Evaluation of Buses in Power Grids by Extended Entropic Degree

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Abstract: Complex network theory has been considered valuable in analyzing the security of networked infrastructures including power grids. However, the traditional pure topological assessment metrics ignore many characteristics in electrical engineering. The most popular metrics for evaluation of nodes or buses in power grids are degree and betweenness. By integration of specific knowledge of electrical engineering, these metrics have been updated to entropic degree and e-betwenness in former studies. However, with consideration of local and global structural features, these two all have defects respectively. This paper made integration of entropic degree and e-betweenness to develop a new concept as extended entropic degree by considering the e-betweenness of line as the weight in entropic degree. The new extended entropic degree can reflect both local and global structural features in evaluation of buses in power grids. This extended entropy degree was tested to verify the effectiveness in IEEE-30, IEEE-118, IEEE-300 and a United Kingdom distribution power grids. The results indicated that this new metric is more reasonable and effective in discovering structural features of buses in power grids.

Key Words: Complex networks; Power system security; Power grid; Degree; Entropic degree; Line Betweenness

1 Introduction

The electric power grid is indispensable in today's society. The reliability and availability of the electric power grid have significant impact for every country's economy and security. The functions in infrastructural systems, such as industrial, commercial, residential sectors and essential services, will be affected by outages of electrical power grid [1].

Throughout the large-scale power outages in recent 20 years, almost all of outages evolved from the initial local failure into a cascading failure [2]. The process of these failures is series, random and unpredictable, which indicated the modern large scale power system's vulnerability. The traditional power grid security analysis method is mainly based on reductionism and uncertainty principle. The simulation analysis about the grids is carried out by means of the mathematical model of structural element and system differential equations [3], which are difficult to explain the large-scale power outage reasonably [3]. With the deepening of research, engineer gradually realized that the large-scale power outage is inseparable with the inherent structure of power grid.

In order to assess the power infrastructure vulnerabilities, power systems engineers became interested in the complex network approach [4]. Complex network theory is proposed to explain the origin of the power grid blackout, and evaluate the vulnerability of the grid structure and the weak link [4]. For example, the concept of degree and betweenness have been applied to the European Union power grid, and the engineer obtained some achievement [5]. However, the evaluation indexes in most of the existing research are based on the pure topological metrics and graph theory, which is not only used for the electrical power grids, but also applied to many real-world complex networks such as topology of food webs, cellular and metabolic networks, the World-Wide Web, the Internet backbone, telephone call graphs [6]. These researches disregard the specific physical and operational features of power grids, therefore the analysis results might be far from the reality in power systems.

To overcome the defects of pure topological approaches in evaluating power grids, especially evaluation of buses in transmission networks, degree and betweenness have been updated to entropic degree and e-betweenness [1][4] by considering the power transmission capacity, power transfer distribution factors (PTDF) and entropic distribution. However, with consideration of local and global structural features, these two all have defects respectively.

This paper made integration of entropic degree and e-betweenness to develop a new concept as extended entropic degree by considering the e-betweenness of line as the weight in entropic degree. The new extended entropic degree can reflect both local and global structural features in evaluation of buses in power grids. This extended entropy degree was tested to verify the effectiveness in IEEE-30, IEEE-118, IEEE-300 and a United Kingdom distribution power grids. The results indicated that this new metric is more reasonable and effective in discovering structural features of buses in power grids.

In this paper, section 2 will make a general review for the development of concepts related to degree and betweenness. In section 3, based on discussion of defects in former definitions, an extended entropic degree will be proposed. In section 4, the extended entropic degree will be tested in IEEE-30, IEEE-118, IEEE-300 and a United Kingdom distribution power grid to verify the rationality of these new metric. Conclusion is summarized in section 5.

