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Combined study of the gluon and ghost condensates $\langle A_\mu^2\rangle$ and $\langle \varepsilon^{abc}\overline{c}^b c^c\rangle$ in Euclidean $SU(2)$ Yang-Mills theory in the Landau gauge

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The ghost condensate $\langle \varepsilon^{abc}\overline{c}^b c^c\rangle$ is considered together with the gluon condensate $\langle A_\mu^2\rangle$ in $SU(2)$ Euclidean Yang-Mills theories quantized in the Landau gauge. The vacuum polarization ceases to be transverse due to the nonvanishing condensate $\langle \varepsilon^{abc}\overline{c}^b c^c\rangle$. The gluon propagator itself remains transverse. By polarization effects, this ghost condensate induces then a splitting in the gluon mass parameter, which is dynamically generated through $\langle A_\mu^2\rangle$. The obtained effective masses are real when $\langle A_\mu^2\rangle$ is included in the analysis. In the absence of $\langle A_\mu^2\rangle$, the already known result that the ghost condensate induces effective tachyonic masses is recovered. At the one-loop level, we find that the effective diagonal mass becomes smaller than the off-diagonal one. This might serve as an indication for some kind of Abelian dominance in the Landau gauge, similar to what happens in the maximal Abelian gauge.

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I. INTRODUCTION

Vacuum expectation values of composite operators, commonly known as vacuum condensates, play an important role in quantum field theory. One can employ them to parametrize certain nonperturbative effects. In the context of gauge theories, the gluon condensate $\langle F_{\mu\nu}^2\rangle$ and quark condensate $\langle \overline{q}q\rangle$ are renowned examples [1].

In the last few years, there has been a growing interest in condensates of dimension two. Most attention was paid to the gluon condensate $\langle A_\mu^2\rangle$ in case of the Landau gauge. We do not intend to give a complete overview of the existing research, we refer e.g. to the papers [2–21] and references therein, covering theoretical, phenomenological, lattice and computational topics concerning the mass dimension two gluon condensate. We have studied this condensate and its generalizations to other gauges, such as the linear covariant, the Curci-Ferrari, and the maximal Abelian gauges. In particular, we developed the so-called LCO formalism, allowing us to construct a renormalizable effective potential obeying a homogenous renormalization group equation for a Local Composite Operator like A_μ^2, see e.g. [8,9,11,12]. The renormalizability properties and relations between various renormalization group functions can be proven to all orders of perturbation theory by making use of the algebraic renormalization technique [22]. According to the LCO construction, an effective tree level gluon mass is dynamically generated due to $\langle A_\mu^2\rangle \neq 0$.

Perhaps less known is the concept of the ghost condensates like $\langle f^{abc}\overline{c}^b c^c\rangle$, $\langle f^{abc}c^b c^c\rangle$ and $\langle f^{abc}\overline{c}^b\overline{c}^c\rangle$. For the benefit of the reader, let us provide here a short overview.

The ghost condensate $\langle \varepsilon^{3bc}\overline{c}^b c^c\rangle$ was first introduced in the maximal Abelian gauge (MAG) in [23–26] in case of the gauge group $SU(2)$. The MAG is a partial nonlinear gauge fixing which is useful for the dual superconductivity picture of low energy QCD. Because of the nonlinearity of the MAG, a quartic ghost interaction needs to be introduced in the action for renormalizability purposes [27,28]. This four-ghost interaction was decomposed by means of an auxiliary field σ, and a one-loop effective potential for the ghost condensate $\langle \varepsilon^{3bc}\overline{c}^b c^c\rangle \sim \langle\sigma\rangle$ was calculated. A nonzero vacuum expectation value $\langle\sigma\rangle$ is favored as it lowers the vacuum energy. It was consequently used to construct an effective mass for the off-diagonal gluons, at one-loop order. The diagonal gluons remained massless. This result was interpreted as analytical evidence for the Abelian dominance hypothesis [24], according to which the low energy regime of QCD should be expressed solely in terms of Abelian degrees of freedom [29]. Lattice evidence of this Abelian dominance in case of the MAG was presented in [30–32]. To our knowledge, there is no analytical proof of the Abelian dominance. An argument that can be interpreted in favor of it, is the fact that the off-diagonal gluons would acquire a mass through a dynamical mechanism. At energies below the scale set by this mass, the off-diagonal gluons should decouple, and in this way

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one should end up with an Abelian theory at low energies. It is worth noticing that lattice simulations of the $SU(2)$ MAG revealed an off-diagonal mass of approximately 1.2 GeV [33,34], while the diagonal gluons behaved masslessly [33] or at least almost masslessly [34].

Returning to the ghost condensation in the MAG, it was shown in [35,36] that, contrary to the claim in [23–26], the induced off-diagonal mass induced was tachyonic, at least at the considered one-loop order. As such, it could not be taken as analytical evidence for the Abelian dominance. Another condensate, namely[1] $\langle\frac{1}{2}A_\mu^\beta A_\mu^\beta + \alpha\overline{c}^\beta c^\beta\rangle$ that could be responsible for a real-valued off-diagonal gluon mass was proposed in [7] and investigated thoroughly in [12] using the LCO formalism.

In [35], it was signalled that the effective potential, obtained using the decomposition of the four-point ghost interaction can cause renormalization problems beyond one-loop order and that the LCO formalism would be more suitable to discuss the ghost condensation. A further aspect of the ghost condensation was pointed out in [37], where it was shown that an alternative decomposition of the quartic ghost interaction led to the two Faddeev-Popov charged ghost condensates $\langle\varepsilon^{3bc}c^b c^c\rangle$ and $\langle\varepsilon^{3bc}\overline{c}^b\overline{c}^c\rangle$, instead of $\langle\varepsilon^{3bc}\overline{c}^b c^c\rangle$. This should be not too big a surprise, as the ghost condensation is an order parameter for a continuous $SL(2,\mathbb{R})$ symmetry present in the MAG [23,24,38]. Said otherwise, a nonvanishing ghost condensate like $\langle\varepsilon^{3bc}\overline{c}^b c^c\rangle$ induces a breakdown of the $SL(2,\mathbb{R})$ symmetry. It turns out in fact that the $SL(2,\mathbb{R})$ rotations interchange the different channels, i.e. $\langle\varepsilon^{3bc}\overline{c}^b c^c\rangle$, $\langle\varepsilon^{3bc}c^b c^c\rangle$, $\langle\varepsilon^{3bc}\overline{c}^b\overline{c}^c\rangle$, in which the ghost condensation might occur.

Later on, the ghost condensation was discussed in the case of the Curci-Ferrari gauge [39], which also possesses the $SL(2,\mathbb{R})$ invariance [38]. This was achieved by decomposing the four-ghost interaction which is present for nonvanishing gauge parameter. This brings one to the Landau gauge, which corresponds to the Curci-Ferrari gauge with vanishing gauge parameter. As there is no longer an interaction to be decomposed, it is less clear how to construct an effective potential for the ghost condensates in this case. However, in [40], it was shown that the LCO formalism does allow to construct an effective potential for the ghost condensates $\langle f^{abc}c^b c^c\rangle$ and $\langle f^{abc}\overline{c}^b\overline{c}^c\rangle$ in the Landau gauge. This study was pursued in [41], where it has been proven that the operators $f^{abc}\overline{c}^b c^c$, $f^{abc}c^b c^c$ and $f^{abc}\overline{c}^b\overline{c}^c$ can be simultaneously coupled to the Yang-Mills action, while preserving the $SL(2,\mathbb{R})$ symmetry, using the LCO setup. It was consequently shown that the condensation[2] can occur in different channels, i.e. $\langle f^{abc}\overline{c}^b c^c\rangle$ and $\langle f^{abc}c^b c^c\rangle$, $\langle f^{abc}\overline{c}^b\overline{c}^c\rangle$. However, the corresponding vacua are equivalent, being connected trough rotations of the broken symmetry. Details on the symmetry breaking and the construction of the potential can be found in [41]. As the ghost condensates carry a color index, let us specifically mention that we have given an argument that the apparent breaking of the global color symmetry should not be observed in the physical sector of the theory [41]. One can also argue that the Goldstone particles associated to the broken continuous $SL(2,\mathbb{R})$ symmetry are unphysical [23,41].

Although the concept of a ghost condensate like $\langle\varepsilon^{abc}\overline{c}^b c^c\rangle$ might seem unusual, it has many features in common with the fermion condensation occurring in models with a four-fermion interaction as, for example, the Gross-Neveu model. Considering the MAG, the ghost condensation and the induced symmetry breaking can be directly compared to what happens in the Gross-Neveu model [42] or in other models with a quartic interaction. Decomposing in fact the quartic interaction via an auxiliary field allows us to construct an effective potential, and to analyze the existence of a possible condensation and the related symmetry breaking. The original setup of the ghost condensation, as it was discussed in [23,24,35,37], is essentially not much different from the analysis of the Gross-Neveu model. Let us also mention that the LCO formalism was first developed to construct a meaningful effective potential for the Gross-Neveu model for any number of fermions at any order of perturbation theory [43]. For a review of the LCO formalism, we refer to e.g. [44].

In this work, we shall continue our study on the gluon and ghost condensates for the gauge group $SU(2)$ in case of the Landau gauge. For the first time, we present a combined study of both condensates, namely $\langle A_\mu^2\rangle$ and $\langle\varepsilon^{abc}\overline{c}^b c^c\rangle$. In Sec. II, we shall discuss the renormalizability issues using the algebraic renormalization. Section III contains a summary of the LCO formalism, the calculation of the one-loop effective potential and the determination of the vacuum configuration. Hereafter, we discuss the consequences of a nonvanishing gluon and ghost condensate. We shall show that the vacuum polarization is no longer transverse, the breaking being directly proportional to the ghost condensate. Moreover, we shall prove that the gluon propagator itself remains transverse. These results will be first discussed by deriving the Slavnov-Taylor identities in the condensed vacuum. Then, we shall illustrate them with explicit one-loop calculations. These issues will be handled in Secs. IV, V, and VI. In Sec. VII, we find another interesting consequence of the ghost condensate. Because of polarization effects on the gluon propagator, the effective dynamical gluon mass generated through $\langle A_\mu^2\rangle$ undergoes a shift which differs for the diagonal and off-diagonal gluons. More precisely, it is found that the effective diagonal gluon mass is smaller than the off-diagonal one. However, unlike the results obtained in the absence of

[1] α is the MAG gauge parameter, while the color index β runs only over the $N(N-1)$ off-diagonal generators of $SU(N)$.

[2] $\langle f^{abc}\overline{c}^b c^c\rangle$ was called the Overhauser condensate, while $\langle f^{abc}c^b c^c\rangle$ and $\langle f^{abc}\overline{c}^b\overline{c}^c\rangle$ the BCS condensates. This nomenclature was based upon a similar kind of phenomenon happening in the theory of superconductivity.

the gluon condensate $\langle A_\mu^2 \rangle$ [35,36], both masses remain now real.

Similarly to what happens in the MAG [12,24,33,34], the fact that the off-diagonal mass is larger than the diagonal one, could be interpreted as evidence that a kind of Abelian dominance might also take place in the Landau gauge. Finally, we note that in [19–21], it was shown that the dual Abelian Meissner effect works in lattice QCD without requiring monopoles from a singular gauge transformation in a study where the lattice Landau gauge was employed. Also some numerical indication of Abelian dominance in this particular gauge, which is *not* of the Abelian gauge fixing type, was presented.

II. RENORMALIZABILITY OF THE LCO FORMALISM INCORPORATING BOTH GLUON AND GHOST CONDENSATES

In this section, we shall prove that the simultaneous introduction of the composite operators A_μ^2 and $gf^{abc}\bar{c}^b c^c$ allows for a multiplicatively renormalizable action, in the Landau gauge.

