HetNets with Range Expansion: Local Delay and Energy Efficiency Optimization

Xiaojie Dong, Fu-Chun Zheng, Senior Member, IEEE, Xu Zhu, Senior Member, IEEE, and Jingjing Luo, Member, IEEE

Abstract—In this paper, we explore the different per-user bandwidths available for each tier of heterogeneous networks (HetNets) and the corresponding downlink range expansion (RE, a key measure to boost small cell association). Based on the practical case of tier-specific path loss exponents, we derive the local delay of HetNets under the random discontinuous transmission scheme (DTX, employed to manage the interference and reduce energy consumption). Using such a new expression for local delay, we then analyze the corresponding energy efficiency (EE). These delay and EE expressions reveal that there exists an optimal value both for the RE bias factor and for the DTX mute probability. A closed-form optimum solution for the RE bias factor is also derived for the low rate regime.

Index Terms—Range expansion, energy efficiency, local delay, random discontinuous transmission scheme, and heterogeneous networks.

I. INTRODUCTION

With the explosive growth of wireless data traffic, ultra dense heterogeneous networks (HetNets) have become a powerful solution. However, the heterogeneity of HetNets may not be fully utilized if the maximum received signal strength (RSS) association rule is always employed, as most of the users may be associated with the macro cells due to their higher transmission power. To this end, the range expansion (RE) scheme can be adopted at BSs [1]. As a main strategy to boost small cell association, range expansion (RE) increases the coverage of small cells (i.e. the boundary of small cells in Fig.1) by using a biased handover threshold. As a result, RE can make small cells serve more end users, hence offloading more traffic off macrocells. Many works have recently been conducted on RE, but these existing results (e.g. [2], [3] and the references therein) normally employ optimization methods to improve the network performances without any closed-form expressions. For example, in [2], the maximization problem of a logarithmic utility function was formulated and solved. However, the procedure is based on iteration, whose complexity and efficiency depend upon the number of iterations needed before reaching convergence. In comparison, the corresponding analysis based on closed-form solutions (such as those in this paper) is clearly more efficient and convenient. Furthermore, as an emerging effective tool, stochastic geometry has been employed to investigate the HetNets. For example, the data rate and outage performance of HetNets with RE were analyzed in [3] by using the Poisson point process (PPP).

In HetNets, small cells can offer a relatively larger per-user bandwidth than macrocells, due to the smaller number of users that they serve, which can significantly improve the end-user data rate. Therefore, letting an end-user associate with the small cells as much as possible via RE can make better use of bandwidth resources and improve the performances of HetNets (both for the uplink and for the downlink), such as the local delay and energy efficiency, which will be analyzed in this paper. On the other hand, users offloaded to small cells may well suffer from a higher level of interference from the macro cell base stations. In reality therefore, there exists a tradeoff between the bandwidth resources and the allowed RE of small cells. To our best knowledge, this has not been addressed in the literature (e.g. [2] and [3]). It is another motivation of this paper to examine such a tradeoff.

In wireless HetNets, transmitting data packets successfully within a specified time is closely related to the quality of service. The local delay was therefore defined as the average time taken until the data packet has been transmitted successfully. The analytical framework of local delay was provided in [4] and [5]. In addition, the local delay was extended to a $K$-tier HetNet with the same per-user bandwidths in [6] and [7]. Obviously, the per-user bandwidths in each tier of HetNets can be very different. More importantly, each tier of HetNets may well have a different path-loss exponent instead of the same path-loss exponent as assumed in [7].

Energy efficiency is another key indicator of system performances in ultra dense HetNets. Many strategies to improve energy efficiency have been proposed. For instance, energy saving can enjoy a huge improvement by jointly optimizing cell activation, RE and multicell multiuser channel assignment according to [8]. Though the cell-specific RE bias is better in permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.
improving energy efficiency, it is hard to formulate and as such a per-tier bias scheme will be employed in this paper. In [9], energy efficiency was analyzed by introducing an additional bias factor to enlarge the range of all small cells.

Different from the above works, we in this paper employ the discontinuous transmission (DTX) scheme to manage the interference and improve the energy efficiency. DTX has proved to be a powerful strategy in managing network interference and in improving network energy efficiency [6] [10].