2 Degree and Betweenness in Complex networks

If each element of the system is abstracted as a node, and the relationship between elements is used as connecting edge, this system can constitute a network. A network whose nodes and edges are numerous may be called complex networks (CNs), such as power networks, cellular

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networks, language networks and so on [7]. Complex network theory is based on the opinion that function may depend on or be influenced by structure, and is to analyze the performance of system from the perspective of network topology. This theory is based on graph theory and statistical physics to study the physical structure of complex systems and structural stability [7].

The origin of the complex networks can retrospect from the "Seven Bridges Problem" which is proposed by Leonhard Euler in 18th century [8]. The classical graph theory can be only used to some regular topological structure such as Euclidean lattice [9]. In late 1950s, two mathematicians, Erdös and Rényi (ER) described a network with complex topology by a random graph theory [9]. There are some limitation in ER random graph model, which cannot provide a reasonable explanation for some evolutionary character such as the process of epidemic, Matthew effect of social wealth and so on [10]. In 1998, in order to describe the transition from a regular lattice to a random graph, Watts and Strogatz (WS) introduced the concept of small-world network which is the so called "six degrees of separation" principle [11]. Then, Barabási and Albert (BA) discovery in the field of many large-scale complex networks are scale-free in 1999 [12]. The discovery of the small-world effect and scale-free feature of complex networks break through the simple regular network and random network model, which revealed the relationship between the collective dynamics and structural feature of complex networks [10].

In these works, degree and betweenness have been considered as important metrics and tools for analyzing the structural features of complex networks. Power grids are widely acknowledged as CNs because of their massive size and the complex interactions existing among individual components [15]. Several researchers have analyzed structural vulnerabilities in the UCTE network and the North American power grid using topological features such as distance, degree and betweenness [16, 17].

In a purely topological model, a power grid is considered to be a network composed of vertices (buses) connected by edges (transmission lines) [4]. The connectivity of a node is traditionally measured by its degree in an unweighted topological and undirected network model, the degree of a vertex i is the number of edges connected to it:

$$k_i = \sum_j m_{ij} \tag{1}$$

Where m_{ij} represent the number of lines connecting i and j.

In a weighted network model, connectivity can also be express by the strength measure as the sum of the weights of the corresponding edges:

$$s_i = \sum_j w_{ij} \tag{2}$$

Where w_{ij} represent the number of lines connecting i and j.

Some researchers [18-21] indicated that in the network the majority of the nodes have low degrees, but there is a continuous hierarchy of high-degree nodes (hubs) which play an important role in the system. They are vulnerable to attacks targeting the high-degree hubs [18]. A node with a large degree is immediately recognized as a major channel of communication, being very visible since it is in direct contact with many other nodes [19].

One example is shown in [18], the result explain that some nodes are less important than the other nodes, since these are less connected (small k), the nodes that apparently are the most important because they have largest number of connections [18].

As well as k, an additional property to be considered is the degree distribution $P(k)$ [20]. The degree distribution of these networks follows a power law $P(k) \sim k^{-\gamma}$ with the exponent γ mostly between 2 and 3 [21]. The degree distribution of the European power grids analyzed are exponential, which means that they are not like the highly skewed scale-free distributions typically found in other complex networks [20].

However, a purely topological degree has several shortcomings that make its application to a power grid problematic. From the point of view of power systems engineering, all physical components of networks (buses and lines) have specific qualitative and quantitative attributes [4]. The characteristics of electrical power grids are disregarded in the pure topological concepts and metrics, which results might be far from the reality in power systems.

2.1 Entropic degree

In a weighted network model, the measurement of the connectivity should reflect three factors: (i) the total strength of connectivity of the edges; (ii) the number of edges connected with the vertex; and (iii) the distribution of total weights among the edges [4]. It is obvious that the definition in Eq. (1) cannot reflect the first factor and the definition in Eq. (2) loses information regarding the second factor. The third factor cannot be reflected by the both equation.

