

Upgrading Conventional Distribution Networks by Actively Planning Distributed Generation Based on Virtual Microgrids

Xiaotong Xu, Fei Xue, Xiaoliang Wang, Shaofeng Lu, Lin Jiang and Ciwei Gao

Abstract—In addition to active energy management, this paper proposes active planning as another critical feature of active distribution networks (ADNs). To develop this set of tasks, this paper introduces a three-layer active planning framework consisting of a physical layer, cyber layer and socioeconomic layer. Furthermore, a three-step developing strategy for ADNs based on virtual microgrid (VM) is put forward. Then, according to this framework, this paper focuses on a specific and fundamental issue that often arises: the optimal allocation of distributed generation (DG). A two-stage scheme based on VMs is a proposed solution. In the first stage, VM boundaries are determined based on the characteristics of network structure. Using the identified VM boundaries as constraints, a bi-level hierarchical optimization method is applied to determine the optimal DG allocation in the second stage. The proposed method is verified in the popular PG&E 69-bus distribution network.

Index Terms—Distributed generation (DG), electrical coupling strength (ECS), active planning, genetic algorithm (GA), virtual microgrids (VMs).

I. INTRODUCTION

CONVENTIONAL distribution networks (CDNs) are facing many challenges from increasing load demand, limited expansion space, environmental issues, and aging infrastructures [1]. Power supply in CDNs has obvious passive characteristics: (1) Lack of distributed energy devices that can be actively controlled; (2) Power supply passively follows variation of demands without flexible operational modes; (3) Passive accommodation of variation in loads and renewable power generation (wind or solar power) depends on modulation of power sources from high voltage transmission networks.

Therefore, a number of studies have been performed that target the development and operation of future distribution networks, such as smart distribution networks [2] and active distribution networks (ADNs) [3] (active distribution systems

[4]). ADNs emphasize the capabilities of active energy management for future distribution networks [5]. ADNs are expected to have the following active characteristics: (1) Abundant controllable distributed energy devices; (2) Active energy management (such as active power flow management, active voltage regulation and demand side management) that can supply power with more flexibility, improved efficiency and better reliability; (3) Accommodation of power variation in loads and renewable power generation by active control of distributed energy devices, and supply of flexibility to variation in high voltage transmission networks.

Many planning strategies for constructing ADNs have already been proposed. In [4], a multi-level model was introduced to characterize high penetration of DG and storage devices. It resolved operation and planning issues by minimizing cost, maximizing reliability and renewable DG penetration. In [6], a multistage coordinated planning of ADN development was proposed. The location, capacity and installation time of new distribution lines, substations, capacitor banks and voltage regulators were determined while minimizing investment costs and considering a variety of active network management schemes during the planning stage. In this work, the allocation of DG was assumed known. In [7], an expansion planning of an active distribution system was proposed. Topology changes, DG integration, rewiring and new load points were determined by applying different methodologies. In [8], a co-optimization model is developed by considering both investment decisions and operation strategies. The model determined the optimal reconfiguration of the ADN and the DG output.

Although plenty of works have targeted ADN planning, most of them have ‘Passive’ features: (1). Passive continuation of conventional centralized decision-making and hierarchical controls via a unique monopoly unit. Future trends in distributed control via multiple control units which have equal positions are not considered. (2). Technical or economical

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objective functions without considering the impact (restrict or support) of planning on the capabilities in active management. (3). Decisions on deployment of DG and decisions on network operation are made by different entities; conflicting interests may impact the effects of active management. (4). More concern is placed on expanded or new ADNs, while little concern is placed on utilizing existing structures and devices to improve active management capabilities.

In the last decade, Cyber-physical systems (CPSs) have been considered in the planning of ADNs, and ADNs are considered CPS systems [9], [10]. The interaction of physical parts and cyber parts in the planning ensures that ADNs are more efficient during operation and more responsive to customers [11]. Energy transformation and consumption are driven by the requirements of social production and life. Therefore, in [12], an extended version of CPS is proposed: extended cyber-physical-socioeconomic system (ECPS). ECPS is a more comprehensive version that can reflect the interaction relationship among different aspects in a system. However, most existing studies for ADN planning only focus on physical systems, few of them have involved novel control decision strategies in cyber systems, but none of them considered future market evolution and novel trading mechanism in socioeconomic systems.

Future power systems may have fully distributed control via decentralized decision-making with swarm intelligence. This is the Web of Cells (WoC) concept proposed by the European Liaison on Electricity grid Committed Towards long-term Research Activities (ELECTRA) project [13]. Similar to the WoC concept, many studies propose to upgrade CDNs by partitioning a distribution network into several smaller units [14]–[20]. In some papers, these partitioned units are called virtual microgrids (VMs) [17], [18]. VMs have similar characteristics to normal microgrids. VMs are autonomous systems that can make a connection with other VMs through intelligent devices such as soft open point (SOP) [21]. This is consistent with the decentralized control of the future power system vision, and can be seen as a possible way to develop ADNs. However, all of these studies consider to construct VMs only in physical systems, but following conventional centralized decision-making mechanism in cyber systems, as well as traditional organization with unique monopoly distribution companies in socioeconomic systems.

The high penetration of DG in distribution networks is a widely accepted trend in future power grid technology. According to some existing policies, such as the Ontario's Standard Offer Program in Canada, customers are allowed to own DG units, and the associated cost of owning DG units, such as investment, operation and maintenance costs, are paid by customers but not local distribution companies [22]. Currently, there are almost no DGs in the majority of CDNs. If DG integration is completely determined by customers and not preplanned well, this may lead to disorder and poor efficiency in future operations, which may conflict with the characteristics required for future distribution networks. Therefore, the reasonable allocation of DG is an important topic.

So far, many DG allocation methods with different objectives

have been proposed [23]–[25]. However all these models followed conventional organization structure and relations from socioeconomic perspective. So decisions on installation of DGs are made by individual customers and the optimization from global perspective is difficult to be accepted and performed by these independent decision-makers.

Based on the discussion above, the contributions of this paper is summarized as:

(1) In addition to active energy management, the concept of active planning as another critical characteristic of ADN is introduced.

(2) A three-layer active planning framework based on ECPS and VMs is proposed to upgrade CDNs to ADNs.

(3) A more specific and fundamental planning issue, the allocation of DG is studied with a two-stage scheme based on VMs.

