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Moduli Fixing in Realistic String Vacua

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Abstract

I demonstrate the existence of quasi–realistic heterotic–string models in which all the untwisted Kähler and complex structure moduli, as well as all of the twisted sectors moduli, are projected out by the generalized GSO projections. I discuss the conditions and characteristics of the models that produce this result. The existence of such models offers a novel perspective on the realization of extra dimensions in string theory. In this view, while the geometrical picture provides a useful mean to classify string vacua, in the phenomenologically viable cases there is no physical realization of extra dimensions. The models under consideration correspond to $Z_2 \times Z_2$ orbifolds of six dimensional tori, plus additional identifications by internal shifts and twists. The special property of the $Z_2 \times Z_2$ orbifold is that it may act on the compactified dimensions as real, rather than complex, dimensions. This property enables an asymmetric projection on all six internal coordinates, which enables the projection of all the untwisted Kähler and complex structure moduli.

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1 Introduction

String theory continues to serve as the most compelling framework for the unification of all the fundamental matter and interactions. Amazingly, preservation of some classical string symmetries necessitates the introduction of a fixed number of degrees of freedom, beyond the observable ones. One interpretation of these additional degrees of freedom is as extra space–time dimensions, which clearly is in contradiction with the observed facts and diminishes from the appeal of string theory. Thus, the need arises to hide the additional dimensions by compactifying the superstring on a six dimensional compact manifold. This in turn leads to the problem of fixing the parameters of the extra dimensions in such a way that it does not conflict with contemporary experimental limits. Additionally, the extra dimensions give rise to light scalar fields whose VEV parameterizes the shape and size of the extra dimensions. This is typically dubbed as the moduli stabilization problem, and preoccupies much of the activity in string theory [1] ever since the initial realization of its potential relevance for particle phenomenology [2, 3].

A particular class of string compactifications that exhibit promising phenomenological properties are the so–called free fermionic heterotic–string models [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. These models are related to $Z_2 \times Z_2$ orbifold compactifications [14], and many of their appealing phenomenological properties are rooted in the underlying $Z_2 \times Z_2$ orbifold structure [14]. Recently, a classification of this class of models was initiated and it was revealed that in a large class of models the reduction to three generations necessitates the utilization of a fully asymmetric shift [15]. This indicates that the geometrical objects underlying these models do not correspond to the standard complex geometries that have been the most prevalent in the literature.

Furthermore, the utilization of the asymmetric shift has important bearing on the issue of moduli fixing in string theory [16], and in this class of string compactifications and in particular. The operation of an asymmetric shift can only occur at special points in the moduli space, and results in fixation of the moduli at the special points. It is therefore of interest to investigate this aspect in greater detail. In the context of the free fermionic models the marginal operators that correspond to the moduli deformations correspond to the inclusion of Thirring interactions among the world–sheet fermions [17, 18, 19, 20]. The allowed world–sheet Thirring interactions must be invariant under the GSO projections that are induced by the boundary condition basis vectors that define the free fermionic string models. Thus, depending on the assignment of boundary conditions in specific models, some of the world–sheet Thirring interactions are forbidden, and their corresponding moduli are projected from the physical spectrum. In this manner the GSO projection induced by the boundary condition basis vectors acts as a moduli fixing mechanism.

In this paper I discuss the existence of quasi–realistic three generation free fermionic models, in which all the untwisted Kähler and complex structure moduli are projected out from the low energy effective field theory. This is a striking result with crucial implications for the premise of extra dimensions in physical string vacua. This result indicates that the interpretation of the additional degrees of freedom needed to obtain a conformally invariant string theory as additional continuous spatial dimensions beyond the observable ones, may in fact not be realized in the phenomenologically relevant cases. Furthermore, the fact that the projection is achieved by an asymmetric orbifold action indicates that the treatment of the moduli problem in the low energy effective supergravity theory is inadequate. The reason being that the effective supergravity theories relate to the additional dimensions as classical geometries, and hence necessarily as left–right symmetric. On the other hand the basic property of string theory is that it allows for the left–right world–sheet asymmetry, which in the context of the phenomenological free fermionic models is instrumental in fixing the moduli.

