Measurement of the shape of the $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ differential decay rate

The LHCb collaboration†

Abstract

A measurement of the shape of the differential decay rate and the associated Isgur-Wise function for the decay $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ is reported, using data corresponding to 3 fb$^{-1}$ collected with the LHCb detector in proton-proton collisions. The $\Lambda_c^+ \mu^- \bar{\nu}_\mu$ final states are reconstructed through the detection of a muon and a $\Lambda_b$ baryon decaying into $pK^-\pi^+$, and the decays $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \mu^- \bar{\nu}_\mu$ are used to determine contributions from $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ decays. The measured dependence of the differential decay rate upon the squared four-momentum transfer between the heavy baryons, $q^2$, is compared with expectations from heavy-quark effective theory and from unquenched lattice QCD predictions.

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1 Introduction

In the Standard Model (SM) of particle physics, quarks participate in a rich pattern of flavor-changing transitions. The relevant couplings form a complex $3 \times 3$ matrix, known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix, characterized by just four independent parameters \cite{1}. A vast body of measurements of individual CKM elements exists, and thus the overall consistency of the SM picture of charged current interactions is highly over-constrained. Decades of experimental work have demonstrated the impressive consistency of experimental data with the CKM paradigm \cite{2,3}; nonetheless, the motivation to probe the CKM matrix remains strong. Effects of physics beyond the SM may be subtle, thus more precise measurements are necessary to unveil them. Semileptonic decays of heavy-flavored hadrons are commonly used to measure CKM parameters, as they involve only one hadronic current, parameterized in terms of scalar functions known as form factors. The number of form factors needed to describe a particular decay depends upon the spin of the initial- and final-state hadrons \cite{4,5}. A precise calculation of these form factors has been elusive for many years as it is not possible in perturbative QCD. Heavy-Quark Effective Theory (HQET) provides the framework to systematically include nonperturbative corrections in computations involving hadrons containing heavy quarks. In particular, in the limit of infinite heavy quark mass, all the form factors describing the semileptonic decay of a heavy-flavored hadron are proportional to a universal function, known as the Isgur-Wise (IW) function \cite{6}. Lattice QCD, namely the use of lattice formulations of QCD in large scale numerical simulations, has emerged in recent years as a technique with well defined and systematically improvable uncertainties which can be applied to a wide range of processes and physical quantities \cite{7}. Predictions from the infinite heavy-quark mass limit are useful as a check of several Lattice QCD calculations \cite{8}.

The decay $A_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ is described by six form factors corresponding to the vector and axial-vector components of the flavor-changing charged current \cite{9}. In HQET, $A_b^0$ decays are particularly simple, as the light $ud$ quark pair has total spin $j = 0$, and thus the chromomagnetic corrections, which are of the order of a few percent for $B$ mesons, are not present \cite{10}. In the static approximation of infinite heavy quark masses, the six form factors characterizing the baryonic semileptonic decay $A_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ can be expressed in terms of the elastic heavy-baryon Isgur-Wise function $\xi_B(w)$ \cite{11}. The scalar invariant $w \equiv v_{A_b^0} \cdot v_{\Lambda_c^+}$ is related to the squared four-momentum transfer between the heavy baryons, $q^2$, by

$$w = (m_{A_b^0}^2 + m_{\Lambda_c^+}^2 - q^2)/(2m_{A_b^0}m_{\Lambda_c^+}),$$

where $v_{A_b^0}$ and $v_{\Lambda_c^+}$ are the four-velocities of the $A_b^0$ and $\Lambda_c^+$ baryons, respectively, and $m_{A_b^0}$ and $m_{\Lambda_c^+}$ are the corresponding invariant masses. Nonperturbative corrections to the static limit can be expressed in terms of an expansion in powers of $1/m_c$ and $1/m_b$, where $m_c$ and $m_b$ represent the $c$- and $b$-quark masses respectively. It has been shown in Ref. \cite{12} that the $1/m_c$ term can be expressed in terms of $\xi_B(w)$ and one dimensionful constant. Moreover, partial cancellations lead to small first-order corrections near $w = 1$ \cite{13}.

In the static approximation the differential decay width of the $A_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ decay is given by

$$\frac{d\Gamma}{dw} = GK(w)\xi_B^2(w),$$

1The inclusion of charge-conjugate modes is implied throughout this paper.
where the constant factor $G$ is given by

$$G = \frac{2}{3} \frac{G_F^2}{(2\pi)^3} |V_{cb}|^2 (m_{A_0^+}/m_{A_0^0})^4 r^2 \text{ with } r = m_{A_0^+}/m_{A_0^0},$$

(3)

where $G_F$ represents the Fermi coupling constant [14], $|V_{cb}|$ is the magnitude of the matrix element describing the coupling of the $c$ quark to the $b$ quark, and the kinematic factor $K(w)$ is given by

$$K(w) = m_{A_0^+} \sqrt{w^2 - 1} \left[3w(1 - 2rw + r^2) + 2r(w^2 - 1)\right].$$

(4)

