Special session - Supporting simulations with strain measurement (CEN WS71)

Probabilistic metric for validation based on strain field data.

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Introduction

Validation is a crucial step in building confidence in the predictions of models which are used to inform decisions with social and economic consequences. The aim of the validation process is to determine the extent to which a model is a reliable representation of the reality with respect to its intended use [1], e.g. strain distribution on a surface of an aerospace component subject to impact. A number of guidelines are available and provide an overview of the validation process, but do not include a prescriptive solution for how to quantify the validity of the model's predictions.

In solid mechanics, it is common for the validation processes to use experimental data such as peak values measured by strain gauges at selected locations; however, this significantly restricts the regions of predictions that can be validated. Optical measurement techniques, e.g. Digital Image Correlation (DIC), provide a vast amount of data and thus allow the expansion of the region of interest and utilisation of more of the model's predictions. The qualitative comparison of these measured and predicted data fields can be achieved by means of a visual inspection, but the desired quantitative comparison is much more challenging, for instance, due to differences in data pitch and density.

The CEN guideline [2] provides a comprehensive validation framework for the evaluation of computational solid mechanics models and is the first guide to incorporate detailed methodologies for data field processing and comparison with the aid of novel image decomposition, or proper orthogonal decomposition, techniques. It is a significant advancement in the field of solid mechanics; however, the validation methodology described in the CEN guide leads to a Boolean result, i.e. valid or invalid model, and does not provide a quantitative outcome. The work described in this paper addresses this gap by presenting a validation metric based on weighted relative error [3] that quantifies the extent to which a model reliably represents reality and discussing its application with the aid of a case study.

Methods

Direct comparison of predicted and measured data fields, for example surface strain maps, can be difficult as variations in data pitch, translation or rotation can affect the outcome of the comparison. Image decomposition, or proper orthogonal decomposition, is insensitive to these variations and allows the compression of the data from a two-dimensional matrix to a one-dimensional vector while capturing global or local shapes present in the data field [4]. The method used in this study was based on fitting a set of continuous orthogonal polynomials to the strain field data and the output was a feature vector consisting of coefficients of the polynomials, which correspond to the particular shapes. Predicted and measured data was processed using the same set of polynomials resulting in equivalent feature vectors which can be directly compared to validate a model's predictions.

Quantitative comparison, as part of a validation process, can be performed by applying a validation metric. Our research concentrated on frequentist approaches, which are based on mapping a discrepancy in the computational response relative to the referent or, for validation purpose, relative to experimental data. In this paper, a novel metric is presented that is based on a weighted relative error and the output can be summarised in a probabilistic statement about the extent of model's validity. The relative errors, e_k are computed between components of the feature vectors and are then assessed against a threshold based on the experimental uncertainty; the sum of weights, w_k of the errors below the uncertainty threshold, e_{unc} is computed to establish the probability of a model being valid, *VM*:

$$VM = \sum w_k \quad \text{for} \quad e_k < e_{unc} \tag{1}$$

Case Study

A case study was used to assess the efficacy of the novel validation metric and the data was taken from an earlier study on I-beam subject to static three-point bending [5], which evaluated the application of the methodology described by the CEN guide [2]. A stereoscopic digital image correlation system was used to acquire measured strain maps, and the ANSYS software package was used to develop a model, by employing an elasto-plastic material model with kinematic hardening, and to obtain predicted strain maps. Two regions of interest were selected along the I-beam, and all data fields were successfully decomposed by Zernike polynomials prior to application of the relative error metric. The new validation results correlate well with previously published conclusions, although the quantitative output and the probabilistic statement are much more informative and consequently can help in building confidence in the model.

Conclusion

Predicted and measured data fields, such as displacement or strain maps, contain a significant amount of information, and through the validation process confidence in the model and its predictions can be built. A number of guides provide overviews of the validation process in engineering disciplines, yet there is a lack of standardised methodologies, i.e. validation metrics, to quantify the extent to which a model is a reliable representation of the reality from the perspective of its intended use. In this work, a novel validation metric addressing this gap in the field of solid mechanics was presented and evaluated with the aid of a case study. The validation metric is based on weighted relative error analysis and can be applied to data fields through the use of image decomposition, by following the data processing methodology proposed in the CEN guideline. The extent of model's validity is quantified and expressed in a probabilistic statement, thus extending the Boolean output of the CEN validation methodology and providing a more informative validation output. This is a significant advance in the field; however further research is necessary to explore applications with responses in three-dimensional spatial and spatio-temporal domains.

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