Contributions: This paper has derived the expressions for local delay and energy efficiency of $K$-tier HetNets under RE and DTX by using stochastic geometry (assuming tier-specific path-loss exponents). The relative per-user bandwidths are employed to change the length of time slots in each tier of HetNets. Based on the derived new expressions, the local delay and energy efficiency with respect to the RE bias factor and the mute probability have been analyzed, revealing the existence of an optimum value for both the RE bias factor and the mute probability. In particular, in the low-rate regime, the optimal bias factor was obtained in a closed form.

II. SYSTEM MODEL

A. Heterogeneous Networks Model

We consider a downlink HetNet (as depicted in Fig. 1), which comprises $K$ independent network tiers of BSs with $K = \{1, 2, ..., K\}$ and a typical mobile user at origin $o \in \mathbb{R}^2$. The set $\Phi$ of the all BSs consists of all the subset $\Phi_i$ of the BSs in each tier. In each tier, the BSs are equipped with a single antenna with density $\lambda_i$ and BS locations follow a homogeneous Poisson point process (PPP). Correspondingly, the transmission power and per-user bandwidth of BSs within Tier $i$ are respectively $P_i$ and $w_i$. In addition, the range extension bias factor $B_i$ is employed to adjust (i.e. enlarge) the coverage of small cell BSs. As shown in Fig. 1, small cells with positive RE (i.e. $B_i > 0$ dB) can serve more users than with the traditional "strongest received signal strength" indicator and therefore can off-load more traffic off macro cells. Assume that the location of a BS in the $i$th tier is $x_i$. Moreover, we adopt a distance-dependent path loss model, $l(x_i) = \|x_i\|^{-\alpha_i}$, where $\|x_i\|$ is the distance between the BS at $x_i$ and the typical user and $\alpha_i$ ($\alpha_i > 2$) is the tier-specific path-loss exponent for the $i$th tier. Furthermore, we adopt the assumption that the power fading coefficients $h_{x_i}$ follow the exponential distribution of unit mean and are spatially and temporally independent. Finally, we neglect the noise as interference is the main problem in dense HetNets.

As in all the existing literature, the average biased received signal strength (RSS) is employed as the association criterion in this paper. The index of the serving BS, assuming in the $k$th tier, is expressed by

$$k = \arg \max_{i \in K} P_i B_i \|x_{i,0}\|^{-\alpha_i},$$

where $x_{i,0}$ is the location of the BS in the $i$th tier offering the strongest biased RSS. Using the bias factor $B_i$, the small cell BSs can have a larger coverage range (i.e. serve more users) and reduce the loads of macro cells.

In this paper, time is divided into discrete slots, and the random discontinuous transmission (DTX) scheme is employed in each time slot at the BS, due to DTX being an effective measure to manage the network interference [6]. The $i$th-tier BSs transmit the data packets with the probability of $1 - \zeta_i$ in each time slot, i.e., it keeps mute with the probability of $\zeta_i \in [0, 1]$. Letting $\Phi_{k,t}$ denote the set of active BSs in the $i$th tier in Time slot $t$, the aggregate interference at the typical user is

$$I_t = \sum_{i \in K} \sum_{BS(x_i) \in \Phi_{k,t} \setminus \{BS(x_{k,0})\}} P_i h_{x_i} \|x_i\|^{-\alpha_i} \mathbf{1}(BS(x_i) \in \Phi_{k,t}),$$

where $BS(x_i)$ is the BS located at $x_i$ and $\mathbf{1}()$ is the indicator function. The received signal-to-interference-ratio (SIR) at the typical mobile user can then be calculated:

$$\text{SIR}_{k,t} = \frac{P_k h_{x_k} \|x_{k,0}\|^{-\alpha_k}}{I_t},$$

where $x_{k,0}$ is the location of the serving BS, as expressed in (1).

B. Performance Metrics

Inspired by [6] and for completeness and comparison, we consider two main performance metrics: local delay and energy efficiency.

