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Higgs–Matter Splitting in Quasi–Realistic Orbifold String GUTs

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Abstract

E_6 grand unification combines the Standard Model matter and Higgs states in the single 27 representation. I discuss how the E_6 structure underlies the quasi–realistic free fermion heterotic–string models. $E_6 \to SO(10) \times U(1)$ breaking is obtained by a GSO phase in the $N = 1$ partition function. The equivalence of this symmetry breaking phase with a particular choice of a boundary condition basis vector, which is used in the quasi-realistic models, is demonstrated in several cases. As a result matter states in the spinorial 16 representation of $SO(10)$ arise from the twisted sectors, whereas the Higgs states arise from the untwisted sector. Possible additional phenomenological implications of this E_6 symmetry breaking pattern are discussed.

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1 Introduction

Grand Unification is well supported by the pattern of observed fermion and gauge boson charges. Additionally, the observed logarithmic running of the Standard Model parameters is compatible with the hypothesis of unification in the gauge sector, and the heavy generation matter sector. Furthermore, the longevity of the proton and the suppression of left–handed neutrino masses also indicate a large unification scale of the order of $10^{16} GeV$.

Among the possible unification scenarios, $SU(5)$ is the most economical. The observation of neutrino oscillations and consequently of neutrino masses, necessitates adding $SU(5)$ singlets and hence the need to go outside $SU(5)$. Matter unification in the framework of $SO(10)$ is most compelling as it accommodates all the matter states of a single generation in the 16 spinorial representation. Then, a priori, one needs only two types of representations to accommodate the Standard Model matter and Higgs spectrum, the spinorial 16 and the vectorial 10 representations. The framework of E_6 grand unification as even further appeal, as, at the expense of adding an additional singlet, it embeds the 16 matter and 10 Higgs $SO(10)$ states into the 27 representation of E_6 [1].

As the observed symmetry at low energies consists solely of the Standard Model symmetry, its embedding into a Grand Unification group necessitates that we break the larger GUT symmetry. Grand Unification introduces additional difficulties with proton decay and neutrino masses. The Grand Unification gauge symmetry breaking and the miscellanea issues typically require the introduction of large representations of the GUT gauge group, like the 126 of $SO(10)$ or the 351 of E_6, and devising complicated symmetry breaking potentials to ensure proton longevity.

By producing a consistent framework for perturbative quantum gravity, while simultaneously giving rise to gauge and matter structures, string theory goes a step beyond conventional Grand Unified Theories (GUTs). In the modern view of string theory, the different ten dimensional string theories, as well as eleven dimensional supergravity, are effective limits of a more fundamental theory, which at present is still unknown. The heterotic limit [2], in particular, gives rise to the Grand Unification structures. Furthermore, the heterotic string is the only effective limit that gives rise to spinorial representations in the perturbative spectrum, and hence is the only limit that can accommodate the $SO(10)$ and E_6 unification pictures [3]. A class of string models that accommodate the conventional GUT structures are the so–called free fermionic models [4, 5, 6, 7, 8, 9, 10], which are related to $Z_2 \times Z_2$ orbifold compactification at special points in the moduli space [11, 12].

String theory offers several additional advantages over conventional GUTs. The replication of fermion families is associated with the properties of the six dimensional compactified manifold. Depending on the properties of this internal manifold, string theory gives rise to novel gauge symmetry breaking mechanism, which can be seen as breaking by GSO projections, or as breaking by Wilson line. Furthermore, string theory gives rise to a doublet–triplet splitting mechanism, in which the color triplets are projected out from the physical spectrum by GSO projections, whereas the electroweak doublets remain. The GUT doublet–triplet splitting problem then has a simple solution without the need to introduce large representations. An explicit realization of the doublet–triplet splitting in string GUT models was introduced in ref. [18].

The doublet–triplet splitting is induced by the breaking of the $SO(10)$ GUT to $SO(6)\times SO(4)$. The $SO(10)$ structure that underlies the three generation free fermionic models is well understood and has been amply exposed in the past. However, the models in fact possess an underlying E_6 structure that, for reasons explained here, has been somewhat obscured in the past. It is the purpose of this paper to remedy this situation and to expose the E_6 structure that underlies the realistic free fermionic models. As discussed above, the characteristic feature of E_6 is the unification of the matter and Higgs states into the 27 representation of E_6. As is typical of string theory, however, the E_6 symmetry is broken directly at the string level by a GSO phase. As in the case of $SO(10)\to SO(6)\times SO(4)$, the string induced breaking $E_6\to SO(10)\times U(1)$ has the additional consequence of projecting the twisted moduli [13], and may prove important for understanding the problem of supersymmetry breaking.

2 Realistic free fermionic models

To elucidate the underlying E_6 structure of the realistic free fermionic models I discuss first the general structure of the three generation models. In the free fermionic formulation [14] of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. In the light–cone gauge the world–sheet field content consists of two transverse left– and right–moving space–time coordinate bosons, $X^\mu_{1,2}$ and $\bar{X}^\mu_{1,2}$, and their left–moving fermionic superpartners $\psi^\mu_{1,2}$, and additional 62 purely internal Majorana–Weyl fermions, of which 18 are left–moving, χ^I, and 44 are right–moving, ϕ^a. In the supersymmetric sector the world–sheet supersymmetry is realized non–linearly and the world–sheet supercurrent is given by $T_F=\psi^\mu\partial X_\mu+i\chi^I y^I\omega^I,\ (I=1,\cdots,6)$. The $\{\chi^I,y^I,\omega^I\}$ $(I=1,\cdots,6)$ are 18 real free fermions transforming as the adjoint representation of $SU(2)^6$. Under parallel transport around a noncontractible loop on the toroidal world–sheet the fermionic fields pick up a phase

$$f\ \to\ -\,\mathrm{e}^{i\pi\alpha(f)}f\ ,\quad \alpha(f)\in(-1,+1]. \tag{2.1}$$

A model in this construction [14] is defined by a set of boundary conditions basis vectors and by a choice of generalized GSO projection coefficients, which satisfy the one–loop modular invariance constraints. The boundary conditions basis vectors b_k

span a finite additive group $\Xi = \sum_k n_i b_i$ where $n_i = 0, \cdots, N_{z_i} - 1$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$ are then obtained by acting on the vacuum state of that sector with the world-sheet bosonic and fermionic mode operators, with frequencies ν_f, ν_{f^*} and by subsequently applying the generalized GSO projections,

$$\left\{ e^{i\pi(b_i F_\alpha)} - \delta_\alpha c^* \begin{pmatrix} \alpha \\ b_i \end{pmatrix} \right\} |s\rangle = 0 \ , \tag{2.2}$$

where $F_\alpha(f)$ is a fermion number operator counting each mode of f once (and if f is complex, f^* minus once). For periodic complex fermions [*i.e.* for $\alpha(f) = 1$)] the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion f, there are two degenerate vacua $|+\rangle$, $|-\rangle$, annihilated by the zero modes f_0 and f_0^* and with fermion number $F(f) = 0, -1$ respectively. In Eq. (2.2), $\delta_\alpha = -1$ if ψ^μ is periodic in the sector α, and $\delta_\alpha = +1$ if ψ^μ is antiperiodic in the sector α.