2 General structure of free fermionic models

In the free fermionic formulation [17] of the heterotic string in four dimensions all the world–sheet degrees of freedom required to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. In the light–cone gauge the world–sheet field content consists of two transverse left– and right–moving space–time coordinate bosons, $X^\mu_{1,2}$ and $\bar{X}^\mu_{1,2}$, and their left–moving fermionic superpartners $\psi^\mu_{1,2}$, and additional 62 purely internal Majorana–Weyl fermions, of which 18 are left–moving, χ^I, and 44 are right–moving, ϕ^a. In the supersymmetric sector the world–sheet supersymmetry is realized non–linearly and the world–sheet supercurrent is given by

$$T_F = \psi^\mu \partial X_\mu + f_{IJK} \chi^I \chi^J \chi^K, \tag{2.1}$$

where f_{IJK} are the structure constants of a semi–simple Lie group of dimension 18. The χ^I $(I = 1, \cdots, 18)$ world–sheet fermions transform in the adjoint representation of the Lie group. In the realistic free fermionic models the Lie group is $SU(2)^6$. The χ^I $I = 1, \cdots, 18$ transform in the adjoint representation of $SU(2)^6$, and are denoted by χ^I, y^I, ω^I $(I = 1, \cdots, 6)$. Under parallel transport around a noncontractible loop on the toroidal world–sheet the fermionic fields pick up a phase

$$f \;\; \to \;\; -\,\mathrm{e}^{i\pi\alpha(f)} f \;\; . \tag{2.2}$$

The minus sign is conventional and $\alpha(f) \in (-1, +1]$. A model in this construction [17] is defined by a set of boundary conditions basis vectors and by a choice of generalized GSO projection coefficients, which satisfy the one–loop modular invariance constraints. The boundary conditions basis vectors $\mathbf{b}_k$ span a finite additive group

$$\Xi = \sum_k n_i b_i \tag{2.3}$$

where $n_i = 0, \cdots, N_{z_i} - 1$. The physical massless states in the Hilbert space of a given sector $\alpha \in \Xi$ are then obtained by acting on the vacuum state of that sector with the world-sheet bosonic and fermionic mode operators, with frequencies ν_f, ν_{f^*} and by subsequently applying the generalized GSO projections,

$$\left\{ e^{i\pi(b_i F_\alpha)} - \delta_\alpha c^* \begin{pmatrix} \alpha \\ b_i \end{pmatrix} \right\} |s\rangle = 0 \tag{2.4}$$

with

$$(b_i F_\alpha) \equiv \{ \sum_{\substack{real+complex \\ left}} - \sum_{\substack{real+complex \\ right}} \} (b_i(f) F_\alpha(f)), \tag{2.5}$$

where $F_\alpha(f)$ is a fermion number operator counting each mode of f once (and if f is complex, f^* minus once). For periodic complex fermions [*i.e.* for $\alpha(f) = 1$)] the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion f, there are two degenerate vacua $|+\rangle$, $|-\rangle$, annihilated by the zero modes f_0 and f_0^* and with fermion number $F(f) = 0, -1$ respectively. In Eq. (2.4), $\delta_\alpha = -1$ if ψ^μ is periodic in the sector α, and $\delta_\alpha = +1$ if ψ^μ is antiperiodic in the sector α. Each complex world–sheet fermion f gives rise to a $U(1)$ ff^* current in the Cartan sub–algebra of the four dimensional gauge group, with the charge given by:

$$Q(f) \ = \ \frac{1}{2}\alpha(f) + F(f) \ . \tag{2.6}$$

Alternatively, a real left–moving fermion may be combined with a real right–moving fermion to form an Ising model operator [21, 22], in which case the $U(1)$ generators are projected out, and the rank of the four dimensional gauge group is reduced. This distinction between complex and real world–sheet fermion has important bearing on the assignment of asymmetric versus symmetric boundary conditions, and hence on the issue of moduli fixing in the $Z_2 \times Z_2$ fermionic models.

