Research Article

# Integrated optimisation model for neutral section location planning and energy-efficient train control in electrified railways


Rui Miao[1,2], Chaoxian Wu[1,2,3], Kuan Zhu[1], Fei Xue[2], Zhongbei Tian[4], Stuart Hillmansen[4], Clive Roberts[4], Shaofeng Lu[1] ✉

[1]Shien-Ming Wu School of Intelligent Engineering, South China University of Technology, 777 Fuxing Ave, Panyu district, Guangzhou, People's Republic of China

[2]School of Advanced Technology, Xi'an Jiaotong-Liverpool University, Suzhou, People's Republic of China

[3]School of Electrical Engineering, Electronics and Computer Science, University of Liverpool, UK

[4]Birmingham Centre for Railway Research and Education, University of Birmingham, UK

✉ E-mail: lushaofeng@scut.edu.cn



**Abstract:** Discontinuously electrified sections, such as neutral sections (NSs) widely exist in modern electrified railways. As a special arrangement of insulator, NSs with no electricity supplies are set up to ensure the two sections are kept electrically separate. This paper proposes a distance-based mixed-integer linear programming (MILP) model to incorporate both NS location planning and energy-efficient train control (EETC) problem and concurrently optimise the NS location and train speed trajectory. The main contribution of this study is that based on the proposed integrated model, a number of case studies are conducted to investigate on the impact mechanism of NS locations on the total energy consumption of train operations. The optimisation results show that the energy saving rate in comparison with the worst cases is ranging from 1.9% to 6.1%% in various scenarios and significant saving rate can be achieved via planning the NS to be located in coasting areas as determined by EETC. In conclusion, the energy-saving effect of the optimal NS location planning on the total energy consumption largely depends on how the NS-triggered forced coasting is located in the journey and NS locations near stations and switching areas of speed limit lead to significant energy increase for bi-directional journeys.


## Nomenclature

*Parameters*

| | |
|---|---|
| $\Delta d$ | preset value for each distance interval, m |
| $\Delta h_i$ | relative altitude difference in the $i$th $\Delta d$, m |
| $\eta_b$ | energy conversion efficiency of braking |
| $\eta_t$ | energy conversion efficiency of traction |
| $A$ | Davis coefficient, kN |
| $A_{max}$ | maximum allowed acceleration in $\Delta d$, m/s$^2$ |
| $B$ | Davis coefficient, kN · s/m |
| $C$ | Davis coefficient, kN · s$^2$/m$^2$ |
| $F_b$ | maximum braking force, kN |
| $F_t$ | maximum traction force, kN |
| $g$ | gravitational constant, m/s$^2$ |
| $i$ | index of the distance interval |
| $K$ | number of $\Delta d$ occupied by NS |
| $L$ | sufficiently large number |
| $m$ | train mass, t |
| $N$ | number of distance intervals $\Delta d$ of the discretised track |
| $n$ | number of $\Delta d$ between station and NS |
| $P_b$ | maximum braking power, kW |
| $P_t$ | maximum traction power, kW |
| $Q$ | maximum number of $\Delta d$ between adjacent NS |
| $T_{total}$ | journey time, s |
| $V_{max}$ | maximum allowed speed in $\Delta d$, m/s |
| $W$ | number of $\Delta d$ between station and NS |

*Variables*

| | |
|---|---|
| $b_{i,d}$ | 0-1 variables to ensure only traction or braking happens at the $i$th $\Delta d$ for the down running direction |
| $b_{i,u}$ | 0-1 variables to ensure only traction or braking happens at the $i$th $\Delta d$ for the up running direction |
| $E_{i,b,d}$ | regenerative energy collected in $i$th $\Delta d$ for the down running direction, kJ |
| $E_{i,b,u}$ | regenerative energy collected in the $i$th $\Delta d$ for the up running direction, kJ |
| $E_{i,t,d}$ | traction energy consumption in the $i$th $\Delta d$ for the down running direction, kJ |
| $E_{i,t,u}$ | traction energy consumption in the $i$th $\Delta d$ for the up running direction, kJ |
| $u_i$ | 0-1 variables to represent the inversion of $z_i$ |
| $v_{i,d}$ | train speed of the down direction when train passes the $i$th $\Delta d$, m/s |
| $v_{i,u}$ | train speed of the up direction when train passes the $i$th $\Delta d$, m/s |
| $x_i$ | 0-1 variables to represent the NS start point |
| $y_i$ | 0-1 variables to represent the NS end point |
| $z_i$ | 0-1 variables to represent the length of NS |

## 1 Introduction

Electrification of railway systems is becoming prevalent around the world especially in an emerging economic entity, such as China. Up until 2018, China has 92,000 km of electrified railway line reaching an electrification rate of 70% [1]. Amongst different electrification systems, 25 kVAC single-phase power supply is preferred and accepted for many overhead traction systems, such as high-speed railways [2]. 25 kVAC railway system is a single-phase load supplied by a medium-voltage three-phase AC grid, leading to an unbalance of the three-phase currents. In order to balance the power supply from different phases, it is proposed to provide each railway section from different phases of power from the main grid. Neutral sections (NSs), also referred to as the 'phase separation sections' or 'dead zone', are the electrical isolation segment between sections. In electrified railways, each phase feeds the catenary for about 30–40 km and NS between each phase is

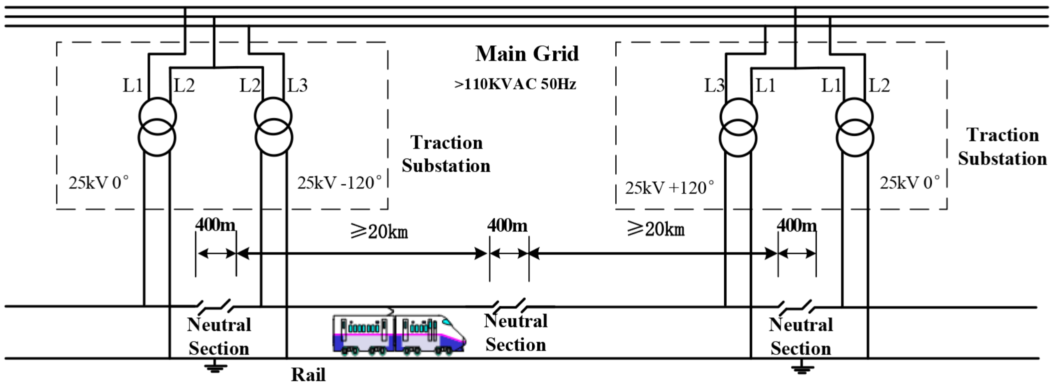


**Fig. 1** *Typical schematic diagram of electrical railway power supply system with NSs. Each phase feeds the catenary for a distance of 20–60 km and the length of NS may vary ranging from a few hundreds metres to more than one thousand metres depending on different systems [3, 4]. NS provides phase separations between different sections*

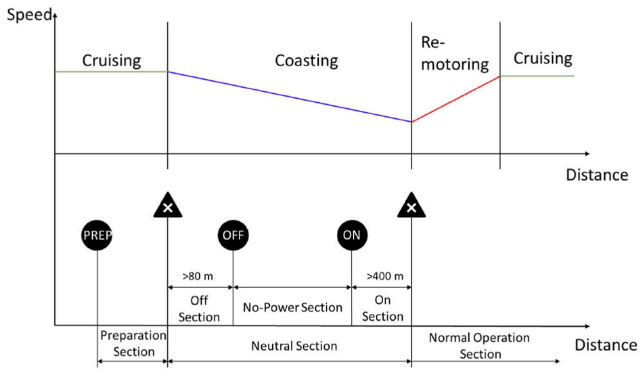


**Fig. 2** *Illustrative schematic diagram of train operations when going through an NS. Before entering NS, the train will be given a warning signal to prepare for power off and main circuit breaker on-board will be cut off until the train leaves NS. Within NS, the train is coasting and re-motoring operation will be applied when the train is leaving NS until the train is back to the original normal speed [4, 7]*

important for isolation and balancing load voltage between phases [3].