2.1 An exemplary model

The model in tables [2.3,2.5] provide an example of a three generation free fermionic model [7]. The model, the full massless spectrum, and the trilevel superpotential are given in ref. [7]. Various phenomenological aspects of this model were analyzed in the literature [15].

The boundary condition basis vectors which generate the realistic free fermionic models are, in general, divided into two major subsets. The first set consist of the NAHE set [16, 17], which is a set of five boundary condition basis vectors denoted $\{\mathbf{1}, S, b_1, b_2, b_3\}$. With '0' indicating Neveu-Schwarz (NS) boundary conditions and '1' indicating Ramond boundary conditions, these vectors are as follows:

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
$\mathbf{1}$	1	1	1	1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	1,...,1	0	0	1	0,...,0

	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$
$\mathbf{1}$	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
S	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
b_1	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
b_2	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
b_3	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

(2.3)

with the following choice of phases which define how the generalized GSO projections are to be performed in each sector of the theory:

$$c\binom{b_i}{b_j} \;=\; c\binom{b_i}{S} \;=\; -\,c\binom{\mathbf{1}}{\mathbf{1}} \;=\; -\,1\ . \tag{2.4}$$

The NAHE set [2.3] is common subset to all the models discussed here, and therefore will be dropped in the following. The gauge group at the level of the NAHE set is

$$SO(10) \times SO(6)^3 \times E_8\ .$$

The $SO(10)$ group gives rise to the universal part of the observable gauge group. The $SO(6)$ groups are flavor dependent symmetries, while the E_8 group is hidden, as the Standard Model states are neutral under this group. The NAHE–set basis vectors b_1, b_2 and b_3 correspond to the three twisted sectors of the $Z_2 \times Z_2$ orbifold. At the level of the NAHE–set the free fermionic models contain 48 chiral generations and correspond to a so–called "orbifold string GUT".

To reduce the number of generations and break the GUT symmetry one introduces three additional basis vectors, typically denoted as α, β and γ. The additional basis vectors that generate the string model of ref. [7] are displayed in table [2.5].

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	0	0	0	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	0	0	1	0	1
γ	0	1	0	1	0	1	0	1	1	0	0	0

(2.5)

$$c\binom{b_i}{\alpha,\beta,\gamma} = -c\binom{\alpha,\beta}{\mathbf{1}} = c\binom{\alpha}{\beta} = c\binom{\gamma}{\alpha} = -c\binom{\gamma}{\beta} = -1\ ,\ (i=1,2,3), \tag{2.6}$$

with the others specified by modular invariance and space–time supersymmetry. The boundary condition basis vector in [2.5] break the gauge group to:

$$SU(3) \times SU(2) \times U(1)^2 \quad \times \quad U(1)^6 \quad \times SU(5) \times SU(3) \times U(1)^2,$$

where the first two $U(1)$s arise from the $SO(10)$ group, the next six $U(1)$s are obtained from $SO(6)^3$, and the remaining two $U(1)$ arise from the hidden E_8 gauge group. Additionally, the basis vectors α, β, γ reduce the number of generations to three. One from each of the twisted sectors b_1, b_2 and b_3. Electroweak Higgs doublets are obtained from the untwisted sector, and the sector $b_1+b_2+\alpha+\beta$. The full spectrum of this model, and detailed phenomenological studies are given in the literature [7, 15].

From the above we see that the model exhibits an underlying $SO(10)$ symmetry, but there is no trace of an E_6 group. It is the purpose of this paper to elucidate the stringy $E_6 \to SO(10) \times U(1)$ breaking, and to show that, just as in the case of the stringy $SO(10) \to SO(6) \times SO(4)$ the stringy breaking of E_6 has additional phenomenological consequences.

3 E_6 origins

To expose the underlying E_6 structure of the free fermionic models, we have to look at subsets of the basis vectors, or of the partition, function that preserve the E_6 symmetry. A good starting point is the subset of basis vectors

$$\{\mathbf{1}, S, 2\gamma, \xi_2 = \mathbf{1} + b_1 + b_2 + b_3\} \ . \tag{3.1}$$

This subset generates an $N = 4$ SUSY vacuum with $SO(12) \times SO(16) \times SO(16)$ gauge group. The NS sector gives rise to the space–time vector bosons that generate $SO(12) \times SO(16) \times SO(8) \times SO(8)$, and the sector ξ_2 complements the hidden gauge group to $SO(16)$ [11]. Adding the basis vectors b_1 and b_2 then breaks $N = 4$ to $N = 1$ supersymmetry. It breaks the gauge group to $SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$, and introduces 24 observable matter multiplets in the spinorial 16 representation of the observable $SO(10)$, from the sectors b_1, b_2 and $b_3 = \mathbf{1} + b_1 + b_2 + \xi_2$, and 16 hidden matter multiplets in the vectorial 16 representation of the hidden $SO(16)$ gauge group from the sectors $b_j + (2\gamma \oplus \xi_2)$ $j = 1, 2, 3$. The symbol $\oplus$ is used here to indicate that there are two sectors that produce the hidden matter representations. One being $b_j + 2\gamma$ and the second $b_j + 2\gamma + \xi_2$. This notation will be used in the following.

Note that we could have projected the enhancing vector bosons from the sector ξ_2 by the choice of GSO phase $c\binom{\xi_2}{S} = -1$, where S is the SUSY generator. The price is that the SUSY generators are projected and the vacuum is tachyon free and nonsupersymmetric. The reason that there are no tachyon is because the only tachyons in the model arise from the NS sector, and the projections of those only depend on δ_S, and not on the phase $c\binom{\xi_2}{S}$. This is reminiscent of the ten dimensional heterotic $SO(16) \times SO(16)$ model, in which modular invariance forces that the GSO phase that breaks $E_8 \times E_8 \to SO(16) \times SO(16)$, also projects out the space–time supersymmetry.