The δ_α in eq. (2.4) is a space–time spin statistics factor, equal to $+1$ for space–time bosons and -1 for space–time fermions, and is determined by the boundary conditions of the space–time fermions $\psi^\mu_{1,2}$ in the sector α, *i.e.*,

$$\delta_\alpha = \mathrm{e}^{i\pi\alpha(\psi^\mu)}.$$

The

$$c^* \begin{pmatrix} \alpha \\ b_i \end{pmatrix}$$

are free phases in the one–loop string partition function. For the Neveu–Schwarz untwisted sector we have the general result that

$$\delta_{NS} c^* \begin{pmatrix} NS \\ b \end{pmatrix} = \delta_b.$$

Hence, the free phases do not play a significant role in the determination of the untwisted moduli. The existence of the untwisted moduli in a model depends solely on the boundary conditions. The free phases of the models will therefore be suppressed in the following.

The boundary condition basis defining a typical realistic free fermionic heterotic string model is constructed in two stages. The first stage consists of the NAHE set, which is a set of five boundary condition basis vectors, $\{1, S, b_1, b_2, b_3\}$ [4, 8, 14, 13]. The gauge group induced by the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ supersymmetry. The space-time vector bosons that generate the gauge group arise from the Neveu–Schwarz sector and from the sector $\xi_2 \equiv 1 + b_1 + b_2 + b_3$. The Neveu-Schwarz sector produces the generators of $SO(10) \times SO(6)^3 \times SO(16)$. The ξ_2-sector produces the spinorial 128 of SO(16) and completes the hidden gauge group to E_8. The NAHE set divides the internal world-sheet fermions in the following way: $\bar{\phi}^{1,\cdots,8}$ generate the hidden E_8 gauge group, $\bar{\psi}^{1,\cdots,5}$ generate the SO(10) gauge group, and $\{\bar{y}^{3,\cdots,6}, \bar{\eta}^1\}$, $\{\bar{y}^1, \bar{y}^2, \bar{\omega}^5, \bar{\omega}^6, \bar{\eta}^2\}$, $\{\bar{\omega}^{1,\cdots,4}, \bar{\eta}^3\}$ generate the three horizontal SO(6) symmetries. The left-moving $\{y, \omega\}$ states are divided into $\{y^{3,\cdots,6}\}$, $\{y^1, y^2, \omega^5, \omega^6\}$, $\{\omega^{1,\cdots,4}\}$ and χ^{12}, χ^{34}, χ^{56} generate the left–moving $N = 2$ world–sheet supersymmetry. At the level of the NAHE set the sectors b_1, b_2 and b_3 produce 48 multiplets, 16 from each, in the 16 representation of SO(10). The states from the sectors b_j are singlets of the hidden E_8 gauge group and transform under the horizontal $SO(6)_j$ $(j = 1, 2, 3)$ symmetries. This structure is common to a large set of realistic free fermionic models.

The second stage of the construction consists of adding to the NAHE set three (or four) additional basis vectors. These additional vectors reduce the number of generations to three, one from each of the sectors b_1, b_2 and b_3, and simultaneously break the four dimensional gauge group. The assignment of boundary conditions to $\{\bar{\psi}^{1,\cdots,5}\}$ breaks SO(10) to one of its subgroups [11]. Similarly, the hidden E_8 symmetry is broken to one of its subgroups, and the flavor $SO(6)^3$ symmetries are broken to $U(1)^n$, with $3 \leq n \leq 9$. For details and phenomenological studies of these three generation string models interested readers are referred to the original literature and review articles [23].