Fig. 1 shows a typical topology of electrical railway power supply system with NSs. For AC electrified railways, NS is an important component and needs to be designed systematically due to its impact on the train operations and the resulted energy consumption. In a typical NS, due to the power cut when the train is passing through, coasting operations will be imposed on train and thus speed loss will be resulted, which gives rise to an impact on safety and energy efficiency of train operations. An illustrative schematic on train operation in NS is presented in Fig. 2. According to the optimal train control theory, once the train operation is altered, the total energy consumption will be changed with fixed journey time as the optimal solution of train operation with minimum energy consumption is unique [5, 6]. It is a complicated problem if the train operation and NS location planning are considered concurrently and this demands an integrated model to tackle.

In previous studies related to NS in electrified railways, safety aspect of NS has been popularly investigated in more advanced power feeding systems to avoid power loss in NS [8], to develop static power switch to ensure proper operation on a medium-voltage level [3] and to conduct a reduction on the number of NSs [8]. On the other hand, given the fact that the train comes across a speed loss within NS, some studies considered the impact of NS planning on the train operations. Tang [9] analysed the effect of the NS on speed loss and running time by setting different NS entering speed values and various gradients. Liu and Wang [10] discussed the factors affecting speed and time loss of the locomotive running through a NS. It calculated the speed and time loss of the train passing different types of NS based on the self-developed simulation software. These studies are focused on the NS impact on the speed loss and thus on the train operations. In the meantime, by using the simulation method the train operation has been considered with the NS location planning optimisation. Song *et al.* [4] applied differential evolution algorithm on optimisation of the layout of non-electric section. The objective of it is to obtain a minimum sum of the delayed time. It focuses on the chasing section and analysed the connection between delayed time and different entering speeds as well as different final speeds when the train is leaving NS. Han *et al.* [7] applied genetic algorithm to analyse the relationship and optimise the NS locations. It demonstrated that the optimisation model can improve the energy efficiency and save running time in comparison with the NS location planning currently adopted in the industry. In a short summary, the above-mentioned papers [4, 7, 9, 10] investigate the impact of NS on train operation due to the speed loss when the train is passing through. Simulation method is applied to model the train operations and the train is operating based on the predetermined operation strategies. When NS planning optimisation is not considered as in [9, 10], the focus is mainly on how NS could impact train operations, such as the time cost of journey and train speed. When NS planning optimisation is applied in [4, 7], the optimisation of location planning of NS becomes the main objective while the train operation is modelled using a simulation-based method. In other words, train operation has been realised by the predetermined strategies not necessarily the optimal strategies with minimum energy consumption.

Based on the above discussion, it is seen that NS location planning could impose a significant impact on train operations including the train running time and energy consumption. The current research gap lies on an integrative study on these two closely related problems: NS location planning and energy-efficient train control (EETC). In general, EETC is looking for a series of operation strategies of train to achieve the minimum energy consumption while satisfying the constraints, such as the journey time, gradients, speed limit, etc. The NS location optimisation will undoubtedly affect the EETC since NS location will in turn affect train operation. Therefore, it is a necessity to integrate both EETC and NS location into one integrated optimisation problem so that an optimal solution can be achieved for EETC and NS location planning. Basically, the presentation of the EETC solution is the train speed trajectory which is commonly represented by speed over distance or speed over time curves [11]. EETC is a typical solution for energy-efficient train operation which has been covered by many influential papers in this field. There are generally four categories of methods to address EETC problem: Pontriyagin's-maximum-principle (PMP)-based method [5, 6], mathematical programming-based method [12–15], dynamic programming [16–19], nature inspired heuristic algorithm-based methods [20, 21]. A comprehensive review on EETC methods has been done in [22].

Due to the great flexibility of incorporating other engineering constraints, such as NS location planning constraint, mathematical programming-based method are widely applied to EETC problems with a consideration of other important components or subsystems to form an integrated optimisation model. This kind of method is

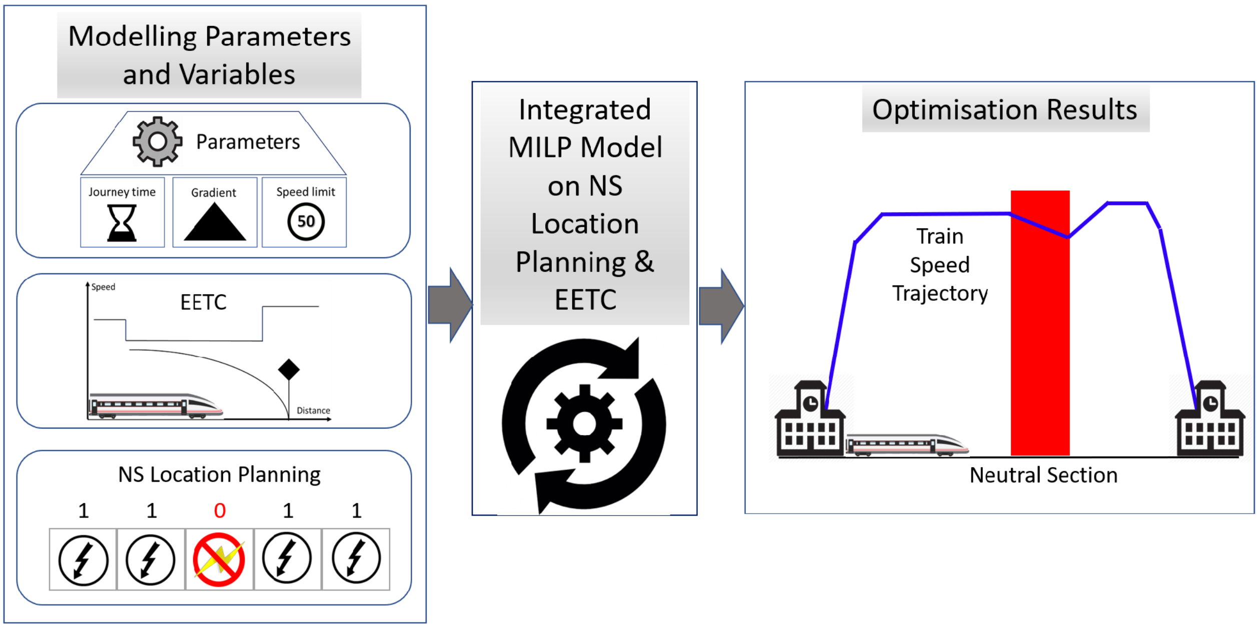


**Fig. 3** *Illustrative schematic diagram of the integrated model. In the model, the NS location is modelled as a series of binary variables, referred to as the NS location variable, to be integrated with the distance-based MILP EETC model directly. With the NS location variable, the model will be able to model the NS-imposed coasting operation on the train during NS section to evaluate the energy cost of the trajectory defined by the EETC model. Optimisation results include the optimal speed trajectory and optimal NS planning scheme*

