Stability of Low-Reynolds-Number Separated Flow Around an Airfoil Near a Wavy Ground

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In this numerical–theoretical study, a linear BiGlobal stability analysis of the steady massively separated flow around a NACA 4415 airfoil was performed at a low Reynolds number ($Re = 200$) and a high angle of attack ($\alpha = 18$ deg) close to a wavy ground, with a focus on the effect of three different types of stationary roughness: 1) a perfectly flat ground, 2) a wavy ground with small-amplitude undulations, and 3) a wavy ground with large-amplitude undulations. On increasing the undulation amplitude $h_0$ of the ground but keeping the mean ground clearance constant, it was found that the lift coefficient increased owing to an increase in the static pressure under the airfoil, which is reminiscent of the conventional ground effect over a flat surface. However, it was also found that the leading flow perturbation was the three-dimensional stationary global mode and not the two-dimensional traveling Kelvin–Helmholtz mode, contrary to the results of previous analogous studies of linear global instability of massively separated flow away from the ground. This study provides new insight into the stability of airfoil-ground flow systems at a low Reynolds number and a high angle of attack, contributing to a better understanding of the ground-effect aerodynamics of small insects and micro air vehicles flying over rough waters or complex terrain.

Nomenclature

$C_l$ = lift coefficient
$C_p$ = static pressure coefficient
$c$ = chord length of airfoil
$H$ = clearance between mean ground profile and trailing edge of airfoil
$h_0$ = peak-to-peak amplitude of ground undulations
$L_z$ = spanwise wavelength
$l$ = wavelength of ground undulations
$q$ = total field vector of velocity and pressure
$\mathbf{q}$ = base-flow vector
$\tilde{q}$ = small perturbation
$\hat{q}$ = amplitude function of perturbation
$Re$ = Reynolds number based on airfoil chord
$St$ = Strouhal number defined as $\omega c/2\pi$
$t$ = dimensionless time
$U_\infty$ = freestream velocity
$x$, $y$ = streamwise and cross-stream directions in Cartesian coordinates
$\alpha$ = angle of attack
$\beta$ = real spanwise wave number
$\omega$ = complex eigenvalue

Subscripts

$i$ = imaginary part
$r$ = real part

I. Introduction

W\text{HEN} an aircraft or a bird flies near the ground, it can experience an increase in aerodynamic efficiency [1], resulting in a higher lift-to-drag ratio [2] and a reduced mechanical power required for sustained flight [3]. This phenomenon is known as the ground effect [4] and has been studied in a variety of applications, ranging from flapping wing animals to unmanned aerial vehicles [5]. However, most of these studies have been carried out at moderate-to-high Reynolds numbers ($Re \sim O(10^3-10^6)$ [6]), at angles of attack $\alpha$ low enough to avoid stall, and with a ground surface that is perfectly flat. These conditions may not always coincide with reality. For example, when small insects or micro air vehicles fly over rough waters or complex terrain, the Reynolds number can drop to $O(10^3-10^4)$ [7] or even $O(10^2)$ [8], the angle of attack can be high enough to induce massive flow separation [9], and the ground may exhibit wavy undulations on its surface. Therefore, it is important to investigate the ground effect under these more realistic conditions.

Surprisingly, only a handful of studies have investigated airfoil flows over a wavy ground. Using Reynolds-averaged Navier–Stokes (NS) simulations, Yang et al. [10] examined the aerodynamic performance of a wing-in-ground-effect vehicle flying at $0.3c$ ($c$ being the chord length) above a wavy ground at $Re = 6 \times 10^7$ and $\alpha = 5$ deg. The pressure under the wing was found to vary periodically in time, producing similarly periodic variations in the aerodynamic forces. However, compared with a flat ground, the wavy ground did not produce a marked difference in lift or drag, resulting in a relatively constant lift-to-drag ratio. By contrast, Wang [11] used the classical slender body theory to analyze the irrotational flow around a slender body moving over a wavy ground. The aerodynamic forces and moments were found to be highly dependent on the amplitude and wavelength of the ground undulations. The ground...
effect was found to be particularly strong when the undulation amplitude became comparable to the ground clearance and/or when the undulation wavelength became comparable to the body length (chord). However, to date, the effect of a wavy ground on the massively separated flow around an airfoil at a low Reynolds number and a high angle of attack has yet to be explored, particularly for undulation wavelengths shorter than the chord length. This is the focus of the present study.