The correspondence of the free fermionic models with the orbifold construction [24] is facilitated by extending the NAHE set, $\{1, S, b_1, b_2, b_3\}$, by at least one additional boundary condition basis vector [14]

$$\xi_1 = (0, \cdots, 0 | \underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}}, 0, \cdots, 0) \ . \tag{2.7}$$

With a suitable choice of the GSO projection coefficients the model possesses an $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ gauge group and $N = 1$ space-time supersymmetry. The matter fields include 24 generations in the 27 representation of E_6, eight from each of the sectors $b_1 \oplus b_1 + \xi_1$, $b_2 \oplus b_2 + \xi_1$ and $b_3 \oplus b_3 + \xi_1$. Three additional 27 and $\overline{27}$

pairs are obtained from the Neveu-Schwarz (NS) $\oplus\ \xi_1$ sector, that correspond to the untwisted sector of the orbifold models.

To construct the model in the orbifold formulation one starts with the compactification on a torus with nontrivial background fields [25]. The subset of basis vectors

$$\{1, S, \xi_1, \xi_2\} \tag{2.8}$$

generates a toroidally-compactified model with $N = 4$ space-time supersymmetry and $\mathrm{SO}(12) \times \mathrm{E}_8 \times \mathrm{E}_8$ gauge group. The same model is obtained in the geometric (bosonic) language by tuning the background fields to the values corresponding to the SO(12) lattice. The metric of the six-dimensional compactified manifold is then the Cartan matrix of SO(12), while the antisymmetric tensor is given by

$$B_{ij} = \begin{cases} G_{ij} & ; i > j, \\ 0 & ; i = j, \\ -G_{ij} & ; i < j. \end{cases} \tag{2.9}$$

When all the radii of the six-dimensional compactified manifold are fixed at $R_I = \sqrt{2}$, it is seen that the left- and right-moving momenta $P^I_{R,L} = [m_i - \frac{1}{2}(B_{ij} \pm G_{ij})n_j]e^{I*}_i$ reproduce the massless root vectors in the lattice of SO(12). Here $e^i = \{e^I_i\}$ are six linearly-independent vielbeins normalized so that $(e_i)^2 = 2$. The e^{I*}_i are dual to the e_i, with $e^*_i \cdot e_j = \delta_{ij}$.

Adding the two basis vectors b_1 and b_2 to the set (2.8) corresponds to the $Z_2 \times Z_2$ orbifold model with standard embedding. Starting from the $N = 4$ model with $\mathrm{SO}(12)\times\mathrm{E}_8\times\mathrm{E}_8$ symmetry, and applying the $Z_2\times Z_2$ twist on the internal coordinates, reproduces the spectrum of the free-fermion model with the six-dimensional basis set $\{1, S, \xi_1, \xi_2, b_1, b_2\}$. The Euler characteristic of this model is 48 with $h_{11} = 27$ and $h_{21} = 3$.

The effect of the additional basis vector ξ_1 of eq. (2.7), is to separate the gauge degrees of freedom, spanned by the world-sheet fermions $\{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1,\cdots,8}\}$, from the internal compactified degrees of freedom $\{y, \omega|\bar y, \bar\omega\}^{1,\cdots,6}$. In the realistic free fermionic models [11, 14] this is achieved by the vector 2γ [14], with

$$2\gamma = (0, \cdots, 0| \underbrace{1, \cdots, 1}_{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3} \bar\phi^{1,\cdots,4}}, 0, \cdots, 0) \ , \tag{2.10}$$

which breaks the $\mathrm{E}_8 \times \mathrm{E}_8$ symmetry to $\mathrm{SO}(16) \times \mathrm{SO}(16)$. The $Z_2 \times Z_2$ twist induced by b_1 and b_2 breaks the gauge symmetry to $\mathrm{SO}(4)^3 \times \mathrm{SO}(10) \times \mathrm{U}(1)^3 \times \mathrm{SO}(16)$. The orbifold still yields a model with 24 generations, eight from each twisted sector, but now the generations are in the chiral 16 representation of SO(10), rather than in the 27 of E_6. The same model can be realized with the set $\{1, S, \xi_1, \xi_2, b_1, b_2\}$, by projecting out the $16 \oplus \overline{16}$ from the ξ_1-sector taking

$$c\begin{bmatrix}\xi_1 \\ \xi_2\end{bmatrix} \to -c\begin{bmatrix}\xi_1 \\ \xi_2\end{bmatrix}. \tag{2.11}$$

This choice also projects out the massless vector bosons in the 128 of SO(16) in the hidden-sector E_8 gauge group, thereby breaking the $E_6 \times E_8$ symmetry to $SO(10) \times U(1) \times SO(16)$. However, the $Z_2 \times Z_2$ twist acts identically in the two models, and their physical characteristics differ only due to the discrete torsion eq. (2.11). This analysis confirms that the $Z_2 \times Z_2$ orbifold on the $SO(12)$ lattice is at the core of the realistic free fermionic models.