referred to as the direct methods which transcript the optimal control problem into a non-linear optimisation problem and solve the problem directly [22]. A comparative study of both direct methods and indirect methods applied on train schedules for EETC has been proposed in [23]. Multiple-phase optimal control using Radau pseudospectral methods were proposed to solve the EETC problem considering various signalling constraints in [13] and timetable constraints in [24]. Non-linear programming methods based on the closed-form expressions for optimal train control problem was proposed in [14] and more complex problems including train scheduling and controlling of leader–follower train pairs. In [25], a distance-based mixed-integer linear programming (MILP) model has once been applied to adaptive train trajectory optimisation problem, the results of which are compared and contrasted with the results obtained from the PMP. In addition, an integrated MILP model combining both train operation and on-board energy storage management was proposed in [26]. In a recent paper [27] by the authors, integration of the train energy-efficient control model with the optimal NS location planning is proposed to investigate how the location of one NS will be able to impact the total energy consumption in a simple case scenario. Based on the previous work, this paper considers a novel EETC problem with the NS location planning as a combined problem of infrastructure planning and train operation planning. A comprehensive extension of the previous study proposed in [27] on different case studies on single and multiple NS planning integrated with EETC problem will be presented.

This paper aims to propose a distance-based MILP model to incorporate both NS location planning and EETC problem and concurrently optimise the NS location planning and train speed trajectory. The main contribution of this paper is two folded: first, this paper proposes a novel integrated MILP model to tackle the NS location planning and EETC for both single-NS case and multi-NS cases; second, this paper conducts an in-depth study on the mechanisms of the impact of NS locations on total energy consumption of train operation.

The remainder of this paper is organised as follows: Section 2 illustrate the details of MILP model used in this study. The NS location planning and EETC optimisation results are shown in Section 3. Conclusion and future work of this research is in Section 4.

## 2 Modelling approach

### 2.1 Model introduction and the objective function

An illustrative schematic diagram of the proposed integrated model is shown in Fig. 3. In the model, the inter-station journey is divided into $N$ intervals and the train is assumed to conduct movement with a constant accelerating or decelerating rate in each interval. Particle train model is adopted with no consideration on the train length and mass distribution along the train. The objective is to achieve two sets of speed trajectories for both directions with a total minimum energy consumption. The distance for every interval is labelled as $\Delta d$. The energy consumed or regenerated on the $i_{\text{th}}$ interval is $\Delta E_i$, where $i$ is the distance interval index.

In this model, both the up running and down running directions are considered and the minimum energy for both directions will be considered. $\Delta E_{i,\mathrm{u}}$ and $\Delta E_{i,\mathrm{d}}$ denotes the energy supplied for the up running and down running on the $i$th interval.

The objective function is shown in the following equation:

$$\text{minimise:} \sum_{i=1}^{N} \Delta E_{i,\mathrm{u}} + \sum_{i=1}^{N} \Delta E_{i,\mathrm{d}} \tag{1}$$

### 2.2 Modelling of NS location planning

On each distance interval, there are three variables related to energy consumption and regenerated during train operations for each direction. The net energy consumption on each interval ($\Delta E_{i,\mathrm{u}}$ and $\Delta E_{i,\mathrm{d}}$) contains two parts, the energy provided for train's traction ($\Delta E_{i,\mathrm{t,u}}$ and $\Delta E_{i,\mathrm{t,d}}$) and the energy collected from the regenerative braking ($\Delta E_{i,\mathrm{b,u}}$ and $\Delta E_{i,\mathrm{b,d}}$). In this model, traction energy supplied by the power network is regarded positive and the regenerative energy collected is negative. For each direction, the following two constraints apply:

$$\Delta E_{i,\mathrm{u}} = \Delta E_{i,\mathrm{t,u}} + \Delta E_{i,\mathrm{b,u}} \tag{2}$$

$$\Delta E_{i,\mathrm{d}} = \Delta E_{i,\mathrm{t,d}} + \Delta E_{i,\mathrm{b,d}} \tag{3}$$

0-1 variables $b_{i,\mathrm{u}}$ and $b_{i,\mathrm{d}}$ are introduced to ensure both traction and braking operations cannot occur simultaneously, for both directions, as shown in (4) and (5) as follows:

$$0 \le \Delta E_{i,\mathrm{t,u}} \le b_{i,\mathrm{u}} L, \quad 0 \le -\Delta E_{i,\mathrm{b,u}} \le (1 - b_{i,\mathrm{u}}) L \tag{4}$$

$$0 \leq \Delta E_{i,\mathrm{t,d}} \leq b_{i,\mathrm{d}}L, \quad 0 \leq -\Delta E_{i,\mathrm{b,d}} \leq (1-b_{i,\mathrm{d}})L \tag{5}$$

where $L$ is a sufficiently large number.

Considering the characteristics of NS, no energy is supplied for the train and the train can only coast without traction and braking force applied in these sections. 0-1 variable $z_i$ is introduced to represent the location of the NS. On the intervals where the NS locate, its corresponding $z_i$ equals to 0 and on the other intervals where electric supplying, $z_i$ equals to 1.

$$\Delta E_{i,\mathrm{u}} \leq z_i L, \quad \Delta E_{i,\mathrm{u}} \geq 0 \times z_i \tag{6}$$

$$\Delta E_{i,\mathrm{d}} \leq z_i L, \quad \Delta E_{i,\mathrm{d}} \geq 0 \times z_i \tag{7}$$

In the proposed model, the distance can vary according to constraints and journey length. It is likely one NS may cover more than one interval. If one NS covers more than one interval in this model, the equations set in (8)–(10) ensure the consecutive intervals are occupied by NS to be zero (occupied by NS) while the others be one.

$$\sum_{i=1}^{N} x_i = 1, \quad \sum_{i=1}^{N} y_i = 1 \tag{8}$$

$$\sum_{i=1}^{N} u_i = K \tag{9}$$

$$x_i - y_i = u_i - u_{i-1}, \quad x_i = y_{i+K} \tag{10}$$

where $K$ refers to the number of intervals occupied by the specific NS, and $K\Delta d$ is the length of NS. $x_i$, $y_i$ are two series of 0-1 variable, with one component of each is 1 and others are zero. The location of 1 in $x_i$ denotes the first interval where the NS start and the location of 1 in $y_i$ denotes an interval before which the NS end. $u_i$ is a series of logic variable, containing a consecutive series of 1 between the location of 1 in $x_i$ and $y_i$. $z_i$ is the inversion of $u_i$ and represents the NS location.

If there are two NSs, to simplify the length and location of NS in this model, the length of NS is set to the distance interval, namely $\Delta d$. In other words, $K$ equals to 1 in this model. Equation (11) sets the total number of NSs and limits the maximum range of the NS location between two stations. The distance between NSs can be constrained by using (12)).

$$\sum_{i=n}^{N-n} z_i = N - 2 \tag{11}$$

$$i \in [1, N-W], \quad z_i + z_{i+W} >= 1 \tag{12}$$

where $n$ refers to the number of distance intervals between station and NS. $W = 1, 2, 3, \ldots, Q$, where $Q$ is the maximum number of distance intervals between NSs based on the electric railway power supply systems requirements.