So far there is no reminiscence of E_6. An alternative way to produce the model of (3.1) is by starting with the set of basis vectors

$$\{\mathbf{1}, S, \xi_1, \xi_2 = \mathbf{1} + b_1 + b_2 + b_3\} \tag{3.2}$$

with

$$\xi_1 = (0, \cdots, 0 | \underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}}, 0, \cdots, 0) \ . \tag{3.3}$$

for a suitable choice of GSO phases, this set generates an $N = 4$ vacuum. The four dimensional gauge group in this model depends on the discrete choice of the GSO phase

$$c\binom{\xi_1}{\xi_2} = \pm 1 \ .$$

Since the overlap of periodic fermions between ξ_1 and ξ_2 is empty, we note from (2.2) that the choice $c\binom{\xi_1}{\xi_2} = -1$ projects all the states from the sectors ξ_1 and ξ_2, whereas the choice $c\binom{\xi_1}{\xi_2} = +1$ retains them in the spectrum. Thus, the choice

$$c\binom{\xi_1}{\xi_2} = +1 \tag{3.4}$$

produces a model with $SO(12) \times E_8 \times E_8$ gauge group, whereas the choice

$$c\binom{\xi_1}{\xi_2} = -1 \tag{3.5}$$

produces a model with $SO(12) \times SO(16) \times SO(16)$ gauge group, and reproduces the spectrum of (3.1). Thus, we note that there are two distinct ways to generate the same model. One is by the mapping $\xi_1 \to 2\gamma$, and the alternative method by the choice of the discrete phase $c\binom{\xi_1}{\xi_2}$.

Adding the basis vectors $\{b_1, b_2\}$ to the set (3.2) corresponds to the $Z_2 \times Z_2$ orbifold projection. This breaks $N = 4$ to $N = 1$ supersymmetry. Setting $c\binom{\xi_1}{\xi_2} = +1$ generates the $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$. The sectors b_j consist of 12 Ramond fermions and produce states in the spinorial 16 representation of $SO(10)$, whereas the sectors $b_j + \xi_1$ produce a matching number of states in the vectorial 10 representation of $SO(10)$. In addition the sectors $b_j + \xi_1$ produce a matching number of $SO(10)$ singlets which are charged under the $U(1)$ in the decomposition $E_6 \to SO(10) \times U(1)$ and a matching number of E_6 singlets. The untwisted NS sector produces 6 vectorial 10 multiplets, a matching number of $SO(10)$ singlets, and a matching number of E_6 singlets. The sector ξ_1 produces 3 16 multiplets and 3 $\overline{16}$ multiplets. Thus, in this case we get a model with 24 multiplets in the 27 representation of E_6 from the twisted sectors and 3 pairs in the $27 + \overline{27}$ from the untwisted sector.

Setting $c\binom{\xi_1}{\xi_2} = -1$ projects the vector bosons from the sectors ξ_1 and ξ_2. Vector bosons therefore are obtained solely from the untwisted sector, which produces the $SO(16) \times SO(16)$ gauge group. Adding the $Z_2 \times Z_2$ twists breaks the gauge group to $SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$. The twisted sectors b_j still produce the 24 multiplets in the spinorial 16 representation of $SO(10)$, but now the sectors $b_j + \xi_1$ produce states in the vectorial 16 representation of the hidden $SO(16)$ gauge group. The same spectrum is reproduced by replacing the basis vector ξ_1 with the basis vector 2γ. In this case the overlap between ξ_2 and 2γ is not empty. Therefore, the projection $\binom{\xi_2}{2\gamma}$ cannot project all the states from ξ_2, but merely halves the spectrum from this sector, whereas the sector 2γ does not produce massless states. The basis vectors b_1 and b_2 reduce the number of supersymmetries and break the gauge symmetry as before. The matter multiplets from the untwisted sector and the twisted sectors b_j remain as before, and the sectors $b_j + (2\gamma \oplus \xi_2)$ now produce the 24 vectorial 16 representations of the hidden $SO(16)$ gauge group.

It is therefore noted that the map

$$\xi_1 \rightarrow 2\gamma \ , \tag{3.6}$$

is in fact equivalent to the discrete choice of GSO phase

$$c\binom{\xi_1}{\xi_2} = +1 \rightarrow c\binom{\xi_1}{\xi_2} = -1, \tag{3.7}$$

and that the later corresponds to the gauge symmetry breaking pattern $E_6 \rightarrow SO(10) \times U(1)$. It is noted that the map (3.6) also requires the phase map

$$c\binom{\xi_1}{\xi_1} \rightarrow -c\binom{2\gamma}{2\gamma} \ . \tag{3.8}$$

So far I discussed the models only at the $N = 4$ level and at the $N = 1$ $Z_2 \times Z_2$ orbifold level. In the following I turn to examine how this structure is manifested in the case of quasi–realistic three generation models. In this regard it should be noted that the original construction of three generation free fermion models, that utilize the NAHE–subset of basis vectors, obscures the underlying E_6 structure of these models. The reason is that these models utilize the vector γ to break the observable $SO(2n) \rightarrow SU(n) \times U(1)$. The vector 2γ, which separates the gauge degrees of freedom from the geometrical degrees of freedom, therefore arises only as a multiple of the vector γ. The vector 2γ also fixes the charges of the chiral generations under $U(1)$'s in the E_8 Cartan subalgebra, which are external to E_6, and hence reduces the NAHE–base generations by 1/2. Thus, the NAHE–base, supplemented with the 2γ, or ξ_1, contains 24 chiral generations, as opposed to the NAHE–set by itself, which contains 48 chiral generations. The remaining reduction to three generations is obtained by the action of the basis vectors $\{\alpha, \beta, \gamma\}$ on the internal free fermions $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\cdots,6}$, each inducing a Z_2 projection on each of the twisted sectors b_j $(j = 1, 2, 3,)$. Hence, reducing the number of generations in each from eight to one. Models that do not contain the vector γ, like $SO(6) \times SO(4)$ models, must explicitly include the vector 2γ, or ξ_1, in the basis, to reduce the number of generations to three.

The model of table [3.9] is constructed to study the map (3.6) in a quasi–realistic model. It should be emphasized that the aim is not to construct a realistic model, but merely to study the map in a model that shares some of the structure of the three generation free fermionic models. In particular the assignment of boundary conditions with respect to the internal world–sheet fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$ is reminiscent of this assignment in the three generation free fermionic models. The model in table [3.9] is generated by the subset of basis vectors $\{\mathbf{1}, S, \xi_1, \xi_2, b_1, b_2\}$, and the additional basis vectors $\{b_4, b_5, \alpha\}$ in table [3.9].

