A simple test for the presence of multidomain behavior during paleointensity experiments

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Detecting and excluding non-ideal behavior during paleointensity experiments is critical to asserting the reliability of data. Our knowledge of detecting non-ideal behavior, in particular the influence of multidomain (MD) grains, has expanded considerably over the past decade and experimental procedures now commonly incorporate checks to detect the effects of MD behavior. However, many older studies were carried out before these checks were devised and provide no quantifiable means of testing for the presence of MD grains. An estimated one third of all entries in the most recent paleointensity database do not include some form of check for MD behavior. The reliability of these results is therefore questionable and can only hinder efforts to understand the evolution of the geomagnetic field and the geodynamo. I propose a simple phenomenological check that can be applied to previous studies, provided that the raw data are available, that will allow the exclusion of MD behavior and provide a means of identifying reliable data. The check is a quantification of the curvature, $k$, of data points on an Arai plot, a feature commonly associated with MD behavior. Analysis of paleointensity data from samples with known grain size indicates that this new parameter is significantly correlated with grain size and with the accuracy of the paleointensity estimates made from both limbs of the curved data. Analysis of 181 samples from five historical data sets indicates that $k$ is significantly correlated with experimentally obtained MD and alteration check parameters, and the accuracy of the paleointensity estimate. A threshold selection value of $k \leq 0.164$ can be defined using the samples with known grain sizes. Applying this cut-off value, combined with a threshold on the quality of the circle fit and a commonly used alteration check, to the historical data yields an accurate result with low scatter. When compared with previously published selection criteria that incorporate experimental checks for non-ideal behavior, the result of applying the criteria proposed here is an improvement. The application of these three criteria rejects over 65% of all inaccurate results and has the highest concentration of accurate results when compared with the other criteria sets tested. Other selection criteria can be subsequently used to improve on this result. While modern studies should always include experimental checks to identify MD behavior, this new criterion will provide a useful tool for future studies and, importantly, a method to assess the reliability of previously published data.


1. Introduction

Determining the strength of the paleomagnetic field from geological materials can provide important constraints on our understanding of the evolution of the geodynamo, Earth’s core, and core–mantle interactions. However, absolute paleointensity experiments can be time consuming, difficult, and prone to high failure rates [e.g., Riisager et al., 2002; Pan et al., 2005; Paterson et al., 2010b]. In the past decade or so, a number of advances have expanded our knowledge of the causes and effects of non-ideal paleointensity behavior [McClelland and Briden, 1996; Valet et al., 1996; Riisager and Riisager, 2001; Selkin et al., 2000; Krása et al., 2003; Draeger et al., 2006; Yamamoto, 2006; Selkin et al., 2007; Fabian, 2009]. Some of these advances suggest the inclusion of additional experimental steps that allow the quantification, and sometimes the correction of, non-ideal behavior [e.g., McClelland and Briden, 1996; Selkin et al., 2000; Krása et al., 2003].
One result of the recent efforts to identify non-ideal paleointensity behavior is that, due to a lack of what are now becoming standard tests, many older data sets are deemed unreliable and often discarded, or given a lower rank during meta-analyses [e.g., Selkin and Tauxe, 2000; Ziegler et al., 2008]. While some non-ideal behavior can be tested for post-experiment, with additional measurements (e.g., remanence anisotropy), others, such as the influence of multidomain (MD) grains and alteration during laboratory heating, cannot be directly tested.

It is difficult to assess the number of studies that do not include tests for MD behavior: To date no paleointensity database records this information. However, nearly one third of the entries in the current PINT database [Biggin, 2010] are from studies using Thellier-type methods [Thellier and Thellier, 1959; Coe, 1967] published before the first suggestion of an experimental procedure that could test for MD effects [McClelland and Briden, 1996]. This is likely to be a minimum estimate as more than half of the entries were published before it was established that such a procedure could be used to detect MD effects [Riisager and Riisager, 1998]. The large proportion of paleointensity data that may be influenced by non-ideal effects will undoubtedly hinder efforts to understand the long-term variation of the geomagnetic field. It is therefore essential to develop new analyses and criteria to assess the fidelity of paleointensity data from previously published studies.

In this paper I propose a new phenomenological criterion that quantifies curvature on an Arai plot and investigates the manifestation of curvature due to the effects of MD grains. This new criterion does not require any additional measurements and can be applied to Thellier-type studies that use the Coe protocol, provided that the data required to construct an Arai plot [Nagata et al., 1963] are available. The criterion makes use of the well documented feature that pseudo-single domain (PSD) and MD grains produce a curved sequence of points when plotted for analysis on an Arai plot [e.g., Levi, 1977; Shcherbakov and Shcherbakova, 2001]. The source of this curvature can be attributed to magnetizations that are unblocked (demagnetized) at temperatures below their respective blocking temperatures, which leads to an excess loss of remanence at lower temperatures compared to remanence gained [Fabian, 2001; Leonhardt et al., 2004b] and due to the progressive stabilization of domain structures during the repeated heatings required during a Thellier-type paleointensity experiment [Shcherbakov and Shcherbakova, 2001; Yu and Tauxe, 2006].