The set of real fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$ correspond to the six dimensional compactified coordinates in a bosonic formulation. The assignment of boundary conditions to this set of internal fermions therefore correspond to the action on the internal six dimensional compactified manifold. Consequently, the assignment of boundary conditions to this set of real fermions determines many of the phenomenological properties of the low energy spectrum and effective field theory. Examples include the determination of the number of light generations [8, 13]; the stringy doublet–triplet splitting mechanism [26]; and the top–bottom–quark Yukawa coupling selection mechanism [27]. In the last two cases, it is the left–right asymmetric boundary conditions that enables the doublet–triplet splitting, as well as the Yukawa coupling selection mechanism [26, 27]. In this paper I discuss how the asymmetric assignment of boundary conditions to the set of internal world–sheet fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$, also fixes the untwisted moduli. It is therefore found that the same condition, namely the left–right orbifold asymmetry, is the one that plays the crucial role, both in the determination of the physical properties mentioned above, as well as in the fixation of the moduli parameters.

The symmetric versus asymmetric orbifold action is determined in the free fermionic models by the pairing of the left–right real internal fermions from the set $\{y, \omega|\bar{y}, \bar{\omega}\}$, into real pairs, that pair a left–moving fermion with a right–moving fermion, versus complex pairs, that combine left–left, or right–right, moving fermions. The real pairs preserve the left–right symmetry, whereas the complex pairs allow for the assignment of asymmetric boundary conditions, that correspond to asymmetric action on the compactified bosonic coordinates.

The reduction of the number of generations to three is illustrated in tables [2.13] and [2.12].

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
α	1	1	0	0	1 1 1 0 0	1	0	0	1 1 0 0 0 0 0 0
β	1	0	1	0	1 1 1 0 0	0	1	0	1 1 0 0 0 0 0 0
γ	1	0	0	1	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 0 0 0

	$y^3\bar{y}^3$	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$y^6\bar{y}^6$	$y^1\bar{y}^1$	$y^2\bar{y}^2$	$\omega^5\bar{\omega}^5$	$\omega^6\bar{\omega}^6$	$\omega^1\bar{\omega}^1$	$\omega^2\bar{\omega}^2$	$\omega^3\bar{\omega}^3$	$\omega^4\bar{\omega}^4$
α	1	0	0	1	0	0	1	0	0	0	0	1
β	0	0	0	1	0	1	1	0	1	0	0	0
γ	1	1	0	0	1	0	0	0	0	1	0	0

(2.12)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,\dots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\dots,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar y^4$	$y^5\bar y^5$	$\bar y^3\bar y^6$	$y^1\omega^5$	$y^2\bar y^2$	$\omega^6\bar\omega^6$	$\bar y^1\bar\omega^5$	$\omega^2\omega^4$	$\omega^1\bar\omega^1$	$\omega^3\bar\omega^3$	$\bar\omega^2\bar\omega^4$
α	1	0	0	0	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	0	0	1	0	1
γ	0	1	0	1	0	1	0	1	1	0	0	0