We shall work in Euclidean space time. The Yang-Mills action in the Landau gauge, $\partial_\mu A_\mu^a = 0$, reads

$$S = S_{\rm YM} + S_{\rm GF}, \qquad S_{\rm YM} = \int d^4x \left(\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a\right),$$
$$S_{\rm GF} = s\int d^4x (\bar{c}^a \partial_\mu A_\mu^a) = \int d^4x (b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b), \tag{2.1}$$

where D_μ^{ab} is the adjoint covariant derivative which is defined by,

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - gf^{abc}A_\mu^c. \tag{2.2}$$

The action (2.1)enjoys the nilpotent BRST symmetry

$$sA_\mu^a = -D_\mu^{ab}c^b, \qquad sc^a = \frac{g}{2} f^{abc}c^b c^c,$$
$$s\bar{c}^a = b^a, \qquad sb^a = 0, \tag{2.3}$$

with

$$sS = 0, \qquad s^2 = 0. \tag{2.4}$$

We introduce two BRST doublets of sources

$$s\tau = J, \qquad sJ = 0, \tag{2.5}$$

$$s\lambda^a = \omega^a, \qquad s\omega^a = 0, \tag{2.6}$$

allowing us to couple the composite operators A_μ^2 and $gf^{abc}\bar{c}^b c^c$ to the action (2.1) in a BRST invariant fashion

$$S' = S_{\rm YM} + S_{\rm GF} + s\int d^4x \left(\frac{1}{2}\tau A_\mu^a A_\mu^a + \frac{\zeta}{2}\tau J + gf^{abc}\lambda^a \bar{c}^b c^c - \frac{\rho}{2}\omega^a \lambda^a\right)$$
$$= \int d^4x \left(\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + b^a\partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b + gf^{abc}\omega^a \bar{c}^b c^c - gf^{abc}\lambda^a b^c c^c + \frac{g^2}{2} f^{abc} f^{cde}\lambda^a \bar{c}^b c^d c^e - \frac{\rho}{2}\omega^a\omega^a + \frac{1}{2}JA_\mu^a A_\mu^a + \tau A_\mu^a \partial_\mu c^a - \frac{\zeta}{2}J^2\right). \tag{2.7}$$

The terms quadratic in the sources ω^a and J are allowed by power counting and are needed to remove the novel divergences appearing in the vacuum correlators $\langle (f^{abc}\bar{c}^a c^b) \times (x)(f^{klm}\bar{c}^l c^m)(y)\rangle$ and $\langle A_\mu^2(x) A_\mu^2(y)\rangle$ for $x \to y$. ρ and ζ are called the LCO parameters.

For completeness, we also have to introduce external sources for the BRST variations of the elementary fields A_μ^a and c^a:

$$S_{\rm ext} = \int d^4x \left(-\Omega_\mu^a D_\mu^{ab} c^b + \frac{g}{2} f^{abc} L^a c^b c^c\right), \tag{2.8}$$

with

$$s\Omega_\mu^a = 0, \qquad sL^a = 0. \tag{2.9}$$

The mass dimension and the ghost number of the fields and sources are listed in Table I.

In principle, the action (2.7) can be supplemented with extra terms in the sources which are allowed by power counting,

TABLE I. Quantum numbers of the field and sources

	A_μ^a	c^a	$\bar{c}^a$	b^a	τ	J	Ω_μ^a	L^a	λ^a	ω^a
dimension	1	0	2	2	2	2	3	4	2	2
ghost number	0	1	−1	0	−1	0	−1	3 − 2	−1	0

$$S'_{\rm ext} = s\int d^4x\Big(\beta\frac{g}{2}f^{abc}\lambda^a\lambda^b c^s + \gamma\lambda^a\partial_\mu A^a_\mu\Big)$$
$$= \int d^4x\Big(\beta g f^{abc}\omega^a\lambda^b c^c + \beta\frac{g^2}{4}f^{abc}f^{cde}\lambda^a\lambda^b c^d c^e + \gamma\omega^a\partial_\mu A^a_\mu + \gamma\lambda^a\partial_\mu D^{ab}_\mu c^b\Big), \quad (2.10)$$

where β and γ are new, independent, dimensionless couplings. The $S'_{\rm ext}$-part of the action has to be introduced to actually prove the renormalizability. This is our next task.

The complete action,

$$\Sigma = S' + S_{\rm ext} + S'_{\rm ext}, \quad (2.11)$$

obeys a few Ward identities.

(i) The Slavnov-Taylor identity

$$S(\Sigma) = \int d^4x\Big(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + b^a\frac{\delta\Sigma}{\delta\bar c^a} + \omega^a\frac{\delta\Sigma}{\delta\lambda^a} + J\frac{\delta\Sigma}{\delta\tau}\Big) = 0. \quad (2.12)$$

(ii) The modified Landau gauge condition

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^a_\mu + g f^{abc}\lambda^b c^c. \quad (2.13)$$

(iii) The modified antighost equation

$$\frac{\delta\Sigma}{\delta\bar c^a} + \partial_\mu\frac{\delta\Sigma}{\delta\Omega^a_\mu} - g f^{abc}\lambda^b\frac{\delta\Sigma}{\delta L^c} = -g f^{abc}\omega^b c^c. \quad (2.14)$$

(iv) The modified ghost Ward identity

$$\mathcal{G}^a(\Sigma) = \Delta^a_{\rm class.}, \quad (2.15)$$

with

$$\mathcal{G}^a = \int d^4x\Big(\frac{\delta}{\delta c^a} + g f^{abc}\bar c^b\frac{\delta}{\delta b^c} + g f^{abc}\lambda^b\frac{\delta}{\delta\omega^c}\Big),$$
$$\Delta^a_{\rm class.} = \int d^4x[g f^{abc}(\Omega^b_\mu A^c_\mu - L^b c^c - \omega^b\bar c^c + (\beta-\rho)\lambda^b\omega^c - \lambda^b b^c)].$$

(v) The modified τ-identity

$$\int d^4x\Big(\frac{\delta\Sigma}{\delta\tau} + c^a\frac{\delta\Sigma}{\delta b^a} - 2\lambda^a\frac{\delta\Sigma}{\delta L^a}\Big) = 0. \quad (2.16)$$

This identity expresses the on-shell BRST invariance of the operator $A^a_\mu A^a_\mu$.

We notice that every term on the right-hand sides of Eqs. (2.13), (2.14), and (2.15), being linear in the quantum fields, represents a classical breaking [22].

We are now prepared to write down the most general local counterterm, $\Sigma_{\rm CT}$, which is compatible with the previous Ward identities and can be freely added to the original action perturbatively. The perturbed action $\Sigma + \eta\Sigma_{\rm CT}$ should obey the same Ward identities as the starting action Σ to the first order in the perturbation parameter η. This corresponds to imposing the following constraints on the counterterm $\Sigma_{\rm CT}$

(i)
$$\mathcal{B}_\Sigma\Sigma_{\rm CT} = 0, \quad (2.17)$$

(ii)
$$\frac{\delta\Sigma_{\rm CT}}{\delta b^a} = 0, \quad (2.18)$$

(iii)
$$\frac{\delta\Sigma_{\rm CT}}{\delta\bar c^a} + \partial_\mu\frac{\delta\Sigma_{\rm CT}}{\delta\Omega^a_\mu} - g f^{abc}\lambda^b\frac{\delta\Sigma_{\rm CT}}{\delta L^c} = 0, \quad (2.19)$$

(iv)
$$\mathcal{G}^a(\Sigma_{\rm CT}) = \int d^4x\Big(\frac{\delta\Sigma_{\rm CT}}{\delta c^a} + g f^{abc}\lambda^b\frac{\delta\Sigma_{\rm CT}}{\delta\omega^c}\Big) = 0, \quad (2.20)$$

(v)
$$\int d^4x\Big(\frac{\delta\Sigma_{\rm CT}}{\delta\tau} - 2\lambda^a\frac{\delta\Sigma_{\rm CT}}{\delta L^a}\Big) = 0, \quad (2.21)$$

where $\mathcal{B}_\Sigma$ is the nilpotent, $\mathcal{B}^2_\Sigma = 0$, linearized Slavnov-Taylor operator, given by

$$\mathcal{B}_\Sigma = \int d^4x\Big(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\times\frac{\delta}{\delta L^a} + b^a\frac{\delta}{\delta\bar c^a} + \omega^a\frac{\delta}{\delta\lambda^a} + J\frac{\delta}{\delta\tau}\Big). \quad (2.22)$$

The most general local counterterm can be written as [22]

$$\Sigma_{\rm CT} = a_0 S_{\rm YM} + \mathcal{B}_\Sigma\Delta^{-1}, \quad (2.23)$$

where Δ^{-1} is an integrated local polynomial of ghost number -1 and dimension 4, given by

$$\Delta^{-1} = \int d^4x\Big[a_1\Omega^a_\mu A^a_\mu + a_2 L^a c^a + a_3 A^a_\mu\partial_\mu\bar c^a + a_4\frac{g}{2}f^{abc}\bar c^a\bar c^b c^c + a_5 b^a\bar c^a + \frac{a_6}{2}\tau A^a_\mu A^a_\mu + a_7\tau\bar c^a c^a - \frac{a_8}{2}\zeta\tau J + a_9\gamma\lambda^a\partial_\mu A^a_\mu + a_{10} g f^{abc}\lambda^a\bar c^b c^c + a_{11}\beta\frac{g}{2}f^{abc}\lambda^a\lambda^b c^c + a_{12} b^a\lambda^a + a_{13}\tau\lambda^a c^a - \frac{a_{14}}{2}\rho\lambda^a\omega^a + a_{15}\omega^a\bar c^a\Big]. \quad (2.24)$$

The constraints (2.18), (2.19), (2.20), and (2.21) imply that

$$a_1 = a_3 = -a_6, \qquad a_{11} = \frac{\rho}{\beta} a_{14}, \qquad (2.25)$$
$$a_2 = a_4 = a_5 = a_7 = a_{10} = a_{12} = a_{13} = a_{15} = 0.$$

Collecting these results, we come to the conclusion that the most general counterterm compatible with the Ward identities (2.17), (2.18), (2.19), (2.20), and (2.21) is eventually given by

$$\Sigma_{\mathrm{CT}} = \int d^4x \bigg\{ \frac{a_0 + 2a_1}{2} [(\partial_\mu A^a_\nu)\partial_\mu A^a_\nu - (\partial_\mu A^a_\nu)\partial_\nu A^a_\mu] + (a_0 + 3a_1)\frac{g}{2} f^{abc}(\partial_\mu A^a_\nu) A^b_\mu A^c_\nu + (a_0 + 4a_1) \times \frac{g^2}{4} f^{abc} f^{cde} A^a_\mu A^b_\nu A^d_\mu A^e_\nu + a_1(\Omega^a_\mu + \partial_\mu \bar{c}^a)\partial_\mu c^a + \frac{a_1}{2} J A^a_\mu A^a_\mu + (a_1 + a_9)\gamma \omega^a \partial_\mu A^a_\mu + \rho a_{14} g f^{abc} \omega^a \lambda^b c^c + \rho a_{14} \frac{g^2}{4} f^{abc} f^{cde} \lambda^a \lambda^b c^d c^e - a_{14} \frac{\rho}{2} \omega^a \omega^a + \gamma a_9 \lambda^a \partial^2 c^a + (a_1 + a_3)\gamma g f^{abc} \lambda^a \partial_\mu (A^b_\mu c^c) - a_8 \frac{\zeta}{2} J^2 \bigg\}. \qquad (2.26)$$

Let us now check that this counterterm can be reabsorbed in the original action (2.11) by means of a multiplicative renormalization of the available parameters, fields and sources, according to

$$\Sigma(\Phi_0, \phi_0, \xi_0) = \Sigma(\Phi, \phi, \xi) + \eta \Sigma_{\mathrm{CT}}(\Phi, \phi, \xi) + O(\eta^2), \qquad (2.27)$$

where

$$\Phi_0 = Z_\Phi^{1/2}\Phi, \qquad \phi_0 = Z_\phi \phi, \qquad \xi_0 = Z_\xi \xi, \qquad (2.28)$$

with $\Phi = \{\Omega^a_\mu, L^a, \tau, J, \lambda^a, \omega^a\}$, $\phi = \{A^a_\mu, b^a, c^a, \bar{c}^a\}$ and $\xi = \{g, \zeta, \rho, \beta, \gamma\}$. From Eqs. (2.26), (2.27), and (2.28) one can check that

$$\begin{aligned}
Z_g &= 1 - \eta \frac{a_0}{2}, \\
Z_A^{1/2} &= Z_b^{-1/2} = Z_L = Z_\omega = 1 + \eta\left(\frac{a_0}{2} + a_1\right), \\
Z_c^{1/2} &= Z_{\bar{c}}^{1/2} = Z_\Omega = Z_g^{-1/2} Z_A^{-1/4}, \quad Z_J = Z_\tau^2 = Z_g Z_A^{-1/2}, \\
Z_\lambda &= Z_g^{-1/2} Z_A^{3/4}, \quad Z_\beta = 1 + \eta\left(\frac{\rho a_{14}}{\beta} - a_0 - 2a_1\right), \\
Z_\rho &= 1 + \eta(a_{14} - a_0 - 2a_1), \\
Z_\zeta &= 1 + \eta(a_8 + 2a_0 + 2a_1), \quad Z_\gamma = 1 + \eta(a_9 - a_0 - a_1).
\end{aligned} \qquad (2.29)$$

allow for the desired multiplicative renormalization. We recall here that the relations found for Z_J and Z_ω, respectively, imply that the anomalous dimensions of the composite operators A^2_μ and $g f^{abc} \bar{c}^b c^c$, are linear combinations of that of A^a_μ and of the beta function $\beta(g^2)$ [9,41].