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	1	1	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
β	1	0	1	0	1 1 1 1 1	0	1	0	0 0 0 0 0 0 0 0
γ	0	0	0	0	1 1 1 0 0	1	1	1	0 0 1 1 0 0 0 0

	$y^3\bar{y}^3$	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$y^6\bar{y}^6$	$y^1\bar{y}^1$	$y^2\bar{y}^2$	$\omega^5\bar{\omega}^5$	$\omega^6\bar{\omega}^6$	$\omega^1\bar{\omega}^1$	$\omega^2\bar{\omega}^2$	$\omega^3\bar{\omega}^3$	$\omega^4\bar{\omega}^4$
α	1	0	0	1	0	0	1	0	0	0	0	1
β	0	0	0	1	0	1	1	0	1	0	0	0
γ	0	1	0	1	0	1	0	1	0	1	0	1

(3.9)

With the choice of generalized GSO coefficients:

$$c\binom{S}{a_j} = \delta_{a_j} \quad , \qquad c\binom{b_{1,2,4,5}}{b_{1,2,4,5}, \xi_1, \xi_2, \alpha} = c\binom{\mathbf{1}}{\xi_1, \xi_2} = \; -1$$
$$c\binom{\mathbf{1}}{\alpha} = c\binom{\xi_1}{\xi_2, \alpha} = \; 1 \; ,$$

with the others specified by modular invariance and space–time supersymmetry. The gauge group of the model arises as follows. The NS sector produces the generators of the observable and hidden gauge groups

$$(SO(6)\times SU(2)_L\times SU(2)_R\times U(1)_{1,2,3})_{\rm O} \times (SO(12)\times SU(2)_{H_1}\times SU(2)_{H_2})_{\rm H}, \quad (3.10)$$

and the sectors ξ_1, and ξ_2, enhance the observable, and hidden, gauge groups of the model, respectively to:

$$\text{observable} : \quad SU(6) \times SU(2)_L \times U(1)^2 \; , \quad (3.11)$$
$$\text{hidden} : \quad E_7 \times SU(2) \; , \quad (3.12)$$

where $SU(2)_R$ and the $U(1)$ combination,

$$U(1)'_6 \;=\; U(1)_1 + U(1)_2 - U(1)_3 \; , \quad (3.13)$$

are embedded in $SU(6)$, and the two orthogonal $U(1)$ combinations are given by

$$U(1)'_1 \;=\; U(1)_1 - U(1)_2 \; , \quad (3.14)$$
$$U(1)'_2 \;=\; U(1)_1 + U(1)_2 + 2U(1)_3 \; . \quad (3.15)$$

Similarly, the hidden $SO(12) \times SU(2)_{H_1}$ are enhanced by the states from the sector ξ_2 to produce the E_7 gauge group.

This model is not a realistic model. It preserves some of the structure of the quasi–realistic string models in the sense that it produces three chiral generations from the sectors, b_1, b_2 and b_3. But the full spectrum is not realistic as it contains additional chiral matter, and the untwisted electroweak Higgs are projected out. The purpose here is to study how the maps (3.6) and (3.7) are related in a model that preserves some of this realistic structure. In the model of (3.9), with the choice of phases above, the sectors $b_j \oplus b_j + \xi_1$ produce three chiral generations in the $(15,1)+(6,2)$ of $SU(6)\times SU(2)_L$. The sectors $b_{4,5}\oplus b_{4,5}+\xi_1$ and $b_4+b_5\oplus b_4+b_5+\xi_1$ produce states in the $(15,1)+(\bar{6},2)$ and the sectors containing α, which breaks the NS $SO(10)$ gauge subgroup, produce states that transform as $(6,1)+(1,2)$ under $SU(6)\times SU(2)_L$ and transform as doublets under the hidden $SU(2)$ gauge group. These sectors are: $b_2+b_4+\alpha$, $b_1+b_4+b_5+\alpha$, $b_1+b_2+b_4+\alpha$, $b_2+b_3+b_5+\alpha$, $b_3+b_5+\alpha$, where I heuristically defined the combination $b_3 = \mathbf{1}+b_1+b_2+\xi_2$. The full massless spectrum of this model is given in appendix A. Table [A.1] contains the states in this model that originate from sectors that preserve the $SO(10)$ symmetry of the NS sector, whereas table [A.2] contains the $SO(10)$ breaking spectrum.

We can now project the vector bosons from the sectors ξ_1 and ξ_2 by the map (3.7) which fixes the phase (3.5). The full massless spectrum of this model is given in appendix B, where tables [B.1], and [B.2], contain the $SO(10)$ preserving, and $SO(10)$ breaking, spectrum, respectively. The GSO projections now project the states from the sectors ξ_1 and ξ_2. The gauge group in this case arises solely from the NS sector and the four dimensional gauge group is that of eq. (3.10). In this case the sectors $b_j \oplus \xi_1$ split. The sectors b_j produce spinorial matter states of the observable gauge group, whereas the sectors $b_j+\xi_1$ produce vectorial matter states of the hidden gauge group. Thus, the would be twisted Higgs states are projected from the physical spectrum by this splitting. Similarly, it is noted from table [B.1] that the spinorial matter states from the sector $S+b_4+b_5$ are projected out from the physical spectrum, and this sector produces vectorial matter states in the observable sector. Therefore, the original E_6 embedding of the spinorial (or matter) and vectorial (or Higgs), representations, which is "remembered" in the $SU(6)$, $(15,1)$ and $(6,2)$, representations, is broken by the choice of GSO projection phase, eq. (3.5). Similarly, the $SO(10)$ breaking spectrum in this model, shown in table [B.2], is split between the sectors that contain, and do not contain, ξ_1, which in the previous model were combined.

We can now perform the map (3.6). Since the overlap between ξ_2 and 2γ is now not empty, we can choose the phase $c\binom{\xi_1}{2\gamma} = -1$. Choosing the opposite phase amounts to redefinition of the charges, and has no physical effect. In the hidden sector of this model the NS sector generate the gauge group

$$(SO(8)\times SU(2)_{H_3}\times SU(2)_{H_4}\times SU(2)_{H_1}\times SU(2)_{H_2})_{\rm H}, \qquad (3.16)$$

and the sector ξ_2 enhances the hidden sector gauge symmetry to $SO(12)\times SU(2)_{H_3}\times SU(2)_{H_4}$, which is identical to that of model B. Thus, in this model the vectorial hidden sector matter states arise from the sectors $b_j + 2\gamma$ and $b_j + 2\gamma + \xi_2$. The symbol $\oplus$ is used to indicate this combination in table [C.2]. Inspecting the spectrum in appendices B and C, we note that the spectrum is indeed identical with the map

$$\xi_1 \leftrightarrow (2\gamma \oplus \xi_2) \ . \tag{3.17}$$

Note that in table [C.2] the states arise from separate sectors and do not combine, as is the case in table [B.2] of model B.