The function $\xi_B(w)$ cannot be determined from first principles in HQET, but calculations based on a variety of approaches exist. The kinematic limit $w = 1$ is special in HQET, as only modest corrections in the $(1/m_b, 1/m_c)$ expansion are expected, due to the absence of hyperfine corrections [15]. Thus it is interesting to express $\xi_B$ as a Taylor series expansion

$$\xi_B(w) = -\rho^2(w - 1) + \frac{1}{2} \sigma^2(w - 1)^2 + \ldots,$$

(5)

where $\rho^2$ is the magnitude of the slope of $\xi_B$ and $\sigma^2$ is its curvature at $w = 1$. Sum rules provide constraints on $\rho^2$ and $\sigma^2$. In particular they require the slope at the zero recoil point to be negative, and give bounds on the curvature and higher-order derivatives [16, 17]. In addition they predict $\sigma^2 \geq 3/5 \rho^2 (\rho^2)^2$ [18] and $\rho^2 \geq 3/4$. Table [1] summarizes theoretical predictions for $\rho^2$ from quenched Lattice QCD, QCD sum rules, and a relativistic quark model.

Recently, state-of-the-art calculations of the six form factors describing the decay $A_0^0 \rightarrow A_0^+ \mu^- \bar{\nu}_\mu$ have been obtained using Lattice QCD with 2 + 1 flavors of dynamical domain-wall fermions [19]. These form factors are calculated in terms of $q^2$. More details on this formalism are given in the Appendix. The resulting theoretical uncertainty attached to a measurement of $|V_{cb}|$ using this form factor prediction is about 3.2%. The precision of this calculation makes this approach an appealing alternative to the ones currently used, all based on $B$-meson semileptonic decays such as $B^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu$. Thus it is important to examine the model’s agreement with measured quantities such as the shape of the $d\Gamma/dq^2$ spectrum.

The experimental knowledge of $A_0^0$ semileptonic decays is quite sparse, as this baryon is too heavy to be produced at the $e^+e^-$ $B$-factories. The only previous experimental study of $\xi_B(w)$ was performed by the DELPHI experiment at LEP, which obtained $\rho^2 = 2.03 \pm 0.46 \text{ (stat)}^{+0.72}_{-1.00} \text{ (syst)}$, with an overall uncertainty of the order of 50% [20].

In this paper we describe a determination of the shape of the $w$ or $q^2$ spectrum of the decay $A_0^0 \rightarrow A_0^+ \mu^- \bar{\nu}_\mu$ and compare it with functional forms related to a single form factor, inspired by HQET, and the Lattice QCD prediction of Ref. [19]. Section 2 presents the experimental procedure and simulated samples, while Sect. 3 describes the method employed to reconstruct $A_0^0 \rightarrow A_0^+ \mu^- \bar{\nu}_\mu$ candidates and to estimate the corresponding kinematic variables $w$ and $q^2$. Section 4 describes the method adopted to isolate the signal, the unfolding procedure used to account for experimental resolution effects, and the efficiency corrections. The fit results for $\xi_B(w)$ corresponding to different functional forms are summarized in Sect. 5. The same analysis procedure is used in Sect. 6 to derive the shape of the differential decay width $d\Gamma/dq^2(A_0^0 \rightarrow A_0^+ \mu^- \bar{\nu}_\mu)$ and compare with the predictions of Ref. [19]. These data are also fitted with a single form-factor parameterization that corresponds to the HQET infinite heavy-quark mass limit.
Table 1: Predictions for the slope at zero recoil of the baryonic Isgur-Wise function $\xi_B$. The evaluation from Ref. [21] includes first-order corrections in HQET.

<table>
<thead>
<tr>
<th>$\rho^2$</th>
<th>Approach</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.35\pm0.13$</td>
<td>QCD sum rules</td>
<td>[22]</td>
</tr>
<tr>
<td>$1.2^{+0.8}_{-1.1}$</td>
<td>Lattice QCD (static approximation)</td>
<td>[23]</td>
</tr>
<tr>
<td>$1.51$</td>
<td>HQET + Relativistic wave function</td>
<td>[21]</td>
</tr>
</tbody>
</table>

## 2 Experimental method

The data used in this analysis were collected with the LHCb detector at the Large Hadron Collider at CERN and correspond to $1 \text{fb}^{-1}$ of integrated luminosity collected at a center-of-mass energy of $7 \text{TeV}$ in 2011 and $2 \text{fb}^{-1}$ collected at a center-of-mass energy of $8 \text{TeV}$ in 2012.

The LHCb detector [24][25] is a single-arm forward spectrometer designed for the study of particles containing $b$ or $c$ quarks. The detector covers the pseudorapidity range $2 < \eta < 5$, where $\eta$ is defined in terms of the polar angle $\theta$ with respect to the beam direction as $-\ln(\tan(\theta/2))$. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the $pp$ interaction region [26], a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about $4 \text{Tm}$, and three stations of silicon-strip detectors and straw drift tubes [27] placed downstream of the magnet. The tracking system provides a measurement of the momentum, $p$, of charged particles with a relative uncertainty that varies from 0.5% at low momentum to 1.0% at $200 \text{ GeV}$ [2]. The minimum distance of a track to a primary vertex, the impact parameter (IP), is measured with a resolution of $(15 + 29/p_T) \mu\text{m}$, where $p_T$ is the component of the momentum transverse to the beam, in GeV. Different types of charged hadrons are distinguished using information from two ring-imaging Cherenkov detectors (RICH) [28]. Photons, electrons and hadrons are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers [29]. The online event selection is performed by a trigger [30], which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction.