III. EFFECTIVE POTENTIAL FOR THE CONDENSATES $\langle A^2_\mu \rangle$ AND $\langle \varepsilon^{abc} \bar{c}^b c^c \rangle$

A. Preliminaries

Let us now discuss how one can obtain the effective potential describing the condensates $\langle A^2_\mu \rangle$ and $\langle \varepsilon^{abc} \bar{c}^b c^c \rangle$. Having proven the renormalizability, we can set to zero all unnecessary external sources, namely $\tau = 0$, $\Omega^a_\mu = 0$, $L^a = 0$ and $\lambda = 0$. In this context, we would like to remark that we had to introduce novel terms in λ in the starting action, Eq. (2.10). These terms, introduced for the algebraic proof of the multiplicative renormalizability, are allowed by power counting and by the symmetries of the model, and they do appear in the most general counterterm, Eq. (2.26). However, they are not needed for the evaluation of the effective potential, so that for our purposes $\lambda \equiv 0$. Considering now the term $\omega \partial A$ in Eq. (2.10), it is apparent that, in the Landau gauge, $\partial A = 0$, such a term is absent. To make this argument a little more formal, one could consider the generating functional $\mathcal{W}(\omega, J)$ and perform the transformation of the Lagrange multiplier $b' = b + \gamma \omega$, with trivial Jacobian.

We are still left with two free LCO parameters ρ and ζ. As shown in [8,11,41], these parameters can be fixed by using the renormalization group equations. We would like to notice that the explicit value of the counterterms $\propto \omega^a$ are not influenced by the presence of J and vice versa. As such, the already determined values of ρ and ζ remain unchanged upon comparison with the cases $\omega^a \equiv 0$ [8,11] and $J \equiv 0$ [41]. More precisely, one still has

$$\rho = \rho_0 + \rho_1 g^2 + \ldots, \qquad \zeta = \frac{\zeta_0}{g^2} + \zeta_1 + \ldots, \qquad (3.1)$$

where

$$\begin{aligned}
\rho_0 &= -\frac{6}{13}, \qquad \rho_1 = -\frac{95}{312\pi^2}, \\
\zeta_0 &= \frac{27}{26}, \qquad \zeta_1 = \frac{161}{52}\frac{3}{16\pi^2},
\end{aligned} \qquad (3.2)$$

in the case of $SU(2)$. The use of dimensional regularization with $d = 4 - \varepsilon$ and of the $\overline{\mathrm{MS}}$ renormalization scheme is understood throughout this paper.

The relevant action is thus given by

$$S = S_{\mathrm{YM}} + S_{\mathrm{GF}} + \int d^4x \left(g f^{abc} \omega^a \bar{c}^b c^c + \frac{1}{2} J A^a_\mu A^a_\mu - \frac{\rho}{2} \omega^a \omega^a - \frac{\zeta}{2} J^2 \right). \qquad (3.3)$$

As the sources J and ω^a appear nonlinearly, the energy interpretation might be spoiled. However, by exploiting a Hubbard-Stratonovich transformation, the starting action incorporating both gluon and ghost condensates can be rewritten as [8,11,41]

$$S = S_{\rm YM} + S_{\rm GF} + \int d^4x\left(\frac{\phi^a\phi^a}{2g^2\rho} + \frac{1}{\rho}\phi^a\varepsilon^{abc}\overline{c}^b c^c + \frac{g^2}{2\rho}(\varepsilon^{abc}\overline{c}^b c^c)^2\right) + \int d^4x\left(\frac{\sigma^2}{2g^2\zeta} + \frac{\sigma}{2g\zeta}A^a_\mu A^a_\mu + \frac{1}{8\zeta}(A^a_\mu A^a_\mu)^2 - \omega^a\frac{\phi^a}{g} - J\frac{\sigma}{g}\right), \quad (3.4)$$

where the sources are now linearly coupled to the fields σ and ϕ^a, while the identifications

$$\langle\phi^a\rangle = -g^2\langle f^{abc}\overline{c}^a c^b\rangle, \qquad \langle\sigma\rangle = -\frac{g}{2}\langle A^2_\mu\rangle, \quad (3.5)$$

hold [8,11,41]. The action (3.4) will be multiplicatively renormalizable and will obey a homogenous renormalization group equation.

B. The one-loop effective potential

For the one-loop effective potential $V^{(1)}(\sigma, \phi)$ itself, we deduce

$$V^{(1)}(\sigma, \phi) = V_{A^2}(\sigma) + V_{\rm gh}(\phi), \quad (3.6)$$

with [8,11]

$$V_{A^2}(\sigma) = \frac{\sigma^2}{2\zeta_0}\left(1 - \frac{\zeta_1}{\zeta_0}g^2\right) + \frac{3(N^2-1)}{64\pi^2}\frac{g^2\sigma^2}{\zeta_0^2} \times\left(\ln\frac{g\sigma}{\zeta_0\overline{\mu}^2} - \frac{5}{6}\right), \quad (3.7)$$

while [41]

$$V_{\rm gh}(\phi) = \frac{\phi^2}{2g^2\rho_0}\left(1 - \frac{\rho_1}{\rho_0}g^2\right) + \frac{1}{32\pi^2}\frac{\phi^2}{\rho_0^2}\left(\ln\frac{\phi^2}{\rho_0^2\overline{\mu}^4} - 3\right), \quad (3.8)$$

where $\phi = \phi^3$, $\phi^a = \phi\delta^{a3}$. This amounts to choosing the vacuum configuration along the 3-direction in color space. For the rest of the paper, it is understood that $N = 2$.

The minimum configuration, describing the vacuum, is retrieved by solving

$$\frac{\partial V^{(1)}(\sigma, \phi)}{\partial\sigma} = \frac{\partial V^{(1)}(\sigma, \phi)}{\partial\phi} = 0, \quad (3.9)$$

or

$$\frac{1}{\zeta_0}\left(1 - \frac{\zeta_1}{\zeta_0}g^2\right) + 2\frac{9}{64\pi^2}\frac{g^2}{\zeta_0^2}\left(\log\frac{g\sigma_*}{\zeta_0\overline{\mu}^2} - \frac{5}{6}\right) + \frac{9}{64\pi^2}\frac{g^2}{\zeta_0^2} = 0, \quad \frac{1}{g^2\rho_0}\left(1 - \frac{\rho_1}{\rho_0}g^2\right) + 2\frac{1}{32\pi^2}\frac{1}{\rho_0^2}\left(\log\frac{\phi_*^2}{\rho_0^2\overline{\mu}^4} - 3\right) + 2\frac{1}{32\pi^2}\frac{1}{\rho_0^2} = 0, \quad (3.10)$$

where (σ_*, ϕ_*) denote the nontrivial solution. For the vacuum energy, we obtain

$$E_{\rm vac} = V^{(1)}(\sigma_*, \phi_*) = -\frac{9}{128\pi^2}\frac{g^2}{\zeta_0^2}\sigma_*^2 - \frac{1}{32\pi^2}\frac{\phi_*^2}{\rho_0^2}. \quad (3.11)$$

One sees thus that nonvanishing condensates will be dynamically favored at one-loop as they both lower the vacuum energy.

For further investigation, it is useful to introduce the variables

$$m^2 = \frac{g\sigma}{\zeta_0}, \qquad \omega = \frac{\phi}{|\rho_0|}. \quad (3.12)$$

hence

$$V_{A^2}(m^2) = \zeta_0\frac{m^4}{2g^2}\left(1 - \frac{\zeta_1}{\zeta_0}g^2\right) + \frac{3(N^2-1)}{64\pi^2}m^4\left(\ln\frac{m^2}{\overline{\mu}^2} - \frac{5}{6}\right), \quad (3.13)$$

while

$$V_{\rm gh}(\omega) = -\rho_0\frac{\omega^2}{2g^2}\left(1 - \frac{\rho_1}{\rho_0}g^2\right) + \frac{\omega^2}{32\pi^2}\left(\ln\frac{\omega^2}{\overline{\mu}^4} - 3\right), \quad (3.14)$$

Let us now try to get an estimate of the vacuum state of the theory. In comparison with the case where only the gluon or ghost condensation is considered, we have now an additional complication, due to the presence of two mass scales. Usually, when only a single scale is present, one chooses the renormalization scale $\overline{\mu}^2$ in such a way that potentially large logarithms vanish in the gap equation. In the present case, two different logarithms show up. In order to keep some control on the expansion, we shall use the RG invariance to explicitly sum the leading logarithms (LL) in the effective potential. To this end, we notice that the potential can be rewritten as

$$V = \zeta_0\frac{m^4}{g^2}\sum_{n=0}^{\infty}a_n\left(g^2\ln\frac{m^2}{\overline{\mu}^2}\right)^n + m^4\left(-\frac{\zeta_1}{2} - \frac{5}{6}\frac{3(N^2-1)}{64\pi^2}\right) + \rho_0\frac{\omega^2}{g^2}\sum_{n=0}^{\infty}b_n\left(g^2\ln\frac{\omega^2}{\overline{\mu}^4}\right)^n + \omega^2\left(\frac{\rho_1}{2} - \frac{3}{32\pi^2}\right), \quad (3.15)$$

for a LL-expansion,[3] where $a_0 = -b_0 = \frac{1}{2}$. For the time being, we shall only consider the part in m^2. The analysis for the part in ω is completely analogous and independent from the m^2-part. We set

$$\overline{\mu}\frac{\partial}{\partial\overline{\mu}}g^2 = \beta(g^2) = -2\sum_{n=0}^{\infty}\beta_n(g^2)^{n+2},$$
$$\overline{\mu}\frac{\partial}{\partial\overline{\mu}}\ln m^2 = \gamma_2(g^2) = \sum_{n=0}^{\infty}\gamma_n(g^2)^{n+1}. \tag{3.16}$$

Since we know that the effective potential should be RG invariant, we find that

$$\overline{\mu}\frac{\partial}{\partial\overline{\mu}}V = 0$$
$$\Downarrow$$
$$(\gamma_0+\beta_0)\sum_{n=0}^{\infty}a_n u^n - (\beta_0 u + 1)\times\sum_{n=0}^{\infty}(n+1)a_{n+1}u^n = 0 + \text{next-to-leading order}. \tag{3.17}$$

We have defined

$$u = g^2\ln\frac{m^2}{\overline{\mu}^2}. \tag{3.18}$$

Setting

$$F(u) = \sum_{n=0}^{\infty}a_n u^n, \tag{3.19}$$

then Eq. (3.17) translates into a differential equation

$$(\gamma_0+\beta_0)F(u) - (\beta_0 u + 1)F'(u) = 0, \tag{3.20}$$

which can be solved to

$$F(u) = \frac{1}{2}(1+\beta_0 u)^{(\gamma_0+\beta_0)/\beta_0}, \tag{3.21}$$

as we have the initial condition $F(0) = \frac{1}{2}$. Using this result, we can write

$$\begin{aligned} V_{A^2}(m^2) &= \zeta_0\frac{m^4(\overline{\mu})}{2g^2(\overline{\mu})}\frac{(1+\beta_0 u)^{\gamma_0/\beta_0}}{(1+\beta_0 u)^{-1}} + m^4(\overline{\mu})\times\left(-\frac{\zeta_1}{2}-\frac{5}{6}\frac{3(N^2-1)}{64\pi^2}\right)+\cdots \\ &= \zeta_0\frac{m^4(m)}{2g^2(m)} + m^4(m)\left(-\frac{\zeta_1}{2}-\frac{5}{6}\frac{3(N^2-1)}{64\pi^2}\right)+\cdots, \end{aligned} \tag{3.22}$$

as the running quantities at scale $\overline{\mu}$ get replaced by their counterparts at scale m.