Another unique aspect of the present study concerns the introduction of three-dimensionality to this problem, through the use of global flow stability analysis. To develop effective control strategies to suppress flow separation and improve aerodynamic performance requires a sound understanding of the instability characteristics of the base flow [12–18]. Linear stability analysis has been used to successfully predict the primary stability of two- and three-dimensional flows [19]. For example, by combining local and global linear stability analyses on the flow around airfoils at a low angle of attack, Theofilis et al. [20] and Lin and Pauley [21] were able to attribute the dominant perturbation to the Kelvin–Helmholtz (KH) mode. Likewise, at a high angle of attack, a similar dominance by the KH mode was found in the massively separated flow around a series of NACA airfoils at a low Reynolds number [22,23]. For steady separated flow around an airfoil at a low Reynolds number, Kitsios et al. [24] observed the existence of three-dimensional stationary modes, which were first reported by Theofilis et al. [25] in a flat-plate boundary layer. However, these three-dimensional stationary modes have recently been found to be less unstable than the two-dimensional KH mode [23]. A similar conclusion was reached by He et al. [26], who analyzed both the primary and secondary instabilities of the unsteady separated flow around a NACA 4415 airfoil at 300 ≤ Re ≤ 1000 and α = 20 deg. The airfoil was positioned at a nondimensional height (based on the chord length) of 0.2 ≤ H ≤ ∞ above two different types of flat ground: a stationary ground and a moving ground. Results from the latter linear stability analysis showed that, consistent with previous studies and regardless of ground type, the two-dimensional KH mode always dominates over the three-dimensional stationary global mode. However, no attempt was made to analyze the stability of the flow at Reynolds-number values below that required for the onset of unsteady vortex shedding or in the presence of a wavy ground.

In this numerical–theoretical study, we perform a linear BiGlobal stability analysis of the steady but massively separated flow around a NACA 4415 airfoil at a low Reynolds number (Re = 200) and a high angle of attack (α = 18 deg), with a focus on the effect of three different types of stationary ground: 1) a perfectly flat ground, 2) a wavy ground with small-amplitude undulations, and 3) a wavy ground with large-amplitude undulations. The aim was to see whether and how the presence of short-wavelength undulations of different amplitudes on the ground can influence the structure and stability of the flow, particularly at a Reynolds-number value below that required for the onset of unsteady vortex shedding.

II. Problem Formulation

A. Wavy Ground Definition

The wavy ground is defined by a set of cosine functions:

\[ h(x) = \frac{h_0}{2} \left(1 + \cos \frac{2\pi x}{l}\right), \quad x \in [0, l] \quad (1) \]

in which \( l \) is the wavelength, and \( h_0/2 \) is the amplitude of the ground undulations. The wave trough is fixed at \( h = 0 \), but the wave crest is set by \( h = h_0 \). When \( h_0 = 0 \), the ground defaults to a flat surface, as shown in Fig. 1. With this ground definition [Eq. (1)], if an airfoil were positioned at a fixed height above the datum\( h = 0 \), increasing \( h_0 \) would have the side effect of reducing the mean ground clearance \( H \), which is defined as the vertical distance between the trailing edge of the airfoil and the ground profile (horizontal dashed lines in Fig. 1). Therefore, to enable an investigation of the sole effect of \( h_0 \), we shift the airfoil up as \( h_0 \) increases, such that \( H \) is kept constant at 0.2. All length scales are nondimensionalized by the chord length \( c \).
III. Results and Discussion

A. Base Flow

Table 2 shows the results of a grid independence study for $H = 0.18$, which requires a higher-resolution mesh than for $H = 0.2$. Convergence is assessed by increasing the polynomial order of the mesh ($p$) and monitoring five indicators: 1) the lift coefficient $C_l$; 2) the drag coefficient $C_d$, and 3–5) the spanwise vorticity $\omega_s = \partial v/\partial x - \partial u/\partial y$ at three different locations around the airfoil, which are near the leading edge $[P_l(0.1,0.6)]$, near the trailing edge $[P_T(1.0,0.22)]$, and in the separation bubble $[P_S(1.1,0.55)]$. The relative error in each of these indicators ($e_{C_l}, e_{C_d}$, and $e_{\omega_s}$) is shown in Table 2. Of all the indicators examined, $\omega_s$ is the most sensitive to changes in grid resolution. Nevertheless, the maximum relative error in $\omega_s$ is still relatively small, at less than 1.1% for $p = 7$. Therefore, to balance the numerical accuracy and the computational cost, we used $p = 7$ in our base-flow simulations and stability analysis.