The paper is organized as follows: Section II introduces the idea of active planning. A three-layer active planning framework and a two-stage strategy for optimizing DG allocation are introduced. Section III introduces the algorithm for partitioning based on a functional community structure. Section IV explains the DG modelling methods and the operating scenarios adopted in the DG allocation process. The algorithm for DG allocation is explained in Section V. In Section VI, the partitioning and DG allocation are verified in the PG&E 69-bus distribution network. The conclusion is drawn in Section VII.

II. ACTIVE PLANNING IN DISTRIBUTION NETWORKS

This paper proposes a new concept called "Active Planning" for ADNs. The planning from CDNs to ADNs should reflect following 'Active' characteristics: (1). It has a guidance from a long-term development perspective to cover all physical, cyber and socioeconomic systems and their relations. (2). It is oriented to maximization of capability in future active energy management. (3). It actively adapts to and utilize the existing characteristics of CDNs, including network structure and devices.

According to characteristics of active planning and ECPS, we propose a three-layer framework for upgrading CDNs to ADNs, based on VMs as presented in Fig. 1. In this framework, VMs are basic units that can determine the decentralized control of ADNs.

In the bottom layer of the three-layer framework (physical system), a CDN is divided into several units. The allocation of new resources and devices, such as DG and other energy storage devices, is optimized to form VMs. Moreover, based on VM systems, the integration of other energy resources (e.g., gas, thermal and cooling) can contribute to form local energy networks. Through the interconnection of these local energy networks, global energy internet could be constructed.

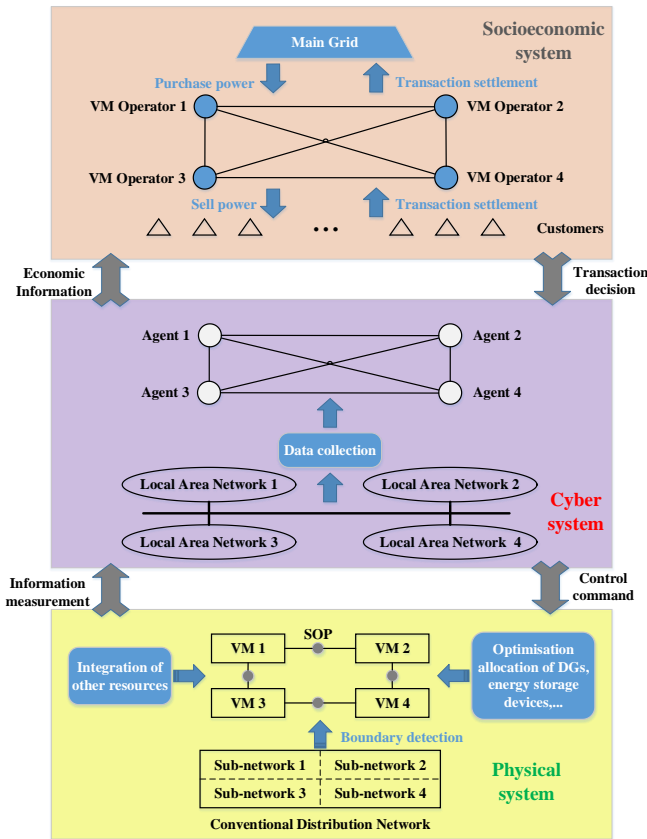


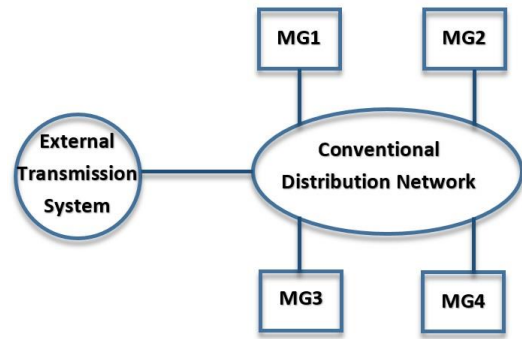
Fig. 1. An active planning framework based on VMs.

In the top layer of the three-layer framework (socioeconomic system), considering the decentralized structure of VMs, each VM should have its own VM operator (VMO) who takes similar responsibilities as an EV aggregator [26]. A VM should be invested and constructed by its corresponding VM operator who will get profits from system operation. So the cost of DG installation and operation will be paid by VM operators, but not customers. Therefore, the optimized DG allocation from system perspective could be easily accepted and performed by VM operators. VM operators could receive the information from cyber systems and send the transaction decision to cyber systems to perform transactive energy control which is similar to networked microgrids [27]. VM operators are independent and autonomous in operating VMs.

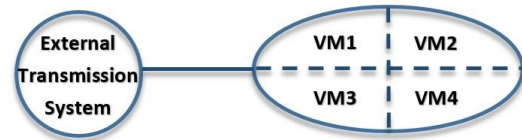
The intermediate layer of this framework is the cyber system, which joins the other two layers together; its ability of information collection and data analysis affects the behavior and performance of the whole system. As the physical system is constructed based on a VM, the measurement layout and information utilization of the cyber system should also be consistent with VM structures. Furthermore, the cyber layer should also support information in decision making of socioeconomic interactions, so its models should be consistent with the socioeconomic relations. Therefore, a multi-agent system could be constructed in this layer. Intelligent agents can help VM operators in decision making.

Before emergence of VMs, there are already some studies about Networked Microgrids (NMGs) or Multiple Microgrids (MMGs) [27]-[29]. NMGs or MMGs are often confounded with

VMs. However, they have quite different characteristics and problems. Fig. 2. indicates the general difference between NMGs and VMs. NMGs are connected through a CDN and VMs are partitioned from a CDN. A distribution network operator for CDN with individual assets and interests are responsible for coordination in NMGs, but VMs are decentralized operated by multiple independent VM operators with equal technical and socioeconomic positions. NMGs are newly constructed normal MGs with clear boundaries, but VMs are virtually partitioned with possibly dynamic boundaries. Compared with limited normal NMGs, the scale of existing CDNs is much larger which may not be impacted much by these NMGs. Therefore, NMGs cannot completely solve the upgrading from CDNs to ADNs.



(a). Networked Microgrids System



(b). Virtual Microgrids System

Fig. 2. Difference between NMGs and VMs.