The same could be done for the ghost condensation part, so that we can write for the LL summed effective potential

$$V = \frac{\zeta_0}{2}\frac{\overline{m}^4}{\overline{g}^2} + \overline{m}^4\left(-\frac{\zeta_1}{2}-\frac{5}{6}\frac{3(N^2-1)}{64\pi^2}\right) - \frac{\rho_0}{2}\frac{\tilde{\omega}^2}{\tilde{g}^2} + \tilde{\omega}^2\left(\frac{\rho_1}{2}-\frac{3}{32\pi^2}\right). \tag{3.23}$$

where it is understood that the barred quantities like $\overline{g}^2$ are considered at scale $\overline{\mu}^2 = m^2$, and the tilded quantities like $\tilde{g}^2$ at scale $\overline{\mu}^2 = \omega$.

For further usage, let us first quote the explicit values of the anomalous dimensions of g^2, m^2 and ω. Using the definitions (3.16) one shall find in e.g. [15] that

$$\beta_0 = \frac{11}{3}\frac{N}{16\pi^2}, \qquad \gamma_0 = -\frac{3}{2}\frac{N}{16\pi^2}. \tag{3.24}$$

Defining

$$\overline{\mu}\frac{\partial}{\partial\overline{\mu}}\ln\omega = \kappa(g^2) = \sum_{n=0}^{\infty}\kappa_n(g^2)^{n+1}, \tag{3.25}$$

one can infer from [11] and the definitions (3.5), (3.6), (3.7), (3.8), (3.9), (3.10), (3.11), and (3.12) that

$$\kappa(g^2) = \frac{1}{2}\frac{\beta(g^2)}{g^2} + \gamma_A(g^2), \tag{3.26}$$

where $\gamma_A(g^2)$ is the anomalous dimension of the gluon field in the Landau gauge as defined in [11]. Hence,

$$\kappa_0 = -\frac{35}{6}\frac{N}{16\pi^2}. \tag{3.27}$$

We are now ready to determine the minimum configuration. The gap equations we intend to solve are given by

$$\begin{aligned} \frac{\partial V}{\partial\overline{m}^2} &= 0 \Rightarrow \frac{\zeta_0}{\overline{g}^2} + \zeta_0(\beta_0+\gamma_0) - 2\left(\frac{\zeta_1}{2}+\frac{5(N^2-1)}{128\pi^2}\right) = 0, \\ \frac{\partial V}{\partial\tilde{\omega}} &= 0 \Rightarrow -\frac{\rho_0}{\tilde{g}^2} - \rho_0(\beta_0+\kappa_0) + 2\left(\frac{\rho_1}{2}-\frac{3}{32\pi^2}\right) = 0. \end{aligned} \tag{3.28}$$

or

$$\begin{aligned} \left.\frac{\overline{g}^2 N}{16\pi^2}\right|_{N=2} &= \frac{9}{37} \approx 0.243, \\ \left.\frac{\tilde{g}^2 N}{16\pi^2}\right|_{N=2} &= \frac{36}{385} \approx 0.094. \end{aligned} \tag{3.29}$$

Using the one-loop $\overline{\text{MS}}$ expression

$$g^2(\overline{\mu}) = \frac{1}{\beta_0\ln\frac{\overline{\mu}^2}{\Lambda^2_{\overline{\text{MS}}}}}, \tag{3.30}$$

[3] A term like $m^4(g^2\ln[\omega^2/\overline{\mu}^4])$ or $\omega^2(g^2\ln[m^2/\overline{\mu}^2])$ shall not occur, as there are no infrared divergences for $m^2 = 0$, $\omega \neq 0$ or $m^2 \neq 0$, $\omega = 0$.

one extracts from the values (3.29) the estimates

$$\overline{m}^2 = e^{37/33}\Lambda^2_{\overline{\rm MS}} \approx 3.07\Lambda^2_{\overline{\rm MS}}, \qquad \tilde{\omega} = e^{35/12}\Lambda^2_{\overline{\rm MS}} \approx 18.48\Lambda^2_{\overline{\rm MS}}, \tag{3.31}$$

while for the vacuum energy, Eq. (3.11), we obtain

$$E_{\rm vac} \approx -1.15\Lambda^4_{\overline{\rm MS}}. \tag{3.32}$$

As the RG improved coupling constants of Eq. (3.29) are relatively small, the performed expansion should have some trustworthiness. Evidently, explicit knowledge of the higher order contributions would be necessary to reach better conclusions about the reliability of the presented values, but this is beyond the scope of this article.

IV. A STUDY OF THE ONE-LOOP GHOST CONTRIBUTION TO THE VACUUM POLARIZATION

We shall now start to investigate the consequences stemming from a nonvanishing condensate $\langle \varepsilon^{3bc}\overline{c}^b c^c\rangle$. Let us begin by making a detailed study of the one-loop ghost contribution to the vacuum polarization. Before starting with the explicit computations, it is worth giving a look at the Ward identity obeyed by the one-loop vacuum polarization stemming from the Slavnov-Taylor identity describing the theory in the condensed vacuum. To focus on the role of the ghost condensate, we switch off, for the time being, the gluon condensate $\langle A^2_\mu\rangle$. Its inclusion can be done straightforwardly. Let us start thus with the action

$$S = S_{\rm YM} + S_{\rm GF} + \int d^4x\left(\frac{\phi^a\phi^a}{2g^2\rho} + \frac{1}{\rho}\phi^a\varepsilon^{abc}\overline{c}^b c^c + \frac{g^2}{2\rho}(\varepsilon^{abc}\overline{c}^b c^c)^2\right), \tag{4.1}$$

where

$$\phi^a(x) = \delta^{a3}\phi_* + \tilde{\phi}^a(x), \qquad \langle\tilde{\phi}^a(x)\rangle = 0. \tag{4.2}$$

Requiring that

$$s\phi^a = s\tilde{\phi}^a = -g^2 s(\varepsilon^{abc}\overline{c}^b c^c), \tag{4.3}$$

will assure that S is left invariant by the following nilpotent BRST transformations

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b, \qquad sc^a = \frac{g}{2}\varepsilon^{abc}c^b c^c,\\ s\tilde{\phi}^a &= -g^2\left(\varepsilon^{abc}b^b c^c + \frac{g}{2}\varepsilon^{abc}\overline{c}^b\varepsilon^{cmn}c^m c^n\right),\\ s\phi_* &= 0, \qquad s\overline{c}^a = b^a, \qquad sb^a = 0, \end{aligned} \tag{4.4}$$

thus

$$sS = 0. \tag{4.5}$$

In order to obtain the Slavnov-Taylor identity, we introduce external sources Ω^a_μ, L^a, F^a coupled to the nonlinear variations of the fields.

$$S_{\rm ext} = \int d^4x\left(-\Omega^a_\mu D^{ab}_\mu c^b + \frac{g}{2}L^a\varepsilon^{abc}c^b c^c + F^a s\tilde{\phi}^a\right), \tag{4.6}$$

The complete action

$$\Sigma = S + S_{\rm ext}, \tag{4.7}$$

obeys the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = 0, \tag{4.8}$$

where

$$\mathcal{S}(\Sigma) = \int d^4x\left(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta F^a}\frac{\delta\Sigma}{\delta\tilde{\phi}^a} + b^a\frac{\delta\Sigma}{\delta\overline{c}^a}\right). \tag{4.9}$$

Because of the absence of anomalies and to the stability of the theory, the Slavnov-Taylor identity (4.8) holds at the quantum level, i.e.

$$\mathcal{S}(\Gamma) = \int d^4x\left(\frac{\delta\Gamma}{\delta\Omega^a_\mu}\frac{\delta\Gamma}{\delta A^a_\mu} + \frac{\delta\Gamma}{\delta L^a}\frac{\delta\Gamma}{\delta c^a} + \frac{\delta\Gamma}{\delta F^a}\frac{\delta\Gamma}{\delta\tilde{\phi}^a} + b^a\frac{\delta\Gamma}{\delta\overline{c}^a}\right) = 0, \tag{4.10}$$

where Γ

$$\Gamma = \Sigma + \hbar\Gamma^1 + \ldots, \tag{4.11}$$

is the generator of the $1PI$ Green functions.

A. Ward identity for the vacuum polarization

Let us now derive the Ward identity for the vacuum polarization at one-loop level following from the Slavnov-Taylor identity (4.10). At one-loop level, one has

$$\Gamma = \Sigma + \hbar\Gamma^1, \tag{4.12}$$

so that the Slavnov-Taylor identity becomes

$$\int d^4x\left(\frac{\delta\Gamma^1}{\delta\Omega^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta\Gamma^1}{\delta A^a_\mu} + \frac{\delta\Gamma^1}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Gamma^1}{\delta c^a} + \frac{\delta\Gamma^1}{\delta F^a}\frac{\delta\Sigma}{\delta\tilde{\phi}^a} + \frac{\delta\Sigma}{\delta F^a}\frac{\delta\Gamma^1}{\delta\tilde{\phi}^a} + b^a\frac{\delta\Gamma^1}{\delta\overline{c}^a}\right) = 0. \tag{4.13}$$

From

$$\begin{aligned} \frac{\delta\Gamma^1}{\delta\Omega^a_\mu} &= [-(D^{ab}_\mu c^b)\cdot\Gamma]^1,\\ \frac{\delta\Gamma^1}{\delta L^a} &= \left[\left(\frac{g}{2}\varepsilon^{abc}c^b c^c\right)\cdot\Gamma\right]^1,\\ \frac{\delta\Gamma^1}{\delta F^a} &= \left[-g^2\left(\varepsilon^{abc}b^b c^c + \frac{g}{2}\varepsilon^{abc}\overline{c}^b\varepsilon^{cmn}c^m c^n\right)\cdot\Gamma\right]^1, \end{aligned} \tag{4.14}$$

where $[\mathcal{O}\cdot\Gamma]$ denotes the generator of the $1PI$ Green functions with the insertion of the composite operator $\mathcal{O}$, it follows that

$$\begin{aligned}0 = \int d^4x\Big(&[-(D^{ab}_\mu c^b)\cdot\Gamma]^1\frac{\delta\Sigma}{\delta A^a_\mu} - (D^{ab}_\mu c^b)\frac{\delta\Gamma^1}{\delta A^a_\mu}\\&+\left[\left(\frac{g}{2}\varepsilon^{abc}c^bc^c\right)\cdot\Gamma\right]^1\frac{\delta\Sigma}{\delta c^a}+\left(\frac{g}{2}\varepsilon^{abc}c^bc^c\right)\frac{\delta\Gamma^1}{\delta c^a}\\&+\left[-g^2\left(\varepsilon^{abc}b^bc^c+\frac{g}{2}\varepsilon^{abc}\bar{c}^b\varepsilon^{cmn}c^mc^n\right)\cdot\Gamma\right]^1\frac{\delta\Sigma}{\delta\tilde{\phi}^a}\\&-g^2\left(\varepsilon^{abc}b^bc^c+\frac{g}{2}\varepsilon^{abc}\bar{c}^b\varepsilon^{cmn}c^mc^n\right)\frac{\delta\Gamma^1}{\delta\tilde{\phi}^a}+b^a\frac{\delta\Gamma^1}{\delta\bar{c}^a}\Big).\end{aligned}\tag{4.15}$$

Acting on both sides of Eq. (4.15) with the test operator

$$\frac{\delta^2}{\delta c^a(x)\delta A^b_\nu(y)},\tag{4.16}$$

and setting all fields and sources equal to zero, $A^a_\mu = \Omega^a_\mu = c^a = L^a = \tilde{\phi}^a = F^a = b^a = \bar{c}^a = 0$, one obtains the Ward identity for the vacuum polarization in the condensed ghost vacuum

$$\partial^x_\mu\frac{\delta^2\Gamma^1}{\delta A^a_\mu(x)\delta A^b_\nu(y)} = \frac{\phi_*}{\rho_0}\left(\frac{\delta^2[\int d^4z(\varepsilon^{3np}b^nc^p)_z\cdot\Gamma]^1}{\delta c^a(x)\delta A^b_\nu(y)}\right).\tag{4.17}$$

The right-hand side of Eq. (4.17) is the one-loop $1PI$ Green function with the insertion of the integrated composite operator $\int d^4z(\varepsilon^{3np}b^nc^p)_z$, with one gluon and one ghost as (amputated) external legs. This Green function, shown in Fig. 1, is nonvanishing.