Figure 4 shows DNS streamlines and spanwise vorticity contours ($-5 \leq \omega_s \leq 5$) around the airfoil at $Re = 200$ and $\alpha = 18$ deg for three different types of ground, from left to right: 1) a flat ground, $h_0 = 0$; 2) a wavy ground with small-amplitude undulations, $h_0 = 0.04$; and 3) a wavy ground with large-amplitude undulations, $h_0 = 0.08$. For all three cases, the flow is steady in time, confirming that Reynolds number is indeed low enough to avoid unsteady vortex shedding. There is massive flow separation behind the airfoil, resulting in a robust separation bubble with two large recirculation zones. As the undulation amplitude $h_0$ of the ground increases, the length of the separation bubble also increases, but its overall structure remains largely unchanged. Figure 5 shows the static pressure coefficient $C_p$ on the airfoil surface. As $h_0$ increases, $C_p$ remains relatively constant on the suction side of the airfoil, but increases slightly on the pressure side, especially near the midchord location, resulting in an increase in the lift coefficient $C_l$, as shown in the legend of Fig. 5.

B. Stability Analysis

Direct stability analysis is performed by finding the least stable eigenvalues of $A$ using the Arnoldi iteration in a time-forward mapping scheme, implemented in Nektar++. Adjoint stability analysis is performed by solving the adjoint operator of Eq. (3) in a time-backward mapping scheme. Both types of stability analysis give the same eigenvalues if the problem is domain independent. We used this feature to demonstrate that our computational domain is sufficiently large. Table 3 shows a sensitivity analysis performed on the size of the computational domain for a sample flow condition ($h_0 = 0.04; \beta = 0$). The agreement in growth rate and frequency between the direct and adjoint modes is good, confirming that the computational domain is large enough for the instability properties to converge.

Figure 6 shows the growth rate ($\omega_0$) and Strouhal number ($Sr \equiv \omega_0/2\pi$) as a function of the spanwise wave number $\beta$ for the flow conditions of Fig. 4. Two different classes of instability modes can be identified: 1) two-dimensional traveling KH modes (denoted by hollow markers), for which $Sr > 0$; and 2) three-dimensional stationary global modes (denoted by filled markers), for which $Sr = 0$ (not plotted). As mentioned earlier, similar modes have been identified before in both steady [24] and unsteady [30] separated flows around an airfoil with [26] and without [23,32] the ground effect. However, in contrast to those earlier studies, in which the KH

### Table 2

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\omega_0$ at $P_1$</th>
<th>$\omega_0$ at $P_2$</th>
<th>$\omega_0$ at $P_3$</th>
<th>$e_{C_l}, %$</th>
<th>$C_l$</th>
<th>$e_{C_d}, %$</th>
<th>$C_d$</th>
<th>$e_{\omega_s}, %$</th>
<th>$\omega_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$-17.674540$</td>
<td>$-2.281345$</td>
<td>$-2.174560$</td>
<td>3.15</td>
<td>0.5166</td>
<td>0.04</td>
<td>1.4172</td>
<td>0.04</td>
<td>$\omega_s$</td>
</tr>
<tr>
<td>7</td>
<td>$-17.673402$</td>
<td>$-2.385018$</td>
<td>$-2.173184$</td>
<td>1.06</td>
<td>0.5168</td>
<td>0.02</td>
<td>1.4166</td>
<td>0.02</td>
<td>$\omega_s$</td>
</tr>
<tr>
<td>9</td>
<td>$-17.670292$</td>
<td>$-2.355664$</td>
<td>$-2.174399$</td>
<td>—</td>
<td>0.5170</td>
<td>—</td>
<td>1.4168</td>
<td>—</td>
<td>$\omega_s$</td>
</tr>
</tbody>
</table>