### 2.3 Train motion analysis

The instant speed between intervals for the up direction and down direction are $v_{i,\mathrm{u}}$ and $v_{i,\mathrm{d}}$. The average speed on every interval is $v_{i,\mathrm{ave,u}}$ and $v_{i,\mathrm{ave,d}}$. The train is treated as a particle on every interval, and for both of the directions there exists a relationship defined by the following equation:

$$v_{i,\mathrm{u}} + v_{i+1,u} = 2v_{i,\mathrm{ave,u}}, \quad v_{i,\mathrm{d}} + v_{i+1,d} = 2v_{i,\mathrm{ave,d}} \tag{13}$$

The train is assumed to move with constant acceleration or deceleration rate on every intervals, its speed and acceleration is shown by the following equation:

$$v_{i+1,u}^2 - v_{i,\mathrm{u}}^2 = 2a_{i,\mathrm{u}}\Delta d, \quad v_{i+1,d}^2 - v_{i,\mathrm{d}}^2 = 2a_{i,\mathrm{d}}\Delta d \tag{14}$$

where $a_{i,\mathrm{u}}$ and $a_{i,\mathrm{d}}$ is positive for acceleration and is negative for deceleration of both directions, and it is limited by maximum requirement in (15). The speed is limited by (16).

$$-A_{\max} \leq a_{i,\mathrm{u}} \leq A_{\max}, \quad -A_{\max} \leq a_{i,\mathrm{d}} \leq A_{\max} \tag{15}$$

$$0 \leq v_{i,\mathrm{u}} \leq V_{\max}, \quad 0 \leq v_{i,\mathrm{d}} \leq V_{\max} \tag{16}$$

The time elapsed on each interval is $\Delta t_{i,\mathrm{u}}$ and $\Delta t_{i,\mathrm{d}}$ for both directions, determined in (17). Sum of the time denotes the total time spent on the journey, restricted by maximum time $T_{\mathrm{total}}$ allowed in (18).

$$\Delta t_{i,\mathrm{u}} = \frac{\Delta d}{v_{i,\mathrm{ave,u}}}, \quad \Delta t_{i,\mathrm{d}} = \frac{\Delta d}{v_{i,\mathrm{ave,d}}} \tag{17}$$

$$\sum_{i=1}^{N} \Delta t_{i,\mathrm{u}} \leq T_{\mathrm{total}}, \quad \sum_{i=1}^{N} \Delta t_{i,\mathrm{d}} \leq T_{\mathrm{total}} \tag{18}$$

The kinetic energy changed in intervals must not exceed the one determined by the maximum force and maximum power of the electric machines. Equations (19) and (20) give the constraints.

$$\begin{aligned} &-F_{\mathrm{b}}\Delta d \leq f_{i,\mathrm{u}}\Delta d + mg\Delta h_i + \frac{1}{2}m(v_{i+1,u}^2 - v_{i,\mathrm{u}}^2) \leq F_{\mathrm{t}}\Delta d, \\ &-P_{\mathrm{b}}\Delta t \leq f_{i,\mathrm{u}}\Delta d + mg\Delta h_i + \frac{1}{2}m(v_{i+1,u}^2 - v_{i,\mathrm{u}}^2) \leq P_{\mathrm{t}}\Delta t_{i,\mathrm{u}} \end{aligned} \tag{19}$$

$$\begin{aligned} &-F_{\mathrm{b}}\Delta d \leq f_{i,\mathrm{d}}\Delta d + mg\Delta h_i + \frac{1}{2}m(v_{i+1,d}^2 - v_{i,\mathrm{d}}^2) \leq F_{\mathrm{t}}\Delta d, \\ &-P_{\mathrm{b}}\Delta t \leq f_{i,\mathrm{d}}\Delta d + mg\Delta h_i + \frac{1}{2}m(v_{i+1,d}^2 - v_{i,\mathrm{d}}^2) \leq P_{\mathrm{t}}\Delta t_{i,\mathrm{d}} \end{aligned} \tag{20}$$

where $F_{\mathrm{t}}$ and $P_{\mathrm{t}}$ stand for the maximum force and power for traction, respectively, $F_{\mathrm{b}}$ and $P_{\mathrm{b}}$ stand for the maximum ones for the braking. $f_{i,\mathrm{u}}$, $f_{i,\mathrm{d}}$ denote the drag force defined by the Davis equation in (21) [25].

$$\begin{aligned} f_{i,\mathrm{u}} &= A + Bv_{i,\mathrm{ave,u}} + Cv_{i,\mathrm{ave,u}}^2, \\ f_{i,\mathrm{d}} &= A + Bv_{i,\mathrm{ave,d}} + Cv_{i,\mathrm{ave,d}}^2 \end{aligned} \tag{21}$$

### 2.4 Energy conversion

The following constraints (22) and (23) are based on the Law of Conservation of Energy during energy conversion. Energy is converted into different forms between electrical, kinetic and potential energies with heat generated along the process. $\eta_{\mathrm{t}}$ refers to the traction energy efficiency and $\eta_{\mathrm{b}}$ refers to the regenerative braking energy efficiency

$$\begin{aligned} &\Delta E_{i,\mathrm{t,u}}\eta_t + \frac{\Delta E_{i,\mathrm{b,u}}}{\eta_{\mathrm{b}}} - f_{i,\mathrm{u}}\Delta d \\ &\quad -mg\Delta h_i - \frac{1}{2}m(v_{i+1,u}^2 - v_{i,\mathrm{u}}^2) \geq 0 \end{aligned} \tag{22}$$

$$\begin{aligned} &\Delta E_{i,\mathrm{t,d}}\eta_t + \frac{\Delta E_{i,\mathrm{b,d}}}{\eta_{\mathrm{b}}} - f_{i,\mathrm{d}}\Delta d \\ &\quad -mg\Delta h_i - \frac{1}{2}m(v_{i+1,d}^2 - v_{i,\mathrm{d}}^2) \geq 0 \end{aligned} \tag{23}$$

Since the energy consumed or regenerated must not exceed the maximum value determined by the maximum allowed traction/ braking force and power, (24)–(27) give the constraints.

$$0 \leq \Delta E_{i,\mathrm{t,u}} \leq \frac{F_{\mathrm{t}}\Delta d}{\eta_t}, \quad 0 \leq \Delta E_{i,\mathrm{t,d}} \leq \frac{F_{\mathrm{t}}\Delta d}{\eta_t} \tag{24}$$

$$-\eta_b F_{\mathrm{b}}\Delta d \leq \Delta E_{i,b,\mathrm{u}} \leq 0, \quad -\eta_b F_{\mathrm{b}}\Delta d \leq \Delta E_{i,b,\mathrm{d}} \leq 0 \tag{25}$$

$$\Delta E_{i,\mathrm{t,u}} \le \frac{P_\mathrm{t}\Delta d}{v_{i,\mathrm{ave,u}}\eta_t}, \quad \Delta E_{i,\mathrm{t,d}} \le \frac{P_\mathrm{t}\Delta d}{v_{i,\mathrm{ave,d}}\eta_t} \tag{26}$$

$$\Delta E_{i,b,\mathrm{u}} \ge -\frac{\eta_b P_\mathrm{b}\Delta d}{v_{i,\mathrm{ave,u}}}, \quad \Delta E_{i,b,\mathrm{d}} \ge -\frac{\eta_b P_\mathrm{b}\Delta d}{v_{i,\mathrm{ave,d}}} \tag{27}$$