In Figure 1, the results obtained using Eqs, (1) and (2) are different:

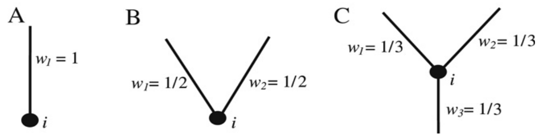

Fig. 1: Same total weight with different connections [4]

$\boldsymbol{k_i^A = 1}$; $\boldsymbol{k_i^B = 2}$; $\boldsymbol{k_i^C = 3}$;

$\boldsymbol{s_i^A = s_i^B = s_i^C = 1}$.

In Figure 2, the results obtained using Eq. (1) and (2) are all same for both cases:

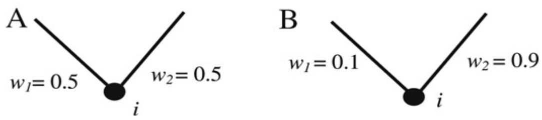

Fig. 2: Different distribution of weight [4]

$k_i^A = k_i^B = 2$; $s_i^A = s_i^B = 1$.

However, for Case A, the weight of two edges is half and half, therefore both edge have the same extent of importance for the node. On the other hand, for Case B, one edge which weight takes 90% is more important than the other. If choose one important edge to attack in the power grid, Case B is more vulnerable than Case A, due to it is effortless to lose their connectivity [4].

In order to reflect all the three factors mentioned above, the engineer uses the concept of entropy to redefine the degree, the line capacity is used as the weight. First, the weight of the edge between vertices i and j is normalized as p_{ij}.

$$p_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} \quad (3)$$

Since $\sum_j p_{ij} = 1$, the entropic degree g_i of i can be defined using entropy as:

$$g_i = \left(1 - \sum_j p_{ij} \log p_{ij}\right) \sum_j w_{ij} \quad (4)$$

The entropic degree is used to identify the connectivity and importance of the bus in the electrical power grids. The higher entropy degree means that the uncertainty of this bus is higher. This concept of entropy degree is applied to the Italian transmission network, where the power-flow limit is considered to be the weight of the transmission lines. There is huge difference about the result between degree and entropy degree. The reason is the entropic degree consider the transition capacity and distributions, the purely topological model cannot reflect these difference [21].

2.2 Electric betweenness (e-betweenness)

In the topology properties of complex networks, the betweenness have been used to identify the critical components in many researches. The definition of betweenness is the sum of the probability for a vertex or edge to belong to a randomly selected geodesic path linking any other pair of vertices [22]. The betweennes of a vertex or an edge can be represented as:

$$B(v) = \sum_i^{N_B} \sum_j^{N_B} \frac{\sigma_{ij}(v)}{\sigma_{ij}}, \quad i \neq j \neq v \epsilon B \quad (5)$$

$$B(l) = \sum_i^{N_B} \sum_j^{N_B} \frac{\sigma_{ij}(l)}{\sigma_{ij}}, \quad l \epsilon L, i \neq j \epsilon B \quad (6)$$

Where $\sigma_{ij}(v)$ and $\sigma_{ij}(l)$ are respectively the number of the geodesic paths between vertices i and j that pass through a vertex v and edge l.

A higher value of the betweenness of a component means a greater number of geodesic paths through the component and that implies a higher criticality of the component. Therefore, the ranking of the betweenness value of the component can be utilized to identify the critical components of a network [1].

However, the concrete engineering characteristics of electrical power grids is disregarded in pure topological concepts and metrics [21]. Therefore, the analysis results from the straight application of the theory in power grids might be far from the reality in power system. Consider about this issue, the engineer redefined the betweenness to electric betweenness. According to the specific features of power grids, the bus electric betweenness is redefined as [21]:

$$\mathcal{T}(v) = \frac{1}{2} \sum_{g \epsilon G} \sum_{d \epsilon D} C_g^d \sum_{l \epsilon L} \left|f_l^{gd}\right| \quad v \neq g \neq d \epsilon B \quad (7)$$

The line electric betweenness is redefined as:

$$\mathcal{T}(l) = max[\mathcal{T}^p(l), \mathcal{T}^n(l)], \quad l \epsilon L \quad (8)$$