Muon candidates are first required to pass the hardware trigger that selects muons with a transverse momentum $p_T > 1.6 \ (1.8) \text{ GeV}$ for the 2011 (2012) data taking period. In the subsequent software trigger, events with one particle identified as a muon are selected if at least one of the final-state particles has both $p_T > 0.8 \text{ GeV}$ and IP larger than $100 \mu\text{m}$ with respect to all of the primary $pp$ interaction vertices (PVs) in the event. In the offline selection, trigger signals are associated with reconstructed particles. Selection requirements can therefore be made on the trigger selection itself and on whether the decision was due to the signal candidate, other particles produced in the $pp$ collision, or a combination of both. This classification of trigger selections can be used for data-driven efficiency determination. Finally, the tracks of two or more of the final-state particles are

\[ ^2 \text{Natural units with } c=\hbar=1 \text{ are used throughout.} \]
required to form a vertex that is significantly displaced from the PVs.

Our study makes use of simulated semileptonic decays, where $pp$ collisions are generated using Pythia \cite{31} with a specific LHCb configuration \cite{32}. Decays of hadronic particles are described by EvtGen \cite{33}, in which final-state radiation is generated using Photos \cite{34}. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit \cite{35} as described in Ref. \cite{36}.

3 Event reconstruction

To isolate a sample of $\Lambda^0_b \rightarrow \Lambda^+_c \mu^- \nu_\mu X$ semileptonic decays, where $X$ represents the undetected particles produced with the $\Lambda^+_c$ in the $c$-quark hadronization, we combine $\Lambda^+_c \rightarrow pK^-\pi^+$ candidates with tracks identified as muons. We consider candidates where a well-identified muon passing the hardware and software trigger algorithms with momentum greater than 3 GeV is found. Charmed baryon candidates are formed from hadrons with momenta greater than 2 GeV and transverse momenta greater than 0.3 GeV. In addition, we require that the average of the magnitudes of the transverse momenta of the hadrons forming the $\Lambda^+_c$ candidate be greater than 0.7 GeV. Kaons, pions, and protons are identified using the RICH system. Each track’s IP significance with respect to the associated primary vertex is required to be greater than $9^\circ$. Moreover, the selected tracks must be consistent with coming from a common vertex: the $\chi^2$ per degree of freedom of the vertex fit must be smaller than 6. In order to ensure that the direction of the parent $\Lambda^0_b$ is well measured, the $\Lambda^+_c$ vertex must be distinct from the primary $pp$ interaction vertex. To this end, we require that the flight-distance significance of the $\Lambda^+_c$ candidate (defined as the measured flight distance divided by its uncertainty) with respect to the associated PV be greater than 100.

Partially reconstructed $\Lambda^0_b$ baryon candidates are formed combining $\mu^-$ and $\Lambda^+_c$ candidates which are consistent with coming from a common vertex, and we require that the angle between the direction of the momentum of the $\Lambda^+_c \mu^-$ candidate and the line from the associated PV to the $\Lambda^+_c \mu^-$ vertex be less than 45 mrad. As the $\Lambda^+_c$ baryon is a $\Lambda^0_b$ decay product with a small but significant lifetime, we require that the difference in the component of the decay vertex position of the charmed hadron candidate along the beam axis and that of the beauty candidate be positive. We explicitly require that the $\Lambda^0_b$ hadron candidate pseudorapidity be between 2 and 5. We measure $\eta$ using the line defined by connecting the associated PV and the vertex formed by the $\Lambda^+_c \mu^-$ system. Finally, the invariant mass of the $\Lambda^+_c \mu^-$ system must be between 3.3 and 5.3 GeV. These selection criteria ensure that the $\Lambda^+_c$ candidates are decay products of $\Lambda^0_b$ semileptonic decays. In particular, the background from directly produced $\Lambda^+_c$ (prompt $\Lambda^+_c$) is highly suppressed. This is quantified by an unbinned extended maximum likelihood fit to the two-dimensional $pK^-\pi^+$ invariant mass and ln(IP/mm) distributions of the $\Lambda^+_c$ candidates, where “/mm” refers to the length unit used to measure the IP. The ln(IP/mm) component allows us to determine the small prompt $\Lambda^+_c$ background. The parameters of the IP distribution of the prompt sample are found by examining directly-produced charm hadrons, as described in Ref. \cite{37}. An empirical probability density function (PDF) derived from simulation is used for the $\Lambda^+_c$ from $\Lambda^0_b$ component. We find $(2.74 \pm 0.02) \times 10^6$

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\footnote{The associated primary vertex to a $\Lambda^0_b \rightarrow \Lambda^+_c \mu^- \nu_\mu X$ candidate is selected as the primary vertex which minimizes the IP significance of the $\Lambda^+_c \mu^-$ system.}
$\Lambda_c^+ \to pK^-\pi^+$ candidates, which can be interpreted as $A_0^0 \to \Lambda_c^+\mu^-\nu_\mu X$ decays, and we determine the prompt $\Lambda_c^+ \to pK^-\pi^+$ fraction to be 1.5%, which can be neglected. The corresponding fit is shown in Fig. 1.