2. Methods

2.1. Quantifying Curvature

Curvature is defined as the degree to which an object (e.g., a line or a plane) deviates from being perfectly flat. In the case of a line, curvature is the reciprocal of the radius of curvature. To quantify the curvature of data on an Arai plot, a best-fit circle of the form \((x-a)^2 + (y-b)^2 = r^2\) is fitted to the data using a least squares approach [Taubin, 1991; Chernov and Lesort, 2005], where \(x\) represents the thermoremanent magnetization (TRM) gained, and \(y\) represents the natural remanent magnetization (NRM) remaining. The best-fit circle is not anchored to any point on the Arai plot. With this approach the data on an Arai plot represent an arc of a much larger circle. The curvature \(k\) of the circle can then be simply calculated as \(\frac{1}{r}\). The higher the value of \(k\) the more curved the arc is (i.e., the smaller the circle), the lower the value of \(k\) the closer the arc is to being a straight line. For a perfectly straight line \(k = 0\). The quality of the best-fit circle to the data can be assessed by determining the sum of the squares of the errors (SSE):

\[
SSE = \sum_{i=1}^{n} \left( \sqrt{(x_i - a)^2 + (y_i - b)^2} - r \right)^2.
\]

Examples of arcs with different values of \(k\) are shown in Figure 1. In Figure 1 the segments of the circles that fall within the coordinate region of an Arai plot have been selected to illustrate the similarity between these circles and real paleointensity data. The appropriateness of fitting circle to paleointensity data is discussed in more detail in the auxiliary material.

To ensure an equal comparison of different samples the data must be appropriately scaled. While normalizing both axes by the same constant, be it the maximum NRM or TRM, preserves the relative relationship for the calculation of the best linear fit, this type of normalization produces different values of \(k\), which depend on the value of the normalization parameter. For example, if two identical and ideal samples recording the same paleointensity were subjected to identical intensity experiments, but with differing laboratory fields, normalization by the initial NRM would produce different TRM coordinates for each sample. This would result in different values of \(k\) despite the linearity of the data being identical. To ensure consistency each axis is normalized by the maximum value of the data on that axis such that \(0 \leq TRM \leq 1\), and \(0 \leq NRM \leq 1\). For the calculation of \(k\) all data points on the Arai plot are used, not just those used to obtain the best linear fit. The implications of this are discussed further in section 6.
2.2. Directionality of Curvature

It should be noted that the definition of \( k \) given above only yields the absolute value of \( k \), that is \( k = \frac{1}{\sqrt{2}} \), where \( r > 0 \). This does not, therefore, distinguish between curved data facing either up or down. When a best-fit linear line is fitted to data, the best-fit line passes through the centroid of the data (\( C = (\bar{x}, \bar{y}) \)). In the case of perfectly linear data the best-fit circle will also pass through the centroid of the data (\( CP = (a, b) \)).

For small degrees of curvature the center point of the circle (\( CP = (a, b) \)) will lie well above \( C \), i.e., \( \bar{x} < a \) and \( \bar{y} < b \). For concave-down data, the converse is true, \( a < \bar{x} \) and \( b < \bar{y} \). In both cases, as the curvature increases \( C \) will tend to \( CP \), and for a circle that falls entirely within the plot \( C \equiv CP \). Therefore, \( k \) can be given a sign of direction to specify if the curvature is concave-up or concave-down based on the position of the center point of the best-fit circle relative to the centroid of the data:

\[
\tilde{k} = \begin{cases} 
\frac{1}{r} & \text{if } (\bar{x} < a) \text{ and } (\bar{y} < b) \\
-\frac{1}{r} & \text{if } (a < \bar{x}) \text{ and } (b < \bar{y}) \\
0 & \text{if } (a = \bar{x}) \text{ and } (b = \bar{y})
\end{cases}
\]

Strictly, there are four possible directions of curvature, but the physical constraints of paleointensity data and analysis preclude the other two (i.e., all magnetizations must be greater than or equal to zero). Most Araki plots exhibiting MD-like behavior data have concave-up, or positive

\[ \text{Slope 1 and slope 2 refer to the low- and high-temperature segments, respectively. N refers to the number of points used to determine the best linear fit.} \]

\[ \text{ZFH refers to experiments that used zero field heating during pTRM acquisition.} \]

\[ \text{The \( \| \) and \( \perp \) refer to the orientation of the laboratory field with respect to the NRM.} \]
data in general, the magnitude of curvature, $k = |\overline{k}|$, is the important parameter; the direction of curvature can give some indication as to the cause of the non-linear behavior.