1) Local Delay: This paper considers the case where a successful transmission happens when the received SIR at the user is larger than some specific SIR threshold $\theta$, otherwise a retransmission will be conducted. Hence, the conditional probability of a successful transmission event $C_F$ is given by

$$P(C_F) = (1 - \zeta_k) P(\text{SIR}_{k,t} > \theta | \mathbf{r}, \Phi),$$

where $r$ is the associated distance, and $1 - \zeta_k$ means that the serving BS must be active when the data packet is transmitted. The local delay is defined as the average number of time slots for a successful transmission over the wireless link. The associated local delay can be expressed by

$$D_k = \mathbb{E}_{\mathbf{r}, \Phi} \left[ \frac{1}{P(C_F)} \right],$$

where $\mathbb{E}_{\mathbf{r}, \Phi} [\cdot]$ is the mean over $\Phi$ and $\mathbf{r}$. To examine the effect of per-user bandwidth on the local delay without changing the unit of the local delay, we now consider the normalized bandwidth. According to the law of total probability, the average local delay is given by

$$D = \sum_{k \in K} \frac{A_k D_k}{W_k},$$
where $A_k$ is the probability of the typical mobile user being associated with the $k$th-tier BS [13] and $W_k$ is the $k$th-tier normalized per-user bandwidth, which can be calculated by

$$W_k = \frac{w_k}{w_1}, \quad k \in \mathcal{K}. \tag{7}$$

In cellular step-up, the bandwidth allocated to a user depends on the cell load which further depend on the biasing factor. We assume that $W$ is the total bandwidth of BS and the per-user bandwidth in the $k$th tier can be calculated as

$$w_k = W\frac{\lambda_k}{A_k\lambda_u}, \tag{8}$$

where $\lambda_u$ is the density of users, sampled from homogeneous PPP, so the average numbers of users per BS in each tier can be expressed as $A_k\frac{\lambda_u}{\lambda_k}$. Specially, we have $W_1 = 1$, implying that 1st-tier (macro cells) bandwidth is regarded as the reference. The normalized bandwidth ensures that the unit of the local delay in (6) is still time slot. Furthermore, when $\{\alpha_k\} = \alpha$, the normalized bandwidth can be expressed by

$$W_k = \frac{w_k}{w_1} = \left(\frac{P_i B_1}{P_k B_k}\right)^{2/\alpha}, \tag{9}$$

which indicates that the normalized per-user bandwidth will decrease when the bias factor increases. Here, $P_i$ and $B_i$ are the same as in (1).

2) Energy Efficiency: The energy efficiency $\eta$ of HetNets is defined as the ratio of average area network to average area power consumption [6]. The linear power consumption model in [11] is employed for each-tier’s BSs, which is given by

$$P_{c,i} = \begin{cases} P_{i,a} + \beta_i B_i, & \text{if} \quad BS(x_i) \in \Phi_{i,t} \\ P_{i,m}, & \text{if} \quad BS(x_i) \notin \Phi_{i,t} \end{cases}, \tag{10}$$

where $P_{i,a}$ and $P_{i,m}$ are respectively the static power of the active BS and the mute BS, and $\beta_i$ is the slope of power consumption for the $i$th-tier BSs. Therefore, the energy efficiency [6], with unit nats/J/Hz, is expressed by

$$\eta = \frac{D^{-1}\log(1+\theta)\sum_{i=1}^K(1-\zeta_i)\lambda_i}{\sum_{i=1}^K \lambda_i [(1-\zeta_i)(P_{i,a} + \beta_i P_i) + \zeta_i P_{i,m}]} \tag{11}$$

### III. LOCAL DELAY

This section will explore characteristics of the local delay with respect to key parameters.

#### A. General Case and Main Results

Based on the above definitions, we have the following results on the local delay.