Where $\mathcal{T}^p(l)$ and $\mathcal{T}^n(l)$ represent respectively the positive electric betweenness and the negative electric betweenness of the l:

$$\mathcal{T}^p(l) = \sum_{g \epsilon G} \sum_{d \epsilon D} C_g^d f_l^{gd}, \quad if\ f_l^{gd} > 0$$

$$\mathcal{T}^n(l) = \sum_{g \epsilon G} \sum_{d \epsilon D} C_g^d f_l^{gd}, \quad if\ f_l^{gd} < 0$$

Where f_l^{gd} is the Power Transfer Distribution Factor (PTDF) on line l when a unit of power injecting at generation bus g and withdrawing at load bus d; C_g^d is the power transmission capacity when power injection at bus g and withdrawing at load bus d, the definition is [21]:

$$C_g^d = min_{l \epsilon L} \frac{P_l^{max}}{\left|f_l^{gd}\right|} \quad (9)$$

Where L is the collection of the transmission line ; P_l^{max} is the maximum value of the transmission limit about the transmission line l.

2.3 Extended entropic degree

By considering the definitions discussed above, we may find following issues:

First, in definition of entropic degree, the general idea is to take into account the distribution of weights in connected links with a specific node. However, the weight of link was selected as the power flow capacity of the line to indicate the strength of connectivity. In fact, different quantitative physical features are related to a transmission line, such as power flow limit, impedance, and length and so on. It is critical to consider which one is appropriate as weight in a weighted network model. This depends on the particular analyzing purpose and specific characteristic of the targeted network. In general model of complex networks, in definition of distance, betweenness and efficiency, there is a common assumption that the transmission of physical quantity is always through the shortest path. This assumption can be further understood that transmission is controllable and the paths are selectable. With this precondition, we can conclude that the capacity of any line could be fully utilized if necessary. And taking the line capacity as the weight in definition of entropic degree can reasonably reflect the strength of connectivity for each line. However, this assumption cannot be accepted for power grids because the power transmission must follow specific physical rules and many different paths all contribute to the power transmission. The paths of transmission are not selectable and their contribution can be quantified by PTDF as discussed. Therefore, even some lines may have large capacities, it doesn't mean that their capacities may be fully utilized in real power transmission. So, to select power flow capacity as weight in definition of entropic degree may not reflect their real positions for the functionality of the network.

Second, the degree or entropic degree only considers the local connectivity of a specific node as a static metric. If the overall structure of network has been changed, degree or entropic degree may keep unchanged unless the connection of links of this node is directly changed. As a part of a large network, the importance and contribution of a node cannot only be determined by its local characteristic. The overall structure can definitely influence the relative positions of nodes in the whole network. Therefore, degree and entropic degree may not be a satisfactory metric to assess importance and contribution of nodes in a network.

Third, on the contrary, betwenness or e-betweenness of nodes make a global statistical evaluation according to the contribution of nodes to overall quantity transmission of the network. Any change in structure of network may influence this metric. However, these metrics do not consider the local characteristics of nodes. If two nodes have equal betweenness but with different number of links, their tolerances to attacks may be totally different.

To overcome the former defects discussed, the concept of line e-betweenness defined in Eq. 8 by introducing PTDF and power transmission capacity can be considered as weight in an extended entropic degree. The weight of the edge between vertices i and j is normalized:

$$w_{ij} = \frac{\mathcal{T}(l)_{ij}}{\sum_j \mathcal{T}(l)_{ij}} \qquad (10)$$

The extended entropic degree for power grids is redefined as:

$$e_i = \left(1 - \sum_j w_{ij} \log w_{ij}\right) \sum_j \mathcal{T}(l)_{ij} \qquad (11)$$

Corresponding to the three defects discussed, e-betweenness of line can indicate the contribution of a line in overall functionality of a power network as it takes into account the specific physical rules in power transmission. For the second issue, any change in overall network structure may change the e-betweennss of a specific line to reflect its global importance. For the third issue, by applying entropy, the distribution of e-betweenness in different links can be quantitatively assessed. Therefore, this new metric may make better contributions to evaluation of buses in power grids.