However, upgrading CDNs to ADNs based on VMs cannot be accomplished at one stroke. This paper proposes a three-step developing strategy as shown in Fig. 3. In step 1, VMs are constructed only in the physical layer. The whole system operation is still performed by centralized control in the cyber layer under supervisory of a unique power distribution company in the socioeconomic layer. Most previous studies about VMs are at step 1 [14]-[20]. With increasing penetration of large-scale Distributed Energy Resources (DER), centralized control may have lots of challenges and difficulties. Therefore, in step 2, decentralized control by multiple agents could be performed in the cyber layer. But all agents belong to the same unique power distribution company in the socioeconomic layer. This is only to improve power supply efficiency and reliability but irrelevant to any socioeconomic issues. In step 3, each agent will represent a VMO in the socioeconomic layer who has independent entitlements and interests. Transactions could be reached between VMOs to perform transactive energy control in the cyber layer [27][28].

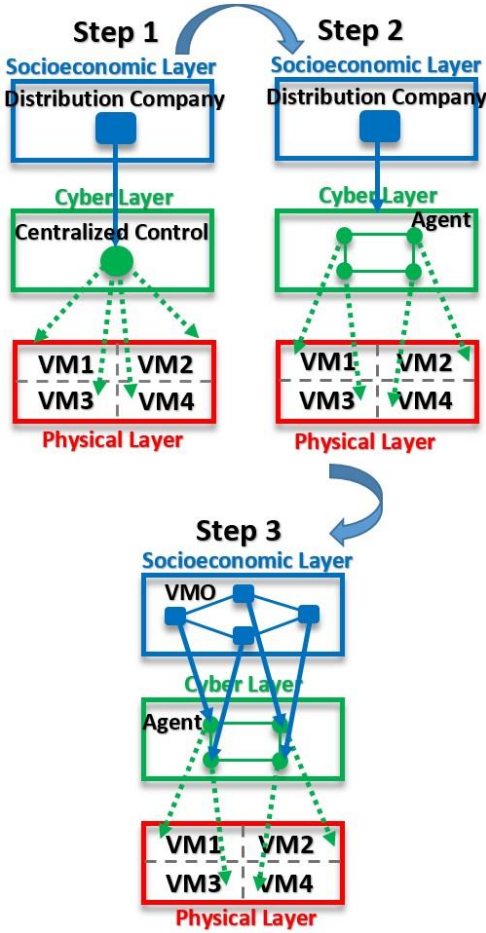


Fig. 3. Three-step developing strategy for ADN based on VMs.

In the physical layer of Fig. 1, a comprehensive planning scheme is required to determine the VM structure and resource allocation of the DGs. Operating states of distribution networks may depend on two planning factors: 1). connections and layout among buses and lines, i.e., network topological structure and corresponding parameters; 2). deployment of distributed energy devices including location, type, and size, such as DG and energy storage devices. For most CDNs, 1 is a passive factor because network structures are established facts without large-scale network upgrading; therefore, they have to be passively accepted. 2 is an active factor since there are still no large-scale penetrations of DG in most CDNs. So a factor of 2 can be actively optimized during planning. Therefore, we propose a two-stage strategy to optimize DG allocation.

Stage 1: To determine VM boundaries according to the functional community structure of the original networks.

Stage 2: Based on the VM boundaries identified in stage 1, to perform DG allocation by optimizing the self-adequacy and capabilities of active management in each VM.

III. VMS PARTITIONING BASED ON FUNCTIONAL COMMUNITY STRUCTURE

A. Electrical Coupling Strength

Complex network theory has been widely applied to issues in power networks [30]–[32], which have typical features of complex networks (such as scale-free and small-world features)

[33]. In complex network theory, an adjacency matrix is frequently used in structure analysis:

$$A_{vw} = \begin{cases} 1 & \text{there is an edge between vertex } v \text{ and } w \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

However, this matrix may not be appropriate to describe structural features of electrical networks because some physical characteristics in electrical engineering cannot be reflected, e.g., electrical distance and transmission capacity [34].

In electrical engineering, the equivalent impedance Z_{vw} between bus v and bus w could represent the electrical distance between them [35].

$$Z_{vw} = Z'_{vv} - 2Z'_{vw} + Z'_{ww}, \quad v, w \in B, \quad (2)$$

where Z'_{vv} , Z'_{ww} and Z'_{vw} are elements in the impedance matrix of power networks.

Considering Power Transfer Distribution Factor (PTDF) and power flow limits of lines, equivalent power transmission capacity between any two buses v and w , can be calculated as [36]

$$C_{vw} = \min \left(\frac{P_{l_{\max}}}{|F'_{vw}|} \right), \quad v, w \in B, l \in L, \quad (3)$$

where $P_{l_{\max}}$ is the maximum power flow limit of line l . F'_{vw} is the power change on line l when a unit power is injected to bus v and withdrawn from bus w .

The working target of power grids is to transmit electrical energy with more power and less losses, so shorter electrical distance and larger transmission capacity could represent tighter electrical coupling between two buses. Therefore, we defined Electrical Coupling Strength (ECS) as [34]:

$$\bar{E}_{vw} = \left| \alpha \bar{Y}_{vw} + j\beta \bar{C}_{vw} \right|, \quad v, w \in B, \quad (4)$$

$$\bar{C}_{vw} = \frac{C_{vw}}{C}, \quad v, w \in B, \quad (5)$$

$$\bar{Y}_{vw} = \frac{Y_{vw}}{Y} = \frac{1/Z_{vw}}{Y}, \quad v, w \in B, \quad (6)$$

where \bar{C} is the average power transmission capacity, and \bar{Y} is the average equivalent admittance, α and β are proportion coefficients, and $\alpha + \beta = 1$. Based on these coefficients, the extent to that these two quantities (the electrical distance and transmission capacity) affect the ECS is adjustable.

With ECS, the traditional adjacency matrix in (1) was upgraded as ECS matrix for power networks in [34]:

$$A_{vw}^E = \begin{cases} \bar{E}_{vw} & \text{there is a transmission line between bus } v \text{ and } w \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

However, in a real power grid, the equivalent impedance and transmission capacity defined in (2) and (3) may exist between any two buses, whether directly connected or not. The coupling strength may be stronger even if there is no line directly connecting them. Therefore the ECS defined in [34] can be improved via

$$A_{vw}^{UE} = \bar{E}_{vw} \quad (\text{for any bus } v \text{ and bus } w). \quad (8)$$

Therefore, unlike a conventional adjacency matrix, which is sparse, **all non-diagonal elements in the improved ECS**

matrix in (8) would be non-zero.