3. Application to Synthetic Samples

Paleointensity data from various studies using extracted natural and synthetic magnetite with known grain sizes have been compiled for this study and a summary of the samples used is given in Table 1. Although these samples contain both natural and synthetic magnetite, for this study these samples will be collectively referred to as synthetic samples. Detailed descriptions of the samples and the experimental procedures are given in the respective references [Levi, 1975, 1977; Muxworthy, 1998; Scherbakov and Scherbakova, 2001; Krása et al., 2003; Xu and Dunlop, 2003; Xu and Dunlop, 2004, and references therein]. Prior to the paleointensity experiments all of the samples underwent thermal stabilization, typically being heated and held at high temperature (~700°C) for a period of time (3- h) to reduce the degree of stress within individual grains and to ensure no or little alteration during the proceeding experiments. All samples have been characterized as low-Ti titanomagnetite (Ti ≤ 0-0.1%) based on Curie temperatures (565–586°C). Both Muxworthy [1998] and Krása et al. [2003] noted that the samples used in their studies may have a small degree of maghemitization. Examples of the Arai plots from these samples are shown in Figure 2 and all of the Arai plots are given in the auxiliary material. The paleointensity experiments performed on these samples were variants of the Coe double heating protocol [Coe, 1967]. In most experiments the laboratory field was applied parallel to the NRM, however, Xu and Dunlop [2004] also carried out experiments in which the applied the field was perpendicular to the NRM. Levi [1975] compared the effects of heating in zero field (zero field heating, ZFH) and heating in-field (field heating, FH) during the partial TRM (pTRM) acquisition steps. The data therefore represent a wide variety of experimental conditions. Sample S11 from Levi [1977] has not been included in this study due to the highly elongated shape of the magnetite crystals, which makes it difficult to compare with other grain sizes.

Visually (Figure 2 and Figures S1–S7 in the auxiliary material) and quantitatively (using the SSE, Table 1) the best-fit arcs shown in Figure 2 fit the data well, with all values of the SSE ≤ 0.0126 (Table 1). This is an indication that least squares circle fitting is appropriate for Arai plot data. All of the samples have positive curvature with $k$ ranging from 0.008 to 1.235 (Table 1).

To assess the accuracy of these samples at recording paleointensities, best-fit linear lines were fitted through two segments of the unnormalized data, one through the first several points and another through the last several points. These fits are referred to as the low-temperature and high-temperature fits, respectively, after the experimental temperatures from which the data are obtained. Each fit consists of at least four consecutive data points and is fitted to the best linear section irrespective of the quality of fit, that is, for some fits the ratio of the standard error of the slope to the value of the slope ($\beta$) is higher than is typically accepted in many paleointensity studies (i.e., $\beta > 0.1$). For a consistent comparison of the results from different samples the
Paleointensity data from the synthetic samples. (a) Accuracy of the paleointensity estimates obtained from the low-temperature (diamonds) and high-temperature (open circles) segments of the Arai plot as a function of the logarithm of grain size. (b) Curvature ($k$) of the Arai plot data as a function of the logarithm of grain size. (c) Accuracy of the paleointensity estimates as a function of curvature. Symbols are the same as in part Figure 3a. $R$ is the Pearson correlation and $p$ is the significance of the correlation.

paleointensity estimates are normalized by the expected values; hereafter simply referred to as the intensity estimate. To assess the accuracy of the intensity estimate the logarithm of the estimate was used in the analysis. When accuracy values are zeros the intensity estimate is exactly the expected value; positive and negative values represent over- and underestimates, respectively. Results of the linear fitting are summarized in Table 1 and samples are considered to yield an accurate estimate if the intensity estimate is within a factor 1.1 (10%) of the expected value (i.e., $\log (1/1.1) \leq \log(\text{Intensity}) \leq \log(1.1)$). The paleointensity estimates from the synthetic samples are symmetric in that the low-temperature segments typically overestimate the expected intensity more than the high-temperature segments underestimates the intensity. Despite the asymmetry of the paleointensity estimates made from the low- and high-temperature segments, the curvature on the Arai plots is symmetric as is evident from the goodness-of-fit of the circles, which are inherently symmetric. A full discussion reconciling these different symmetries is given in the auxiliary material.

Three studies incorporated experimental checks for MD behavior [Muxworthy, 1998; Krása et al., 2003; Yu and Dunlop, 2003]. Due to the laboratory field being parallel to the NRM the check values are generally low [cf. Yu and Tauxe, 2005], except where the data suffer from noise (e.g., sample MB). The additivity check of Krása et al. [2003] is the only MD check parameter that is consistently correlated with the logarithm of grain size (in $\mu$m). With data from only 15 samples it is difficult to assess the efficacy of pTRM tail checks; additional data are required, measured with various angles of laboratory field, to adequately assess these commonly used MD checks.

As is expected, and commonly reported [e.g., Chauvin et al., 2005], paleointensity estimates from the low-temperature segments overestimate the true intensity and estimates from the high-temperature segments are underestimated (Table 1, Figure 3). A comparison between the accuracy of both segments and grain size indicates that the accuracies of both segments are significantly correlated with grain size (Figure 3a); unless otherwise stated correlations with grain size refer to correlations with the logarithm of grain size). A comparison between curvature and grain size also yields a significant correlation ($R = 0.736, p \ll 0.01$), which confirms curved Arai plots are associated with large grains of magnetite (Figure 3b). Not only is $k$ correlated with grain size, but it is also significantly correlated with the accuracy of the low-temperature segment ($R = 0.906, p \ll 0.01$) and with the accuracy of the high-temperature segment ($R = -0.841, p \ll 0.01$; Figure 3c). For a sample with constrained grain size $k$ is a good proxy for grain size and the inaccuracies that larger grains can produce during a paleointensity experiment.