(2.13)

together with a consistent choice of one–loop GSO phases [9]. Both models produce three generations and the $SO(10)$ GUT gauge group is broken to $SU(3) \times SU(2) \times U(1)^2$. The model of table [2.12] produces three pairs of $SU(3)$ color triplets from the untwisted sector, whereas that of table [2.13] produces the corresponding three pairs of electroweak Higgs doublets [9, 26]. The set of boundary conditions in table [2.12] is symmetric between the left and right movers, whereas that of table [2.13] is asymmetric. Hence, these two models demonstrate that the reduction to three generations in itself does not require asymmetric boundary conditions. This seems to be in contradiction to the conclusion reached in [15]. However, I note that in [2.12] the gauge group is broken in two stages, whereas in [15] the $SO(10)$ symmetry remained unbroken. In particular, the breaking pattern of the hidden gauge group $SO(16) \to SO(4) \times SO(12)$ in [2.12] was a case not considered in the classification of [15]. This breaking pattern is, however, allowed if the $SO(10)$ GUT symmetry is broken to $SO(6) \times SO(4)$, as in [2.12]. The conclusion is therefore that, whereas the classification of ref. [15] found that the reduction to three generations necessitates the use of an asymmetric shift in the scanned space of models, more complicated symmetry breaking patterns allow for the reduction also with symmetric actions. Hence, it is concluded that the reduction to three generations in itself does not necessitate the use of an asymmetric shift, in agreement with the findings of ref. [28]. However, the distinction between the symmetric versus asymmetric boundary conditions is manifested in the models of table [2.12] versus table [2.13] at the more detailed level of the spectrum and the phenomenological consequences. Hence, the world–sheet left–right symmetric model of table [2.12] produces $SU(3)$ color triplets and does not give rise to untwisted–twisted–twisted Standard Model fermion mass terms, whereas the asymmetric model of table [2.13] does. As I discuss in the following this distinction has a crucial implication also for the issue of moduli fixing in these models.

To translate the fermionic boundary conditions to twists and shifts in the bosonic formulation we bosonize the real fermionic degrees of freedom, $\{y,\omega|\bar{y},\bar{\omega}\}$. Defining,

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = i\sqrt{\frac{1}{2}}(y_i - i\omega_i) = ie^{-iX_i} \tag{2.14}$$

with similar definitions for the right movers $\{\bar{y},\bar{\omega}\}$ and $X^I(z,\bar{z}) = X^I_L(z) + X^I_R(\bar{z})$. With these definitions the world–sheet supercurrents in the bosonic and fermionic formulations are equivalent,

$$T_F^{int} = \sum_i \chi_i y_i \omega_i = \sum_i \chi_i \xi_i \eta_i = i \sum_i \chi_i \partial X_i.$$

The momenta P^I of the compactified scalars in the bosonic formulation are identical with the $U(1)$ charges $Q(f)$ of the unbroken Cartan generators of the four dimensional gauge group, eq. (2.6). The internal coordinates can be complexified by forming the combinations ($X = X_L + X_R$)

$$Z_k^\pm = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^\pm = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1,2,3) \tag{2.15}$$

where the $Z_k^\pm$ are the complex coordinates of the six compactified dimensions, now viewed as three complex planes, and $\psi_k^\pm$ are the corresponding superpartners.

3 Moduli in free fermionic models

The relevant moduli for our discussion here are the untwisted Kähler and complex structure moduli of the six dimensional compactified manifold. Additionally, the string vacua contain the dilaton moduli whose VEV governs the strength of the four dimensional interactions. The VEV of the dilaton moduli is a continuous parameter from the point of view of the perturbative heterotic string, and its stabilization requires some nonperturbative dynamics, or some input from the underlying quantum M–theory, which is not presently available. The problem of dilaton stabilization is therefore not addressed in this paper, as the discussion here is confined to perturbative heterotic string vacua. Since the moduli fields correspond to scalar fields in the massless string spectrum, the moduli space is determined by the set of boundary condition basis vectors that define the string vacuum and encodes its properties. The first step therefore is to identify the fields in the fermionic models that correspond to the untwisted moduli. The subsequent steps entail examining which moduli fields survive successive GSO projections and consequently the residual moduli space.