A similar map operates in models which utilize the vector γ and hence break the $SO(10)$ symmetry to $SU(5) \times U(1)$ or to $SU(3) \times SU(2) \times U(1)^2$. Supplementing the NAHE–set basis vectors with the set of basis vectors in table [3.18]

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	0	0	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
β	0	0	0	0	1 1 1 1 1	0	1	0	0 0 0 0 0 0 0 0
γ'	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0 0 0 1 1 1

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
b_4	1	0	0	1	0	0	1	1	0	0	1	1
b_5	0	0	1	1	1	0	0	1	0	1	0	1
γ'	0	1	0	0	0	1	0	0	1	0	0	0

(3.18)

and the choice of the additional generalized GSO coefficients:

$$c\binom{\mathbf{1}}{b_4, b_5, \gamma'} = c\binom{b_4, b_5, \gamma'}{b_1, b_2, b_3} = c\binom{\gamma'}{b_4, b_5} = -c\binom{b_4}{b_5} = +1$$
$$c\binom{S}{a_j} = \delta_{a_j}$$

The gauge group of this model is:

$$(SU(5) \times U(1) \times U(1)_{1,2,3} \times U(1)_{4,5,6})_{\rm O} \times (SO(10) \times SO(6))_{\rm H} \ , \tag{3.19}$$

where $U(1)_{1,2,3}$ are embedded in the observable E_8, whereas $U(1)_{4,5,6}$ are from the complexified world-sheet real fermions, $(\bar{y}^3 + i\bar{y}^6)$, $(\bar{y}^1 + i\bar{\omega}^5)$, $(\bar{\omega}^2 + i\bar{\omega}^4)$. Space–time vector bosons in this model arise solely from the Neveu–Schwarz sector. The model contains several additional combinations of basis vectors that may a priori give rise to vector bosons. These include the sectors: $2\gamma'$, $\xi_2 = \mathbf{1} + b_1 + b_2 + b_3$ and the sector $S + b_1 + b_2 + b_3 + b_4 + b_5 \pm \gamma'$. All the states from these sectors are projected out and hence there is no enhancement of the NS gauge group in this model. In the case of the sector $2\gamma'$ the projection depends on $\gamma \cdot b_j$. If $\gamma' \cdot b_j$ = even & $\gamma' \cdot b_i$ = odd, with $i \neq j$, vector bosons from $2\gamma'$ are projected out. This may occur because γ' must contain periodic internal fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}$. The reason is that

γ' breaks the $SO(10)$ symmetry and simultaneously halves the number of generations by fixing the $U(1)_{1,2,3}$ charges. In models with only periodic boundary conditions the later function is performed by the vector 2γ, which does not break the gauge group. Thus, in NAHE–based models with 1/2 boundary conditions, we must assign in γ periodic boundary conditions to internal fermions to insure that full $SO(10)$ spinorial 16 representations remain in the physical spectrum. This means that we have the freedom to choose appropriate boundary conditions that project the vector bosons from $2\gamma'$. Additionally, with the choice of phases above the vector bosons from the sector $S + b_1 + b_2 + b_3 + b_4 + b_5 \pm \gamma'$ are projected out.

The model of [3.18] then contains three generations of $SO(10)$ chiral 16 representations, decomposed under $SU(5) \times U(1)$ from the sectors $b_{1,2,3}$; three generations of the hidden $SO(16)$ vectorial 16 representation, decomposed under the hidden $SO(10) \times SO(6)$ gauge group, The sectors $b_2 + b_5$, $b_1 + b_4$ and $S + b_1 + b_2 + b_4 + b_5$ produce states that are $E_8 \times E_8$ singlets, and are charged with respect to the complexified internal fermions, $\{\bar{y}^3\bar{y}^6; \bar{y}^1\bar{\omega}^5; \bar{\omega}^2\bar{\omega}^4\}$. The sectors $b_3 \pm \gamma'$; $S+b_2+b_3+b_5 \pm \gamma'$; $S+b_2+b_3+b_4+b_5 \pm \gamma'$; $S+b_1+b_3+b_4 \pm \gamma'$; $S+b_1+b_3+b_4+b_5 \pm \gamma'$; $S+b_1+b_2+b_4+b_5 \pm \gamma'$, produce fractionally charged matter states that transform as $4+\bar{4}$ of the hidden $SO(6)$ gauge group. Note that in this model the entire set of untwisted geometrical moduli are projected out due to the specific pairing of the left–moving real fermions into complex pairs [13]. Additionally, the twisted moduli from the sectors $b_{1,2,3}$ are projected out as well [13]. The NS sector in this model produces, in addition to the gravity and gauge multiplets, scalar states that are charged with respect to $U(1)_{4,5,6}$. I note that this is not a realistic model as it does not contain the Higgs representations that are needed to break the GUT and electroweak symmetries.

I now turn to show how the map (3.6) is implemented in this model. The map is induced by the substitution $\gamma' \to \gamma$, with γ given in table [3.20]

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0 1 1

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
γ	0	1	0	0	0	1	0	0	1	0	0	0

(3.20)

Additionally, modular invariance requires the phase modification

$$c\binom{\gamma}{\gamma} = -c\binom{\gamma'}{\gamma'} \ . \tag{3.21}$$

All other GSO phases are identical in the two models. The gauge group in this model arises as follows. In the observable sector the gauge group remains as in (3.19). In the hidden sector the NS sector produces the gauge bosons of the

$$SU(4) \times U(1) \times SO(4) \times SO(4) \tag{3.22}$$

subgroup, and the sector $\xi_2 = \mathbf{1} + b_1 + b_2 + b_3$ produces the vector bosons that complete the hidden gauge group to $SO(10) \times SO(6)$. Thus, the four dimensional gauge group is identical in the two models. The sectors $b_{1,2,3}$, $b_2 + b_5$, $b_1 + b_4$ and $S + b_1 + b_2 + b_4 + b_5$ are not affected by this map, and therefore trivially produce the same spectrum. The three hidden $SO(16)$ vectorial representations are now obtained from the sectors $b_{1,2,3} + (2\gamma \oplus \xi_2)$, and are decomposed under the unbroken hidden $SO(10) \times SO(6)$ gauge group. Thus the spectrum from these sectors is identical to the one found in the model of [3.18]. Finally, the exotic fractionally charged states are obtained in this model from the sectors $b_3 \pm \gamma \oplus \xi_2$; $S + b_2 + b_3 + b_5 \pm \gamma \oplus \xi_2$; $S + b_2 + b_3 + b_4 + b_5 \pm \gamma \oplus \xi_2$; $S + b_1 + b_3 + b_4 \pm \gamma \oplus \xi_2$; $S + b_1 + b_3 + b_4 + b_5 \pm \gamma \oplus \xi_2$; $S + b_1 + b_2 + b_3 + b_5 \pm \gamma \oplus \xi_2$, and, as in the previous model, transform as $4 + \bar{4}$ of the hidden $SO(6)$ gauge group. Hence, we see that the entire spectrum of the two models is identical, with the substitutions

$$\begin{aligned} 2\gamma' &\rightarrow (2\gamma \oplus \xi_2) \ , \\ \gamma' &\rightarrow (\gamma \oplus \xi_2) \ , \end{aligned} \tag{3.23}$$

in sectors that preserve, and break, the observable $SO(10)$ symmetry, respectively.

