Performance Analysis of Improved Energy Detection in Full Duplex Non-Time-Slotted Cognitive Radio

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Abstract—The spectral efficiency of cognitive radio (CR) can be improved by employing full-duplex (FD) systems which enables simultaneous data transmission and spectrum sensing during a given time period, over the same idle channel. Improved Energy Detection (IED) has shown a significant improvement compared to classical energy detection (CED) in half-duplex (HD). Time-slotted cognitive radio networks (CRN) are considered in most of the current works where an assumption is taken that the primary user (PU) and secondary user (SU) are perfectly synchronized. However, in real scenario PU can access and leave the frequency bands in an unsequenced manner and SU is not synchronized with the PU activity which is termed as non-time-slotted access. This paper investigates the IED scheme in full duplex cognitive radio (FDCR) for time-slotted as well as non-time-slotted access. The results demonstrate the significance of using IED scheme over existing methods, hence indicating remarkable improvement in the system performance. Based on the proposed scheme, simulation results show an agreement with analytical results, therefore validating the proposed scheme.

Index Terms—Spectrum Sensing, Full Duplex Cognitive Radio, Energy Detection, Improved Energy Detection, Non-Time-Slotted.

I. INTRODUCTION

With a rapid development in Wireless Communications Technology and the advent of 5G, there is a massive increase in subscription of wireless services which has caused an increasing demand for higher data rates and seamless connectivity. This rapid development has lead to the problem of spectrum becoming a limited commodity. However, analyses of usage of spectrum have exposed that substantial portions of the spectrum are not significantly exploited [1]. To solve this problem, spectrum allocation needs to be dynamic for efficient usage of the spectrum band. Dynamic Spectrum Access/Cognitive Radio (DSA/CR) systems are investigated to be a reliable approach to solve this problem [2]. DSA/CR system intents to increase the productivity of spectrum by granting the permission to secondary users to use licensed spectrum bands in a non-interfering manner which is being temporarily unused by the primary users [3].

Conventionally, Half-Duplex radios are used by the most of existing Cognitive Radio Networks (CRNs) to exploit the white spaces. To sense a channel, Half - Duplex Cognitive Radio (HDCR) has to follow a Listen-Before-Talk (LBT) protocol [4] where the channel needs to be sensed before a transmission can take place. However, HDCR has two critical drawbacks. First, as simultaneous transmission and reception is not possible it employs a time-slotted approach wherein one-time slot is utilized to sense the channel and in next time slot, it transmits to the secondary user. This approach may result into data loss. Moreover, to avoid imperfect sensing, HDCR spends a significant amount of its time for robust sensing. Second, HDCR utilizes two different channels, one for data transmission and another for reception which requires more spectral resources. FD systems facilitate to sense and transmit simultaneously in the given time-slot over the same idle channel, as a result, FD antennas will cause a self-interference into the system. However, recent advancements in self-interference reduction techniques [5] - [7] have made it possible to develop the FD concept in practice. Using FD concept in CR one can improve the spectral efficiency of the secondary network.

The spectrum sensing in FDCR using Energy Detection (ED) has been explored in [8]. Improved Energy Detection (IED) algorithm which outperforms Classical Energy Detection (CED) for HD is proposed in [9] , which owes its performance by considering of average test statistics, as well as test statistics of the previous event rather than just considering instantaneous test statistics like Energy Detection. However, performance analysis of IED in FDCR still remains unexplored. Time-slotted CRNs are considered in most of the existing works, where the considered assumption is that PUs and SUs are synchronized [10] - [14]. PU’s state changes only at the onset of a new sensing time-slot in time-slotted CRN as shown in Fig. 1(a). Therefore, it becomes easy for the SU to begin transmission in intial short duration if the sensed channel is found idle. In [10], authors have taken in consideration the SUs choice to detect and get access to channels utilizing a Markov decision process. In [15] authors have analyzed the performance of non-time slotted CRN in a non-cooperative scenario using ED. While, ED in non-time-slotted FDCR under cooperative sensing scenario was proposed in [16] and [17]. Additionally, Some work on MAC protocol in FD non-time slotted has been done in [19] and
Finally, paper is concluded in Section V. is assessed and compared with energy detection in Section IV. Section III. The analytical and simulation performance of IED detection probability for IED in non-time-slotted is obtained in slotted FDCR as well as the new analytical expression of the slotted FDCR. The performance analysis of IED in demonstrate the system model for time slotted and non-time-slotted FDCR under a cooperative scenario with residual self-interference is obtained.