Our goal is the study of the ground-state semileptonic decay $A_0^0 \to \Lambda_c^+\mu^-\nu_\mu$, thus we need to estimate the contributions from $\Lambda_c^+ \rightarrow \Lambda_c^+\pi\pi$ states. Theoretical predictions suggest that the inclusive rate $A_0^0 \to \Lambda_c^+\mu^-\nu_\mu X$ is dominated by the exclusive channel $A_0^0 \to \Lambda_c^+\mu^-\nu_\mu$ [38,39]. The residual contribution is expected to be accounted for by the $A_0^0 \to \Lambda_c^+(2595)\mu^-\nu_\mu$ and $A_0^0 \to \Lambda_c^+(2625)\mu^-\nu_\mu$ channels. Other modes, such as $A_0^0 \to \Sigma_c^+\mu^-\nu_\mu$, are suppressed in the static limit and to order $1/m_Q$, where $m_Q$ represents the heavy quark mass ($m_c$ or $m_b$) [40], with an additional stronger suppression factor of the order $(m_d - m_u)/m_c$ rather than $(m_d - m_u)/m_{QCD}$ [9].

We use $A_0^0 \to \Lambda_c^+\pi^+\pi^-\mu^-\nu_\mu$ decays to infer contributions from the excited $\Lambda_c^+$ modes, where the $\Lambda_c^+$ candidates are selected as $pK^-\pi^+$ combinations whose invariant mass is within ±20 MeV of the nominal $\Lambda_c^+$ mass. The $\Lambda_c^+\mu^-\nu_\mu$ candidates are combined with pairs of opposite-charge pions that satisfy criteria similar to those used to select the pions from the $\Lambda_c^+$ decay. The minimum transverse momentum of these pions is required to be 0.2 GeV and the transverse momentum of the $\Lambda_c^+\pi^+\pi^-$ system is required to be greater than 1.5 GeV. Lastly, the $\chi^2$ per degree of freedom of the vertex fit for the $\Lambda_c^+\pi^+\pi^-$ system must be smaller than 6.

The resulting spectrum, measured as the mass difference $m(pK^-\pi^+\pi^-\pi^+) - m(pK^-\pi^+\pi^-)$ added to the known $\Lambda_c^+$ mass [14], is shown in Fig. 2. We see peaks corresponding to the $\Lambda_c^+(2595)^+$, $\Lambda_c^+(2625)^+$, $\Lambda_c^+(2765)^+$, and $\Lambda_c^+(2880)^+$ resonances. The $\Lambda_c^+(2595)^+$ is only a few MeV above the kinematic threshold and thus it is not well described by a Breit-Wigner function. The baseline fit for this resonance uses a PDF consisting of the sum of two bifurcated Gaussian functions. As a check, we use an S-wave relativistic Breit-Wigner convolved with a Gaussian function with standard deviation $\sigma = 2$ MeV that accounts for the detector resolution. While the second parameterization is more accurate, the fits to the invariant mass spectra in different kinematic bins are more stable with the baseline parameterization. We fit the $\Lambda_c^+(2625)^+$ signal with a double Gaussian PDF with shared mean, as the natural width is expected to be well below the measured detector.

![Figure 1](image-url)

**Figure 1:** (a) The ln(IP/mm) distribution and (b) $pK^-\pi^+$ invariant mass for $\Lambda_c^+$ candidate combinations with a muon. The red (dashed-dotted) curves show the combinatorial $\Lambda_c^+$ background, the green (dashed) curves the $\Lambda_c^+$ from $A_0^0$ and the blue-solid curve the total yields.
Table 2: Measured raw yields for the four $\Lambda^+_c\mu^-\bar{\nu}_\mu$ final states and the inclusive $\Lambda^+_c\mu^-\bar{\nu}_\mu X$.

<table>
<thead>
<tr>
<th>Final state</th>
<th>Yield</th>
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<tbody>
<tr>
<td>$\Lambda_c(2595)^+\mu^-\bar{\nu}_\mu$</td>
<td>$8569 \pm 144$</td>
</tr>
<tr>
<td>$\Lambda_c(2625)^+\mu^-\bar{\nu}_\mu$</td>
<td>$22965 \pm 266$</td>
</tr>
<tr>
<td>$\Lambda_c(2765)^+\mu^-\bar{\nu}_\mu$</td>
<td>$2975 \pm 225$</td>
</tr>
<tr>
<td>$\Lambda_c(2880)^+\mu^-\bar{\nu}_\mu$</td>
<td>$1602 \pm 95$</td>
</tr>
<tr>
<td>$\Lambda^+<em>c\mu^-\bar{\nu}</em>\mu X$</td>
<td>$(2.74\pm0.02)\times10^6$</td>
</tr>
</tbody>
</table>

resolution. The shape of the combinatoric background PDF is inferred from wrong-sign (WS) candidates, where a $\pi^+\pi^+$ or $\pi^-\pi^-$ pair is combined with $\Lambda^+_c$ instead of $\pi^+\pi^-$. In addition, we observe peaks corresponding to two higher mass resonances, with masses and widths consistent with the $\Lambda_c(2765)^+$ and $\Lambda_c(2880)^+$ baryons [14]. In order to determine their yields, we fit the two signal peaks with single Gaussian PDFs with unconstrained masses and widths. The measured yields for the four $\Lambda^+_c$ final states, as well as the $\Lambda^+_c\mu^-\bar{\nu}_\mu X$ final state, are presented in Table 2.