It ought to be remarked, however, that the map $\gamma \rightarrow \gamma'$ does not always exist in the case of the three generation standard–like models. The reason being that there are such cases in which the modular invariant constraints are not preserved by the map. Such example are provided by the models of refs [7, 8]. In these models the assignment in the basis vectors $\{\alpha, \beta\}$, and γ, is such that the product $\alpha \cdot \gamma$ among the world–sheet fermions that produce the observable E_8 gauge group is $3/2$. This means that product between these basis vectors in the hidden sector has to be half–integral as well. Thus, as the map $\gamma' \rightarrow \gamma$ removes an even number of half–integral boundary conditions, it cannot preserve the modular invariance constraints. Nevertheless, also in these models, the Higgs and matter sectors still preserve their E_6 origins, as they originate from sectors that preserve the $SO(10)$ symmetry. Similarly, the models of refs. [20, 21] do not originate from an $N = 4$ $SO(12) \times E_8 \times E_8$ vacuum, but rather from $N = 4$ $SO(16) \times E_7 \times E_7$, and $SO(28) \times E_8$, respectively. Therefore, in this cases the overlap between ξ_1 and ξ_2 is not empty, and there is no equivalence between the map and the discreet choice of the phase. However, the models of ref. [20, 21] do not produce realistic spectra, as discussed there. The model of ref. [4] provides an example of a quasi–realistic three generation free fermionic model, in which the equivalence between the map and the discreet choice of the phase is applicable.

4 Conclusions

I demonstrated in this paper that the utilization of the vector 2γ in a large class of quasi–realistic free fermionic models is equivalent to setting the GSO projection coefficient between the two spinorial generators of the observable and hidden $SO(16)$

group factors ξ_1 and ξ_2 to

$$c\binom{\xi_1}{\xi_2} = -1.$$

Although, the equivalence was illustrated in several concrete model, I conjecture that it is in fact a general equivalence, and arises from modular properties of the $N = 1$ partition function. Thus, this equivalence applies to the larger class of free fermionic models. It would be of further interest to examine whether it applies on other classes of string compactification, and what are the precise modular properties that it reflects.

This results in the projection of the states from the sectors ξ_1 and ξ_2 and has important phenomenological implications. At the $N = 4$ level it results in the breaking of the $E_8 \times E_8$ gauge group to $SO(16) \times SO(16)$. In the $N = 1$ ten dimensional level it implies the breaking of $N = 1$ supersymmetry. This result arises in ten dimensions because of the identity $S = \mathbf{1} + \xi_1 + \xi_2$, where S is the supersymmetry generator. A question of interest in this respect is whether this phase plays a role in supersymmetry breaking in lower dimensions. In ref. [12] it was argued that free phases in the partition function may in certain cases be interpreted as vacuum expectation values of background fields in the effective field theory description of the string vacuum. A question of interest from this point of view is whether the GSO phase $c\binom{x_1}{\xi_2}$ admits such an interpretation.

In the $N = 1$ model the choice of the GSO phase (3.5) results in the breaking of $E_6 \rightarrow SO(10) \times U(1)$. In this case the 27 multiplet of E_6 splits into spinorial matter states from the twisted sectors, and vectorial matter states from the untwisted sector. The would vectorial matter states from the twisted sectors are mapped to vectorial hidden matter states, whereas the untwisted spinorial states are projected out. In this way, while the E_6 symmetry is broken, the models possess an underlying E_6 grand unifying structure. The mapping of the twisted observable vector states into hidden matter states, also results in the projection of the twisted moduli in these models. An additional consequence of this breaking is that the $U(1)$ which is embedded in E_6 becomes anomalous [19], which may be an additional indication for the relevance of this symmetry breaking pattern for supersymmetry breaking. To summarize, understanding the role of the phase $c\binom{xi_1}{\xi_2}$ may hold the key to understanding some of the key questions in the relation between string theory and the particle data.

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A Model with enhanced symmetries

SEC	$SU(6)\times SU(2)_L\times$	Q'_1	Q'_2	$E_7\times SU(2)_{H_1}$
Neveu –	$(15,1)$	2	2	$(1,1)$
Schwarz	$(\overline{15},1)$	-2	-2	$(1,1)$
$\oplus$	$(15,1)$	-2	2	$(1,1)$
ξ_1	$(\overline{15},1)$	2	-2	$(1,1)$
	$(15,1)$	0	-2	$(1,1)$
	$(\overline{15},1)$	0	2	$(1,1)$
	$(1,1)$	∓ 2	± 6	$(1,1)$
	$(1,1)$	± 2	0	$(1,1)$
	$(1,1)$	± 2	± 6	$(1,1)$
	$6\times(1,1)$	0	0	$(1,1)$
$b_1\oplus\xi_1$	$(15,1)$	1	1	$(1,1)$
	$(6,2)$	1	1	$(1,1)$
	$(1,1)$	-3	-3	$(1,1)$
	$4\times(1,1)$	± 1	∓ 3	$(1,1)$
$b_2\oplus\xi_1$	$(15,1)$	-1	1	$(1,1)$
	$(6,2)$	-1	1	$(1,1)$
	$(1,1)$	3	-3	$(1,1)$
	$4\times(1,1)$	± 1	± 3	$(1,1)$
$b_3\oplus\xi_1$	$(15,1)$	0	-1	$(1,1)$
	$(6,2)$	0	-1	$(1,1)$
	$(1,1)$	0	6	$(1,1)$
	$4\times(1,1)$	0	± 1	$(1,1)$
$b_4\oplus\xi_1$	$(15,1)$	1	1	$(1,1)$
	$(\bar{6},2)$	-1	-1	$(1,1)$
	$(1,1)$	-3	-3	$(1,1)$
	$4\times(1,1)$	± 1	∓ 3	$(1,1)$
$b_5\oplus\xi_1$	$(15,1)$	-1	1	$(1,1)$
	$(\bar{6},2)$	1	-1	$(1,1)$
	$(1,1)$	0	-3	$(1,1)$
	$4\times(1,1)$	± 1	∓ 3	$(1,1)$
$S+$	$(15,1)$	0	-1	$(1,1)$
$b_4+b_5\oplus$	$(\bar{6},2)$	0	1	$(1,1)$
ξ_1	$(1,1)$	0	6	$(1,1)$
	$4\times(1,1)$	± 1	0	$(1,1)$

(A.1)

$SO(10)$ preserving spectrum in the model of table [3.9], with $c\binom{\xi_1}{\xi_2}=+1$. The symbol $\oplus$ is used to denote that the states arise from the two sectors a & $a+\xi_1$. Here $SO(10)$ preserving means that these states arise from sectors that do not contain the basis vector α.

