

# BAYESIAN REGRESSION OVER SPARSE FATIGUE CRACK GROWTH DATA FOR NUCLEAR PIPING

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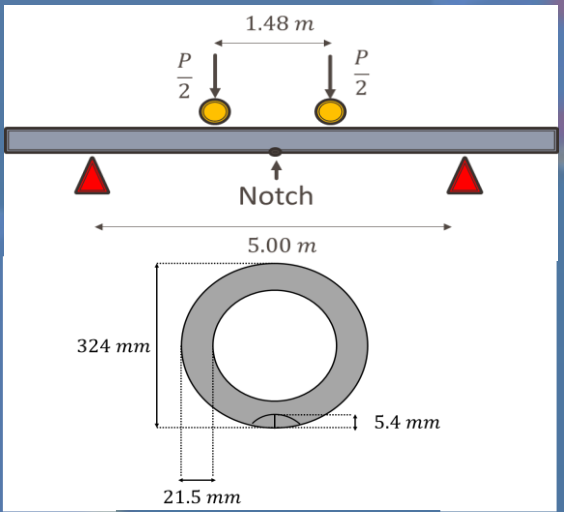


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## INTRODUCTION

- Research objectives: To perform probabilistic model calibration and quantify the uncertainty over the sparse data;
- A set of crack data obtained via a 4-point bending test on a Carbon-Steel nuclear piping [1];
- Test was conducted over 40000 periodic stress cycles,  $N_{cycles}$ ;
- Each stress cycle has stress range:  $\Delta P = 156 MPa$ ;
- 24 readings of crack depth,  $z$ , obtained for 24 distinct  $N_{cycles}$ .



## PROBLEM

- Crack growth assumed to follow Paris-Erdogan Law [2]:

$$\frac{dz}{dN_{cycles}} = C (\Delta K)^m \quad (1)$$

- This can be linearized to:

$$\log \left[ \frac{dz}{dN_{cycles}} \right] = m \cdot \log[\Delta K] + \log[C] \quad (2)$$

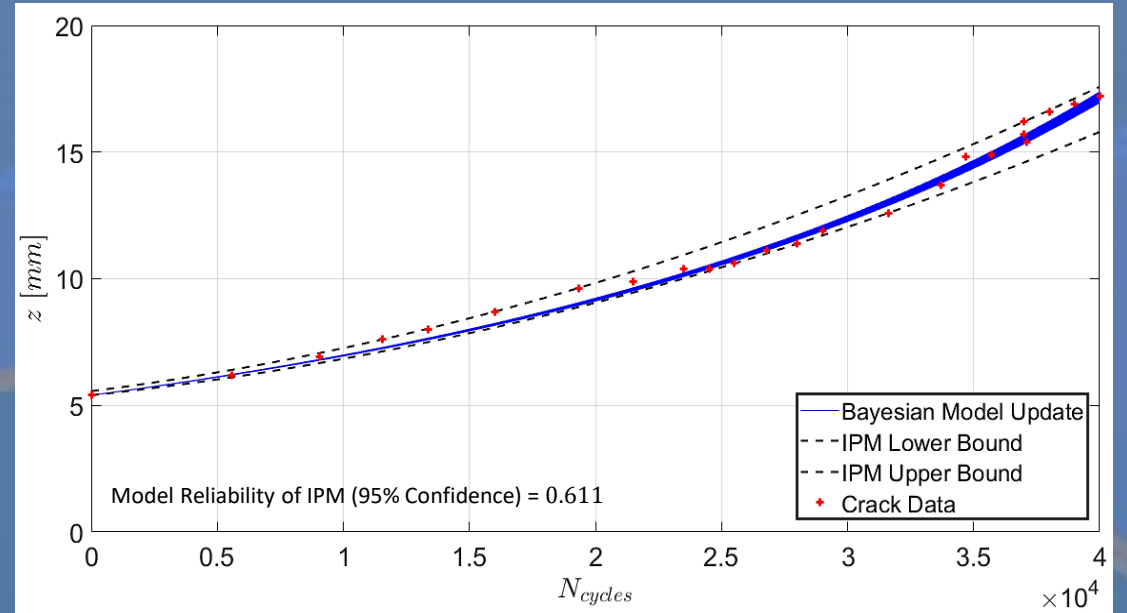
where:

$$\Delta K = \left( \frac{z}{\sqrt{r_m t}} \right)^{\frac{1}{8}} \cdot \Delta P \cdot \sqrt{\pi \cdot z} \quad (3)$$

## METHODOLOGY

- Bayesian regression technique for uncertainty quantification over the sparse data in log-space;
- Epistemic parameters to be inferred:  $\theta = \{\log[C], m\}$ ;
  - Prior PDF: 2D Uniform distribution with correlation coefficient of -0.999 defined by a Gaussian Copula.  $\log[C]$  has bounds  $[-50, 0]$  while  $m$  has bounds  $[0, 10]$ ;
  - Likelihood function is Gaussian with standard deviation:  $\sigma = 0.0191$ ;
  - Model used for Bayesian updating is defined by Eq. (2) ;
  - 1000 samples generated via TMCMC [3].
- Compare the results with 2<sup>nd</sup>-order polynomial Interval Predictor Model [4] in the original real space.

## RESULTS



## CONCLUSION

- Bayesian regression / model updating results showed that all possible trajectories lie within IPM;
- Bayesian model updating results yield tighter bounds and this is attributed to the choice of  $\sigma$  in the Likelihood function;
- Further works: To compare the results from Bayesian regression using different models of  $\Delta K$  and to also include Kriging as a form of validation.

## REFERENCES

[1] R. Rastogi, S. Ghosh, A. K. Ghosh, K. K. Vaze, and P. K. Singh (2016). Fatigue crack growth prediction in nuclear piping using Markov chain Monte Carlo simulation. *Fatigue and Fracture of Engineering Materials and Structures*, 40(1), 145–156. doi: 10.1111/ffe.12486

[2] P. Paris and F. Erdogan (1963). A Critical Analysis of Crack Propagation Laws. *Journal of Basic Engineering*, 85(4), 528–533. doi: 10.1115/1.3656900

[3] J. Ching and Y. C. Chen (2007). Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection, and Model Averaging. *Journal of Engineering Mechanics*, 133(7). doi: 10.1061/(ASCE)0733-9399(2007)133:7(816)

[4] M. Campi, G. Calafiore, and S. Garatti (2009). Interval predictor models: Identification and reliability. *Automatica*, 45(2), 382–392. doi: 10.1016/j.automatica.2008.09.004