Community detection is a typical issue in the research of complex networks. Nevertheless, community is defined from a topological perspective according to the density of connections that can be called **topological community structures**. However, partitioning VMs is accomplished from the perspective of network functionality. VMs should have some partitioned areas that have strong internal coupling strength to perform power transmission function. Hence, we could call them **functional community structures**. The Newman Fast Algorithm is a popular method in the detection of topological community structures [37], [38] according to traditional adjacency matrix. Based on that algorithm, then a specific partitioning for a network can be quantitatively evaluated by so-called modularity. As a general rule: the bigger the modularity, the better the partitioning result. The modularity is, therefore, expressed as

$$Q = \frac{1}{2m} \sum_{vw} \left[A_{vw} - \frac{k_v k_w}{2m} \right] \delta(C_v, C_w), \quad (9)$$

$$m = \frac{1}{2} \sum_{vw} A_{vw}, \quad (10)$$

where A_{vw} is the element in the v th row and w th column of the adjacency matrix, k_v and k_w are the total number of edges connecting to vertex v and w , respectively, and C_v and C_w are the communities that vertex v and w respectively belong to. The symbol $\delta(C_v, C_w)$ is the Kronecker delta and is equal to 1 if vertex v and w are in the same community, otherwise, it is equal to 0. m is the total number of edges in the network.

However, considering the specific electrical characteristics, it is not appropriate to detect functional community structures directly by this modularity. Therefore, in this paper, modularity is upgraded based on an improved ECS matrix that the ECS matrix characterized by Equation (8) as a so-called electrical modularity:

$$Q_e^U = \sum_{v,w \in B} \left[\frac{A_{vw}^{UE}}{2M} - \frac{A_v^{UE}}{2M} \cdot \frac{A_w^{UE}}{2M} \right] \delta(C_v, C_w), \quad (11)$$

$$A_v^{UE} = \sum_{i \in B} A_{vi}^{UE}, \quad (12)$$

$$M = \frac{1}{2} \sum_{vw} A_{vw}^{UE}, \quad (13)$$

where A_{vw}^{UE} is the element in the v th row and w th column of the improved ECS matrix. A_v^{UE} is the sum of ECS associated with bus v . M is half of the sum of ECS in the network.

Based on the electrical modularity defined in (11), the following detailed partitioning process is similar to the process found in [34]. Although the partitioning processes are similar, there is a major difference that only the coupling strength between directly connected buses are considered in [34], but in this paper, the ECS between any two buses are considered (all non-diagonal elements are non-zero).

IV. MODELLING OF DISTRIBUTED GENERATION, LOADS AND OPERATING SCENARIOS

In this section, the modelling of loads and DGs, including dispatchable and non-dispatchable DGs, is explained. Based on

these models, adopted operating scenarios and relevant probabilities are also discussed.

A. Modelling of Dispatchable DG

As the power generated by dispatchable DG can be adjusted, the output power of dispatchable DG is fully controllable within its capacity limit [17], [39].

B. Modelling of Non-dispatchable DG

Wind power generation is considered as renewable-based non-dispatchable DG in this paper. Considering the fluctuating and intermittent nature of wind power, most of the wind turbine generator modelling is based on probability density functions (PDFs) [22], [40]. In this paper, output power of wind turbine generators is determined from historical wind speed data, which is modeled by the Johnson SB PDF.

$$\begin{cases} f(x) = \frac{\psi}{\lambda \sqrt{2\pi z(1-z)}} \exp\left(-\frac{1}{2}\left(\gamma + \psi \ln\left(\frac{z}{1-z}\right)\right)^2\right) \\ \zeta \leq x \leq \zeta + \lambda \\ z = \frac{x - \zeta}{\lambda} \end{cases}, \quad (14)$$

where ψ and γ are shape parameters; ζ is the location parameter; λ is a scale parameter.

TABLE I
PARAMETERS OF WIND TURBINE GENERATORS

Scenario	W1	W2	W3	W4
Season	Spring	Fall	Summer	Winter
γ	0.40832	0.1866	0.48423	-0.0199
ψ	0.46673	0.49059	0.55561	0.48906
λ	0.97881	0.98015	0.97956	0.95746
ζ	-0.0765	-0.00616	-0.00874	0.005568
Probability	1/4	1/4	1/4	1/4

Because wind speed has seasonal characteristics, four seasons are considered. The parameters of a Johnson SB PDF, with relevant probability of different seasons, is listed in Table I [16]. If we assume equally partitioned season, the probability of each season is 1/4.

C. Modelling of Loads

A load model can be constructed based on Weibull PDF.

$$f(x) = \frac{\varphi}{\sigma} \left(\frac{x - \mu}{\sigma}\right)^{\varphi-1} e^{-\left(\frac{x - \mu}{\sigma}\right)^{\varphi}}, \quad (15)$$

where φ is the shape parameter; σ is the scale parameter; μ is the location parameter.

Based on the load data of IEEE-RTS in [39], the difference in seasons and between weekday and weekend is also considered. Therefore, 8 scenarios are modeled. For each scenario, a Weibull PDF represents the deviation of actual load data from mean load value. Parameters of the Weibull PDF and probability are shown in TABLE II [16]. With 5 weekdays and 2 weekend days of each week, for each season, the probability of a weekend is $1/4 \times 2/7 = 1/14$ while the probability of a weekday is $1/4 \times 5/7 = 5/28$.

TABLE II
PARAMETERS OF LOADS

SEASON	SCENARIO	φ	σ	μ	PROBABILITY
SPRING	L1	2.4226	0.09934	-0.08812	5/28
	L2	1.7979	0.05353	-0.04758	1/14
FALL	L3	5.247	0.22676	-0.20872	5/28
	L4	5.1698	0.16188	-0.14876	1/14
SUMMER	L5	8.2088	0.21547	-0.20307	5/28
	L6	17.046	0.29313	-0.28402	1/14
WINTER	L7	8.2088	0.21547	-0.20307	5/28
	L8	17.046	0.29313	-0.28402	1/14

V. DG ALLOCATION BASED ON VM PARTITIONING RESULTS

In this paper, allocation of DG in VMs is performed, including so-called dispatchable and non-dispatchable DG. For each type, DG location, number and size need to be optimized. In this section, a bi-level hierarchical optimization model is constructed accordingly.