4. Defining a Threshold for Data Selection

Given the good correlations of $k$ with grain size and accuracy, can a threshold value be defined to exclude the effects of MD grains? Only two samples (MGH1 and Yu_s1) fall within the SD grain size range for magnetite ($\sim 0.03 \sim 0.08 \mu m$) [Dunlop and Özdemir, 1997]. These two samples have a maximum curvature of 0.118 (Table 1) and this may provide a suitable threshold value. It has been noted, however, that small PSD grains are capable of producing accurate results [Shcherbakov and Shcherbakova, 2001] and this is reaffirmed here (e.g., samples S4–9 and Xu_s1 (⊥), Table 1). Nine samples yield accurate intensity estimates from both segments and suggest a threshold value of $k \leq 0.164$. One sample (S8 (ZFH)) yields an accurate estimate from one segment, but not both. This may suggest an upper maximum of $k \sim 0.270$, but two samples yield inaccurate results with values of $k$ in this range (i.e., S3 and Xu_s3 (∥)). Therefore a threshold value of $k \leq 0.164$ is suggested to exclude inaccuracies produced by the effects of MD and large PSD grains. An example of the an arc with $k = 0.164$ is shown in Figure 1. The maximum grain size that satisfies this criterion is $0.6 \mu m$ (Xu_s1 (⊥), Table 1).

5. Application to Real Data

To test the effectiveness of $k$ at excluding MD behavior a paleointensity data set comprised of 181 estimates from five
historical volcanoes, where the expected intensity of the geomagnetic field is known from direct observations, was compiled. The data set includes 18 data from the 1950, 1979, and 1983 eruption of Mt. Etna, Italy [Biggin et al., 2007], 52 data from the 1914 and 18 data from the 1946 eruption of Sakurajima, Japan [Yamamoto and Hoshi, 2008], 46 data from the 1993 eruption of Láscar, Chile [Paterson et al., 2010b], 29 data from the 1943 eruption of Paricutín, Mexico, and 18 data from the 1944 eruption of Vesuvius, Italy [Muxworthy et al., 2011]. These studies used the Ce paleointensity protocol [Coe, 1967] and incorporated pTRM and pTRM tail checks. Full details are reported by Biggin et al. [2007], Yamamoto and Hoshi [2008], Paterson et al. [2010b], and Muxworthy et al. [2011]. A number of samples reported by Muxworthy et al. [2011] had multiple components of magnetization with overlapping blocking spectra as evidenced on the vector component diagrams of the NRM; these were not included in the data set. All remaining samples have single components of magnetization. Using the best-fits reported, the effects of applying several previously published selection criteria have been investigated. Five sets of selection criteria were tested on this data set: SELCRIT2 from Biggin et al. [2007] is a modification of the criteria proposed by Selkin and Tauxe [2000]; PICRIT03 from Kissel and Laj [2004]; ThellierTool ‘B’ and ThellierTool ‘A’ are the default criteria of the ThellierTool v4.2 software from Leonhardt et al. [2004a]; and the selection criteria for pyroclastic lithic clasts proposed by Paterson et al. [2010b]. The definitions and threshold values used by these criteria sets are presented in the respective references along with the details of the selected fits, but summary tables of the criteria definitions, the threshold values used, and the best linear fit data are given in the auxiliary material; the results of applying these criteria are given in Table 2. Histograms of the intensity estimates of the unselected data and the tested criteria sets are shown in Figure 4.

17 The unselected data yield an accurate average, but with a large scatter $\delta B_n > 26\%$ ($\delta B_n$ (%) is the sample size adjusted within-site scatter of Paterson et al. [2010a], which represent the maximum within-site scatter at the 95% confidence level; details of the calculation of $\delta B_n$ (%) are given in the auxiliary material). With the exception of the ThellierTool ‘A’ criteria, applying any of the criteria sets maintains the accuracy of the mean estimate, but all criteria sets fail to appreciably reduce the scatter of the estimates. This is largely due all criteria sets failing to exclude the most deviant sample, VM1F (Intensity = 2.92, Figure 5a). The histograms in Figure 4 indicate that the intensity estimates have a wide distribution that is skewed by the overestimate of sample VM1F. This sample exhibits obvious curvature, but the best linear fit passes selection and none of the previously published selection can exclude this sample. Many investigators may exclude this sample based on visual inspection; however, a more objective reasoning for excluding samples such as this is required.