For instance, for the Fourier transform of the component $a = b = 3$, one finds

$$\left(\frac{\delta^2[\int d^4x_1(\varepsilon^{3np}b^nc^p)_{x_1}\cdot\Gamma]^1}{\delta c^3\delta A^3_\nu}\right) = \mathcal{M}^{33}_\nu(p,\omega),\tag{4.18}$$

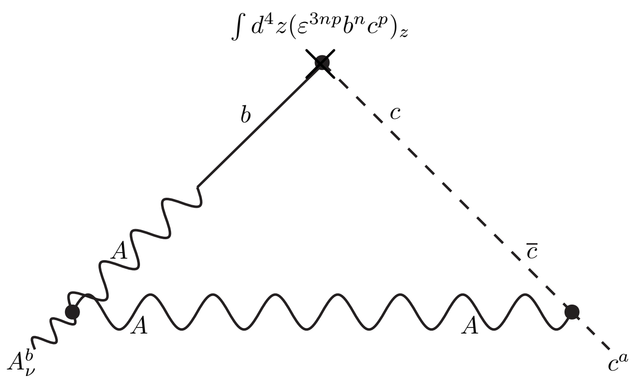

FIG. 1. The Green function appearing in the right-hand side of Eq. (4.17).

with

$$\begin{aligned}\mathcal{M}^{33}_\nu(p,\omega) = -2g^2\int\frac{d^dk}{(2\pi)^d}&\\\times\frac{(k^2\delta_{\nu\rho}+p_\nu p_\rho-2k_\nu p_\rho)p_\tau}{(k^4+\omega^2)((p-k)^2+m^2)}&\\\times\left(\delta_{\rho\tau}-\frac{(p-k)_\rho(p-k)_\tau}{(p-k)^2}\right),&\end{aligned}\tag{4.19}$$

where use has been made of the mixed $b - A$ propagator

$$\langle b^a(k)A^b_\mu(-k)\rangle = \frac{k_\mu}{k^2}.\tag{4.20}$$

From the identity

$$\frac{k^2}{k^4+\omega^2} = \frac{1}{k^2} - \frac{\omega^2}{(k^4+\omega^2)k^2},\tag{4.21}$$

it follows that

$$\mathcal{M}^{33}_\nu(p,\omega) = \mathcal{M}^{33}_\nu(p,0) + \omega^2\tilde{\mathcal{M}}^{33}_\nu(p,\omega),\tag{4.22}$$

where $\mathcal{M}^{33}_\nu(p,0)$ is logarithmic divergent, while $\tilde{\mathcal{M}}^{33}_\nu(p,\omega)$ is ultraviolet finite. One should certainly notice that the right-hand side of Eq. (4.17) is proportional to the ghost condensation ϕ_*.

In summary, the Ward identity (4.17) shows at a formal level that the one-loop vacuum polarization in the condensed ghost vacuum is not transverse. This will be explicitly checked in Sec. VI at one-loop order. We recognize that, in the absence of the ghost condensation, the well known result of the transversality of the vacuum polarization is recovered.

We observe that, due to Lorentz invariance, one may write

$$\frac{\phi_*}{\rho_0}\mathcal{M}^{33}_\nu(p,\omega) = a^{33}(p,\omega)p_\nu,\tag{4.23}$$

where $a^{33}(p,\omega)$ is a suitable scalar quantity. The Ward identity (4.17) becomes thus

$$p_\mu\Pi^{33}_{\mu\nu}(p,\omega) = a^{33}(p,\omega)p_\nu,\tag{4.24}$$

where $\Pi^{33}_{\mu\nu}(p,\omega)$ stands for the vacuum polarization. Equation (4.24) can be recast into the form

$$p_\mu(\Pi^{33}_{\mu\nu}(p,\omega) - a^{33}(p,\omega)\delta_{\mu\nu}) = 0,\tag{4.25}$$

which is suitable for analyzing the location of the pole of the complete gluon propagator $\mathcal{G}_{\mu\nu}(p)$ in the condensed vacuum.

V. WARD IDENTITY FOR THE GLUON PROPAGATOR

Having discussed the breakdown of the transversality of the vacuum polarization due to the ghost condensation, one might wonder what happens with the gluon propagator itself in the ghost condensed vacuum.

A. Consequences of the ghost condensation on the pole of the propagator

In order to have a better understanding of the Ward identity (4.17), let us discuss the resulting modification on the location of the pole of the gluon propagator $\mathcal{G}_{\mu\nu}(p)$. From Eq. (4.25), it follows that

$$\Pi^{33}_{\mu\nu}(p,\omega) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi^{33}(p,\omega) + a^{33}(p,\omega)\delta_{\mu\nu}. \tag{5.1}$$

Therefore, for the complete one-loop propagator $\mathcal{G}^{33}_{\mu\nu}(p)$ one has

$$\mathcal{G}^{33}_{\mu\nu}(p) = \frac{P_{\mu\nu}(p)}{p^2+m^2} - \frac{P_{\mu\rho}(p)}{p^2+m^2}\Pi^{33}_{\rho\sigma}(p,\omega)\frac{P_{\sigma\nu}(p)}{p^2+m^2}, \tag{5.2}$$

where

$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \tag{5.3}$$

is the transversal projector. Thus

$$\begin{aligned}\mathcal{G}^{33}_{\mu\nu}(p) &= \frac{P_{\mu\nu}(p)}{p^2+m^2} - \frac{P_{\mu\rho}(p)}{p^2+m^2}P_{\rho\sigma}(p)\Pi^{33}(p,\omega)\frac{P_{\sigma\nu}(p)}{p^2+m^2} - \frac{P_{\mu\rho}(p)}{p^2+m^2}a^{33}(p,\omega)\delta_{\rho\sigma}\frac{P_{\sigma\nu}(p)}{p^2+m^2} \\ &= \frac{P_{\mu\nu}(p)}{p^2+m^2} - \frac{P_{\mu\nu}(p)}{p^2+m^2}\frac{\Pi^{33}(p,\omega)}{p^2+m^2} - \frac{P_{\mu\nu}(p)}{p^2+m^2}\frac{a^{33}(p,\omega)}{p^2+m^2} = \frac{P_{\mu\nu}(p)}{p^2+m^2}\left(1 - \frac{(\Pi^{33}(p,\omega)+a^{33}(p,\omega))}{p^2+m^2}\right) \\ &\simeq \frac{P_{\mu\nu}(p)}{p^2+m^2}\frac{1}{1+\frac{(\Pi^{33}(p,\omega)+a^{33}(p,\omega))}{p^2+m^2}}.\end{aligned} \tag{5.4}$$

Finally

$$\mathcal{G}^{33}_{\mu\nu}(p) \simeq \frac{P_{\mu\nu}(p)}{p^2+m^2+\Pi^{33}(p,\omega)+a^{33}(p,\omega)}, \tag{5.5}$$

from which one sees that the location of the pole is indeed affected by the presence of the quantity $a^{33}(p,\omega)$.

Analogously, for the components (α,β) of the gluon propagator, one shall find that

$$\mathcal{G}^{\alpha\beta}_{\mu\nu}(p) \simeq \delta^{\alpha\beta}\frac{P_{\mu\nu}(p)}{p^2+m^2+\Pi(p,\omega)+a(p,\omega)}, \tag{5.6}$$

with

$$\Pi^{\alpha\beta}_{\mu\nu}(p,\omega) = \delta^{\alpha\beta}\left(\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi(p,\omega) + a(p,\omega)\delta_{\mu\nu}\right). \tag{5.7}$$

Eqs. (5.5) and (5.6) express the physical meaning of the Ward identity (4.17), namely, due to the violation of the transversality of the vacuum polarization in the condensed vacuum, the location of the pole of the propagator is affected by the quantity $\mathcal{M}_\nu(p,\omega)$ appearing in the right-hand side of Eq. (4.17). We also notice that the gluon propagator, being proportional to the projector $P_{\mu\nu}(p)$, (5.3), remains transverse. This will be formally proven in the following subsection.

B. Transversality of the gluon propagator

As the gluon propagator is the connected two-point function, we should consider the generator Z^c of connected Green functions, obtained from the $1PI$ quantum action Γ by means of a Legendre transformation. Therefore, we introduce sources I^a_b, J^a_μ and K^a_c, respectively, for the fields b^a, A^a_μ and c^a. The effective action (4.11) obeys the Ward identity

$$\frac{\delta\Gamma}{\delta b^a} = \partial_\mu A^a_\mu + g^2\varepsilon^{ade}F^d c^e. \tag{5.8}$$

At the level of Z^c, this identity is translated into

$$I^a_b = \partial_\mu\frac{\delta Z^c}{\delta J^a_\mu} + g^2\varepsilon^{ade}F^d\frac{\delta Z^c}{\delta K^e_c}, \tag{5.9}$$

from which one deduces

$$0 = \partial^x_\mu\frac{\delta^2 Z^c}{\delta J^a_\mu(x)\delta J^b_\mu(y)}\bigg|_{F,I,J,K=0} = \partial^x_\mu\mathcal{G}^{ab}_{\mu\nu}(x,y), \tag{5.10}$$

meaning that the gluon propagator does remain transverse in the ghost condensed vacuum.

VI. ONE-LOOP EVALUATION OF THE GHOST CONTRIBUTION TO THE VACUUM POLARIZATION

In this section, we shall discuss the one-loop contribution to the vacuum polarization coming from diagrams that contain internal ghost lines, denoted by $(\Pi^{ab}_{\mu\nu}(p,\omega))_{\rm gh}$.

In order to evaluate the one-loop ghost contribution to the vacuum polarization, let us first remind the form of the tree level gluon and ghost propagators in the nontrivial vacuum, given by $\langle A^2_\mu\rangle \neq 0$ and $\langle\varepsilon^{abc}\bar{c}^b c^c\rangle \neq 0$. From [8,11], the gluon propagator reads

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \delta^{ab}\frac{1}{k^2+m^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right),\qquad a,b=1,2,3. \tag{6.1}$$

Here we see the meaning of the parameter m^2, defined in Eq. (3.12). It corresponds to a dynamically generated tree level gluon mass parameter. At higher orders, quantum polarization effects will affect the value of that mass parameter. In the absence of the ghost condensation, this was discussed in [13,14].

For the ghost propagator corresponding to the Overhauser vacuum given in Eq. (4.2), we have [41]

$$\langle\bar{c}^3(k)c^3(-k)\rangle = \frac{1}{k^2},\qquad \langle\bar{c}^\alpha(k)c^\beta(-k)\rangle = \frac{\delta^{\alpha\beta}k^2-\omega\varepsilon^{\alpha\beta}}{k^4+\omega^2},\qquad \alpha,\beta=1,2, \tag{6.2}$$

where $\varepsilon^{\alpha\beta}=\varepsilon^{3\alpha\beta}$. Here we see that the behavior of the ghost propagator is changed by the presence of the nonvanishing ghost condensate: there is a clear distinction between the diagonal and off-diagonal part of the propagator.