![Figure 2](image-url)

Figure 2: The mass difference $m(pK^-\pi^+\pi^+\pi^-)-m(pK^-\pi^+)$ added to the known $\Lambda^+_c$ mass, $m_{PDG}(\Lambda^+_c)$ [14], for candidates with $pK^-\pi^+$ invariant mass within $\pm20$ MeV of the known $\Lambda^+_c$ mass in candidate semileptonic decays for the entire $w$ range: data are shown as black dots, the combinatoric background is shown as a blue solid line, and the gray histogram shows the WS spectrum, obtained by combining a $\pi^+\pi^+$ or $\pi^-\pi^-$ pair with $\Lambda^+_c$ instead of $\pi^+\pi^-$. The signal fits are identified as follows: (a) for $m < 2700$ MeV, the $\Lambda_c(2595)^+$ as magenta dashed line, and the $\Lambda_c(2625)^+$ as green long-dashed line, (b) for $m > 2700$ MeV, the $\Lambda_c(2765)^+$ as magenta dashed line, and the $\Lambda_c(2880)^+$ as green long-dashed line.
The measured contributions from the two heavier $\Lambda_c^{++}$ final states, shown in Table 2, are smaller than those from $A_b^0 \rightarrow A_c(2595)^+\mu^+\nu_\mu$ and $A_b^0 \rightarrow A_c(2625)^+\mu^-\nu_\mu$ decays. No theoretical prediction for nonresonant $A_b^0 \rightarrow \Lambda_c^+\pi^+\pi^-\nu_\mu X$ exists, but we estimate systematic uncertainties due to the subtraction of this component with an alternative fit of the $A_b^0 \rightarrow \Lambda_c^+\pi^+\pi^-\nu_\mu X$ spectrum from candidate $A_b^0$ semileptonic decays, where we derive both the yield and shape of the combinatoric background from the WS sample.

The kinematical quantities $q^2$ and $w$ in the decay $A_b^0 \rightarrow \Lambda_c^+\mu^-\nu_\mu$ can be calculated if the magnitude of the $A_b^0$ momentum is known. The $A_b^0$ flight direction can be inferred from the primary and secondary vertex locations, and this input, combined with the constraints from energy and momentum conservation, implies the following relationship for $p_{A_b^0}$

\[
\left(\frac{\hat{p}_{A_b^0} \cdot \vec{p}_{A_c^+\mu^-}}{E_{A_c^+\mu^-}}\right)^2 - 1 - \frac{\left(\frac{m_{A_b^0}^2 + m_{A_c^+\mu^-}^2}{E_{A_c^+\mu^-}^2}\right)}{2} = 0,
\]

where the unit vector $\hat{p}_{A_b^0}$ is the direction of the $A_b^0$ baryon, $\vec{p}_{A_c^+\mu^-}$ is the momentum of the $A_c^+\mu^-$ pair, $E_{A_c^+\mu^-}$ is the energy of the $A_c^+\mu^-$ pair, $m_{A_c^+\mu^-}$ is the invariant mass of the $A_c^+\mu^-$ pair, $m_{A_b^0}$ is the nominal mass of the $A_b^0$ baryon, and $\Lambda_c^+$ identifies the $pK^-\pi^+$ combination. This is a quadratic equation, reflecting the lack of knowledge of the neutrino orientation in the $A_b^0$ rest frame with respect to the $A_b^0$ direction in the laboratory. The solution corresponding to the lower value of $p_{A_b^0}$, which is correct between 50% and 60% of the time depending upon the kinematics of the final state, is chosen in the $q^2$ and $w$ determination as simulation studies have shown that this choice introduces the smallest bias. The $w$ resolution is estimated from simulated data in different $w$ intervals. The distributions of differences between reconstructed and generated $w$ are fitted with double-Gaussian functions and the effective standard deviations are found to be between 0.01 and 0.05. The overall $w$ resolution is estimated with a fit with a triple-Gaussian function, and has an effective standard deviation $\sigma$ equal to 0.028.

**4 The spectral distribution $dN_{\text{corr}}/dw(A_b^0 \rightarrow \Lambda_c^+\mu^-\nu_\mu)$**

The $A_b^0 \rightarrow \Lambda_c^+\mu^-\nu_\mu X$ candidates are separated into 14 equal-size bins of reconstructed $w$ in the full kinematic range $1 \leq w \leq 1.43$. The parameters of the PDFs describing the signal and background components are determined from the fit to the overall $pK^-\pi^+$ mass spectrum. The contributions from semileptonic decays including higher-mass baryons in the final state is evaluated by fitting the $\Lambda_c^+\pi^+\pi^-$ mass spectra with two different methods. In the first, we fit for the four resonances shown in Fig. 2 using a PDF derived from the WS sample to model the background, and then use the simulation to correct for efficiency. In the second, we determine the signal yields of the $\Lambda_c^{++}$ states by subtracting the WS background and treating the residual smooth component of the spectrum as originating from a semileptonic decay $A_b^0 \rightarrow \Lambda_c^+\mu^-\nu_\mu X$. The second method provides an estimate of the systematic uncertainty introduced by the contribution from nonresonant $\Lambda_c^+\pi^+\pi^-$
Various kinematic distributions have been studied in simulation and data and we find where $N \approx 25$ that they are all in good agreement.

