td>3.15E-03</td>
<td>3.79E-03</td>
<td>1.85E-03</td>
</tr>
<tr>
<td>SMC</td>
<td>1.04E-02</td>
<td>2.87E-03</td>
<td>2.08E-03</td>
<td>9.96E-04</td>
</tr>
<tr>
<td>POSMC</td>
<td>7.21E-03</td>
<td>1.24E-03</td>
<td>6.97E-04</td>
<td>4.71E-04</td>
</tr>
</tbody>
</table>

5.5 Comparative studies

The integral of absolute error (IAE) indices of each approach calculated in different cases are summarized
in Table 2, where $\text{IAE}_x = \int_0^T |x - x^*|\,dt$ and $x^*$ is the reference of variable $x$. The simulation time $T=30$
s. It shows that POSMC owns the lowest IAE indices (in bold) in all cases compared to those of other

Figure 8: Perturbation estimation performance of SMSPO and SMPO during FRT.
Plant-model mismatch

| Peak value $|P_e|$ (p.u.) | POSMC | SMC | FLC | VC |
|--------------------------|-------|-----|-----|----|
| %Rs                      |       |     |     |    |
| 0.85                     | 1.04  | 1.06| 1.08| 1.12|
| 0.875                    |       |     |     |    |
| 0.9                      |       |     |     |    |
| 0.925                    |       |     |     |    |
| 0.95                     |       |     |     |    |

Figure 9: Peak value of active power $|P_e|$ obtained under a 0.25 p.u. voltage drop at power grid with 20% variation of the stator resistance $R_s$ and mutual inductance $L_m$ of different approaches, respectively.

methods. In particular, its $\text{IAE}_{Q_1}$ obtained in random variation of wind speed is merely 33.51%, 18.39%, and 14.61% to that of SMC, FLC, and VC, respectively; Additionally, its $\text{IAE}_{V_{dc1}}$ obtained in voltage drop at power grid is just 58.45%, 37.46%, and 20.65% to that of SMC, FLC, and VC, respectively.

The overall control efforts of different controllers needed in three cases are given in Fig. 10. One can easily conclude that the overall control efforts of POSMC are the least in all cases except of FRT, this is resulted from its merits that the over-conservativeness of control efforts is only involved in the observer loop and excluded from the controller loop. Therefore, POSMC outperforms other methods with greater robustness enhancement as well as more reasonable control efforts.

6 Conclusions

This paper proposes a robust sliding-mode controller scheme called POSMC to achieve an optimal power extraction of DFIG in various operation conditions. A perturbation is firstly defined to aggregate the wind turbine nonlinearities, generator parameter uncertainties, and wind speed randomness, which is then rapidly estimated by nonlinear perturbation observers and fully compensated by POSMC, so that a consistent and robust control performance under different operation conditions can be achieved.
Simulation results have demonstrated that POSMC can optimally extract the wind energy during wind speed variations and effectively suppress the power oscillations during FRT, together with suitable control efforts thanks to the perturbation compensation.

Compared to other typical nonlinear robust approaches, POSMC can be readily implemented in practice as it only requires the measurement of rotor speed (by an additional rotor speed measuring apparatus) and reactive power (read directly from current power measurement platform), hence the construction costs of measurement apparatus is quite low. Moreover, as POSMC is a decentralized control scheme, no central controller is needed in the face of large-scale wind farms.

Appendix

System parameters [7, 11, 33]:

\[
\omega_b = 100\pi \text{ rad/s}, \quad \omega_n = 1.0 \text{ p.u.}, \quad \omega_{r,\text{base}} = 1.29, \quad v_{s,\text{nom}} = 1.0 \text{ p.u.}
\]

DFIG parameters:

\[
P_{\text{rated}} = 5 \text{ MW}, \quad R_s = 0.005 \text{ p.u.}, \quad R_r = 1.1R_s, \quad L_m = 4.0 \text{ p.u.}, \quad L_{ss} = 1.01L_m, \quad L_{rr} = 1.005L_{ss}, \quad L_s' = L_{ss} - L_m^2/L_{rr}, \quad T_t = L_{rr}/R_r, \quad R_1 = R_s + R_2, \quad R_2 = (L_m/L_{rr})^2R_t.
\]

Wind turbine parameters:

\[
\rho = 1.225 \text{ kg/m}^3, \quad R = 58.59 \text{ m}^2, \quad v_{\text{wind,\text{nom}}} = 12 \text{ m/s}, \quad \lambda_{\text{opt}} = 6.325, \quad H_m = 4.4 \text{ s}, \quad D = 0 \text{ p.u.}
\]

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References