### 2.5 Piecewise linear

This model has non-linear relationship between speed-related variables on each distance interval for both directions, as a result, SOS2 (special ordered sets of type 2) is employed to realise the piecewise linear. SOS2 is a method proposed by IBM and is used in many other proposed models [25, 26]. The maximum train speed is set at 80 m/s (288 km/h).

$$v = 80\alpha_1 + 79\alpha_2 + 78\alpha_3 + \cdots + 2\alpha_{79} + 1\alpha_{80} \tag{28}$$

$$v^2 = 80^2\alpha_1 + 79^2\alpha_2 + 78^2\alpha_3 + \cdots + 2^2\alpha_{79} + 1^2\alpha_{80} \tag{29}$$

$$v^{-1} = 80^{-1}\alpha_1 + 79^{-1}\alpha_2 + \cdots + 2^{-1}\alpha_{79} + 1^{-1}\alpha_{80} \tag{30}$$

where $\alpha_k$ is a series of variables of SOS2. Adjacent two of them have a positive value, with a sum of 1, and others equal to zero. To realise this in MILP, $\lambda_k$ is introduced in the following constraints:

$$0 \le \alpha_k \le 1 \tag{31}$$

$$\sum_{k=1}^{80} \alpha_k = 1, \tag{32}$$

$$\alpha_{k+1} + \alpha_k - \lambda_k \ge 0 \tag{33}$$

$$\sum_{k=1}^{79} \lambda_k = 1. \tag{34}$$

## 3 Result and discussion

### 3.1 Introduction on case studies

In this section, four case studies will be proposed to demonstrate the robustness of the model and provides an insightful understanding on the integrated optimisation model of both NS location planning and EETC. It is aimed at further understanding a general relationship between various NS locations and minimum energy consumption based on the NS setup and thus to offer in-depth analysis on the mechanism of the impact of NS location on EETC and tried to provide a general principle in designing the NS locations under various engineering scenarios.

Table 1 lists the parameters of traction systems of a typical high-speed train adopted in the modelling. Four case studies are conducted and their general information is provided as follows:

• *Scenario 1:* a simple case scenario without gradient and speed limit is conducted and the result and discussion are shown in Section 3.2.
• *Scenario 2:* a case study with speed limit on a level track is conducted and presented in Section 3.3, in order to further understand the impact of speed limit on the problem.
• *Scenario 3:* integrated NS planning and EETC problem on a complex case scenario with speed limit and gradient is demonstrated in Section 3.4. The impact of gradients on the optimisation model is also discussed in the same section.
• *Scenario 4:* a case study with two NSs containing speed limit and gradient will be conducted in Section 3.5.

### 3.2 One NS planning on a level track and without speed limit

In this section, the energy consumption of train operation with one NS planned will be investigated and no influence of the gradient of the track and the speed limit will be present over the journey. This case shows the minimum energy consumption with a NS at various locations between two stations. The length of the track is 21 km and the length of the NS is assumed to 525 m. The parameters for the train used in this part are listed in Table 1. With the changing location of NS, its corresponding minimum energy, which is the sum of both directions, is shown in Figs. 4 and 5. For both directions, the time of the case in Fig. 4 is fixed in 500 s and the time of the case in Fig. 5 is fixed in 457 s. For both cases with different journey time, the energy consumption increases when the NS moving towards the station. This is because when the NS moving approaching the stations, the forced coasting operation is applied in the acceleration or deceleration area as defined by the normal optimal train control strategy, and this leads to the more increase of energy consumption compared to the cases with the NS location placed in the coasting and cruising zones.

**Table 1** Parameters based on a typical high-speed train adopted in modelling

| Parameters | value |
|---|---|
| mass *m*, t | 480 |
| maximum force $F_t$, $F_b$, kN | 250 |
| maximum power $P_t$, $P_b$, kW | 9,376 |
| traction efficiency $\eta_\mathrm{t}$ | 0.9 |
| braking efficiency $\eta_\mathrm{b}$ | 0.7 |
| Davis coefficient *A*, kN | 10.689 |
| Davis coefficient *B*, kN · s/m | 0.28906 |
| Davis coefficient *C*, kN · $\mathrm{s}^2/\mathrm{m}^2$ | 0.011282 |

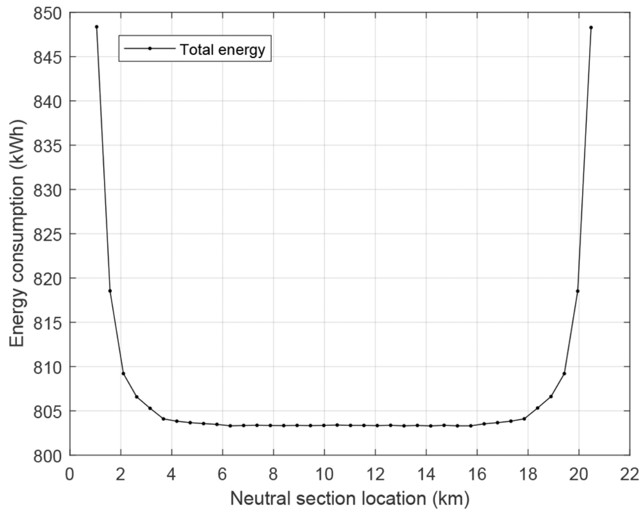


**Fig. 4** *Relationship between minimum energy consumption and the location of NS for a 21 km journey and a total time of 500 s. The location of NS has less impact on the minimum energy consumption of train operations, indicated by the flat area along the curve. Maximum energy consumption is found for both sides of the journey. Due to the simplicity of the model constraints, the curve demonstrates a high degree of symmetry*

Considering up and down running directions in the same model, the total energy is the sum of the energy consumption in both directions. The minimum consumption happens where the NS locates in the intermediate part of the inter-station journey and the energy consumption increases when the NS moves towards two ends of the journey. For the shorter time cases as shown in Fig. 4, no solution can be found in the area closing to the stations (0–4 and 18–21 km), which means the NS is not possible in these positions within this fixed time (457 s), where the train does not have enough time to adjust the speed before coasting in the NS section so as to arrive at the station on time. This observation implies that setting NS at acceleration and deceleration areas may result in significant speed loss and thus journey time increase. In some extreme cases, this would lead to feasibility issues of train operations, for example the train is not able to arrive at the next station on time.

Energy consumption is minimum for the NS locations in the intermediate three locations, in the former case with time in 500 s.