Finally, using simulated samples of signal events, we correct the unfolded spectrum for reconstructed distribution. The spectra before and after unfolding are shown in Fig. 3.

We have performed closure tests with different simulation models of the $A^+_b \rightarrow A^+_c \mu^- \nu_\mu$ dynamics, and verified that this unfolding procedure does not bias the reconstructed distribution. The spectra before and after unfolding are shown in Fig. 3.

Finally, using simulated samples of signal events, we correct the unfolded spectrum for $w$-dependent acceptance and selection efficiency to obtain the distribution $dN_{corr}/dw$. Various kinematic distributions have been studied in simulation and data and we find that they are all in good agreement.
5 The shape of $\xi_B(w)$ for $\Lambda^0_b \to \Lambda^+_c \mu^- \bar{\nu}_\mu$ decays

In order to determine the shape of the Isgur-Wise function $\xi_B(w)$, we use the square root of $dN_{\text{corr}}/dw$ divided by the kinematic factor $K(\langle w \rangle)$, defined in Eq. 4, evaluated at the midpoint in the seven unfolded $w$ bins. We derive the IW shape with a $\chi^2$ fit, where the $\chi^2$ is formed using the full covariance matrix of $dN_{\text{corr}}/dw$.

We use various functional forms to extract the slope, $\rho^2$, and curvature, $\sigma^2$, of $\xi_B(w)$. The first functional form is motivated by the $1/N_c$ expansion [44], where $N_c$ represents the number of colors, and has an exponential shape parameterized as

$$\xi_B(w) = \exp[-\rho^2(w - 1)].$$

The second functional form, the so called “dipole” IW function, which is more consistent with sum-rule bounds [17], is given by

$$\xi_B(w) = \left(\frac{2}{w + 1}\right)^{2\rho^2}.$$

Finally, we can use a simple Taylor series expansion of the Isgur-Wise function and fit for the slope and curvature parameters using the Taylor series expansion introduced in Eq. 5. Figure 4 shows the measured $\xi_B(w)$ and the fit results with this parameterization. Table 3 summarizes the slope and curvature at zero recoil obtained with the three fit models. Note that the curvature is an independent parameter only in the last fit, while in the first two models it is related to the second derivative of the IW function.
Figure 4: (a) The Isgur-Wise function fit for the decay $A_0^b \rightarrow \Lambda_c^+ \mu^- \nu$ with a Taylor series expansion in $(w - 1)$ up to second order. The black dots show the data and the solid (blue) line shows the fitted function with the second-order Taylor series expansion model. The vertical scale is in arbitrary units. (b) The correlation between slope $\rho^2$ and curvature $\sigma^2 / 2$: the three ellipses correspond to the $1\sigma$, $2\sigma$, and $4\sigma$ contours.

As the slope of the IW function is the most relevant quantity to determine $|V_{cb}|$ in the framework of HQET [13], we focus our studies on the systematic uncertainties on this parameter. We consider several sources of systematic uncertainties, which are listed in Table 4. The first two are determined by changing the fit models for $\Lambda_c^+$ and $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ signal shapes in the corresponding candidate mass spectra. The software trigger efficiency uncertainty is estimated by using an alternative procedure to evaluate this efficiency using the trigger emulation in the LHCb simulation. In order to assess systematics associated with the bin size, we perform the analysis with different binning choices. The sensitivity to the $A_0^b \rightarrow \Lambda_c^+ \mu^- \nu$ form factor modeling is assessed by reweighting the simulated $w$ spectra to correspond to different $\xi_B$ functions with slopes ranging from 1.5 to 1.7. The “phase space averaging” sensitivity is estimated by comparing the fit to the expression for $dN_{corr}/dw$ with the fit to $1/K(\langle w \rangle) \sqrt{dN_{corr}/dw}$. The uncertainty associated with the $A_0^b \rightarrow \Lambda_c^+ \mu^- \nu$ modeling is evaluated by changing the relative fraction of $\Lambda_c^+ \pi^+ \pi^-$ versus $\Lambda_c^+ \pi^0 \pi^0$ of the $\Lambda_c^+$ spectrum by $\pm 20\%$. Finally, we use the alternative evaluation of the fraction of $A_0^b \rightarrow \Lambda_c^+ \pi^+ \pi^- \mu^- \nu$ which includes the maximum possible nonresonant component to assess the sensitivity to residual $\Lambda_c^+$

Table 3: Summary of the values for the slope and curvature of the Isgur-Wise function with different parameterizations. The quoted uncertainties are statistical only. The models marked with “*” have only the slope at zero recoil as a free parameter, thus the curvature is derived from the fitted $\rho^2$.

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\rho^2$</th>
<th>$\sigma^2$</th>
<th>correlation coefficient</th>
<th>$\chi^2$/DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential*</td>
<td>$1.65 \pm 0.03$</td>
<td>$2.72 \pm 0.10$</td>
<td>100%</td>
<td>5.3/5</td>
</tr>
<tr>
<td>Dipole*</td>
<td>$1.82 \pm 0.03$</td>
<td>$4.22 \pm 0.12$</td>
<td>100%</td>
<td>5.3/5</td>
</tr>
<tr>
<td>Taylor series</td>
<td>$1.63 \pm 0.07$</td>
<td>$2.16 \pm 0.34$</td>
<td>97%</td>
<td>4.5/4</td>
</tr>
</tbody>
</table>
components in the subtracted spectrum. The total systematic uncertainty in $\rho^2$ is 0.08.