18 The investigated criteria sets variably exclude accurate results and accept inaccurate results: As many as 78% of all accurate results are rejected (ThellierTool ‘A’) and as many as 80% of all inaccurate results are accepted (SELCRIT2, Table 2). The selection criteria of Paterson et al. [2010b] yield the highest concentration of accurate results (Accurate-to-inaccurate, Table 2). These selection criteria were defined using the data from Láscar, which constitutes ~25% of the data set. The best of the previously published criteria sets yield accurate, but imprecise mean estimates with relatively low concentrations of accurate results.

19 The calculation of curvature for the 181 samples indicates that $k$ is weakly, but significantly correlated with accuracy, $\beta$, pTRM checks $\delta CK$, DRAT, and CD RAT, and with all of the pTRM tail checks tested (at the 0.05 significance level). The correlation of $k$ and $\beta$ is not surprising, to some extent both are a measure of non-linearity. The quality factor ($q$) is correlated with $k$ through $\beta$ ($q \propto \frac{1}{k}$). The fact that $k$ is correlated with pTRM checks may indicate that progressive alteration is producing curved Arai plots [e.g., Kosterov and Prévote, 1998], or that MD effects are manifesting in pTRM checks [e.g., Biggin and Thomas, 2003; Leonhardt et al., 2004b]. Two of three pTRM checks tested ($\delta TR$ and $\delta TR^*$) are weakly, but significantly correlated with the accuracy of the paleointensity estimate. Despite these correlations extremely low threshold values ($\delta TR < 2.35$ and $\delta TR^* < 2.9$) are required to exclude sample VM1F, which as noted previously passes all selection criteria. This highlights the necessity of an objective means of discriminating against non-linear behavior.

20 Before assessing the effectiveness of $k$, a criterion to remove the possible influence of chemical alteration during the experiment, $\delta CK \leq 10$, is applied. This is the selection criterion used by Paterson et al. [2010b] and is less strict than used by other criteria sets (e.g., ThellierTool). This criterion removes 14 samples (3 accurate results) and only
Data that have large $SSE$ are either too noisy or highly non-linear and cannot be approximated by a circle or a straight line. The quality of circle fits is more variable for the historical data set than for the synthetic samples, which is likely due to a reduced influence of alteration, chemical remagnetizations, and other non-ideal influences in the latter data set. For the historical data set, the $SSE$ varies from $4.0 \times 10^{-6}$ to 0.120, with 146 samples having $SSE \leq 0.0126$ (auxiliary material). As is the case with $k$, the well controlled conditions used to measure the synthetic samples may also reduce the variability of the $SSE$; although the maximum $SSE$ from the synthetic samples is 0.0126 this may be too strict a criterion for natural samples. After the application of the $k$ and $\delta CK$ criteria five samples that have $SSE > 0.0126$ remain; three of these are inaccurate. Selecting only data with $SSE \leq 0.0126$ further improves the final result (Table 2). The sample size adjusted within-site scatter is less than 25% and this criteria set accepts the highest concentration of accurate results: a ratio of 1.00. The histogram of intensity estimates (Figure 4) indicates a reduced spread of results with a higher concentration of results close to the expected value. These selection criteria are the only ones that yield an accurate mean estimate with a high precision around the corrected result. If the no $\delta CK$ criterion is applied a similar result is achieved (Mean = 1.10, $n = 77$, $\delta B_n (\%) = 19.59$, $n$ accurate = 38, Most deviant intensity = 1.98). The criteria set proposed here includes only three parameters ($k \leq 0.164$, $\delta CK \leq 0.0126$, and $\delta CK \leq 10$), the other criteria sets include as many as 10 criteria. Other selection criteria can applied after the three criteria used here to improve the overall result (e.g., $f \geq 0.4$, $g \geq 5.5$, and $CDRAT \leq 14.5$; Mean = 1.07, $n = 59$, $\delta B_n (\%) = 13.77$, $n$ accurate = 33, Most deviant intensity = 1.56, accurate-to-inaccurate = 1.27). The efficacy of these parameters will not be fully explored here, but this illustrates how the use of $k$ can complement existing selection criteria.

The final criteria set used in this study excludes 63% of all inaccurate results. Only one other set of selection criteria excludes more inaccurate results (ThellierTool ‘A’), but this set yields an inaccurate mean with an unacceptably large scatter of results. Although the criteria used here have the second highest rejection rate of accurate results (Table 2) the relative proportion of accurate-to-inaccurate results is increased, which maintains the accurate and well constrained mean result (Table 2, Figure 4). The accurate mean, reduced scatter of data, high rejection rate of inaccurate data, and the concentration of accurate data are a strong demonstration that parameters based on curvature are suitable for improving paleointensity results.

6. Discussion

6.1. Common MD Features

There are a number of experimental procedures that have been qualitatively demonstrated to enhance or reduce curvature of Arai plot data, such as the orientation of the laboratory field. $k$ provides a quantitative means of assessing these effects.