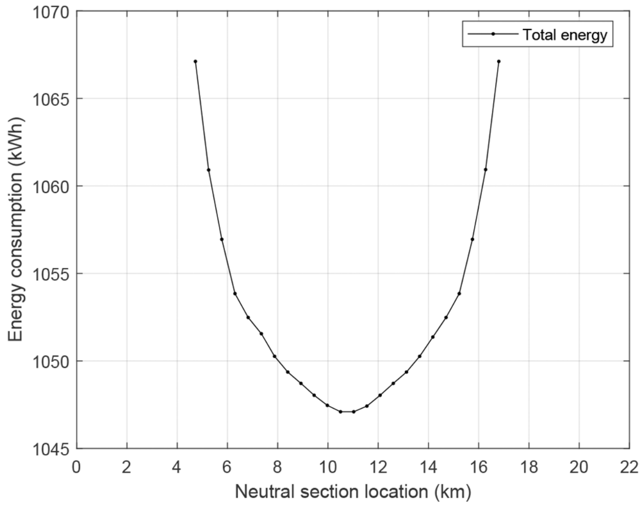


**Fig. 5** *Relationship between minimum energy consumption and the location of NS for a 21 km journey and a total time of 457 s. Due to the requirement of less journey time, the trains need to conduct more acceleration and deceleration to clime up to a higher level of cruising speed, thus the location of NS imposes more impact on energy consumption. It is due to this reason, when NS is located at locations closes to stations, it is likely that the optimal solution cannot be achieved as the train is unable to reach the destination on time due to significant speed loss within those NSs*

**Table 2** Results and comparison of Scenario 1: simple scenario without speed limit and gradients

| Journey time, s | Max energy consumption, kWh | Mean energy consumption, kWh | Min energy consumption, kWh | Saving rate[a] % |
|---|---|---|---|---|
| 500 | 848.36 | 807.19 | 803.30 | 5.3, 0.5 |
| 457 | 1067.10 | 1052.80 | 1047.10 | 1.9, 0.5 |

[a]Saving rate contains two values, the first of which is the saving rate compared with the max energy consumption and the second is saving rate
compared with the mean energy consumption.

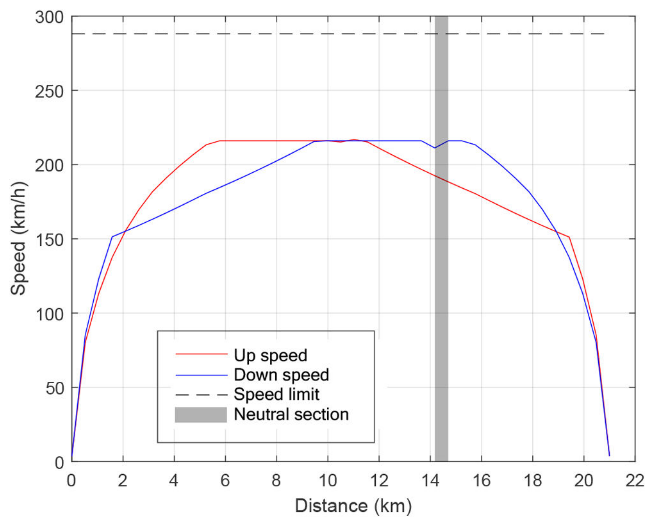


**Fig. 6** *Speed trajectory with NS fixed at 14 km for a journey time of 500 s*

For the latter case, it has two intervals in the same minimum consumption with time in 457 s. More tests have been done and it can be concluded that for longer running time or lower average speed, the available range of locations for minimum energy consumption is wider, and is in the middle part of the path. In other words, for high-speed railways, applying a NS in the middle between two stations gives the minimum energy consumption. The comparison of the energy consumption is tabulated in Table 2. For

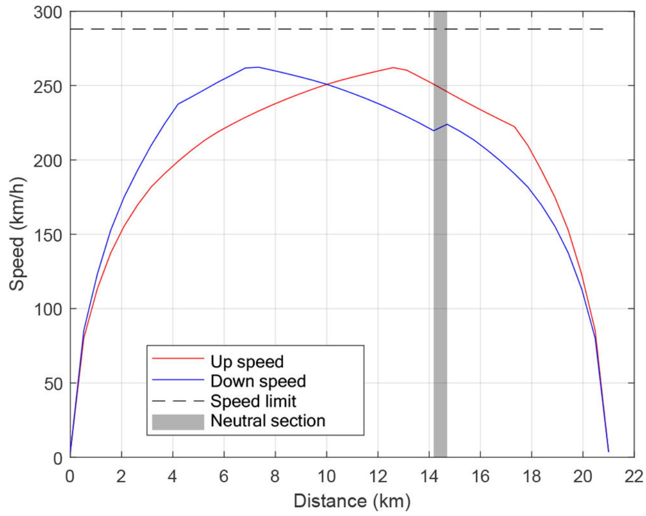


**Fig. 7** *Speed trajectory with NS fixed at 14 km for a journey time of 457 s*

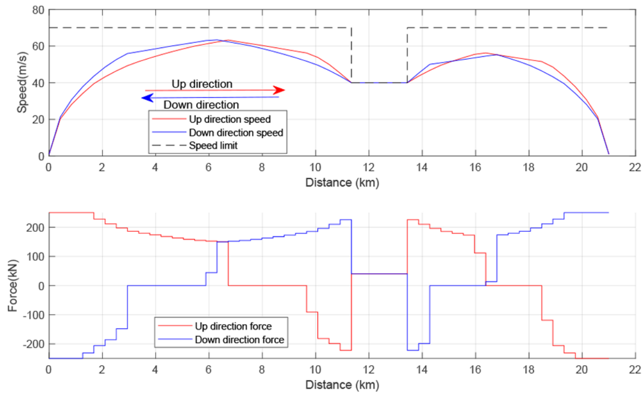


**Fig. 8** *Optimal speed trajectory and corresponding traction forces profiles with no NS on a 21 km level-track journey with speed limits*

the case with a journey time being 500 s, the mean value of the energy consumption with various NS locations in this range is 807.19 kWh, and the optimal value is 803.30 kWh, which reduces 0.5% of energy consumption is saved compared to the mean value in this example. The maximum value of the energy consumption within the range is 1067.10 kWh. Compared to this, the optimal result can reduce 5.3% of energy consumption. Similar results can also be seen in the case with journey time being 457 s, where the optimal solution brings the highest energy efficiency.

To investigate why different journey times result in the different optimal range of the NS location, two case studies with the NS location fixed at 14 km for journeys with 500 and 457 s are conducted. The optimal speed trajectory for both cases are presented in Figs. 6 and 7. For both cases, it is seen that the train in NS undergoes a forced coasting operation. For the 500 s case, NS is located at the coasting section for the up direction and cruising section for the down direction. For the 457 s case, NS is located in the coasting section for the up direction and acceleration section for the down direction. It can be speculated that the impact due to the NS location is higher when the NS is located at the acceleration section than at the cruising area. In the 500 s case, NS is located in the coasting/cruising section, while it is coasting/acceleration section in 457 s, the impact of NS on the increase of minimum energy for the former case (500 s) would be less. Later studies in Section 3.3 will more clearly reveal a general principle that NS would be much preferred to be located at coasting or cruising sections, where speed loss will be minimum, for both directions to achieve the minimum impact on the optimal train operations defined by EETC, so as to reduce the increment of the minimum energy consumption.