**Theorem 1:** The local delay in a $K$-tier PPP distributed HetNet with RE and random DTX is given by

$$D = \sum_{k=1}^K \int_0^\infty \frac{2\pi \lambda_k}{(1-\zeta_i)W_k} \exp \left\{ -\pi \sum_{i=1}^K r^{2\alpha_i} \lambda_i (\bar{P}_i B_i)^{\frac{2}{1-\zeta_i}} \left(1-(1-\zeta_i)Z(\zeta_i, \alpha_i, \theta, \bar{B}_i)\right) \right\} r dr, \tag{12}$$

where $\bar{P}_i = P_i/P_k$, $\bar{B}_i = B_i/B_k$ and $Z(\zeta_i, \alpha_i, \theta, \bar{B}_i) = C(\alpha_i)^{\theta/\alpha_i} \bar{B}_i^{\frac{\alpha_i}{1-\alpha_i}} - \frac{1}{\sqrt{2\pi}} F_1(1, \alpha_i^{-1} \frac{\bar{B}_i}{\bar{B}_k} - \frac{1}{\theta})$ with $C(\alpha_i) = 1/\sin(2/\alpha_i)$ and $F_1(\cdot)$ denotes the Gauss hypergeometric function. The proof is given in the Appendix.

From Theorem 1, we observe that larger per-user bandwidth will indeed reduce the local delay. RE of small cells can ensure more users to be served with the larger available per-user bandwidth. Therefore, the per-user bandwidth advantage of small cell BSSs can be utilized fully by increasing the bias factor. Moreover, a sufficient condition is that the bias factors satisfy $(1-\zeta_i)Z(\zeta_i, \alpha_i, \theta, \bar{B}_i) < 1, \forall i, k \in \mathcal{K}$, ensuring that RE leads to a finite local delay.

The local delay expression in (12) can be simplified in some special cases. First, if we assume that BSs in each tier have the same path-loss exponent $\{\alpha_k\} = \alpha$ and mute probability $\{\zeta_i\} = \zeta$ and use the function $\int_0^\infty 2re^{-cr^2} dr = 1/c$, Theorem 1 can be simplified as

$$D = \frac{1}{(1-\zeta)} \sum_{k=1}^K \lambda_k \frac{1}{W_k} g_k, \tag{13}$$

where $g_k = \sum_{i=1}^K \lambda_i (\bar{P}_i B_i)^{\frac{2}{1-\zeta}} (1-(1-\zeta)Z(\zeta, \alpha, \theta, \bar{B}_i))$.

#### B. Analysis of Range Expansion

**Proposition 1:** In the low-rate regime $(\theta \to 0)$, the optimal bias factor that maximizes the local delay is given by

$$B_k = \frac{1}{P_k} \left(\frac{b_0}{b_2} + \sqrt{\frac{b_0^2}{b_2^2} + \frac{b_1}{b_2}}\right)^2, \tag{14}$$

where $b_2 = \lambda_k$, $b_1 = \sum_{i=1, i \neq k}^K \lambda_i v_i$, $b_0 = \sum_{i=1}^K \lambda_i v_i$, and $v_i = (P_i B_i)^{2/\alpha}$.

**Proof:** In the low-rate regime $(\theta \to 0)$, we have $g_k = \sum_{i=1}^K \lambda_i (\bar{P}_i B_i)^{\frac{2}{1-\zeta}}$ and the local delay can be approximated as

$$D \sim \sum_{i=1}^K \lambda_i v_i^2.$$ 

Through derivative of the local delay in (15) with respect to $v_i$, we obtain $\frac{\partial D}{\partial v_i} \sim 2\lambda_k v_k \sum_{i=1}^K \lambda_i v_i - \lambda_k \sum_{i=1}^K \lambda_i v_i^2$.

Therefore, the optimal $v_k^*$ can be calculated as

$$v_k^* = \frac{b_0}{b_2} + \sqrt{\frac{b_0^2}{b_2^2} + \frac{b_1}{b_2}}. \tag{16}$$

Then, the optimal bias factor can be obtained immediately. \qed

Formula (14) shows that the optimal bias factor $B_k^*$ decreases as the $k$th-tier density and transmission power of BSSs increase. This is because increasing density and transmission power can already expand the coverage of small cells, reducing the need for RE. Specifically, when $\lambda_k \to \infty$, the optimal bias factor tends to zero dB. That is, in the extremely dense scenario of small cells, the typical user will be associated with small cells without having to employ RE. From (11), we can observe that the optimal bias factor in (14) can maximize energy efficiency. In addition, the sufficient condition restricting the bias factor to ensure a finite local delay becomes $\frac{\partial D}{\partial v_i} < (2\theta/(\alpha-2))^{-1}$. Therefore, when the data rate increases,
the bias factor of small cell BSs should decrease. The reason is that the threshold $\theta$ restricts the effectively-serving range of the BS.