At one-loop level, the relevant interaction vertex which has to be taken into account in order to evaluate $(\Pi^{ab}_{\mu\nu}(p,\omega))_{\rm gh}$, is the ghost-antighost-gluon vertex, $\varepsilon^{abc}\partial_\mu\bar{c}^aA^b_\mu c^c$.

A. Evaluation of the off-diagonal component $(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh}$

Let us consider the off-diagonal component of the ghost contribution to the vacuum polarization, given by

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh} = g^2\varepsilon^{\alpha mn}\varepsilon^{\beta pq}\int\frac{d^dk}{(2\pi)^d}(p-k)_\mu \times k_\nu\langle\bar{c}^mc^q\rangle_{p-k}\langle\bar{c}^pc^n\rangle_k. \tag{6.3}$$

A little algebra results in

$$\varepsilon^{\alpha mn}\varepsilon^{\beta pq}\langle\bar{c}^mc^q\rangle_{p-k}\langle\bar{c}^pc^n\rangle_k = -\frac{1}{(p-k)^2}\times\frac{(\delta^{\alpha\beta}k^2+\omega\varepsilon^{\alpha\beta})}{k^4+\omega^2}-\frac{1}{k^2}\times\frac{(\delta^{\alpha\beta}(p-k)^2-\omega\varepsilon^{\alpha\beta})}{(p-k)^4+\omega^2}, \tag{6.4}$$

hence

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh} = -g^2\int\frac{d^dk}{(2\pi)^d}(p-k)_\mu k_\nu\frac{1}{(p-k)^2}\times\frac{(\delta^{\alpha\beta}k^2+\omega\varepsilon^{\alpha\beta})}{k^4+\omega^2} - g^2\int\frac{d^dk}{(2\pi)^d}(p-k)_\mu k_\nu\frac{1}{k^2}\times\frac{(\delta^{\alpha\beta}(p-k)^2-\omega\varepsilon^{\alpha\beta})}{(p-k)^4+\omega^2}, \tag{6.5}$$

Use has been made of the property

$$\varepsilon^{\alpha\delta}\varepsilon^{\delta\beta} = -\delta^{\alpha\beta}. \tag{6.6}$$

In the second integral in Eq. (6.5), we make the change of variables

$$k_\mu \to p_\mu - k_\mu, \tag{6.7}$$

to find that

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh} = -g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\times\frac{k^2(p_\mu k_\nu+p_\nu k_\mu-2k_\mu k_\nu)}{(p-k)^2(k^4+\omega^2)} - g^2\omega\varepsilon^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\frac{(p_\mu k_\nu-k_\mu p_\nu)}{(p-k)^2(k^4+\omega^2)}. \tag{6.8}$$

Since the last term in Eq. (6.8) vanishes due to its antisymmetry, we arrive at

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh} = -g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\times\frac{k^2(p_\mu k_\nu+p_\nu k_\mu-2k_\mu k_\nu)}{(p-k)^2(k^4+\omega^2)}, \tag{6.9}$$

which coincides in fact with the expression found in [36].

Before calculating $(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh}$, we notice, by making use of the relation

$$\frac{k^2}{k^4+\omega^2} = \frac{1}{k^2}-\frac{\omega^2}{(k^4+\omega^2)k^2}, \tag{6.10}$$

that

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh} = (\Pi^{\alpha\beta}_{\mu\nu}(p,0))_{\rm gh}+\omega^2J^{\alpha\beta}_{\mu\nu}(p,\omega), \tag{6.11}$$

where

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,0))_{\rm gh} = -g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\times\frac{(p_\mu k_\nu+p_\nu k_\mu-2k_\mu k_\nu)}{(p-k)^2k^2}, \tag{6.12}$$

and

$$J^{\alpha\beta}_{\mu\nu}(p,\omega) = g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\frac{(p_\mu k_\nu + p_\nu k_\mu - 2k_\mu k_\nu)}{(p-k)^2(k^4+\omega^2)k^2}. \tag{6.13}$$

One observes that only the term $(\Pi^{\alpha\beta}_{\mu\nu}(p,0))_{\rm gh}$ is ultraviolet divergent, while the ω-dependent part $J^{\alpha\beta}_{\mu\nu}(p,\omega)$ is convergent by power counting. The divergent part of $(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh}$ is thus the same as if it were computed in the absence of the ghost condensation.

To calculate explicitly $(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh}$, we shall concentrate on

$$\pi_{\mu\nu} = \int\frac{d^dk}{(2\pi)^d}\frac{k^2(p_\mu k_\nu + p_\nu k_\mu - 2k_\mu k_\nu)}{(p-k)^2(k^4+\omega^2)}. \tag{6.14}$$

Using

$$\frac{k^2}{k^4+\omega^2} = \frac{1}{2}\left(\frac{1}{k^2+i\omega}+\frac{1}{k^2-i\omega}\right), \tag{6.15}$$

it follows that

$$\pi_{\mu\nu} = \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2+i\omega}\right)\frac{p_\mu k_\nu - k_\mu k_\nu}{(p-k)^2} + \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\times\left(\frac{1}{k^2+i\omega}\right)\frac{p_\nu k_\mu - k_\mu k_\nu}{(p-k)^2} + (\omega\to-\omega). \tag{6.16}$$

Setting

$$\tilde{\pi}_{\mu\nu} = \int\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2+i\omega}\right)\frac{p_\mu k_\nu - k_\mu k_\nu}{(p-k)^2}, \tag{6.17}$$

then clearly

$$\pi_{\mu\nu} = \frac{1}{2}\tilde{\pi}_{\mu\nu} + \frac{1}{2}\tilde{\pi}_{\nu\mu} + (\omega\to-\omega). \tag{6.18}$$

Introducing a Feynman parameter and employing dimensional regularization, one shall find after some calculation that [4] for Eq. (6.18)

$$\pi_{\mu\nu} = \frac{1}{16\pi^2}\left[\frac{\delta_{\mu\nu}}{d}\left(\frac{2}{3}\frac{\omega^3}{p^4}\arctan\frac{p^2}{\omega} + \frac{1}{9}\left(\frac{12p^2}{\varepsilon}+13p^2-\frac{6\omega^2}{p^2}\right) - \frac{p^2}{3}\ln\frac{p^4+\omega^2}{\bar{\mu}^4} + 2\omega\arctan\frac{\omega}{p^2} - \frac{\omega^2}{p^2}\ln\frac{\omega^2}{p^4+\omega^2}\right) + p_\mu p_\nu\left(-\frac{2}{3}\frac{\omega^3}{p^6}\arctan\frac{p^2}{\omega^2} + \frac{1}{18}\left(\frac{12}{\varepsilon}+10+12\frac{\omega^2}{p^4}\right) - \frac{1}{6}\ln\frac{p^4+\omega^2}{\bar{\mu}^4} + \frac{1}{2}\frac{\omega^2}{p^4}\ln\frac{\omega^2}{p^4+\omega^2}\right)\right]. \tag{6.19}$$

As such, we obtain at last

$$(\Pi^{\alpha\beta}_{\mu\nu}(p,\omega))_{\rm gh} = \frac{-g^2\delta^{\alpha\beta}}{16\pi^2}\left[\frac{\delta_{\mu\nu}}{4}\left(\frac{2}{3}\frac{\omega^3}{p^4}\arctan\frac{p^2}{\omega} + \frac{1}{9}\left(\frac{12p^2}{\varepsilon}+13p^2-\frac{6\omega^2}{p^2}+3p^2\right) - \frac{p^2}{3}\ln\frac{p^4+\omega^2}{\bar{\mu}^4} + 2\omega\arctan\frac{\omega}{p^2} - \frac{\omega^2}{p^2}\ln\frac{\omega^2}{p^4+\omega^2}\right) + p_\mu p_\nu\left(-\frac{2}{3}\frac{\omega^3}{p^6}\arctan\frac{p^2}{\omega^2} + \frac{1}{18}\left(\frac{12}{\varepsilon}+10+12\frac{\omega^2}{p^4}\right) - \frac{1}{6}\ln\frac{p^4+\omega^2}{\bar{\mu}^4} + \frac{1}{2}\frac{\omega^2}{p^4}\ln\frac{\omega^2}{p^4+\omega^2}\right)\right]. \tag{6.20}$$

According to the analysis of the Slavnov-Taylor identities, the contribution to the vacuum polarization, that is induced by the ghost condensation, displays a violation of the transversality, as it is apparent from the above expression (6.20).

The result (6.20) can be compared with that of [36]. In order to do so, we observe that the $\overline{\rm MS}$ renormalization was not performed in [36]. However, a careful examination reveals that our results are in perfect agreement with those of [36], keeping in mind that

$$-\gamma + \ln 4\pi - \ln p^2 \xrightarrow{\overline{\rm MS}} -\ln\frac{p^2}{\bar{\mu}^2} \equiv -\frac{1}{2}\ln\frac{p^4}{\bar{\mu}^4},$$
$$\frac{\pi}{2} - \arctan\frac{1}{x} = \arctan x, \qquad \forall x > 0. \tag{6.21}$$

[4] The property that $(1/2i)\ln(1+ix)/(1-ix) = \arctan x$ was employed.

B. Evaluation of the diagonal component $(\Pi^{33}_{\mu\nu}(p,\omega))_{\rm gh}$

Let us take a look at the diagonal component $(\Pi^{33}_{\mu\nu}(p,\omega))_{\rm gh}$. From expressions (6.2) , one has

$$\begin{aligned}(\Pi^{33}_{\mu\nu}(p,\omega))_{\rm gh} &= g^2\varepsilon^{\alpha\beta}\varepsilon^{\delta\gamma}\int\frac{d^dk}{(2\pi)^d}\times\frac{(p-k)_\mu k_\nu}{((p-k)^4+\omega^2)(k^4+\omega^2)}\times(\delta^{\alpha\gamma}(p-k)^2-\omega\varepsilon^{\alpha\gamma})(\delta^{\delta\beta}k^2-\omega\varepsilon^{\delta\beta})\\ &= 2g^2\int\frac{d^dk}{(2\pi)^d}\frac{(p-k)_\mu k_\nu}{((p-k)^4+\omega^2)(k^4+\omega^2)}\times(-(p-k)^2k^2+\omega^2)\end{aligned} \tag{6.22}$$

One can check that

$$\frac{(p-k)^2k^2}{((p-k)^4+\omega^2)(k^4+\omega^2)} = \frac{1}{(p-k)^2k^2} - \omega^2\left(\frac{k^2}{((p-k)^4+\omega^2)(p-k)^2(k^4+\omega^2)} + \frac{1}{(p-k)^2(k^4+\omega^2)k^2}\right), \quad (6.23)$$

leading to

$$(\Pi^{33}_{\mu\nu}(p,\omega))_{\text{gh}} = (\Pi^{33}_{\mu\nu}(p,0))_{\text{gh}} + \omega^2 J^{33}_{\mu\nu}(p,\omega), \quad (6.24)$$

where

$$(\Pi^{33}_{\mu\nu}(p,0))_{\text{gh}} = -2g^2\int\frac{d^dk}{(2\pi)^d}\frac{(p-k)_\mu k_\nu}{(p-k)^2k^2}, \quad (6.25)$$

and

$$J^{33}_{\mu\nu}(p,\omega) = 2g^2\int\frac{d^dk}{(2\pi)^d}\frac{(p-k)_\mu k_\nu}{(k^4+\omega^2)}\left(\frac{1}{((p-k)^4+\omega^2)} + \frac{k^2}{((p-k)^4+\omega^2)(p-k)^2} + \frac{1}{(p-k)^2k^2}\right). \quad (6.26)$$

Again, we observe that only the term $(\Pi^{33}_{\mu\nu}(p,0))_{\text{gh}}$ is ultraviolet divergent, while the ω-dependent part $J^{33}_{\mu\nu}(p,\omega)$ is convergent by power counting. The divergent part of $(\Pi^{33}_{\mu\nu}(p,\omega))_{\text{gh}}$ is thus the same as if it were computed in the absence of the ghost condensation.