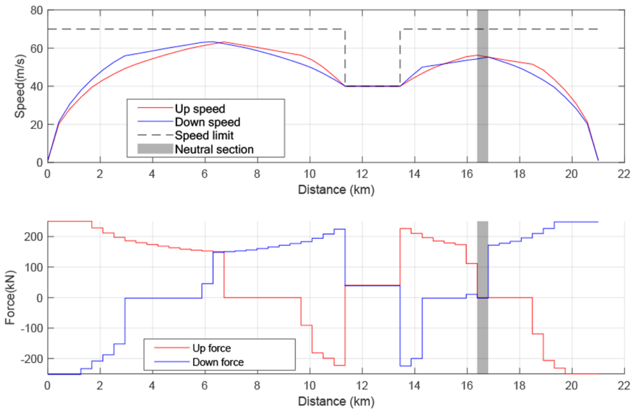


**Fig. 9** *Optimal speed trajectory and corresponding traction forces profiles with one NS on a 21 km level-track journey with speed limits. The optimal NS location is planned at a distance range from 16.38 to 16.8 km which is also a coasting operation distance range defined by the optimal train control strategy shown in Fig. 8*

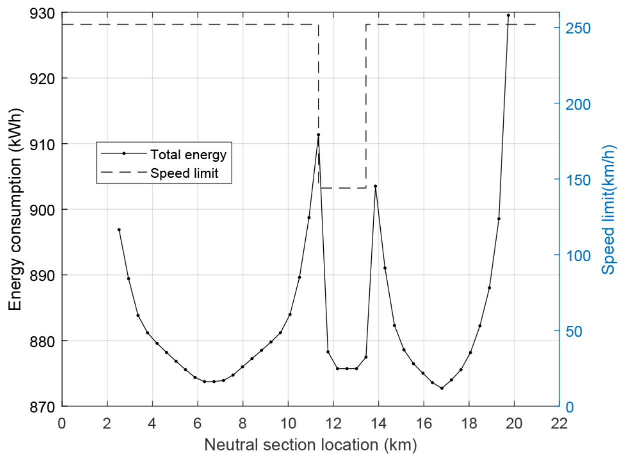


**Fig. 10** *Minimum energy consumption versus different NS location along the journey. The optimal NS location is found to be in the centre of distance range (16.8 km) with a constant speed limit where the coasting operation is conducted for the optimal train control strategy with no NS planned along the journey as shown in Fig. 8. Four peaks of energy consumption are resulted for four NS positions, namely 11.34, 13.44 km, the beginning and ending spots of the journey, respectively*

**Table 3** Result comparison of Scenario 2: level track with speed limit and Scenario 3: complex track with speed limit and gradient

| Scenario | Max energy consumption, kWh | Mean energy consumption, kWh | Min energy consumption, kWh | Saving rate[a] % |
|---|---|---|---|---|
| 2 | 929.55 | 882.64 | 872.74 | 6.1, 1.1 |
| 3 | 599.84 | 573.80 | 568.54 | 5.2, 0.9 |

[a]Saving rate contains two values, the first of which is the saving rate compared with the max energy consumption and the second is the saving rate
compared with the mean energy consumption.

### 3.3 One NS location planning with speed limits on the level track

Figs. 8 and 9 demonstrate the optimisation results for cases with and without NS planned with speed limit imposed on a 21 km level track.

Fig. 10 shows the minimum energy consumption versus different NS locations for a level track with changing speed limits. The length of NS is 420 m. NS is planned to be located in the 21 km inter-station journey. Modelling parameters are listed in Table 1. As shown in Fig. 10, it can be understood that less energy is consumed when NS is located in the middle of one speed limit section, i.e. a distance range with a constant speed limit. For example, the minimum energy consumption occurs when the NS is planned at 16.38–16.8 km. This is in the coasting section for both directions with zero and very low traction forces, as shown in Fig. 9. The forced coasting operation applied in these sections has minimum effect on the traction and speed trajectory leading to the minimum energy consumption. In the meantime, more energy is resulted when NS is planned at locations where the train is likely to conduct acceleration and deceleration operations, such as the beginning and ending of the journey for both running directions and the changing spot of speed limit. For example, the maximum energy is resulted at the distance range of 10.92–11.34 km which is one of the switching points of the speed limits and the train for both directions conducts braking and motoring operations. Table 3 shows the energy efficiency of the proposed NS location and control strategy of both scenarios. Without the model and control strategy, the maximum and mean energy consumption of various NS locations are higher than the resulted minimum energy consumption.

The mechanism behind the results would be that NS should impose the minimum impact on the optimal train control strategies under the same journey time constraints to achieve a more energy efficient train control strategy. The more impact imposed by NS, the higher energy consumption will be resulted by unnecessary motoring or braking caused by the speed loss due to the forced coasting at NS. Thus, the higher energy consumption will be resulted when the NS is planned in the traction and braking sections, which makes the speed trajectory to stray away from the optimal trajectory. These areas are usually close to the station or somewhere near the speed limit switching point. Note that our study also finds out with the increase of the journey time, less impact will be resulted by the speed limit change as the train does not need to accelerate or deceleration at proximity of speed limit switch point.

This important finding would provide some general guidance for engineering practice when planning the NS. If train operation can be known in advance, certain range of the journey should be avoided so that NS is not located in the areas where the train is likely to conduct more acceleration and deceleration than coasting and cruising.

### 3.4 Single NS location planning in a complex case

For cases with more complex engineering constraints like gradient and speed limit, the optimisation model can achieve the optimal solution with the optimal NS location and the optimal train speed trajectory. Speed limit and gradients are shown in the top figure of Fig. 11, while the energy consumption versus different NS locations is shown in the bottom figure. The optimal NS location and the optimal speed trajectory has been demonstrated in Fig. 12. In Fig. 11, each point on the curve refers to that when put the NS on the location of its $x$-axis, the $y$-axis reflects its optimal energy consumption. This energy consumption is the sum of the energy from both directions over the entire journey. It shows that the minimum energy consumption distribution can be complex under the complex constraint including speed limit, gradient and fixed operation time.

The most energy-efficient location of the NS is where the coasting operation is executed in the optimal speed trajectory without NS for both directions. If such a section does not exist, it is suggested to avoid the area where the train is highly likely to conduct traction or braking. Locations to be avoided for the NS location planning during the construction of the railway includes:

- Areas near the stations where the traction or braking operation for both directions are expected.
- Areas where the speed limit changes, where the train may conduct traction to increase speed or decrease the speed.

In this study, the impact of gradient is found to be insignificant. This is very much due to the fact that for high-speed train, the gradients are much less influential on the train operation and thus

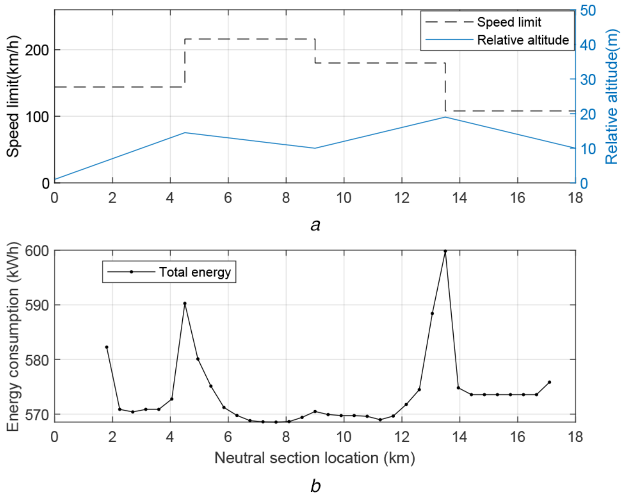


**Fig. 11** *Neutral section location planning optimisation result in a complex case scenario. The top figure demonstrates the gradient and speed limit and the bottom figure shows the energy consumption in responding to the position of each NS location planning scheme. Two peaks appear at the position of around 4 and 14 km where speed limits go through the significant changes*