A. Bi-level Optimization Model

The resources for active management during operation depend on their allocation during planning. The active planning proposed in this paper maximizes the capabilities of resources for active energy management in system operation. The impact that planning decisions have on active management, during system operation, should be evaluated. Therefore, a bi-level optimization model consisting of an outer planning optimization and an inner operating optimization is shown in Fig. 4.

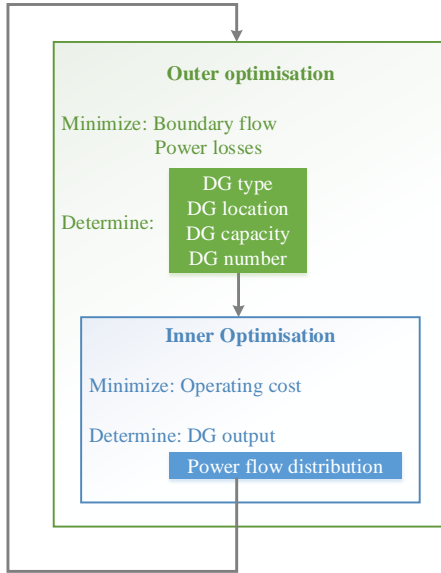


Fig. 4. Bi-level hierarchical optimization.

In the inner optimization, active energy management is represented by Optimal Power Flow (OPF) which adjusts the output power of DG to minimize generation cost.

$$OF_{inner} = \sum_{i=1}^{N_b} (Cost_{nd_DG} \cdot Power_{nd_DG,i}^{real} + Cost_{d_DG} \cdot Power_{d_DG,i}^{real}), \quad (17)$$

where N_b is the total bus number. $Cost_{nd_DG}$ is the operating and maintenance cost of non-dispatchable DG and $Cost_{d_DG}$ is the operating and maintenance cost of dispatchable DG. $Power_{nd_DG,i}^{real}$ and $Power_{d_DG,i}^{real}$ are real output power of non-dispatchable DG and dispatchable DG on bus i . If there is no corresponding DG on bus i , then $Power_{nd_DG,i}^{real} = 0$, and $Power_{d_DG,i}^{real} = 0$.

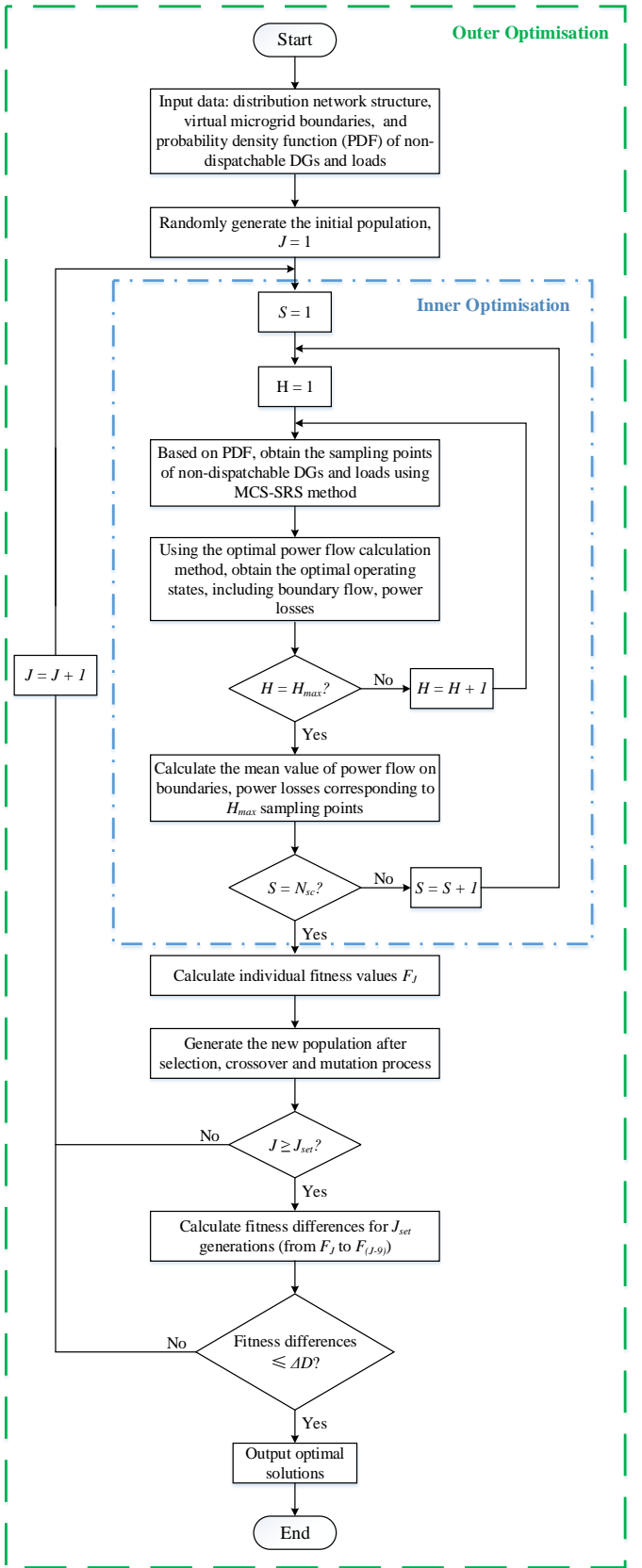


Fig. 5. Flowchart of the DG allocation optimization

The outer planning optimization determines the optimal DG allocation (DG type, location, number and capacity). The effect of active management is represented by power losses managed by inner OPF, so one target for this optimal planning is to

minimize power losses by allocating optimal resources. Furthermore, as most studies consider self-adequacy as one critical feature of VM [16]-[18], another target is to minimize the power flow on boundaries. That is to say, the self-adequacy of each VM should be guaranteed by reasonable DG allocation and provide better independency and security in system operation.

$$OF_{outer} = \sum_{j=1}^{N_{sc}} (P_{boundaries,j} + P_{loss,j}) \cdot T_j \quad (16)$$

where N_{sc} is the total scenario number. $P_{boundaries,j}$ is the total active power of all the VM boundaries in scenario j . $P_{losses,j}$ is the total power loss in scenario j . T_j is the probability of scenario j . Based on the results of the method introduced in section III, the VM boundaries are detected. Then by minimizing power flow on boundaries, VMs may keep independence and flexibility for operation.