The relative error in $\omega_0$ is the maximum value found across the three sampling locations.
mode was always found to dominate over the stationary mode, here, we found the opposite behavior: the KH mode is more stable than the stationary mode. The former is dominant at small wave numbers ($\beta \approx 0$), whereas the latter is dominant at large wave numbers ($\beta \approx 3$). As $h_0$ increases, the frequency of the KH mode decreases. The growth rates of both the KH and stationary modes also decrease, but to a lesser extent, with the former mode being slightly more stable than the latter mode. All the growth rates of the perturbations are negative, implying that both the KH and stationary modes decay in the long-time asymptotic limit.

Figure 7 shows the three components of the perturbation amplitude function $\hat{q}$ at $\beta = 1$ (KH mode) and $\beta = 3$ (stationary mode) for $h_0 = 0.04$; these components are normalized by their respective maximum velocities. As mentioned earlier, the KH mode is stable at this Reynolds number ($Re = 200$). However, a recent work by He et al. [26] on the separated flow around a NACA 4415 airfoil above a flat ground has shown that, when the Reynolds number exceeds roughly 500, the KH mode becomes unstable, destroying the steady separation bubble behind the airfoil and producing large-scale vortex shedding downstream at a well-defined frequency. Thus, at higher values of Reynolds number, the KH mode is expected to cause the airfoil wake to transition from a spatial amplifier of extrinsic perturbations to a self-excited oscillator with an intrinsic natural frequency [33]. Such a transition, which is often referred to as a Hopf bifurcation [34], has been reported before not only in airfoil wakes [35], but also in cylinder wakes [36], crossflowing jets [37], and low-density jets [38–40]. To examine the evolution of the optimal perturbations, we performed a transient growth analysis by estimating the energy gain $G$ over a time horizon $\tau$. Figure 8 shows that $G(\tau)$ grows to $O(10)$ over a short time horizon, and then decays linearly, indicating the presence of convective instability [23].

Unlike the spatially growing stationary mode identified by He et al. [23], here, the spatial structures of the stationary mode dominate the wake behind the airfoil, and then decay downstream. The streamwise perturbation is consistently stronger than the other two components. The perturbation functions for the other two values of $h_0$ (0 and 0.08) are qualitatively similar to that for $h_0 = 0.04$ (Fig. 7). Figure 9 shows a three-dimensional reconstruction of the least stable stationary mode at $\beta = 3$, as represented by the $\lambda_2$ criterion and the streamwise vorticity (transparent isocontours). It can be seen that the vortical structure of the perturbation is most complex behind the trailing edge of the airfoil, with two long stable vortices developing downstream. The streamwise velocity of the stationary mode is visualized by dashed and solid lines ($-0.01 \leq \hat{u} \leq 0.01$) in a vertical plane at $x = 4$. The energy gain $G(\tau)$ over a time horizon $\tau$.
IV Conclusions

A linear BiGlobal stability analysis of the steady massively separated flow around a NACA 4415 airfoil has been performed at a low Reynolds number \( (Re = 200) \) and a high angle of attack \( (\alpha = 18 \, \text{deg}) \), with a focus on the effect of three different types of stationary ground: 1) a perfectly flat ground, 2) a wavy ground with small-amplitude undulations, and 3) a wavy ground with large-amplitude undulations. For a fixed ground clearance \( (H = 0.2) \), it was found that increasing the undulation amplitude \( h_0 \) of the ground leads to a higher lift coefficient, similar to the classical ground effect over a flat surface. However, it was also found that the leading flow perturbation is the three-dimensional stationary global mode and not over a flat surface. Increasing \( h_0 \) was found to reduce the growth rates and frequencies of the KH and stationary modes. This work shows that the spanwise periodicity length of the leading stationary disturbances in the wake of an airfoil is of \( O(2c) \).

This study provides new insight into the stability of airfoil–ground flow systems at a low Reynolds number and a high angle of attack, contributing to an improved understanding of natural flyers and micro air vehicles. Possible directions for future work include adopting a dynamic mesh so that the effect of a time-dependent phase relationship between the airfoil and the wavy ground can be explored.

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References


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