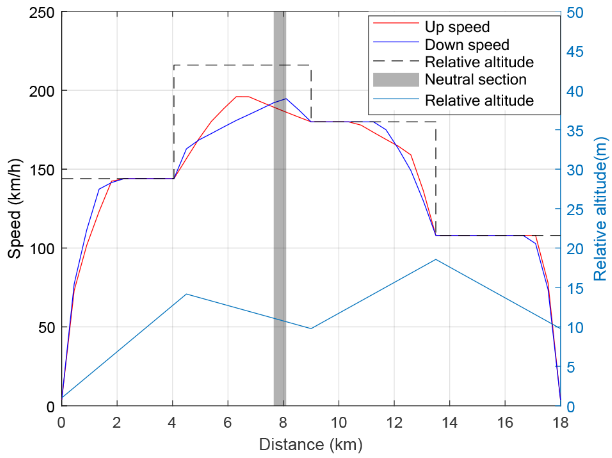


**Fig. 12** *Optimal speed trajectories for both up and down directions and the optimal NS location as shown in the grey bar. The optimal NS location is set the coasting section for both directions, thus the forced coasting due to NS will not lead to lower energy consumption compared to cases with other NS locations*

the NS location will be less impacted by the gradient of the location along the journey. For example, depending on the location of NS, the flat, the uphill and downhill gradients can be found. For the case when the NS is located on the downhill section, it can be predicted to have less impact on the speed loss in comparison to the cases of uphill section. However, since the total energy is considered based on both directions, the impact of gradient on the NS location will be counteracted on both directions leading to much insignificant impact from the gradient on the total energy with different NS location. A brief investigation has been done on the 21 km and 500 s journey with a different set of gradients and no speed limits. The minimum energy profile over different NS locations with two different journey gradient profiles has been demonstrated in Fig. 13. The detailed energy efficiency comparison is also tabulated in Table 4, where the energy consumption is different due to the different gradients introduced while the energy-saving rate remains the same, namely 4.0% reduction compared with maximum energy consumption and 0.3% reduction compared with the mean value, which shows an insignificant influence of route gradient on the NS location planning.

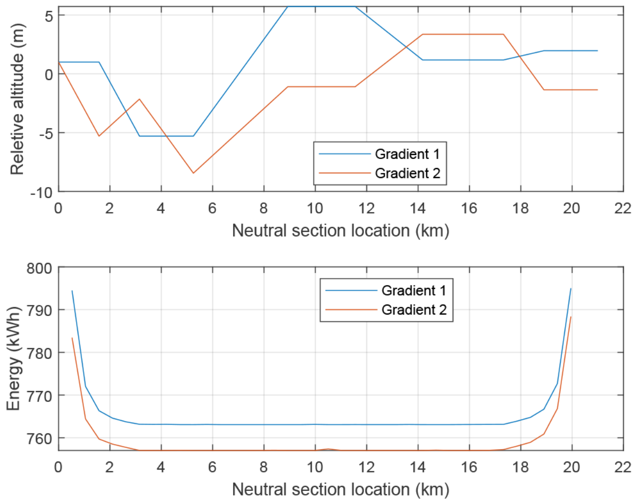


**Fig. 13** *Minimum energy profile over different NS location with two different journey gradient profiles. The total journey time is 500 s and total journey length is 21 km. It is seen that while the total energy consumption is affected by the gradient profile, the gradient change along the journey has insignificant impact on the energy profile, as reflected by the very similar change pattern. This is a different observation from Figs. 10 and 11, where the speed limit profile is seems to impose significant impact on the total energy consumption profile*

**Table 4** Result comparison of cases with different gradients and no speed limit

| Gradient | Max energy consumption kWh | Mean energy consumption kWh | Min energy consumption kWh | Saving rate[a] % |
|---|---|---|---|---|
| 1 | 795.01 | 765.56 | 763.09 | 4.0, 0.3 |
| 2 | 788.40 | 759.33 | 757.03 | 4.0, 0.3 |

[a]Saving rate contains two values, the first of which is the saving rate compared with the max energy consumption and the second is saving rate
compared with the mean energy consumption.

### *3.5 Two-NS location planning in a real-world long track case*

A two-NS location planning problem has been studied over a long track with both speed limit and gradients constraints. The parameters in Table 1 are adopted. It has 150 intervals and the length of the NS is 400 m. The minimum distance between NSs is set 20 km. The length of the journey is 60 km. As shown in Fig. 1, a schematic diagram of the distance between two NSs has been shown.

Fig. 14 shows the optimal speed trajectory and corresponding traction and braking forces with two NSs presented. The running time for both running directions is fixed at 1235 s. As can be observed in this figure, the distance between NSs is 25.2 km. Distance range of 22.8–23.2 km is the optimal location of the first NS. For the up running direction, it is where the train begins to coast. For the down running direction, it is the coasting section of the train. Another optimal NS is in 48.4–48.8 km. On the contrary, it is where the train begins to coast for the train on the down running direction and it is the coasting section for the up running direction. The traction effort of the train is reduced to be zero when the train is passing through the two neutral sections.

## 4 Conclusions

In this paper, the backgrounds, significance, motivation and objective of the study on an integrated optimisation model for both NS location planning and EETC has been discussed, and it is followed by the mathematical modelling approach, a number of case studies and discussion on the optimisation results. Both single and multiple NS location planning with different engineering constraints has been covered on bi-directional journeys between

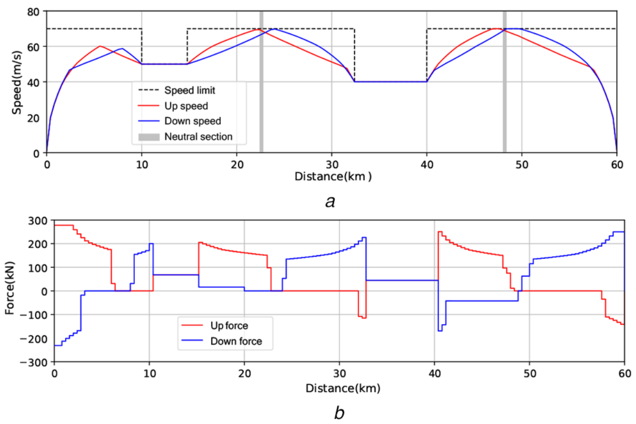


**Fig. 14** *Optimal speed trajectory and traction forces for up and down directions with optimal two-NS location planning results*

***(a)*** It shows the optimal speed trajectories with two-NS location planning results for both up and down directions. The planned optimal NS locations are in the location with a constant speed limit where the train speed on both directions are conducting coasting mode, ***(b)*** It shows the corresponding train traction and braking forces for both directions with different train operation modes presented

two stations. General mechanisms between NS location planning and total energy cost are illustrated and discussed. The main conclusions are drawn as follows.

• The impact of NS location planning is due to the forced coasting operation on the train within the NS location. When the train is going through the NS, coasting operation will be applied leading to speed loss of the train and possible deviation from the optimal train operation.
• The significance of NS location planning on the total energy consumption largely depends on how the NS-triggered forced coasting is located along the inter-station train speed trajectory. The NS location is preferred to be located to certain areas where trains conduct coasting or cruising as determined by optimal train control strategies defined by EETC.
• It is found that as affected by normal train operations, NS location near the stations and switch areas of speed limit causes much more energy increase for the bi-directional journeys than others areas. In general, gradients have little influence on the train energy consumption.

In future, the integrated model can be further extended by incorporating more electrical constraints on the setup of NS location, e.g. minimum distance between each NS. A network-wide NS location planning is also of interest for the next step of the study. All of these relies on the availability of more field data and more awareness on the huge energy-saving potential on a close integration of the infrastructure planning and operation planning in railway industry.

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