For dispatchable DG, the output power can be controlled by any value that is within the rated capacities. A wind turbine generator, a possible resource for non-dispatchable DG in active management, is supposed to be adjusted to any output power within the limits determined by the real-time wind speed.

B. Constraints

In practical engineering, investment in planning is often limited by the actual conditions and design targets. So this limitation is approximately modeled as a constraint of total DG capacity in the whole network:

$$\sum_{i=1}^{N_b} C_{DG,i} \leq R. \quad (18)$$

where N_b is the total bus number and $C_{DG,i}$ is the DG capacity on bus i . If there is no DG on bus i , $C_{DG,i}=0$. R is the total DG capacity limitation.

Similar to normal microgrids, VMs may also have two operating modes connected to the main grid or islanding operation. To guarantee power supply to critical loads in islanding operations, the total capacity of dispatchable DG in each VM should be larger than the total peak value of critical loads:

$$\sum_{i=1}^{N_{bVM}} C_{d_DG,i} \geq K\% \cdot C_{tot_L}, \quad (19)$$

where N_{bVM} is the total bus number in any VM, $C_{d_DG,i}$ is the dispatchable DG capacity of bus i . If there is no dispatchable DG on bus i , $C_{d_DG,i}=0$. $K\%$ is proportional coefficient, which is the percentage of peak critical loads to the total peak loads. C_{tot_L} is total loads in the VM.

Constraints in Power flow calculation:

$$P_{DG,i} - P_{load,i} = |V_i| \sum_{k=1}^{N_b} |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \quad (20)$$

$$Q_{DG,i} - Q_{load,i} = |V_i| \sum_{k=1}^{N_b} |V_k| (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$$

where $P_{DG,i}$ and $Q_{DG,i}$ are the active and reactive DG output power on bus i . $P_{load,i}$ and $Q_{load,i}$ are the active and reactive loads on bus i . V_i and V_k are the voltage on bus i and bus k . G_{ik} and B_{ik}

are the real and imaginary parts of the i th row and k th column in the admittance matrix. θ_{ik} is the voltage phase angle difference, and $\theta_{ik} = \theta_i - \theta_k$.

Assuming bus 1 is the slack bus, the voltage and angle on the slack bus are

$$\begin{aligned} V_{j,1} &= 1 \\ \delta_{j,1} &= 0 \end{aligned} \quad (21)$$

The bus voltage limitation is

$$V_i^{\min} \leq V_{j,i} \leq V_i^{\max}, \quad \forall i \in \{1, 2, 3, \dots, N_b\}, \quad (22)$$

where V_i^{\min} and V_i^{\max} are the minimum and maximum voltage limitation on bus i .

Feeder power flow limitation

$$P_{ik} \leq P_{ik}^{\max} \quad \forall i, k \in \{1, 2, 3, \dots, N_b\}, \quad (23)$$

where P_{ik}^{\max} is the maximum power flow limitation on the feeder connecting bus i and bus k .

C. Optimization Algorithm

With large-scale decision variables and their complex relationships, the outer planning optimization is performed by a Genetic Algorithm (GA). The inner operating optimization can be implemented as an Optimal Power Flow model to determine output power of all DG given the allocation of DGs from outer planning decision. The flow chart of the optimization algorithm is shown in Fig. 5. J is iteration number. S is the number of scenarios and H is the sampling point number of a Monte Carlo Simulation (MCS). H_{max} is the total number of sampling points. J_{set} and AD are setting values of GA.

In GA, each possible solution is called a chromosome. Each variable is a gene of the chromosome, and the number of genes should be equal to the number of variables. In this paper, DG location, number, type and capacity are optimized and determined. The following three vectors are used to represent each chromosome,

$$\begin{aligned} chromosome &= [T_{yDG,1} \ T_{yDG,2} \ \dots \ T_{yDG,i} \ \dots \ T_{yDG,N_b}]; \\ & [C_{d_DG,1} \ C_{d_DG,2} \ \dots \ C_{d_DG,i} \ \dots \ C_{d_DG,N_b}]; \\ & [C_{nd_DG,1} \ C_{nd_DG,2} \ \dots \ C_{nd_DG,i} \ \dots \ C_{nd_DG,N_b}] \end{aligned} \quad (24)$$

where $T_{yDG,i}$ is the type of DG on bus i . Three numbers are defined to indicate the type and presence of DG.

$$T_{yDG,i} = \begin{cases} 0 & \text{There is no DG allocated on bus } i \\ 1 & \text{Dispatchable DG is allocated on bus } i \\ 2 & \text{Non-dispatchable DG is allocated on bus } i \end{cases} \quad (25)$$

As in [42], a flowchart for the GA implementation describes the optimization process, which is shown in Fig. 5.

With uncertainty in wind power and loads, probabilistic power flow (PPF) is implemented for the inner operating optimization [43]. The MCS based on simple random sampling (MCS-SRS) is one of the most popular and effective PPF methods. The deterministic power flow calculation of the sampling points, that is randomly selected according to PDFs, is repeated several times. In this paper, MCS-SRS method is used to generate the points of non-dispatchable DG available power and loads according to relevant PDFs; the selected points

of non-dispatchable DG represent the maximum power limit they can supply at that moment. Therefore, the optimal power flow calculation is implemented to optimize the power output of these DGs by using the tool provided by MATPOWER, which is a package for solving steady-state power system simulation and optimization. After the optimal power flow calculation, the optimal decision from inner optimization for each scenario can be utilized in calculating the fitness result of GA in outer optimization. A detailed process is shown in Fig. 5.

VI. CASE STUDY

In this section, the active planning method is tested on the PG&E 69-bus distribution network, where the data are available in [44].

A. Partitioning Results

To calculate the power transmission capacity, we assume the maximum power limit for all lines is the same due to same material and linewidths. The proportion coefficients α and β in the definition of improved ECS are both adopted as 0.5.