The rest of the paper is ordered as follows. Section II demonstrate the system model for time slotted and non-time slotted FDCR. The performance analysis of IED in time-slotted FDCR as well as the new analytical expression of the detection probability for IED in non-time-slotted is obtained in Section III. The analytical and simulation performance of IED is assessed and compared with energy detection in Section IV. Finally, paper is concluded in Section V.

II. SYSTEM MODEL

A. Time-Slotted FDCR

In time-slotted FDCR networks, it is assumed that the networks of PU and SU is perfectly synchronized. The SU sensing span is divided into four time-slot where in time-slot 1 and 4 the PU is in ON state while in time-slot 2 and 3 the PU is in OFF state as shown in Fig. 1(a). As the PU state remains unchanged in a particular given time-slot it is simple for SU to sense the channel and if PU signal is found absent, SU can begin the transmission. A two-antenna full-duplex secondary user (SU) is assumed during simultaneous sensing and data transmission as shown in Fig. 1(b). Here the self-interference cancellation is assumed to be imperfect due to residual distortion.

B. Non-Time-Slotted FDCR

As mentioned above, in the time-slotted scenario PU and SU is assumed to be synchronized, yet the PU can connect and leave the licensed bands in an arbitrary manner within a periodic spectrum sensing frame and the SU might be unaware of definite time frame slot of the PU transmission. This phenomenon is quoted as non-time-slotted CRNs in literature. The studied non-time-slotted scenario is conferred in Fig. 2(a), where the activity of PU can shift at any time instant within the SU spectrum sensing frame. The precise modeling of the PU activity is a key challenging condition of CRN. Here, the

Fig. 1: (a) Time slotted model in which the PU’s state changes only at the onset of the new sensing frame, and (b) Two-antenna full duplex secondary user.

[20]. Also, [21] describes about use of improved energy detector in cognitive radio which is different than improved energy detection algorithm presented in this paper. In this paper, performance analysis of IED in FDCR for time-slotted as well as non-time-slotted case is presented. The analytical expression of the probability of detection for IED in non-time-slotted FDCR under a cooperative scenario with residual self-interference is obtained.

OFF and ON represents the states where the PU signal is absent and present respectively. The primary activity is assumed to follow continuous-time Markov model in existing works of literature. However, a discrete distribution is followed by the alternating ON and OFF periods [18]. So a discrete-time Markov process (ON - OFF) with parameter δ and μ is taken by which the primary activity is modeled. A geometrical distribution is considered for modelling the length of the ON and OFF duration’s with the mean length \( L_{ON} = \frac{1}{\mu} \) and \( L_{OFF} = \frac{1}{\delta} \). Here, the probability of the PU being ON is represented by \( P_{ON} = \frac{\delta}{\delta + \mu} \) and similarly when the PU is in the OFF state is given by \( P_{OFF} = \frac{\mu}{\delta + \mu} \). According to primary user activity, the two states which occur during the sensing can be labeled as Steady State(SS) and Unsteady State(US). If the samples obtained by sensing the spectrum are from the identical state of the PU, than it is in SS while in the US the beginning sensing samples of PU are from one state and the rest of the samples are from the another state of the PU. As shown in Fig. 2(a), time-slot 2 and 4 are in steady state while time-slot 1 and 3 are in unsteady state. In SU sensing time slots there are samples from alternate states, so two random variables \( A \) and \( B \) are defined where \( A \) characterize the randomness of OFF state in US under hypothesis \( H_0 \) and similarly, \( B \) describes the randomness of ON state in US under hypothesis \( H_1 \). \( A \) and \( B \) follow the probability mass function as given in [18], \( P_A(a) \mid \mu_0 = \frac{1}{N-a+1} \) and \( P_B(b) \mid \mu_1 = \frac{1}{N-b+1} \). The probability of steady state given \( H_0 \) and \( H_1 \) are: [17]:

\[
P_{SS \mid H_0} = \frac{1}{1 + \delta \sum_{a=1}^{N}(1 - \mu)^{(N-a-1)}(1 - \delta)(a-N)}
\]

\[
P_{SS \mid H_1} = \frac{1}{1 + \mu \sum_{b=1}^{N}(1 - \delta)^{(N-b-1)}(1 - \mu)(b-N)}
\]
If the mean length $L_{ON}$ and $L_{OFF}$ are equal. The steady state probabilities changes to:

$$P_{SS|H_0} = \frac{1}{1 + (N - 1)\delta}$$  \hspace{1cm} (3)

$$P_{SS|H_1} = \frac{1}{1 + (N - 1)\mu}$$  \hspace{1cm} (4)

The probability of unsteady-state duration given $H_0$ is $P_{US|H_0} = (1 - P_{SS|H_0})$ and under $H_1$ is $P_{US|H_1} = (1 - P_{SS|H_1})$.