The value of $\rho^2$ obtained from the Taylor series expansion is

$$\rho^2 = 1.63 \pm 0.07 \pm 0.08,$$

which is consistent with Lattice calculations [23], QCD sum rules [22], and relativistic quark model [21] expectations. The measured slope is compatible with theoretical predictions and with the bound $\rho^2 \geq 3/4$ [16]. The measured curvature $\sigma^2$ is compatible within uncertainties with the lower bound $\sigma^2 \geq 3/5(\rho^2 + (\rho^2)^2)$ [18].

6 Comparison with unquenched lattice predictions

The lattice QCD calculation in Ref. [19] uses a helicity-based description of the six form factors governing $A_0^b \rightarrow \Lambda$ transitions introduced in Ref. [45]. The calculation uses state-of-the-art techniques encompassing the entire $q^2$ region. The stated uncertainties on the predicted width are therefore larger than what is expected in a high-$q^2$ region, but remain rather small, namely 6.3%. This corresponds to a 3.2% theoretical uncertainty on $|V_{cb}|$, thus raising the prospect of an additional precise independent determination of $|V_{cb}|$.

The simplest check on this theoretical prediction consists of a comparison of the predicted shape of $d\Gamma/dq^2$ and the measured data. Thus we measure the distribution $dN_{\text{corr}}/dq^2$ with the same procedure adopted to derive $dN_{\text{corr}}/dw$, including efficiency corrections and the unfolding procedure, with the same number of bins used to determine the raw and unfolded yields. We produce seven corrected yields and their associated covariance matrix, where the nondiagonal terms are related to the unfolding procedure. We then perform a $\chi^2$ fit to the seven experimental $dN_{\text{corr}}/dq^2$ data points using the theoretical functional shape given in Eq. 85 of Ref. [19], which also provides the nominal values of the form factor parameters, and thus we leave only the relative normalization floating. This fit uses a covariance matrix that combines experimental and theoretical

Table 4: Summary of the systematic uncertainties on the slope parameter $\rho^2$. The total uncertainty is obtained by adding the individual components in quadrature.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma(\rho^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal fit for $A_c^+$</td>
<td>0.02</td>
</tr>
<tr>
<td>Signal PDF for $A_{c^+}$</td>
<td>0.02</td>
</tr>
<tr>
<td>Software trigger efficiency</td>
<td>0.02</td>
</tr>
<tr>
<td>w binning</td>
<td>0.03</td>
</tr>
<tr>
<td>SVD unfolding regularization</td>
<td>0.03</td>
</tr>
<tr>
<td>Phase space averaging</td>
<td>0.03</td>
</tr>
<tr>
<td>$A_0^b \rightarrow A_c^+ \mu^- \nu_\mu$ modeling</td>
<td>0.03</td>
</tr>
<tr>
<td>$A_0^b \rightarrow A_c^{*+} \mu^- \nu_\mu$ modeling</td>
<td>0.03</td>
</tr>
<tr>
<td>Additional components of the semileptonic spectrum</td>
<td>0.02</td>
</tr>
<tr>
<td>$A_0^b$ kinematic dependencies</td>
<td>0.02</td>
</tr>
<tr>
<td>Total</td>
<td>0.08</td>
</tr>
</tbody>
</table>
uncertainties, which yields a $\chi^2$ equal to 2.0 for 6 degrees of freedom, and a corresponding p-value of 92%. This shows that the predicted shape is in good agreement with our measurement.

The form factor decomposition in Ref. [19] does not allow a straightforward extrapolation to the HQET limit of infinite heavy-quark masses. However, we know that in the static limit all the form factors are proportional to a single universal function. In order to assess how well our data are consistent with the static limit, we perform a second $\chi^2$ fit assuming that all the form factors are proportional to a single $z$-expansion function [46]. Fits with different pole masses used in the six form factors determined in Ref. [19] are performed. The overall shape does not change appreciably; the pole mass of 6.768 GeV is preferred. The two fit parameters are the coefficients $a_0$ and $a_1$, giving the strength of the first two terms in the $z$-expansion. The resulting fitted shape is shown in Fig. 5. This fit has a $\chi^2$ equal to 1.85 for 5 degrees of freedom, with a corresponding p-value of 87%. Note that the shape obtained with a single form-factor is very similar to the one predicted in Ref. [19]. This is consistent with the HQET prediction [15] that the shape of the differential distribution is well described by the static approximation, modulo a scale correction of the order of 10%, reflecting higher-order contributions. Further details of this fit and the fit using the Lattice QCD calculation can be found in the Appendix.

![Graph](image)

**Figure 5:** Comparison between the fit to the seven experimental data points using either the Lattice QCD calculation of Ref. [19], shown as grey points with a shaded area corresponding to the binned $1\sigma$ theory uncertainty, or a single form factor fit in the $z$-expansion, shown as the solid blue curve. The data points, modulo a scale factor, are shown as black points with error bars.