SEC	$SU(6) \times SU(2)_L$	Q'_1	Q'_2	$E_7 \times SU(2)_{H_1}$
$S + b_2 + b_4 +$	$(6,1)$	0	-2	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	-2	0	$(1,2)$
$b_1 + b_4 + b_5 +$	$(6,1)$	-1	1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	1	3	$(1,2)$
$b_1 + b_2 + b_4 +$	$(6,1)$	-1	1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	-1	-3	$(1,2)$
$b_2 + b_3 + b_5 +$	$(6,1)$	0	-1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	1	0	$(1,2)$
$S + b_3 + b_5 +$	$(6,1)$	1	1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	1	-3	$(1,2)$
$S + b_1 + b_2 + b_3 + b_4 +$	$(6,1)$	1	1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	-1	3	$(1,2)$
$b_2 + b_3 + b_5 +$	$(6,1)$	0	-1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	1	0	$(1,2)$
$S + b_3 + b_5 +$	$(6,1)$	1	1	$(1,2)$
$\alpha \oplus \xi_1$	$(1,2)$	1	-3	$(1,2)$

(A.2)

$SO(10)$ breaking spectrum in the model of table [3.9], with $c\binom{\xi_1}{\xi_2} = +1$.

B Model with $c\binom{\xi_1}{\xi_2} = -1$

SEC	$SU(4)\times$ $SU(2)_L \times SU(2)_R$	Q'_6	Q'_1	Q'_2	$SO(12)\times$ $SU(2)_{H_1} \times SU(2)_{H_2}$
Neveu – Schwarz	$(6,1,1)$	±2	±2	±2	$(1,1,1)$
	$(6,1,1)$	±2	∓2	±2	$(1,1,1)$
	$(6,1,1)$	∓2	0	±4	$(1,1,1)$
	$(1,1,1)$	0	±2	±6	$(1,1,1)$
	$(1,1,1)$	±4	∓2	∓2	$(1,1,1)$
	$(1,1,1)$	±4	0	∓4	$(1,1,1)$
	$(1,1,1)$	0	±4	0	$(1,1,1)$
	$(1,1,1)$	0	±2	±6	$(1,1,1)$
	$(1,1,1)$	±4	±2	∓2	$(1,1,1)$
	$6\times(1,1)$	0	0	$(1,1)$	
b_1	$(4,2,1)$	1	1	1	$(1,1,1)$
	$(\bar4,1,2)$	1	1	1	$(1,1,1)$
$b_1+\xi_1$	$(1,1,1)$	0	-1	3	$(12,1,1)$
	$(1,1,1)$	0	-1	3	$(1,2,2)$
b_2	$(4,2,1)$	1	-1	1	$(1,1,1)$
	$(\bar4,1,2)$	1	-1	1	$(1,1,1)$
$b_2+\xi_1$	$(1,1,1)$	0	1	3	$(12,1,1)$
	$(1,1,1)$	0	1	3	$(1,2,2)$
b_3	$(4,2,1)$	-1	0	2	$(1,1,1)$
	$(\bar4,1,2)$	-1	0	2	$(1,1,1)$
$b_3+\xi_1$	$(1,1,1)$	1	0	1	$(12,1,1)$
	$(1,1,1)$	1	0	1	$(1,2,2)$
b_4	$(\bar4,2,1)$	-1	-1	-1	$(1,1,1)$
	$(\bar4,1,2)$	1	1	1	$(1,1,1)$
$b_4+\xi_1$	$(1,1,1)$	0	-1	3	$(12,1,1)$
	$(1,1,1)$	0	1	-3	$(1,2,2)$
b_5	$(\bar4,2,1)$	-1	1	-1	$(1,1,1)$
	$(\bar4,1,2)$	1	-1	1	$(1,1,1)$
$b_5+\xi_1$	$(1,1,1)$	0	-1	-3	$(12,1,1)$
	$(1,1,1)$	0	1	3	$(1,2,2)$
$S+b_4+b_5$	$(6,1,1)$	-2	0	-2	$(1,1,1)$
	$(1,2,2)$	2	0	2	$(1,1,1)$
	$(1,1,1)$	4	0	-2	$(1,1,1)$
	$(1,1,1)$	0	0	6	$(1,1,1)$
	$4\times$ $(1,1,1)$	0	±2	0	$(1,1,1)$

(B.1)

$SO(10)$ preserving spectrum in the model of table [3.9], with $c\binom{\xi_1}{\xi_2} = -1$.

SEC	$SU(4)\times$ $SU(2)_L\times SU(2)_R$	Q'_6	Q'_1	Q'_2	$SO(12)\times$ $SU(2)_{H_1}\times SU(2)_{H_2}$
b_1+b_4+ $b_5+\alpha+\xi_1$	$(4,1,1)$	0	-1	3	$(1,2,1)$
b_1+b_4+ $b_5+\alpha$	$(1,2,1)$ $(1,1,2)$	0 -2	1 -1	3 1	$(1,1,2)$ $(1,1,2)$
b_1+b_2+ $b_4+\alpha+\xi_1$	$(4,1,1)$	1	-1	1	$(1,2,1)$
b_1+b_2+ $b_4+\alpha$	$(1,2,1)$ $(1,1,2)$	0 -2	-1 -1	-3 1	$(1,1,2)$ $(1,1,2)$
$S+b_2+$ $b_4+\alpha$	$(4,1,1)$	1	0	-2	$(1,1,2)$
$S+b_2+$ $b_4+\alpha+\xi_1$	$(1,2,1)$ $(1,1,2)$	0 -2	-1 0	0 -2	$(1,1,2)$ $(1,1,2)$
b_2+b_3+ $b_5+\alpha+\xi_1$	$(4,1,1)$	-1	0	2	$(1,2,1)$
b_2+b_3+ $b_5+\alpha$	$(1,2,1)$ $(1,1,2)$	2 0	0 -2	2 0	$(1,1,2)$ $(1,1,2)$
$S+b_1+$ b_3+b_4+ $b_5+\alpha+\xi_1$	$(4,1,1)$	-1	-1	-1	$(1,1,2)$
$S+b_2+b_3+$ $b_4+b_5+\alpha$	$(1,2,1)$ $(1,1,2)$	2 0	-1 2	-1 -3	$(1,2,1)$ $(1,2,1)$
$S+b_3+b_5+\alpha$	$(\bar{4},1,1)$	1	1	1	$(1,1,2)$
$S+b_3+b_5+$ $\alpha+\xi_1$	$(1,2,1)$ $(1,1,2)$	2 0	-1 2	-1 -3	$(1,2,1)$ $(1,2,1)$

(B.2)

$SO(10)$ breaking spectrum in the model of table [3.9], with $c\binom{\xi_1}{\xi_2}=-1$.