III. PERFORMANCE ANALYSIS OF IED IN FDCR

A. Time-Slotted

A conventional cognitive spectrum sensing system has been considered that tests a binary hypothesis for HDCR depending upon the presence of Primary User (PU) in a particular channel. The hypothesis at the receiver can be formulated as below:

$$y(n) = \begin{cases} 
  d(n) + w(n) & H_0 \\
  h_p x(n) + d(n) + w(n) & H_1
\end{cases}$$

The hypothesis $H_0$ refer to the case when PU is absent and $H_1$ refer to the case when PU is present. Here $n = 1, 2, 3, ..., N$ be $N$ samples of the transmitted signal, $y(n)$ represents the received signal at $n^{th}$ instant, $x(n)$ represents the transmitted signal, $h_p$ is the channel coefficient between the PU and SU sensing antenna, $d(n)$ represents residual distortion and $w(n)$ indicates additive Gaussian thermal noise with variance $\mathcal{E}\{w(n)^2\} \triangleq \sigma_w^2$. Furthermore, $\mathcal{E}\{d(n)^2\} \triangleq \gamma_{12}\sigma_w^2$, where $\gamma_{12}$ is the effective Interference-to-Noise Ratio (INR) after cancellation and Signal-to-Noise Ratio (SNR) can be given as $\gamma_{FD} \triangleq h_p^2 E_s/\sigma_w^2$. The overall receiver SNR can be expressed as $\gamma_{FD} = \frac{\gamma_{2}}{\gamma_{FD}\gamma_{T}}$. The test statistics can be used to compute probability of false alarm ($P_f$) and probability of detection ($P_d$).

$$T_i(y_i) = \frac{2}{N_o} \sum_{n=1}^{N} |y_i[n]|^2$$  \hspace{1cm} (5)

where $T_i(y_i)$ is the test statistics computed in the i-th sensing event and is then compared to a pre-defined energy threshold $\lambda$ and decision is taken, $N_o$ indicates the power spectral density of self-interference plus noise. The test statistic of IED owe its performance utility by taking into consideration the test statistic of instantaneous events, test statistic of previous event and the average test statistic of past L sensing events while M represents the total sensing events where the primary signal was really present. The Probability of Detection for IED as a function of SNR can be expressed as (6) as given in [9]:

$$P_d^{IED} = P_d^{CED} + P_d^{CED} \ast (1 - P_d^{CED}) \ast \varepsilon(\gamma_{FD})$$  \hspace{1cm} (6)

where

$$P_d^{CED} = Q\left(\frac{Q^{-1}(P_{fa})\sqrt{2N - N\gamma_{FD}}}{\sqrt{2N(1 + \gamma_{FD})}}\right)$$  \hspace{1cm} (7)

$$\varepsilon(\gamma_{FD}) = Q\left(\frac{Q^{-1}(P_{fa})\sqrt{2N - \frac{MN\gamma_{FP}}{L}}}{\sqrt{\frac{2N}{L}(1 + \frac{M}{L}(1 + \gamma_{FD})^2 - 1)}}\right)$$  \hspace{1cm} (8)

While in low SNR regime the equation can be simplified to:

$$P_d^{CED} = Q\left(\frac{Q^{-1}(P_{fa})}{\sqrt{N/2\gamma_{FD}}}\right)$$  \hspace{1cm} (9)

$$\varepsilon(\gamma_{FD}) = Q\left(\frac{Q^{-1}(P_{fa})\sqrt{N/2\gamma_{FD}}}{\sqrt{2N}}\right)$$  \hspace{1cm} (10)

which involves the Q-function that is defined by $Q(x) = \frac{1}{\sqrt{2\pi}}\int_{x}^{\infty} \exp\left(-\frac{t^2}{2}\right)dt$. Here, the test statistic differs from that of HD and the effective receiver SNR ($\gamma_{FD}$) also changes.