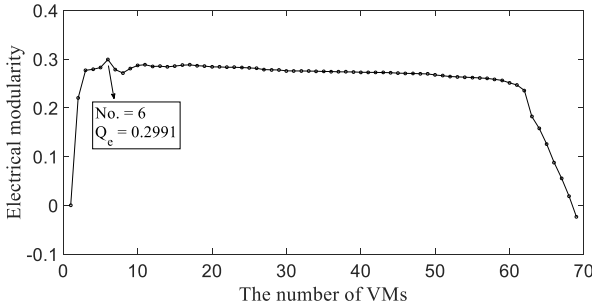


Fig. 6. Variation of electrical modularity with the number of VMs

Fig. 6 shows the variation of the electrical modularity with different number of partitioned VMs. The maximum electrical modularity is 0.2991 when the number of VMs is 6. Because a larger electrical modularity indicates a better partitioning results, this result with maximum electrical modularity is used to determine the VM boundaries, as shown in Fig. 7 and Fig. 8.

In [16], a self-adequate microgrid system with dynamic boundaries is proposed. The boundaries are determined by the lines of least power flow. In [16], a partitioning process is developed that is based on a network with preset conditions of DG allocation. However, as discussed before, a more common factor in most CDNs is that the large-scale penetration of DG has not occurred yet. Therefore, a more reasonable approach is to optimize the DG allocation after boundary detection. Additionally, the electrical modularity of the partitioning that results in [16] is 0.2689, which is smaller than the result in this paper (0.2991). Therefore, the partitioning result of this paper is more consistent with the structural characteristics of the network.

B. DG Allocation for different scenarios

To directly reveal the relation between planning and capabilities of active energy management, three different scenarios are constructed:

Scenario 1: Conventional distribution network which has no DG penetration and all power is supplied by the main grid.

Scenario 2: DG allocation without inner optimization by OPF of figure 2. This is to test the planning without considering impacts on capabilities and effects of active energy management.

Scenario 3: DG allocation by bi-level optimization model of figure 2. This is to test how active planning can improve the capabilities and effects of active energy management.

B-1: Scenario 1

The total power loss in this mode without DG deployment is 0.0514 kW.

B-2: Scenario 2

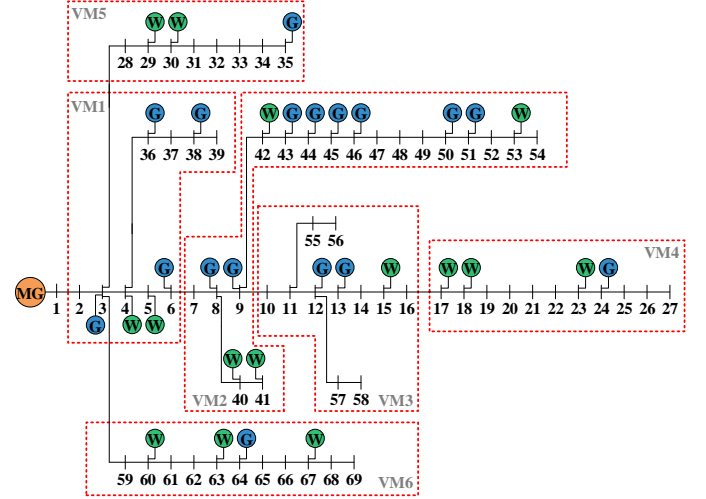


Fig. 7. Optimized DG allocation in scenario 2.

Two types of DG, wind turbine generators and biomass generators, are considered. Wind turbine generators and biomass generators represent non-dispatchable and dispatchable DGs, respectively. Thus, we assume that all buses can be selected as possible locations of DGs. The total capacity limitation of DG allocation R is assumed to be 4000 kW, and in each VM, the critical load accounts for 30% (which is the value of K in Equation (19)) of the total load. Considering different design requirements, these parameters can be adjusted to different values. The possible candidate capacities of all DGs are 50 kW, 100 kW, 150 kW, 200 kW. The maintenance and operating costs of wind turbine generators and biomass generators are 0.01 \$/kWh and 0.025 \$/kWh [45]. The parameter settings for optimization are shown in Table III.

TABLE III
PARAMETER SETTING FOR OPTIMIZATION

Population size	Mutation rate	Crossover rate	N_{sc}	G_{set}	ΔD
100	0.6	0.001	8	10	10^{-8}

In this case, active energy management represented by OPF is not performed. Wind turbine generators generate the maximum power according to weather conditions while biomass generators produce constant power according to their capacity rating. In the inner optimization, Newton Raphson power flow, instead of optimal power flow calculation, is applied to get DG allocation results. The power loss after DG optimization is 0.0148 kW, which is smaller than scenario 1 (0.0514 kW).

TABLE IV
CAPACITIES OF BIOMASS GENERATORS IN SCENARIO 2

Location (bus No.)	3	6	8	9	12	13

Capacity (kW)	200	50	150	50	150	50
Location (bus No.)	24	35	36	38	43	44
Capacity (kW)	100	50	200	200	100	150
Location (bus No.)	45	46	50	51	64	
Capacity (kW)	200	150	50	50	100	

DG allocation result is shown in Fig. 7. G and W stand for biomass generators and wind turbine generators, respectively. The capacities of wind turbine generators and biomass generators are listed in Table IV and Table V, respectively. The total capacities of wind turbine generators and biomass generators are 1650 kW and 2000 kW, respectively. The capacity of DGs with renewable resources accounts for 45% of the total DG capacity.

TABLE V
CAPACITIES OF WIND TURBINE GENERATORS IN SCENARIO 2

Location (bus No.)	4	5	15	17	18
Capacity (kW)	100	50	100	50	150
Location (bus No.)	23	29	30	40	41
Capacity (kW)	150	150	50	150	150
Location (bus No.)	42	53	60	63	67
Capacity (kW)	50	150	100	150	100

B-3: Scenario 3

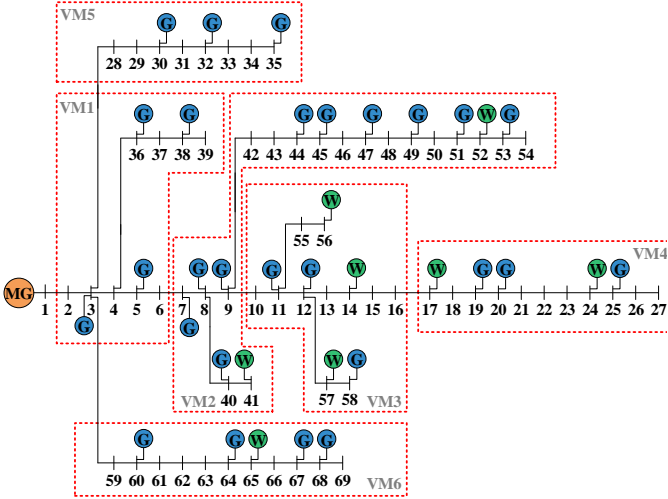


Fig. 8. Optimized DG allocation in scenario 3.