Unlike in the off-diagonal case, we shall not determine the diagonal part of the polarization tensor, $(\Pi^{33}_{\mu\nu}(p,\omega))_{\text{gh}}$, for general incoming momentum p. To obtain the full expression for $(\Pi^{33}_{\mu\nu}(p,\omega))_{\text{gh}}$, very tedious calculations would be required. To illustrate how complicated things can become, we have collected some more details in the Appendix A.

VII. MASS SPLITTING

We now come to another consequence due to the presence of the ghost and gluon condensation. We recall once more that the condensate $\langle A^2_\mu\rangle$ gives rise to an effective dynamical mass in the gauge fixed action [8,11], as it is apparent from eqns.(3.4) and from the propagator in Eq. (6.1). In the absence of the ghost condensation, this mass is the same for all colors. In the presence of the ghost condensate $\langle\varepsilon^{3bc}\bar{c}^bc^c\rangle$, the interesting phenomenon of the splitting of the diagonal and off-diagonal gluon masses takes place, due to quantum corrections induced by the vacuum polarization.

A. Identification of the mass term

As we have seen in the previous section, the ω-dependent part $J^{ab}_{\mu\nu}(p,\omega)$ of the ghost contribution to the vacuum polarization is free from ultraviolet divergences. Furthermore, according to Eqs. (6.13) and (6.26), $J^{ab}_{\mu\nu}(p,\omega)$ attains a finite value at $p=0$. This allows us to interpret $J^{ab}_{\mu\nu}(0,\omega)$ as a contribution to the gluon mass in the effective action. Consider in fact the one-loop two-point function part of the $1PI$ effective action, namely

$$\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(p)\Pi^{ab}_{\mu\nu}A^b_\nu(-p), \quad (7.1)$$

where $\Pi^{ab}_{\mu\nu}$ stands for the complete one-loop vacuum polarization. Let us consider, in particular, the ω-dependent part of the ghost contribution

$$\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(p)J^{ab}_{\mu\nu}(p,\omega)A^b_\nu(-p), \quad (7.2)$$

which can be rewritten as

$$\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(p)(J^{ab}_{\mu\nu}(p,\omega) - J^{ab}_{\mu\nu}(0,\omega))A^b_\nu(-p) + \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(p)J^{ab}_{\mu\nu}(0,\omega)A^b_\nu(-p). \quad (7.3)$$

The second term in expression (7.3) is interpreted as an induced mass. In the next section, we shall see that this term will be responsible for the splitting of the masses of the diagonal and off-diagonal components of the gluon field.

To avoid any confusion, we mean with mass thus the $1PI$ effective mass obtained from the vacuum polarization at zero momentum.

One could also study the pole mass. In the absence of the ghost condensation, the pole of the (Euclidean) gluon propagator was calculated in the LCO formalism in [13,14]. In principle, a study along the lines of [13,14] might be done also in the present case, but we would need knowledge of the polarization tensor at nonvanishing momentum. We mention again that, in case of $(\Pi^{33}_{\mu\nu}(p,\omega))_{\text{gh}}$, this is a highly complicated task (see Appendix A). A determination of the pole mass might be useful in order to facilitate a numerical comparison with other values in the unsplitted case, but this is beyond the aim of the current paper.

1. Evaluation of $(\Pi^{ab}_{\mu\nu}(0))_{\text{gh}}$

For the one-loop ghost contribution to the vacuum polarization at zero momentum we get

$$(\Pi^{ab}_{\mu\nu}(0))_{\text{gh}} = \frac{1}{d}\delta_{\mu\nu}(\Pi^{ab}_{\rho\rho}(0))_{\text{gh}}, \quad (7.4)$$

with

$$(\Pi^{ab}_{\rho\rho}(0))_{\text{gh}} = -g^2\varepsilon^{amn}\varepsilon^{bpq}\int\frac{d^dk}{(2\pi)^d}k^2\langle\bar{c}^mc^q\rangle_k\langle\bar{c}^pc^n\rangle_k. \quad (7.5)$$

2. Evaluation of the off-diagonal components $(\Pi^{\alpha\beta}_{\mu\nu}(0))_{\rm gh}$

Let us begin by evaluating the off-diagonal components[5] $(\Pi^{\alpha\beta}_{\rho\rho}(0))_{\rm gh}$, α, $\beta = 1, 2$, namely

$$(\Pi^{\alpha\beta}_{\rho\rho}(0))_{\rm gh} = -g^2\varepsilon^{\alpha mn}\varepsilon^{\beta pq}\int\frac{d^dk}{(2\pi)^d}k^2\langle\overline{c}^m c^q\rangle_k\langle\overline{c}^p c^n\rangle_k. \tag{7.6}$$

From Eq. (6.9) with $p_\mu \equiv 0$, one derives

$$\begin{aligned}(\Pi^{\alpha\beta}_{\rho\rho}(0))_{\rm gh} &= 2g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^4+\omega^2)}\\ &= 2g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\frac{k^4}{k^2(k^4+\omega^2)},\end{aligned} \tag{7.7}$$

hence

$$\begin{aligned}(\Pi^{\alpha\beta}_{\rho\rho}(0))_{\rm gh} &= 2g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2}-\frac{\omega^2}{k^2(k^4+\omega^2)}\right)\\ &= -2\omega^2g^2\delta^{\alpha\beta}\int\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2(k^4+\omega^2)}\right)\\ &= -\delta^{\alpha\beta}\frac{\omega g^2}{16\pi}.\end{aligned} \tag{7.8}$$

3. Evaluation of the diagonal component $(\Pi^{33}_{\mu\nu}(0))_{\rm gh}$

It remains to evaluate the diagonal component $(\Pi^{33}_{\mu\nu}(0))_{\rm gh}$. Setting $p_\mu \equiv 0$ in the expression (6.22), one derives

$$(\Pi^{33}_{\rho\rho}(0))_{\rm gh} = -g^2\int\frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^4+\omega^2)^2}(-2k^4+2\omega^2). \tag{7.9}$$

One can check that this can be rewritten as

$$(\Pi^{33}_{\rho\rho}(0))_{\rm gh} = -2g^2v^2\int\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2(k^4+\omega^2)}+\frac{2k^2}{(k^4+\omega^2)^2}\right). \tag{7.10}$$

Observe that the integrals in the left hand side of eq.(are ultraviolet finite. Making use of

$$\begin{aligned}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k^4+\omega^2)} &= \frac{1}{32\pi}\frac{1}{\omega},\\ \int\frac{d^4k}{(2\pi)^4}\frac{k^2}{(k^4+\omega^2)^2} &= \frac{1}{64\pi}\frac{1}{\omega},\end{aligned} \tag{7.11}$$

we obtain

$$(\Pi^{33}_{\rho\rho}(0))_{\rm gh} = -\frac{g^2\omega}{8\pi}. \tag{7.12}$$

[5] The mixed component $(\Pi^{3\beta}_{\rho\rho}(0))_{\rm gh}$, $\beta = 1, 2$, is easily seen to vanish.

We see thus that $(\Pi^{33}_{\rho\rho}(0))_{\rm gh} \neq (\Pi^{\alpha\beta}_{\rho\rho}(0))_{\rm gh}$, implying that the ghost condensate $\langle\varepsilon^{abc}\overline{c}^b c^c\rangle$ removes in fact the degeneracy of the gluon mass. Notice, in particular, that the diagonal component of the vacuum polarization at zero momentum is twice the off-diagonal part, a fact already observed in [36] in the case of the Curci-Ferrari gauge, see Eq. (28) of [36].

4. Determination of the pure gluon component $(\Pi^{ab}_{\mu\nu}(0,\omega))_{\rm gl}$

We shall also need the contributions to the vacuum polarization which are coming from one-loop diagrams built up without ghosts. Because of the form of the gluon propagator given in Eq. (6.2), these diagrams give the same contribution to both diagonal and off-diagonal masses. It is important to keep in mind that the polarization is not solely determined from the usual Yang-Mills interactions, by employing the massive gluon propagator, Eq. (6.1). We draw attention to the fact that one should calculate with the action (3.4) describing the condensed vacuum, where, next to (4.2), one has

$$\sigma(x) = \sigma_* + \tilde{\sigma}(x), \qquad \langle\tilde{\sigma}(x)\rangle = 0. \tag{7.13}$$

For example, there will be novel contributions coming from the extra four-point interaction, that adds to the snail diagram, which is no longer vanishing when the massive propagator of Eq. (6.1) is employed. Furthermore, there is a $1PI$ diagram generated from the $(\tilde{\sigma}AA)$-vertex, also contributing to $(\Pi^{ab}_{\mu\nu}(0,\omega))_{\rm gl}$.

Here, we shall not enter into the details of the calculation, as the relevant diagrams have already been evaluated in [13,14]. We shall only quote the result at $p^2 = 0$,

$$(\Pi^{ab}_{\mu\nu}(0))_{\rm gl} = \frac{g^2N}{16\pi^2}m^2\left(-\frac{7}{96}+\frac{17}{16}\ln\frac{m^2}{\overline{\mu}^2}\right)\delta^{ab}\delta_{\mu\nu}, \tag{7.14}$$

where we have dropped already the divergent part in $1/\varepsilon$, as we are assured that the theory is renormalizable.

B. Interpretation of the mass splitting

As we have seen, the ghost condensate $\langle\varepsilon^{abc}\overline{c}^b c^c\rangle$ induces a splitting in the gluon masses through quantum effects. Also, as observed in [35,36], the contribution of the ghost condensate to the effective gluon mass is negative. For the splitting of the gluon masses at one-loop order, we may write

$$\begin{aligned}m^2_{\rm diag} &= m^2+\delta m^2-\frac{g^2\omega}{32\pi},\\ m^2_{\rm off-diag} &= m^2+\delta m^2-\frac{g^2\omega}{64\pi},\end{aligned} \tag{7.15}$$

where δm^2 stands for the contribution to the vacuum polarization at zero momentum following from one-loop diagrams built up with gluons only. Explicitly, from Eq. (7.14),

$$\delta m^2 = \frac{g^2}{16\pi^2} m^2 \left(-\frac{7}{48} + \frac{17}{8} \ln \frac{m^2}{\overline{\mu}^2} \right), \tag{7.16}$$

where we have set $N = 2$.

We could also use the RG invariance here to sum the leading logarithms. Contributions $\propto \omega$ shall only be multiplied by powers of $g^2 \ln(\omega/\overline{\mu}^2)$ as the limit $m^2 \to 0$ exists, thus terms like $g^2 m^2 \ln(\omega/\overline{\mu}^2)$ cannot appear. The same holds true for contributions $\propto m^2$. Similar to what happened in section 3, running quantities will get replaced by their values at the mass scale m^2 or ω. More precisely,

$$\begin{aligned} m^2_{\text{diag}} &= \overline{m}^2 + \frac{\overline{g}^2}{16\pi^2} \overline{m}^2 \left(-\frac{7}{48} \right) - \frac{\tilde{g}^2 \tilde{\omega}}{32\pi}, \\ m^2_{\text{off-diag}} &= \overline{m}^2 + \frac{\overline{g}^2}{16\pi^2} \overline{m}^2 \left(-\frac{7}{48} \right) - \frac{\tilde{g}^2 \tilde{\omega}}{64\pi}, \end{aligned} \tag{7.17}$$

Let us take a look at the numbers. Substituting the quantities quoted in Eq. (3.31), we arrive at

$$m^2_{\text{diag}} \approx 1.66 \Lambda^2_{\overline{\text{MS}}}, \qquad m^2_{\text{off-diag}} \approx 2.34 \Lambda^2_{\overline{\text{MS}}}, \tag{7.18}$$

We notice that

$$m^2_{\text{diag}} < m^2_{\text{off-diag}}, \tag{7.19}$$

In the MAG, the gap existing between the diagonal and off-diagonal gluon masses was interpreted as analytical evidence for the Abelian dominance [12,24,33,34]. Analogously, we could interpret the result (7.18) and (7.19) as a certain indication that a kind of Abelian dominance might take place in the Landau gauge too. Of course, the numbers[6] in Eq. (7.18) should be interpreted with care: a difference between both masses shows up, but this difference is far to small to see it as the ultimate proof of Abelian dominance in the Landau gauge. Even in the MAG, the existing difference in diagonal and off-diagonal mass is taken only as an indication. Moreover, it is interesting to observe that in the Landau gauge, a distinction can arise between the diagonal and the off-diagonal sectors of the gauge group, albeit the gauge fixing itself respects the global $SU(2)$ invariance. The MAG already makes a distinction between the diagonal and off-diagonal part of the gauge group from the beginning.[7] As we have discussed in [12], in the MAG the condensate $\langle \frac{1}{2} A^a_\mu A^a_\mu + \alpha \overline{c}^a c^a \rangle$, which is the counterpart of $\langle A^2_\mu \rangle$ in the Landau gauge, provides a mass only for off-diagonal gluons, thereby already giving an indication of Abelian dominance in the low energy region without the inclusion of $\langle \varepsilon^{3bc} \overline{c}^b c^c \rangle$. The combined effect of the ghost condensation as well as of $\langle \frac{1}{2} A^a_\mu A^a_\mu + \alpha \overline{c}^a c^a \rangle$ are currently under investigation in the MAG.