IV. NUMERICAL RESULTS

In this section, we carry out numerical analyses of the local delay and energy efficiency to verify and complement the above analysis. A two-tier HetNet structure, consisting of macro cells (Tier 1) and pico (small) cells (Tier 2), is used. We focus on the RE and the random DTX scheme of pico cells according to (11) and (12). In addition, the related parameters are $P_1 = 20W$, $P_2 = 0.13W$, $P_{1,a} = 130W$, $P_{2,a} = 6.8W$, $P_{1,m} = 0.75P_{1,a}$, $P_{2,m} = 0.75P_{2,a}$, $\beta_1 = 4.7$, and $\beta_2 = 4.0$ [11], which are similar to the actual base station parameters and have now been widely adopted for simulations.

Moreover, there exists a trade-off between the minimum local delay and the maximum energy efficiency. As a result of such a trade-off, an optimal bias factor exists and needs to be determined to minimize the local delay and maximize the energy efficiency. For the low-rate regime, we have obtained the optimal bias factor’s expression.

In addition, with the increase of picocells’ density, the optimal bias factor in terms of the local delay and the energy efficiency will decrease. When the density of picocells tends to infinity (i.e. an extremely dense scenario), the bias factor becomes useless. The physical meaning here is that in extremely dense HetNets users generally are associated with small cells anyway (i.e. unlikely with a macro-cell), eliminating the role of RE.

Fig. 2. The local delay and energy efficiency as a function of the mute probability, where $\alpha = 4$, $B_1 = 0dB$, $\theta = 1$, $\lambda_1 = 1/(\pi500^2)$, $\lambda_2 = 10\lambda_1$.

Fig. 3. The local delay and energy efficiency as a function of bias factor, where $\theta = 0.1$, $\alpha = 4$, $B_1 = 0dB$, $\zeta_1 = \zeta_2 = 0.5$, $\lambda_1 = 1/(\pi500^2)$.

Fig. 4. The optimal local delay and energy efficiency as a function of pico cells’ density, where $\alpha = 4$, $B_1 = 0dB$, $\zeta_1 = \zeta_2 = 0.9$, $\lambda_1 = 1/(\pi500^2)$.

In Fig. 4, we show that the local delay and energy efficiency as a function of pico cells’ density. We compare the optimal bias factor $B_{opt}$ from the numerical results with the optimal bias factor $B_{2*}^2$ in (14). The local delay and the energy efficiency match well between the two cases, which verify the accuracy of (14). Furthermore, any increase in pico cells’ density can make the local delay decrease and the energy efficiency increase with the optimal bias factor. The reason is that, when the density of small cell increases, the association distances between BSs and users decrease. Users also tend to connect to small cells with larger bandwidth resources, so the local delay decreases. The results under different $\lambda_1$ values (macro BS density), although omitted here, have shown similar trends.
V. CONCLUSION

In this letter, the local delay and energy efficiency of HetNets with RE under the random DTX have been derived by employing different per-user bandwidth in each tier. The results indicated that the local delay and energy efficiency of HetNets can be optimized with respect to the mute probability and the bias factor. As a result, utilizing the bandwidth advantage of the small cell BSs, the optimal bias factor in the low-rate regime was obtained as a closed form. Future works include the scenarios of user mobility and clustered BS distribution.