B. Non-Time-Slotted

A binary hypothesis can be formed for co-operative spectrum sensing in non-time-slotted scenario where hypothesis $H_0$ and $H_1$ refer to case where the primary channel is in the OFF and ON state respectively. The hypothesis at the $k^{th}$ SU receiver can be given as below:

$$y(n) = \begin{cases} 
  d_k(n) + w_k(n) & H_0 \\
  h_p x_k(n) + d_k(n) + w_k(n) & H_1
\end{cases}$$

where $x_k(n) \sim \mathcal{N}(0, \sigma_z^2)$ is the PT signal sample, $h_p \sim \mathcal{CN}(0, \sigma_{h_p}^2)$ denotes the coefficient of channel between the
\[ P_{f_k} = P_{SS|n_k} Q \left( \frac{\sqrt{N}}{\gamma_k + 1} \left( \frac{\lambda}{\sigma_{n_k}^2} - \gamma_k - 1 \right) \right) + \frac{P_{US|n_k}}{N-1} \sum_{a=1}^{N-1} Q \left( \frac{\frac{N\lambda}{\sigma_{n_k}^2} - (N-a)\gamma_k - N\gamma_i - N}{\sqrt{(N-a)(\gamma_k + 1)^2 + a(\gamma_k + 1)^2}} \right) \]  
\[ P_{d_k} = P_{SS|h_k} Q \left( \frac{\sqrt{N}}{\gamma_{sk} + 1} \left( \frac{\lambda}{\sigma_{n_k}^2} - \gamma_{sk} - \gamma_k - 1 \right) \right) + \frac{P_{US|h_k}}{N-1} \sum_{b=1}^{N-1} Q \left( \frac{\frac{N\lambda}{\sigma_{n_k}^2} - b\gamma_k - N\gamma_i - N}{\sqrt{b(\gamma_k + 1)^2 + (N-b)(\gamma_k + 1)^2}} \right) \]  

\[ P_d^{IED} = P_\{T_i(y_i) > \lambda \} H_i + P_\{T_i(y_i) \leq \lambda \} H_i \ast P_\{T^{avg}_i(y_i) > \lambda \} H_i \ast P_\{T_{i-1}(y_i-1) > \lambda \} H_i \]  
\[ P_d^{CED} = P_d^{CED} + P_d^{CED} \ast (1 - P_d^{CED}) \ast \xi \]  

\[ \xi = P_\{T_i^{avg}(y_i) > \lambda \} H_i = Q \left( \frac{\lambda - \mu_{avg}}{\sigma_{avg}} \right) \]  

\[ Var[\|y_k(n)\|^2_{OF}] = E[\|d_k(n) + w_k(n)\|^2] = \lambda^2 \sigma_{x_k}^2 + \sigma_{n_k}^2 \]  
\[ E[\|y_k(n)\|^2_{ON}] = E[\|h_{pk}x_k(n) + d_k(n) + w_k(n)\|^2] = \sigma_{n_k}^2 \gamma_k + \gamma_{sk} + 1 \]  
\[ Var[\|y_k(n)\|^2_{OF}] = E[\|d_k(n) + w_k(n)\|^2] = \sigma_{n_k}^4 (\gamma_k + 1)^2 \]  

\[ Var[\|y_k(n)\|^2_{ON}] = E[\|h_{pk}x_k(n) + d_k(n) + w_k(n)\|^2] = \sigma_{n_k}^4 (\gamma_k + 1)^2 \]  

\[ \mu_{avg} = \frac{M}{L} E[\|y_k(n)\|^2_{OF}] + \frac{L - M}{L} E[\|y_k(n)\|^2_{ON}] \]  

\[ \mu_{avg} = \sigma_{n_k}^2 \left( \frac{M}{L} (\gamma_k + \gamma_{sk} + 1) + \frac{L - M}{L} (\gamma_k + 1) \right) \]  

\[ \sigma_{avg}^2 = \sigma_{n_k}^2 \left( \frac{M}{L} (\gamma_k + \gamma_{sk} + 1)^2 + \frac{L - M}{L^2} (\gamma_k + 1)^2 \right) \]  

\[ \xi = Q \left( \frac{\lambda - \sigma_{n_k}^2 \left( \frac{M}{L} (\gamma_k + \gamma_{sk} + 1) + \frac{L - M}{L} (\gamma_k + 1) \right)}{\sqrt{\sigma_{n_k}^2 \left( \frac{M}{L} (\gamma_k + \gamma_{sk} + 1)^2 + \frac{L - M}{L^2} (\gamma_k + 1)^2 \right)}} \right) \]  

\[ \xi = Q \left( \frac{L \left( \frac{\lambda}{\sigma_{n_k}^2} - \gamma_k - 1 \right) - M \gamma_{pk}}{\sqrt{M(\gamma_{pk} + \gamma_{sk} + 1)^2 + (L - M)(\gamma_k + 1)^2}} \right) \]  