6.1.1. Orientation of the Laboratory Field

It has been frequently noted that the magnitude of pTRM tail checks (e.g., $\delta TR$, $DRAFT_{Tail}$) depends on the orientation of the laboratory field with respect to the NRM [e.g., Leonhardt et al., 2004b; Yu and Tauxe, 2005; Biggin, 2006].
Xu and Dunlop [2004] compared the effects of applying the laboratory field both parallel and perpendicular to the NRM. They noted that curvature was qualitatively enhanced when the laboratory field was parallel to the NRM. This finding is quantitatively confirmed by $k$ (Table 1), which is at least 33% larger when the laboratory field is parallel to the NRM. The difference in curvature ($k_{parallel} - k_{perpendicular}$) may be correlated with grain size ($R = 0.891, p = 0.109$), but there is insufficient data to prove the significance of this correlation.

$k$ will be a useful addition to modern paleointensity studies: In a Coe paleointensity experiment, when the field is applied perpendicular to the NRM, pTRM tail checks are enhanced, but curvature is reduced, conversely if the field is applied parallel to the NRM, curvature is enhanced and pTRM tail checks are reduced. Therefore failure of one or both checks is a strong indication of the presence of MD grains. This implies that using both pTRM tail checks and curvature criteria should provide a more robust means of discriminating against MD behavior than using only one of these checks. A fuller analysis, involving more than four samples is required to further characterize the dependence of $k$ on the orientation of the laboratory field.

### 6.1.2. Field Heating Versus Zero-Field Heating

Levi [1975] compared two versions of the Coe protocol on three synthetic samples (S2, S7, and S8). The first method was that of Coe [1967] in which pTRMs were acquire by first heating to temperature in zero magnetic field and cooling to room temperature in an applied field (zero-field heating, ZFH). The initial assumption of Levi [1975] was that the ZFH method would reduce the effect of high temperature viscous magnetization. His results, however, indicated that ZFH enhanced the curvature seen on the Arai plots. The Arai plots for sample S8 are shown in Figures 2a and 2b. The remaining Arai plots are given in the auxiliary material. Quantitatively, ZFH does enhance curvature: $k$ increases from 0.284 to 0.530, from 0.054 to 0.670, and from 0.017 to 0.270, for samples S2, S7, and S8, respectively. Biggin [2006] also noted this effect and made the recommendation that all intensity experiments should use FH. Although the ratio of the curvatures from the two types of experiments ($\frac{k_{FH}}{k_{ZFH}}$) has an apparent trend with grain size, the correlation is not significant ($R = 0.991, p = 0.087$). This is largely due to a insufficient number of data ($n = 3$); further experiments are required to confirm this relationship.

### 6.2. Refining the Threshold for Data Selection

Most paleointensity studies choose criteria thresholds values in an arbitrary fashion [e.g., Tarduno and Cottrell, 2005; Hill et al., 2008] or define/optimize the thresholds using the data to which they are then applied [e.g., Ben-Yosef
et al., 2008; Gee et al., 2010; Paterson et al., 2010b]. Although each of these studies intuitively aims for appropriate values (i.e., low values of MD and alteration checks, low scatter about the best linear fit and large fractions), each study uses unique threshold values. A distinct advantage of the approach used in this study is that k has been defined independently of the data to which it is applied and is a step toward a universal set of selection criteria for paleointensity studies that use the Coe protocol. Additional data will further constrain the threshold value suggested here, with grain sizes $< 2 \mu m$ being of most interest to refine k. Further influences such as the orientation of the laboratory field and experimental protocol should also be fully investigated.

[28] The threshold value for k determined from the synthetic samples may be too strict for natural samples, as suggested by synthetic samples S8 (ZFHF) and Yu_s2 (Table 1, section 4). Twenty-five samples from the historical data set have $0.164 < k < 0.230$, 14 of which yield accurate paleointensity estimates. An example of an accurate result from a sample with large k is given in Figure 5b. This sample has a noticeable degree of curvature, but yields an accurate estimate. This suggests that samples with a relatively high degree of curvature, likely produced by PSD-sized grains, can still produce accurate results. The accuracy of these samples is likely to depend on the fraction of NRM used to calculate the best linear fit [e.g., Biggin and Thomas, 2003; Chauvin et al., 2005]. A more detailed study of samples with grains in the small PSD-size range (0.1–2 \mu m) is needed to investigate these effects and to refine the threshold value of k.

6.3. Other Methods of Calculating Curvature

[29] Attempts have been made to quantify the curvature of data on an Arai plot, but with varying or limited success. Both Coe et al. [1978] and Biggin and Thomas [2003] defined the curvature of data as the maximum percentage deviation of the slope of any segment of the best linear fit, spanning greater than 50% of the length of the fitted line. Using this method to estimate curvature, for all the data points from the synthetic data yields a low correlation between grain size and curvature ($R = 0.355$, $p = 0.029$). The weak correlation is largely due to sample W6, which has a value 3.5 times larger than the next largest value. Removing W6 improves the correlation ($R = 0.684$, $p \ll 0.01$), but it remains lower than the correlation between grain size and k for all the data. This method of quantifying curvature is sensitive to extremely curved data and data with large amounts of noise; these effects are reduced in the calculation of k, which makes it a more appropriate parameter for quantifying curvature.