### 7 Conclusions

A precise measurement of the shape of the Isgur-Wise function describing the semileptonic decay $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \nu_{\mu}$ has been performed. The measured slope is consistent with theoretical models and the bound $\rho^2 \geq 3/4$ [16]. The measured curvature $\sigma^2$ is consistent with the lower-bound constraint $\sigma^2 \geq 3/5(\rho^2 + (\rho^2)^2)$ [18]. The shape of $d\Gamma /dq^2$ is studied and found to be well described by the unquenched lattice QCD prediction of Ref. [19], as well
as by a single form-factor parameterization. Further studies with a suitable normalization channel will lead to a precise independent determination of the CKM parameter $|V_{cb}|$.

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Appendix: Analytical expression for $d\Gamma/dq^2$

This Appendix describes the formalism used in the $d\Gamma/dq^2$ fits. In particular, we give the expression of $d\Gamma/dq^2$ in terms of the form factor basis chosen in Ref. [19], the so-called “helicity form factors.” In addition, we show the corresponding expression used to model the static limit.

The Lattice QCD calculations reported in Ref. [19] predict the differential decay width

<table>
<thead>
<tr>
<th>$f$</th>
<th>$J^P$</th>
<th>$m_{\text{pole}}^J(\Lambda_b \to \Lambda_c)$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_+, f_\perp$</td>
<td>$1^-$</td>
<td>6.332</td>
</tr>
<tr>
<td>$f_0$</td>
<td>$0^+$</td>
<td>6.725</td>
</tr>
<tr>
<td>$g_+, g_\perp$</td>
<td>$1^+$</td>
<td>6.768</td>
</tr>
<tr>
<td>$g_0$</td>
<td>$0^-$</td>
<td>6.276</td>
</tr>
</tbody>
</table>

Table 5: Masses of the relevant form factor poles in the physical limit (in GeV).
\[ \frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \sqrt{s_+ s_-} \left( 1 - \frac{m_t^2}{q^2} \right)^2}{768 \pi^3 m_{\Lambda_c}^3} \times \left\{ 4 \left( m_c^2 + 2q^2 \right) \left( s_+ [g_\perp(q^2)]^2 + s_- \left[ f_\perp(q^2) \right]^2 \right) \\
+ 2 \frac{m_c^2 + 2q^2}{q^2} \left( s_+ \left[ (m_{\Lambda_c} - m_X) g_+(q^2) \right]^2 + s_- \left[ (m_{\Lambda_c} + m_X) f_+(q^2) \right]^2 \right) \\
+ \frac{6m_c^2}{q^2} \left( s_+ \left[ (m_{\Lambda_c} - m_X) f_0(q^2) \right]^2 + s_- \left[ (m_{\Lambda_c} + m_X) g_0(q^2) \right]^2 \right) \right\}, \quad (10) \]

where \( g_\perp, f_\perp, g_+, f_+, g_0, \) and \( f_0 \) represent the six form factors necessary to describe this decay, \( X \equiv \Lambda_c \) denotes the final-state baryon, \( m_t \) represents the mass of the muon, \( q^2 \) is the squared four-momentum transfer between the heavy baryons, and \( s_\pm = (m_{\Lambda_c} \pm m_X)^2 - q^2 \).

The six form factors are cast in terms of the \( z \)-expansion [46] up to first order, and have the functional form

\[ f(q^2) = \frac{1}{1 - q^2/(m_{\text{pole}}^2)^2} \times \left[ a_0^f + a_1^f z^f(q^2) \right], \quad (12) \]

where \( z^f(q^2) \) is given by

\[ z^f(q^2) = \sqrt{\frac{t_+^f - q^2}{t_+^f - t_0}} - \sqrt{\frac{t_+^f - q^2}{t_+^f - t_0}}, \quad (13) \]

\[ t_0 = (m_{\Lambda_c} - m_X)^2, \quad (14) \]

and \( t_+^f \) is given by

\[ t_+^f = (m_{\text{pole}}^f)^2, \quad (15) \]

and the pole masses used in the calculations are shown in Table [5]. The parameters \( a_0^f \) and \( a_1^f \) for the six form factors describing this decay are given in Table VIII of Ref. [19].

In the static limit all the helicity form factors are proportional to a single universal function. Thus, we use a common \( z \)-expansion parameterization

\[ \frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2 \sqrt{s_+ s_-} \left( 1 - \frac{m_t^2}{q^2} \right)^2 g_\perp(q^2)}{768 \pi^3 m_{\Lambda_c}^3} \times \left\{ 4 \left( m_c^2 + 2q^2 \right) \left( s_+ + s_- \right) \right. \\
+ \frac{4}{q^2} \left[ s_+ (m_{\Lambda_c} - m_X)^2 + s_- (m_{\Lambda_c} + m_X)^2 \right] \left[ 2m_c^2 + q^2 \right] \right\}, \quad (16) \]

where the choice of \( g_\perp \) reflects the choice of the pole mass used in the single \( z \)-expansion fit given in Sect. [6]. We have performed the fits with various choices of pole masses and
examined the effects on the shape $d\Gamma/dq^2$ and found the shape did not vary significantly, though it was found that the parameters defining $g_\perp$ yielded the optimal fit. In this case, the fit parameters are the coefficients $a_0$ and $a_1$ in the $z$-expansion parameterization of $g_\perp(q^2)$, which has the form shown in Eq. [12].

References


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