IV. NUMERICAL RESULTS

In this section, we illustrate the performance of Improved Energy Detection in time-slotted as well as non-time-slotted FDCR with Monte Carlo Simulations. A system with a PU transceiver and a SU FDCR transceiver is taken into consideration. The results are obtained for the simultaneous transmission and sensing case in FD while the sensing-only case is not considered as it is essentially same as the conventional HDCR. The number of iterations for Monte Carlo simulations is taken
to be 1,00,000 and The decision threshold is set to meet the target probability of false alarm. The number of secondary users for co-operative scenario in non-time-slotted case is taken as 10 and OR rule is used at the fusion center. The results are obtained by assuming that both primary transmitter and secondary user transmit a Gaussian signal. The performance of ED and IED is assessed by demonstrating the complementary Receiver Operating Characteristics (ROCs) \( P_m = 1 - P_d \) versus \( P_f \).

Fig. 3 shows the complementary ROCs \( P_m \) vs \( P_f \) depicting comparison in performance of ED and IED in time-slotted scenario for SNR \( \gamma_2 = \{4, 6\} \) dB. Here, number of samples \( N = 5 \), interference-to-noise ratio \( \gamma_{12} = 6 \) dB, number of previous sensing events \( L = 3 \) and number of sensing events where a primary signal is actually present \( M = 1 \) are the tuned parameters. As clearly evident, IED outperforms ED sensing technique and a significant reduction in probability of miss detection can be observed. For example, at \( \gamma_2 = 4 \) dB and \( P_f = 0.4 \), the probabilities of missed detection for IED and ED are 0.0841 and 0.2782 respectively.

Fig. 4 shows the analytical and simulation curves of ED and IED in time-slotted case for probability of miss detection in terms of INR \( \gamma_2 = \{4, 5, 6\} \) dB, \( L = 3, M = 1 \).
in terms of residual self-interference levels at two different values of SNR ($\gamma_2$ = 4dB and 6dB with $N = 5, L = 3, M = 1$ and probability of false alarm kept constant as 0.05. It can be inferred by looking at the two extremes that full-duplex sensing could be considered ideal if the INR ($\gamma_{I2}$) was suppressed below -15 dB, while detection is almost impossible if the INR is above 15dB.

Fig.5 represents the standard ROC curves for ED and IED scheme in non-time-slotted scenario with two different values of self interference suppression factor ($X_i = \{0.3, 0.9\}$). Here, number of samples ($N = 1000$), average SNR ($\gamma_p$ = 10 dB), average INR ($\gamma_i$ = 0 dB), mean length for ON and OFF duration ($L_{ON} = L_{OFF} = 3000$), $M = 1$ and $L = 3$ are the considered parameters. For $X = 0.3$ and $P_f = 0.1$ the probability of detection for ED and IED is 0.4569 and 0.5788 respectively, which shows a significant improvement of IED as compared to ED. It can be observed that as the SIS factor decreases the sensing performance increases and vice versa.

Fig. 6 shows the simulation and analytical curve in non-time-slotted case for detection probability of IED as a function of SNR, for different values of INR = 0dB, 6dB and 9dB with $X = 0.3, L = 3, M = 1, N = 1000, L_{ON} = L_{OFF} = 2000$. The target probability of false alarm ($P_f$) for all the SUs is taken to be 0.01. It can be observed that with the increase in average INR, there is decrease in detection probability.

The comparison of computational cost of ED and IED can be analyzed as follows. The computation of test statistics which is common in both method requires N multiplication operation and $(N - 1)$ addition operations. In IED method the extra computation is to calculate average test statistics which requires the last $(L - 1)$ test statistics values to be stored in memory and to compute $(L - 1)$ sums and one division. The increased computation cost of IED algorithm is negligible as compared to the maximum eigenvalue-based detection proposed in [17] for non-time-slotted which requires calculation of covariance matrix and singular value decomposition. Here, the fading channel is considered to be Rayleigh while other fading channels like Nakagami-m and $\kappa - \mu$ can be considered for future work. Also the primary user activity in non-time-slotted FDCR is modelled by discrete geometric distribution while other distribution like discrete Pareto can also be considered for the further work.

V. CONCLUSION

This paper studied Improved Energy Detection algorithm for spectrum sensing in full-duplex cognitive radio. Moreover, in this work, the performance of IED in spectrum sensing considering the impact of primary user activity in cooperative FD-CRNs has been analyzed. Here, each SU has the capability to sense and transmit simultaneously. The analytical expression for IED has also been derived for full-duplex cognitive radio in non-time-slotted scenario. It is demonstrated in the obtained analytical and simulation results that IED outperforms the well-known energy detection in time slotted as well as non-time slotted scenario when sensing the PU channel during SU transmission.