[30] Recently, Fabian and Leonhardt [2010] also proposed a method of quantifying curvature as a domain state proxy. Their proposed proxy equates to

$$P_{DS} = \frac{|F_{low} - F_{exp}| + |F_{high} - F_{exp}|}{2F_{exp}}, \hspace{1cm} (3)$$

where $F_{low}$ and $F_{high}$ are the paleointensity estimates from the low-temperature and high-temperature portions of the Arai plot data, respectively, and $F_{exp}$ is the expected intensity. The correlation between $P_{DS}$ and grain size is high ($R = 0.709, p \ll 0.01$), but not as high as with k. It should be noted, however, that $P_{DS}$ can only be used in controlled situations where the expected paleointensity is known.

is not the case with k, which is a generally applicable parameter. All methods of estimating curvature based on the slopes of both limbs of the data are subject to a number of pitfalls: the subjective nature of choosing the segments; the effects of poor data distribution; and the inherent difficulty in fully describing curved data with a straight line.

6.4. Multidomain or Multicomponent?

[31] Both Yu and Dunlop [2002] and Shcherbakov and Zhidkov [2006] illustrated that an Arai plot for a sample with two components of magnetization can exhibit curvature. Examples of this are shown in Figure 6a. In these examples, an original NRM has been partially overprinted in a field perpendicular to the NRM with varying strengths. As the strength of the overprinting field increases, relative to the original field strength, the curvature progresses from concave-down to concave-up (Figures 6a and 6b). A viscous remanent magnetization may also manifest on an Arai plot in a similar fashion, but is likely to affect lower temperatures. The normalization of Figure 6a has been chosen for the calculation of k, but distorts the appearance of the slopes; all high temperature components have the same slope. In the absence of directional data confirming the presence of multiple components of magnetization, which may be the case when working with older data sets, it is recommended to err on the side of caution and exclude any data with an unacceptably large degree of curvature.

6.5. Multidomain or Alteration?

[32] Curvature on an Arai plot may not be due to MD effects, but may arise from the effects of laboratory induced alteration [e.g., Kosterov and Prévot, 1998]. Alteration, however, is a poorly understood aspect of paleointensity determinations and at present there are no control experiments from which to estimate variations of curvature due to alteration. Therefore a rigorous assessment of the effects of alteration cannot be made in the same fashion as is the case for assessing the influence of MD grains. Qualitatively speaking, however, alteration has the potential to produce both positive and negative curvature. It would be expected that alteration where remanence carrying phases are preferentially destroyed would produce concave-up ($k > 0$) curvature due to a relatively rapid loss of NRM compared with TRM gained. In the case where new remanence carriers are formed in addition to the pre-existing magnetic phases, relatively more TRM would be gained compared with NRM lost, which would produce concave-down ($k < 0$) curvature.

[33] Forty-seven samples from the historical data set (~26%) have negative curvature, which may be the result of alteration. Twenty-five accepted results have $k < 0$; 13 accurate and 12 inaccurate. Most of these samples have small k (23 of 25 samples have $k < 0.087$), which represents small deviation from near linear data, likely due to experimental noise. The largest curvature is from an accurate sample (ET3_023E2), which has $k = -0.154$. Of the 22 rejected results with $k < 0$, only four samples fail the alteration check. Seventeen samples fail the k criterion, 10 of which also fail the SSE criterion. The large negative curvature is likely due to the effects of alteration and the large SSE values suggest that the resultant curvature is either noisy or highly non-linear.
A number of samples analyzed in section 5 have accurate linear low-temperature segments, but suffer from alteration at higher temperatures (e.g., LV15B1, Figure 5c), which produces a high degree of curvature. Sample LV15B1 is an example of a sample that has undergone progressive alteration producing a curved Arai plot (Figure 5c). The last three pTRM checks increasingly fail, but are acceptable at low-temperatures. Removing the last two data points reduces the curvature to an acceptable degree \( (k = 0.088) \), removing all three points affected by alteration further reduces the curvature \( (k = 0.045) \). Similarly, pTRM checks for sample LV30D1 (Figure 5d) fail at high temperatures. In this case \( \approx 15\% \) of the NRM is lost with nearly zero pTRM acquisition. The result of this alteration is to produce a high degree of curvature in a negative direction. If the last three points are removed the curvature is reduced to \(-0.112\).

A careful and detailed consideration of each individual sample may therefore improve the overall result of using \( k \) as a selection criterion. This analysis would require pTRM checks to test for alteration, and where alteration is evident at high-temperatures \( k \) could be recalculated. After the application of the \( \\
\delta C K \) and \( S S E \) criteria, of the 41 accurate results rejected by \( k \geq 24 \) would be included after a careful analysis, however 22 of the 53 inaccurate results rejected would also be included. The overall effect is a slight improvement to the result \( (\text{Mean} = 1.07, n = 120, \delta B_k (\%) = 15.06, n \text{ accurate} = 61, \text{Most deviant intensity} = 1.57) \). Additional selection criteria can be subsequently applied to improve this result. This approach, however, may be subjectively applied and care must be taken not to accept results affected by unidentified progressive alteration. Therefore, although strict, the case and less subjective nature of the blanket application of a \( k \) threshold may be preferred.