In this case, the proposed bi-level optimization is applied to obtain the optimal allocation of the DGs. The parameter setting is the same with that in scenario 2. The output of DGs is optimized via an optimal power flow to represent active energy management. By considering impacts on capabilities and effects of this active management, the DG allocation result is quite different from that of scenario 2. Fig. 8 presents the optimal DG distribution, and the number of biomass generators (27) is much bigger than that of wind turbine generators (8). According to the capacities of DG, listed in TABLE VI and TABLE VII, the total capacity of DGs, including wind turbine generators and biomass generators, is 4000 kW, which does not exceed the total DG limitation (4000 kW). Capacities of wind turbine generators and biomass generators are 1000 kW and 3000 kW, respectively. The capacity of wind turbine generators accounts for 25% of the total DG capacity. The power loss of scenario 3 is 0.0041 kW, which is much smaller than that in scenario 2 (0.0148 kW) and scenario 1 (0.0514 kW). The capability of active energy management supported by active planning is much better in scenario 3. Compared to the optimization results in scenario 2, a better optimization result is

achieved as the value of objective in scenario 3 (0.0693 kW) is approximately 11 times smaller than that in scenario 2 (0.7638 kW). Active planning in scenario 3 can support better resources allocation to guarantee more effective active management and better self-adequacy in operation. However, the proportion of DG with renewable resources in this case (25%) is smaller than that in scenario 2 (45%).

TABLE VI
CAPACITIES OF BIOMASS GENERATORS IN SCENARIO 3

Location (bus No.)	3	5	7	8	9	11	12
Capacity (kW)	150	150	150	100	150	100	150
Location (bus No.)	19	20	25	30	32	35	36
Capacity (kW)	50	100	50	100	50	50	150
Location (bus No.)	38	40	44	45	47	49	51
Capacity (kW)	200	100	100	100	150	150	150
Location (bus No.)	53	58	60	64	67	68	
Capacity (kW)	150	50	50	150	100	50	

TABLE VII
CAPACITIES OF WIND TURBINE GENERATORS IN SCENARIO 3

Location (bus No.)	14	17	24	41	52	56	57	65
Capacity (kW)	150	150	100	50	150	150	100	150

C. Comparison of optimization results with different proportion of renewable DG

According to the analysis of scenario 3, a significant improvement of the objective function can be achieved by active energy management, but the proportion of renewable power generation is smaller than that in scenario 2. An essential function of ADNs is to accommodate more renewable power generation, but the proportion of non-dispatchable DG may impact the capabilities of active management significantly. To quantitatively assess the impact of this conflict, a constraint is set as:

$$\sum_{i=1}^{N_b} C_{nd_DG,i} \geq T\% \cdot C_{tot_DG} \quad (26)$$

where $C_{nd_DG,i}$ is the non-dispatchable DG capacity on bus i . If there is no DG on bus i , $C_{nd_DG,i}=0$, where $T\%$ is a proportional coefficient for non-dispatchable DG. C_{tot_DG} is the total DG capacity, including the capacity of dispatchable and non-dispatchable DG.

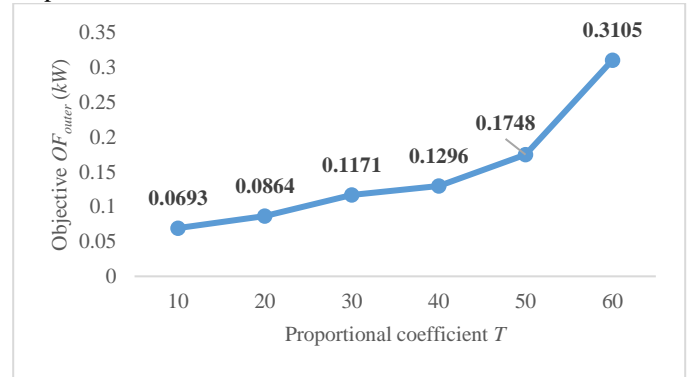


Fig. 9. Objective with different proportional coefficient T .

Five cases with different values of T are selected to find optimal allocation results of DG by applying a bi-level optimization method when T is equal to 10, 20, 30, 40, 50 and 60. Corresponding results of objective functions are presented in Fig. 9. Lower objective value indicates improved capabilities of active management. The value of objective becomes bigger as T increases. When the value of T is less than 50, the change

in the value of the objective is small. A rapid increase of objective can be found when T increases from 50 to 60. Therefore, the capability of active energy management is affected by the proportion of renewable power generation. To balance the conflict between proportion of renewable generation and active energy management, the value of T should be carefully selected.

VII. CONCLUSION

ADN has been considered a promising direction of development to solve problems in CDNs. Active management is one of the most important characteristics of ADNs, but how planning may impact the capabilities of active management has not been widely studied. Hence, in this paper, an active planning framework that considers all physical, cyber and socioeconomic factors and relations are presented. Moreover, a three-step developing strategy based on VMs is put forward. More specifically, a two-stage planning strategy for optimizing DG allocation is introduced. The first stage determines the VM boundaries based on structural characteristics of CDNs. With the VM boundaries as an important constraint, the second phase is to optimize the DG allocation by using a bi-level optimization method.

The effectiveness of the proposed method is verified in the case study. From the structural point of view, our partitioning result is better compared to other early studies as it has a larger electrical modularity. Based on this partitioning result, the allocation for both dispatchable and non-dispatchable DG is determined. Compared to the original distribution network, that does not consider DG allocation, the power losses reduced much after DG allocation. Additionally, by comparing the DG allocations, with and without considering active management by OPF, we found that the autonomy and efficiency of distribution networks are improved a lot by considering active energy management in planning. Although a better performance is achieved via active management, it does not accommodate renewable energy solutions. To balance this conflict, under the premise of active energy management in ADNs, a proportional coefficient should be selected as a constraint on the capacities of DG with renewable resources. In future research, active energy management can be extended from OPF to other methods, such as network control, energy storage and demand response. Better index for capability and effect of active energy management (not only power losses) could be developed. With allocated resources from active planning, transactive energy control for optimal ADN operation is expected to be performed among independent VM operators.

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