APPENDIX

Proof of Theorem 1

Proof: Using the probability generating functional of the PPP [12], the associated local delay conditioned on the distance \( r \) is given by

\[
D_k(r) = \frac{1}{1 - \zeta_k} \exp \left[ \sum_{i=1}^{K} \pi \lambda_i (1 - \zeta_i) \left( \frac{P_i B_i}{P_k B_k} \right)^{\frac{2}{\alpha_i}} r^{-\frac{2\alpha_i}{\alpha_k}} Z(\zeta_k, \alpha_k, \theta, B_k) \right].
\]  \( (17) \)

Based on the association rule of the strongest biased RSS, the probability distribution function of the associated distance \( r \) between the user and its associated BS is

\[
f_k(r) = \frac{2\pi \lambda_k}{A_k} \exp \left[ -\pi \sum_{i=1}^{K} \lambda_i \left( \frac{P_i B_i}{P_k B_k} \right)^{\frac{2}{\alpha_i}} r^{-\frac{2\alpha_i}{\alpha_k}} \right] r. \]  \( (18) \)

According to the integral of \( D_k(r) \) with respect to \( r \) and the law of total probability in (6), the local delay in (12) can be determined by

\[
D = \sum_{i=1}^{K} \frac{A_k}{W_k} \int_{0}^{\infty} D_k(r)f_k(r)dr. \]  \( (19) \)

\( \square \)

REFERENCES


Xiaojie Dong received the B.S. in Electronic Information Engineering from Heilongjiang University, China in 2016 and the MEng in Information and Communication Engineering from Harbin Institute of Technology, Shenzhen, China in 2019. His research interests include Stochastic geometry and Heterogeneous networks.

Fu-Chun Zheng obtained the BEng (1985) and MEng (1988) degrees in radio engineering from Harbin Institute of Technology, China, and the PhD degree in Electrical Engineering from the University of Edinburgh, UK, in 1992.

From 1992 to 1995, he was a post-doctoral research associate with the University of Bradford, UK, Between May 1995 and August 2007, he was with Victoria University, Melbourne, Australia, first as a lecturer and then as an associate professor in mobile communications. He was with the University of Reading, UK, from September 2007 to July 2016 as a Professor (Chair) of Signal Processing. He has also been a distinguished adjunct professor with Southeast University, China, since 2010. Since August 2016, he has been a distinguished professor with Harbin Institute of Technology (Shenzhen), China and the University of York, UK. He has been awarded two UK EPSRC Visiting Fellowships - both hosted by the University of York (UK): first in August 2002 and then again in August 2006. Over the past two decades, Dr Zheng has also carried out many government and industry sponsored research projects - in Australia, the UK, and China. He has been both a short term visiting fellow and a long term visiting research fellow with British Telecom, UK. Dr Zheng’s current research interests include signal processing for communications, multiple antenna systems, green communications, and ultra-dense networks.

He was an editor (2001-2004) of IEEE Transactions on Wireless Communications. In 2006, Dr Zheng served as the general chair of IEEE VTC 2006-S, Melbourne, Australia (www.ieeevtc.org/vtc2006spring) - the first ever VTC held in the southern hemisphere in VTC’s history of six decades. More recently he was the executive TPC Chair for VTC 2016-S, Nanjing, China (the first ever VTC held in mainland China: www.ieeevtc.org/vtc2016spring).
Xu Zhu (S’02-M’03-SM’12) received the B.Eng. degree (with the first-class Hons.) in electronics and information engineering from Huazhong University of Science and Technology, Wuhan, China, in 1999, and the Ph.D. degree in electrical and electronic engineering from the Hong Kong University of Science and Technology, Hong Kong, in 2003. She joined the Department of Electrical Engineering and Electronics, the University of Liverpool, Liverpool, U.K. in 2003 as an academic member, where she is currently a Reader. She has over 160 peer-reviewed publications on communications and signal processing. Her research interests include MIMO, channel estimation and equalization, resource allocation, cooperative communications, green communications, etc. She has acted as a chair for various international conferences, such as Symposium Co-Chair of IEEE ICC 2016 and 2019, Vice-Chair of the 2006 and 2008 ICARN International Workshops, Program Chair of ICSAI 2012, and Publicity Chair of IEEE IUCC-2016. She has served as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and a Guest Editor for several international journals such as Electronics.

Jingjing Luo received the B.S. and Ph.D. degrees from the Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan, China, in 2010 and 2015, respectively. From 2016 to 2018, she was a Post-Doctoral Fellow with the Department of Information Engineering, Chinese University of Hong Kong. Since 2018, she has been an Assistant Professor with the Department of Electronic and Information Engineering, Harbin Institute of Technology, Shenzhen, China. Her research interests include online learning and optimization in wireless networks.