C Model with 2γ

SEC	$SU(4)\times$ $SU(2)_L \times SU(2)_R$	Q'_6	Q'_1	Q'_2	$SO(12)\times$ $SU(2)_{H_1}\times SU(2)_{H_2}$
Neveu – Schwarz	$(6,1,1)$	±2	±2	±2	$(1,1,1)$
	$(6,1,1)$	±2	∓2	±2	$(1,1,1)$
	$(6,1,1)$	∓2	0	±4	$(1,1,1)$
	$(1,1,1)$	0	±2	±6	$(1,1,1)$
	$(1,1,1)$	±4	∓2	∓2	$(1,1,1)$
	$(1,1,1)$	±4	0	∓4	$(1,1,1)$
	$(1,1,1)$	0	±4	0	$(1,1,1)$
	$(1,1,1)$	0	±2	±6	$(1,1,1)$
	$(1,1,1)$	±4	±2	∓2	$(1,1,1)$
	$6\times(1,1)$	0	0	$(1,1)$	
b_1	$(4,2,1)$	1	1	1	$(1,1,1)$
	$(\bar{4},1,2)$	1	1	1	$(1,1,1)$
$b_1+(2\gamma\oplus\xi_2)$	$(1,1,1)$	0	-1	3	$(12,1,1)$
	$(1,1,1)$	0	-1	3	$(1,2,2)$
b_2	$(4,2,1)$	1	-1	1	$(1,1,1)$
	$(\bar{4},1,2)$	1	-1	1	$(1,1,1)$
$b_2+(2\gamma\oplus\xi_2)$	$(1,1,1)$	0	1	3	$(12,1,1)$
	$(1,1,1)$	0	1	3	$(1,2,2)$
b_3	$(4,2,1)$	-1	0	2	$(1,1,1)$
	$(\bar{4},1,2)$	-1	0	2	$(1,1,1)$
$b_3+(2\gamma\oplus\xi_2)$	$(1,1,1)$	1	0	1	$(12,1,1)$
	$(1,1,1)$	1	0	1	$(1,2,2)$
b_4	$(\bar{4},2,1)$	-1	-1	-1	$(1,1,1)$
	$(\bar{4},1,2)$	1	1	1	$(1,1,1)$
$b_4+(2\gamma\oplus\xi_2)$	$(1,1,1)$	0	-1	3	$(12,1,1)$
	$(1,1,1)$	0	1	3	$(1,2,2)$
b_5	$(\bar{4},2,1)$	-1	1	-1	$(1,1,1)$
	$(\bar{4},1,2)$	1	-1	1	$(1,1,1)$
$b_5+(2\gamma\oplus\xi_2)$	$(1,1,1)$	0	-1	-3	$(12,1,1)$
	$(1,1,1)$	1	1	3	$(1,2,2)$
$S+b_4+b_5$	$(6,1,1)$	-2	0	-2	$(1,1,1)$
	$(1,2,2)$	2	0	2	$(1,1,1)$
	$(1,1,1)$	4	0	-2	$(1,1,1)$
	$(1,1,1)$	0	0	6	$(1,1,1)$
	$4\times\ (1,1,1)$	0	±2	0	$(1,1,1)$

(C.1)

$SO(10)$ preserving spectrum in the model of table [3.9], with the substitution $\xi_1 \to 2\gamma$.

SEC	$SU(4)\times$ $SU(2)_L\times SU(2)_R$	Q'_6	Q'_1	Q'_2	$SO(12)\times$ $SU(2)_{H_1}\times SU(2)_{H_2}$
$b_1+b_4+b_5+$ $\alpha+(\xi_2+2\gamma)$	$(4,1,1)$	1	-1	1	$(1,2,1)$
b_1+b_4+ $b_5+\alpha$	$(1,2,1)$ $(1,1,2)$	0 -2	1 -1	3 1	$(1,1,2)$ $(1,1,2)$
$b_1+b_2+b_4+$ $\alpha+(\xi_2+2\gamma)$	$(4,1,1)$	1	-1	1	$(1,2,1)$
b_1+b_2+ $b_4+\alpha$	$(1,2,1)$ $(1,1,2)$	0 -2	-1 -1	-3 1	$(1,1,2)$ $(1,1,2)$
$S+b_2+$ $b_4+\alpha$	$(4,1,1)$	1	0	-2	$(1,1,2)$
$S+b_2+b_4+$ $\alpha+(\xi_2+2\gamma)$	$(1,2,1)$ $(1,1,2)$	0 -2	-1 0	0 -2	$(1,1,2)$ $(1,1,2)$
$b_2+b_3+b_5+$ $\alpha+(\xi_2+2\gamma)$	$(4,1,1)$	-1	0	2	$(1,2,1)$
b_2+b_3+ $b_5+\alpha$	$(1,2,1)$ $(1,1,2)$	2 0	0 -2	2 0	$(1,1,2)$ $(1,1,2)$
$S+b_1+b_3+$ $b_4+b_5+\alpha+$ $(\xi_2+2\gamma)$	$(4,1,1)$	-1	-1	-1	$(1,1,2)$
$S+b_2+b_3+$ $b_4+b_5+\alpha$	$(1,2,1)$ $(1,1,2)$	2 0	-1 2	-1 -3	$(1,2,1)$ $(1,2,1)$
$S+b_3+$ $b_5+\alpha$	$(\bar{4},1,1)$	1	1	1	$(1,1,2)$
$S+b_3+b_5+$ $\alpha+(\xi_2+2\gamma)$	$(1,2,1)$ $(1,1,2)$	2 0	-1 2	-1 -3	$(1,2,1)$ $(1,2,1)$

(C.2)

$SO(10)$ breaking spectrum in the model of table [3.9], with the substitution $\xi_1 \to 2\gamma$.

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