In the absence of the gluon condensation $\langle A^2_\mu \rangle$, thus $m^2 \equiv 0$, Eq. (7.15) shows that the diagonal and off-diagonal gluon fields attain an effective mass which is *tachyonic*. This fact was first observed in [35] in the case of the MAG and later on confirmed in the Curci-Ferrari gauge [36].

VIII. CONCLUSION

We have studied simultaneously the condensation of the mass dimension two local composite operators A^2_μ and $f^{abc} \overline{c}^a c^b$ in the case of $SU(2)$ Yang-Mills gauge theory in the Landau gauge. This extended the already existing research on the gluon condensate $\langle A^2_\mu \rangle$ [8,9,11] and the ghost condensate $\langle \varepsilon^{3bc} \overline{c}^a c^b \rangle$ [40,41].

Employing the LCO formalism to construct the one-loop effective potential, we have shown that both condensates are dynamically favored as they lower the vacuum energy. The renormalizability of the resulting theory has been proven to all orders by means of the algebraic renormalization technique [22]. We also presented a study of some effects induced by the ghost condensation. We have shown, by analyzing the Slavnov-Taylor identities in the ghost condensed vacuum and by explicit one-loop calculations, that the vacuum polarization is no longer transverse, whereas the gluon propagator is.

In the LCO formalism, the nonvanishing condensate $\langle A^2_\mu \rangle$ gives rise to an effective tree level gluon mass, thus the lowest order gluon propagator gets modified, as it is apparent from Eq. (6.1). Likewise, the ghost condensate $\langle \varepsilon^{3bc} \overline{c}^a c^b \rangle$ influences the ghost propagator, given in Eq. (6.2). We determined the one-loop correction to the effective gluon mass and found that the ghost condensate induces a splitting between the diagonal and off-diagonal sector. The effective off-diagonal gluon mass turns out be larger than the diagonal one. This might be interpreted as a first analytical indication for a possible kind of Abelian dominance in the Landau gauge, analogously to what was done in the case of the maximal Abelian gauge in [12,24,33,34]. It is worth mentioning that, recently, some evidence for the Abelian dominance in the Landau gauge by lattice numerical simulations was announced in the works [19,20], where the appearance of an Abelian dual Meissner effect in this gauge has been pointed out.

Finally, we hope that our results could stimulate further lattice numerical studies of the ghost propagator. It would be very interesting if, somehow, one would be able to simulate the Overhauser vacuum (4.2). This could allow one to investigate the diagonal and off-diagonal part of ghost propagator, which turns out to be affected by the ghost condensation, see Eqs. (6.2). At the time of finishing this work, we have been informed about the results appearing in [45]. In this paper [45], a numerical study of the ghost condensation in the Overhauser channel for $SU(2)$

[6] We are unable to provide an estimate in terms of GeV, as, to our knowledge, an explicit value of $\Lambda_{\overline{\text{MS}}}^{N=2;N_f=0}$ is not available in the existing literature.

[7] As a matter of fact, only the off-diagonal gauge freedom is fixed. A supplementary condition has to be imposed on the diagonal part of the gauge freedom, as for instance an Abelian Landau gauge, as used in [12,28,33,34].

lattice gauge theory in the Landau gauge was performed. The data was fit to the theoretical prediction given in Eq. (6.2), and assuming a small value of the ghost condensate, the fitted power law behavior tended to be $\sim p^{-4}$, in accordance with the theoretical prediction following from Eq. (6.2). This is promising as it is first numerical indication that a ghost condensation might occur in the Landau gauge, although to obtain more conclusive results, simulations at larger physical volumes will be certainly necessary.

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APPENDIX A: $(\Pi^{33}_{\mu\nu}(p,\omega))_{gh}$.

In this Appendix, we shall outline some details concerning the evaluation of $(\Pi^{33}_{\mu\nu}(p,\omega))_{gh}$, given in Eq. (6.22).

We shall concentrate on

$$\pi'_{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{(p-k)_\mu k_\nu}{((p-k)^4+\omega^2)(k^4+\omega^2)} \times (-(p-k)^2k^2+\omega^2). \tag{A1}$$

Using the decomposition (6.15) twice, one can write

$$\pi'_{\mu\nu} = \frac{1}{4}\int \frac{d^dk}{(2\pi)^d} \frac{(p-k)_\mu k_\nu}{k^2(p-k)^2}(-(p-k)^2k^2+\omega^2) \left(\frac{1}{(p-k)^2+i\omega}\frac{1}{k^2+i\omega}+\frac{1}{(p-k)^2+i\omega}\frac{1}{k^2-i\omega}\right) + (\omega\to-\omega), \tag{A2}$$

meaning that we must calculate

$$\tilde{\pi}'_{\mu\nu} = \omega^2 \int \frac{d^dk}{(2\pi)^d} \frac{(p-k)_\mu k_\nu}{k^2(p-k)^2}\left(\frac{1}{(p-k)^2+i\omega}\frac{1}{k^2+i\omega} + \frac{1}{(p-k)^2+i\omega}\frac{1}{k^2-i\omega}\right) - \int\frac{d^dk}{(2\pi)^d}(p-k)_\mu k_\nu \times\left(\frac{1}{(p-k)^2+i\omega}\frac{1}{k^2+i\omega}+\frac{1}{(p-k)^2+i\omega} \times\frac{1}{k^2-i\omega}\right). \tag{A3}$$

Therefore, let us study the basic types of integrals we shall need to perform the full calculation, being

$$I^{\pm}_{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{(p-k)_\mu k_\nu}{k^2(p-k)^2}\left(\frac{1}{(p-k)^2+i\omega}\frac{1}{k^2\pm i\omega}\right), \tag{A4}$$

$$J^{\pm}_{\mu\nu} = \int \frac{d^dk}{(2\pi)^d}(p-k)_\mu k_\nu\left(\frac{1}{(p-k)^2+i\omega}\frac{1}{k^2\pm i\omega}\right). \tag{A5}$$

Let us begin with $I^{\pm}_{\mu\nu}$. We shall have to employ the generalized Feynman trick,

$$\frac{1}{ABCD} = \int_0^1 dxdydz\frac{6}{[xA+yB+zC+(1-x-y-z)D]^4}, \tag{A6}$$

which leads to

$$I^{\pm}_{\mu\nu} = 6\int_0^1 dxdydz\int\frac{d^dk}{(2\pi)^d}\frac{p_\mu k_\nu - k_\mu k_\nu}{[yp^2-2yp\cdot k+zp^2-2zp\cdot k+zi\omega+k^2\pm(1-x-y-z)i\omega]^4}. \tag{A7}$$

The substitution

$$K = k - yp - zp, \tag{A8}$$

allows to conclude that

$$I^{\pm}_{\mu\nu} = 6\int_0^1 dxdydz\int\frac{d^dK}{(2\pi)^d} \times \frac{-\frac{\delta_{\mu\nu}}{d}K^2 + p_\mu p_\nu(y+z)(1-(y+z))}{[K^2+\Delta^\pm]^4}, \tag{A9}$$

where

$$\Delta^\pm = p^2(-y^2-z^2-2yz+y+z) + zi\omega \pm (1-x-y-z)i\omega. \tag{A10}$$

Both K-integrations showing up are finite, and can be directly computed without any regularization, leading to

$$I^{\pm}_{\mu\nu} = \int_0^1 dxdydz\left(-\frac{\delta_{\mu\nu}}{32\pi^2}\frac{1}{\Delta^\pm}+\frac{p_\mu p_\nu}{16\pi^2} \times (y+z)(1-(y+z))\frac{1}{(\Delta^\pm)^2}\right). \tag{A11}$$

From the previous expression, it might be clear that the triple integral in (x, y, z) is far from being trivial and would give rise to a very complicated final result. We shall not attempt to calculate it any further.

As a check of the computation of $(\Pi^{33}_{\mu\nu}(p, \omega))_{\rm gh}$, we could determine its pole structure and compare it with the output determined by the symbolic language FORM [46], in which case the integrals were calculated by expanding them in the external momentum p. The $1/\varepsilon$ part is completely determined by the integrations of the type $J_{\mu\nu}$, as $I_{\mu\nu}$ is finite. We introduce once more a Feynman parameter to write

$$J^{\pm}_{\mu\nu} = \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \times \frac{p_\mu k_\nu - k_\mu k_\nu}{(xp^2 - 2xp\cdot k + k^2 + xi\omega \pm (1-x)i\omega)^2}, \tag{A12}$$

Substituting $K = k - xp$, one finds

$$J^{\pm}_{\mu\nu} = \frac{1}{16\pi^2}\int_0^1 dx\left[-\delta^{\pm}\frac{\delta_{\mu\nu}}{d}\left(-\frac{4}{\varepsilon} + 2\ln\frac{\delta^{\pm}}{\overline{\mu}^2} - 1\right) + x(1-x)p_\mu p_\nu\left(\frac{2}{\varepsilon} - \ln\frac{\delta^{\pm}}{\overline{\mu}^2}\right)\right], \tag{A13}$$

after the necessary simplifications, where we defined

$$\delta^{\pm} = -x^2p^2 + xp^2 + xi\omega \pm (1-x)i\omega. \tag{A14}$$

We remind that

$$(\Pi^{33}_{\mu\nu}(p,\omega))_{\rm gh} = \frac{g^2}{2}[(I^+_{\mu\nu} + I^-_{\mu\nu} + (\omega\to-\omega)) + (J^+_{\mu\nu} + J^-_{\mu\nu} + (\omega\to-\omega))], \tag{A15}$$

while simplifying the $J^{\pm}_{\mu\nu}$-part leads to

$$(\Pi^{33}_{\mu\nu}(p,\omega))_{\rm gh} = \frac{g^2}{2}[(I^+_{\mu\nu} + I^-_{\mu\nu} + (\omega\to-\omega))] - \frac{g^2}{2}\frac{1}{16\pi^2}\times\left[\frac{\delta_{\mu\nu}}{d}\left(\frac{8p^2}{3\varepsilon} + \text{finite}\right) + p_\mu p_\nu\left(\frac{4}{3\varepsilon} + \text{finite}\right)\right], \tag{A16}$$

and we conclude that

$$(\Pi^{33}_{\mu\nu}(p,\omega))^{\rm div}_{\rm gh} = -\frac{g^2}{16\pi^2}\delta_{\mu\nu}\frac{p^2}{3\varepsilon} - \frac{g^2}{16\pi^2}p_\mu p_\nu\frac{2}{3\varepsilon}. \tag{A17}$$

This result is in accordance with the result obtained by using FORM. Let us finally mention that the complete result of $(\Pi^{33}_{\mu\nu}(p,\omega))_{\rm gh}$, is not very transparent.

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