In order to further verify and analyze the new entropic degree, some real power grid models which are IEEE-30, IEEE-118, IEEE-300 and a United Kingdom distribution power grid are simulated by MATLAB. Meanwhile, the results of data are shown and analyzed in the following section.

3 Case study

In this section, the extended entropic degree is applied to the IEEE-30, IEEE-118, IEEE-300 and a United Kingdom distribution power grid systems to verify the rationality of these new metric. The evaluation indexes which are degree, strength, entropy degree and new entropy degree for power grids are calculated through MATLAB. Furthermore, these results are compared and analyzed, and some results are obtained and discussed in the following paragraphs.

3.1 Variation analysis

The IEEE 118 which represents a portion of the American Electric Power System (in the Midwestern US), is exemplified to compare and analyze. The contrast between old entropic degree and extended entropic degree is shown in below.

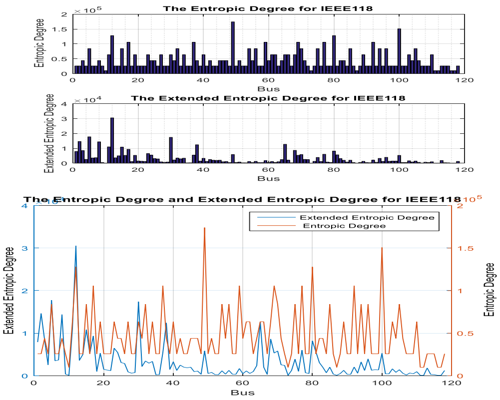

Fig. 3: The different between entropic degree and extended entropic degree in IEEE 118

Upon comparing the above two figures, the results demonstrate that the two types of entropic degree reveal a huge different result. The value of entropic degree is same in the nodes which are connected with equal number of branch, however, for the extended entropic degree the results existed a difference in these nodes. Meanwhile, the importance of some buses has changed for the extended entropic degree. The similar result is reflected in a United Kingdom distribution power grid UKGDS_01.

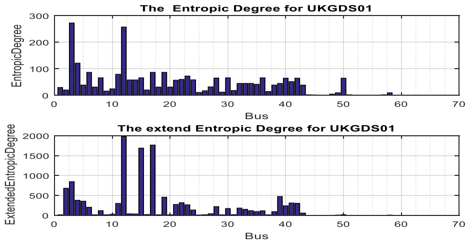

Fig. 4: The difference between entropic degree and extended entropic degree in UKGDS01

The reason is that the weight is various. According to the definition of two entropic degree in Eq. 4 and Eq. 11, the capacity and line betweenness are considered as the weight for two different entropic degree.

Table 1: Data of buses which are connected three branches in IEEE 118

From	To	Entropy degree	Extended Entropy degree	Line electric betweenness	Capacity
4	23	43871	5350	1924	9900
	70			889	
	72			959	
5	23		3045	9842	
	26			1070	
	27			164	
8	30		12253	4404	
	37			569	
	65			3988	

In this table, some nodes are chosen, which nodes are connected with same number of branches. It is clear to find that the value of the entropic degree is same for these nodes. Nevertheless, there are enormous variation in the extended entropic degree. The cause is that the capacity is equal for these nodes, but the value of line betweenness is different. To compare these two weights, the definition of line betweenness in Eq. 8 shown that the PTDF and power transmission capacity are considered. The PTDF is calculated by the admittance matrix which reflects the power grid structure, and the transmission capacity indicates the functional characteristics. Therefore, the extended entropic degree reflects the structural and functional characteristics of the power grids. This extended index can better reflect the importance and contribution of each bus, which is better than the entropic degree. For resilience of power grid, the extended entropic degree has the ability to prepare and prevent the power grid before the system encounters a disturbance event.