**6.6. Curvature of the Best Linear Fit**

The analysis of curvature proposed in this study involves all of the data points on the Arai plot and, as illustrated above, a number of factors may give rise to curvature from data with an accurate linear segment. An alternative approach would be to calculate the curvature of the best linear fit used to estimate the palaeointensity. Analysis of the synthetic sample data indicates that the curvature for the high-temperature segment is correlated with grain size and accuracy, \( R = 0.347, p = 0.033 \), and \( R = -0.577, p \ll 0.01 \), respectively. The curvature of the low-temperature segment, however, is uncorrelated with grain size, but correlated with accuracy, \( R = 0.316, p = 0.054 \), and \( R = 0.409, p = 0.011 \), respectively. It may be expected that this poor correlation is related to noisy data in some samples (e.g., samples MD and W6, Figures 2c and 2e, respectively). If samples with poor linear fits \( (\beta > 0.1) \) are excluded, curvature for the low-temperature segment remains uncorrelated with grain size and the correlation with accuracy weakens \( (R = 0.349, p = 0.032) \). If the same criterion is applied to the high-temperature linear fits the correlation of curvature with grain size increases \( (R = 0.480, p < 0.01) \), but the correlation with accuracy decreases \( (R = -0.445, p < 0.01) \). In both cases \( k \), determined from all the data, remains significantly correlated with grain size \( (R \geq 0.736, p \ll 0.01) \) and accuracy of both segments \( (|R| \geq 0.786, p \ll 0.01) \). The curvature determined from all data points therefore provides a better means of selecting palaeointensity data. Applying a \( k \) selection criterion based on all data points may be strict and exclude accurate results that are unaffected by MD influences (44 accurate results are rejected in the analysis in section 5). This is the case with most selection criteria \( (15–63 \text{ accurate results were rejected by the selection criteria investigated}) \) and in the case of \( k \) this approach is validated by the accuracy and low scatter of the result from the historical data set.

**6.7. A MD Model Comparison**

A number of models exist that can replicate MD-like palaeointensity behavior, including curved Arai plots [e.g., Fabian, 2001; Leonhardt et al., 2004b; Biggin, 2006]. Figure 7 is a comparison of curvature determined from the synthetic samples and that predicted by the phenomenological model of
Comparison of curvature from the synthetic samples and predicted by the phenomenological model of Biggin [2006]. Symbols are the same as in Figure 3a. Details of the parameters of Models P1 and P2 are outlined in the text.

Biggin [2006]. A full description of the model and model parameters is given by Biggin [2006] and will not be given here. Two models are compared to real data: Model 1 has a narrow blocking/unblocking range ($\alpha_3 = 0.01$) and the temperature steps are spaced to provide approximately evenly spaced NRM demagnetization steps, Model 2 has a slightly broader blocking/unblocking range ($\alpha_3 = 0.02$) and the temperature steps are evenly spaced, both models use ten temperature steps and mimick progressively more MD-like behavior by varying the $\lambda$ parameter from 0.01 to 0.5. The remaining model parameters are $\alpha_1 = 1$, $\alpha_2 = 0.8$, $\alpha_4 = 0$, and in both cases the laboratory field was applied parallel to the NRM. All best linear fits were determined using the first six points for the low-temperature segments and the last four points for the high-temperature segments (the average number of points used for the low-temperature fits for the synthetic samples is ~8, for the high-temperature fits it is ~5, Table 1). These two models predict envelopes of accuracy and curvature that encompass most of the experimental data (Figure 7) and other variations of model parameters can be used to predict intermediate behavior. This confirms that this phenomenological model can quantitatively predict some MD behavior.

7. Conclusions

Quantification of curvature on an Arai plot provides a means of assessing the influence of MD behavior on a paleointensity estimate. Curvature of paleointensity data is correlated with both grain size and the accuracy of the paleointensity estimate and provides a criterion with which to select data. This new criterion, $k$, presents a number of advantages over existing methods. Most notably, this check does not require additional experimental steps, which allows it to be applied to older data sets that may otherwise be discarded due to questionable reliability. A threshold value for $k (< 0.164)$ is defined independently of real data sets using data from samples with known grain sizes. When $k$ and two other selection criteria are applied to a real data set measured using the Coe paleointensity protocol the result is an improvement compared with that obtained from applying typically used paleointensity selection criteria. The application of the proposed criteria in combination with some previously used criteria can further improve the final result. As more data become available the threshold value for $k$ can be further refined and specific experiments can be undertaken to better characterize the possible variation of $k$ with different experimental protocols. Modern paleointensity studies should always include experimental checks for MD behavior, but more data from samples with known grain sizes are required to adequately constrain suitable threshold values for data selection. Based on the analysis undertaken in this study the use of the $k$ in modern paleointensity studies will prove beneficial and in situations where no other MD checks are available $k$ provides the only means of assessing the reliability of the data.

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References


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