3.2 Branch cut test

Theoretically, in accordance with the definition of two kinds of entropic degree in Eq. 4 and Eq. 11, when the branch is cutting the entropic degree will just be changed in the buses which are connected with the severed branch. However, the value of extended entropic degree will be changed for not only the buses which are related with the severed branch, but also some other buses, which may not be connected with the amputating circuits.

The IEEE 30 bus test case is chosen with the purpose of verifying this conjecture. The IEEE 30 bus test system is similar with the IEEE 118 system which represents a portion of the American Electric Power System. Firstly, one branch is cut which is the branch 9-10. The result is shown in Figure 5.

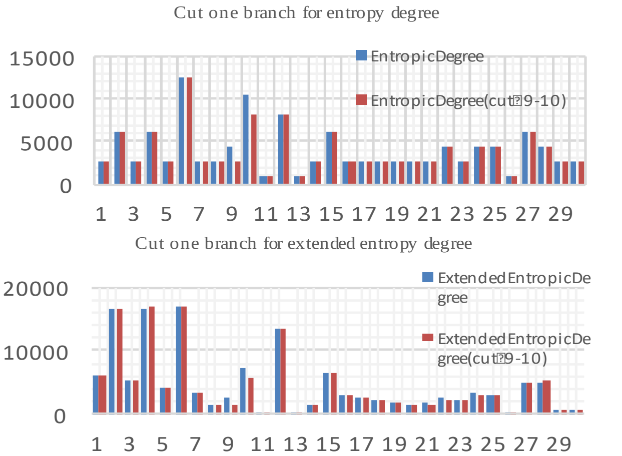

Fig. 5: Cut one branch for different entropic degree

In the above figure, it is clear to find that the value of entropic degree just changed in the bus 9 and 10 which is connected with the branch 9-10. Nevertheless, for the extended entropic degree, the value not only changed in the bus 9 and 10, but also changed in the other buses which are not connected with the bus 9 and 10, which conforms to the conjecture. In order to further illustration, two branches and three branches are severed. Through comparing and analysing the results it is obvious to obtain that the entropic degree just change in the buses which are connected with the cut branches, for the extended entropic degree, the value not only changed in these directly connected buses, but also the other buses also changed which are not connected with these branches. The extended entropic degree incarnates the influence of the structural changes for the power grid system, however, the entropic degree cannot reasonably reflect this point. This result illustrates that the extended entropic degree is better than the entropic degree to reflect the structural characteristic of electrical power grid. This paper only change the structure of the grid to simulate the power grid failure, the extended entropic degree of bus is higher indicates that this bus is easy to be attacked in cascading failure. The cascading failure models will be focused on in the future work.

4 Comparing with degree and degree distribution

4.1 Degree and Extended entropic degree

Traditionally, the degree reflects the importance of the nodes in the pure topology of complex networks. This metric disregarded the characteristics of electrical power grids, which cannot be accepted by the electrical engineer. Therefore, the engineer made improvements for degree, the entropic degree is proposed which considered the transition capacity and distributions. The Figure 6 showed the difference which is about degree and old entropic degree in IEEE118 test system. It is clear to find that there is a little discrepancy between each bus for degree and the value of old entropic degree.

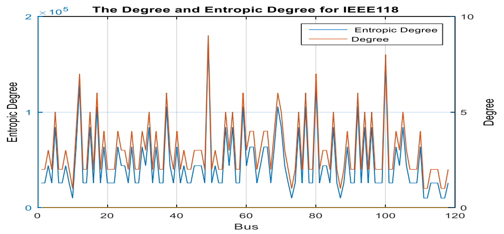

Fig. 6: The difference between degree and old entropy degree

In above figure, the trend of these two metrics is similar. In other words, it means that the result of the identification of critical bus is similar for the entropic degree and degree. As the improvement for degree, the entropic degree only indicated that the important bus is more and more important. This difference is worth pondering, then the extended entropic degree is compared with the degree. The result is shown as below.

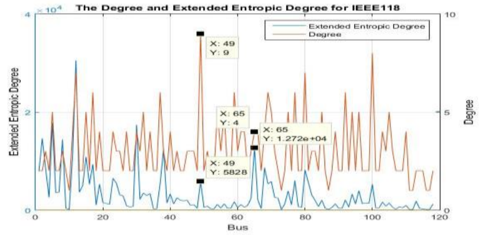

Fig. 7: The difference between degree and extended entropic degree

In Figure 7, a huge difference is obtained between degree and extended entropic degree. The importance of some buses has changed. In order to compare the difference between degree and extended entropic degree, the bus 49 and 65 are chosen as an example. The degree of bus 45 is larger than bus 65, which means the importance of bus 45 is higher than 65. However, the extended entropic degree of 45 is smaller than bus 65, therefore the importance has changed for these two buses. The reason is the line electric betweenness of bus 65 is great than bus 45. The line electric betweenness, as weight, reflects the responsibility of branch. Higher line electric betweenness signifies that the responsibility of branch is higher. In consequence, the importance of the bus 65 is higher than bus 45. Through comparing, the extended entropic degree which is used to measure the importance of bus, is more rational than degree.

Similar results have occurred in the IEEE 30 test system. Obviously, that is because the extended entropic degree considers the power grid structure and the functional characteristics of transmission capacity. The extended entropic degree better reflects the difference and importance between each bus.

4.2 Entropic degree distribution

The degree distribution $p(k)$ is an additional property on the complex network to identify the scale-free [20]. The cumulative degree distribution of nodes is $P_{cum}(K)$. That indicates the percentage that the number of nodes whose degree greater than or equal K, accounting for the total nodes [23]. The cumulative probability of the node degrees is defined as,

$$P_{cum}(K) = \sum_{k>K} p(k) \qquad (12)$$

In the previous research [24], degree distribution follows a power-law $p(k) \sim k^{-\gamma}$, however, cumulative degree distribution is an exponential distribution for the power grid of the western United States.

Through the MATLAB, the function of cumulative extended entropic degree distribution is calculated by curve fitting, and each equation is

IEEE 30:

$$P_{cum}(ED) = 1.159 * \exp(-0.0002494 * ED)$$

IEEE 118:

$$P_{cum}(ED) = 1.195 * \exp(-0.0003478 * ED)$$

Where the ED is the average value of each region of extended entropic degree. According to the two equations, the cumulative extended entropic degree distribution conforms the exponential distribution. However, in the Figure 10, the function of cumulative extended entropic degree distribution for the IEEE 300 test system, which is equal to

$$P_{cum}(ED) = 768.5 * ED^{-0.984}$$

This distribution accords with the power-law.

Because of these results, the extended entropic degree distribution for electrical power grid expresses different feature from traditional degree distribution. The extended entropic degree distribution for small scale networks may belong to exponential distribution, however, it may change to power-law distribution with the increase of the network scale.

5 Conclusion

Electric power grid, which plays a key role in any country's economy and security. The infrastructural systems for electric power delivery play a major role because they are widely distributed and indispensable to modern society. The security of power grids has attracted widespread attention.

Complex network theory is very useful for analyzing the security of network. As popular metrics for nodes in complex networks, degree and betweenness all have defects in application to power grids. Although these two metrics have been further updated for power grids, some critical defects still exist. To solve this problem, an extended entropic degree for power grids has been proposed by integrating entropic degree and e-betweenness. This new metric can take into account special physical rules in power transmission and combine local and global features together in evaluation of buses.

Simulation results of IEEE systems and a real distribution system have preliminary indicate its effectiveness and special characteristics. As degree and betweenness are widely utilized in many different works in complex networks, there will be great possibilities to apply this new metric and reveal many new features. The future work will focus on its distribution and related applications in cascading failure models.

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