The four dimensional fermionic heterotic string models are described in terms of two dimensional conformal and superconformal field theories of central charges $C_R = 22$ and $C_L = 9$, respectively. In the fermionic formulation these are represented in terms of world–sheet fermions. A convenient starting point to formulate such a fermionic vacuum is a model in which all the fermions are free. The free fermionic formalism facilitates the solution of the conformal and modular invariance constraints in terms of simple rules [17]. Such a free fermionic model corresponds to a string vacuum at a fixed point in the moduli space. Deformations from this fixed point are then incorporated by including world-sheet Thirring interactions among the world–sheet fermions, that are compatible with the conformal and modular invariance constraints. The coefficients of the allowed world–sheet Thirring interactions correspond to the untwisted moduli fields. For symmetric orbifold models, the exactly marginal operators associated with the untwisted moduli fields take the general form $\partial X^I \bar{\partial} X^J$, where X^I, $I = 1, \cdots, 6$, are the coordinates of the six–torus T^6. Therefore, the untwisted moduli fields in such models admit the geometrical interpretation of background fields [25], which appear as couplings of the exactly marginal operators in the non–linear sigma model action, which is the generating functional for string scattering amplitudes [29, 30]. The untwisted moduli scalar fields are the background fields that are compatible with the orbifold point group symmetry.

It is noted that in the Frenkel–Kac–Segal construction [31] of the Kac–Moody current algebra from chiral bosons, the operator $i\partial X^I$ is a $U(1)$ generator of the Cartan sub–algebra. Therefore, in the fermionic formalism the exactly marginal operators are given by Abelian Thirring operators of the form $J^i_L(z)\bar{J}^j_R(\bar{z})$, where $J^i_L(z)$, $\bar{J}^j_R(\bar{z})$ are some left– and right–moving $U(1)$ chiral currents describe by world–sheet fermions. It has been shown that [18] Abelian Thirring interactions preserve conformal invariance, and are therefore marginal operators. One can therefore use the Abelian Thirring interactions to identify the untwisted moduli in the free fermionic models. The untwisted moduli correspond to the Abelian Thirring interactions that are compatible with the GSO projections induced by the boundary condition basis vectors, which define the string models.

The set of Abelian Thirring operators, and therefore of the untwisted moduli fields, is consequently restricted by progressive boundary condition basis vector sets. The minimal basis set is given by the basis that contains only the two vectors $\{1, S\}$.

$$
\begin{aligned}
1 &= \{\psi^{1,2}, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \\
&\quad \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\psi}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}, \qquad (3.1) \\
S &= \{\psi^{1,2}, \chi^{1,\dots,6}\}. \qquad (3.2)
\end{aligned}
$$

This set generates a model with $N = 4$ supersymmetry and $SO(44)$ right–moving gauge group and correspond the a Narain model at an enhanced symmetry point. Accordingly, we can identify the six χ^I with the fermionic superpartners of the six compactified bosonic coordinates. Therefore, each pair $\{y^I, \omega^I\}$ is identified with the fermionized version of the corresponding left–moving bosonic coordinate X^I, *i.e.* $i\partial X^I_L \sim y^I \omega^I$. The two–dimensional action for the for the Abelian Thirring interactions is

$$
S = \int d^2 z h_{ij}(X) J^i_L(z) \bar{J}^j_R(\bar{z}) \ , \qquad (3.3)
$$

where $J^i_L (i = 1, \cdots, 6)$ are the chiral currents of the left–moving $U(1)^6$ and $\bar{J}^j_R (j = 1, \cdots, 22)$, are the chiral currents of the right–moving $U(1)^{22}$. The couplings $h_{ij}(X)$, as functions of the space–times coordinates X^μ, are four dimensional scalar fields that are identified with the scalar components of the untwisted moduli fields. In the simplest model with the two basis vectors of eq. (3.2) the 6×22 fields $h_{ij}(X)$ in eq. (3.3) are in one–to–one correspondence with the 21 and 15 components of the background metric G_{IJ} and antisymmetric tensor B_{IJ} $(I,J=1,\cdots,6)$, plus the 6×16 Wilson lines A_{Ia}. The $h_{ij}(X)$ fields therefore parameterize the $SO(6,22)/SO(6)\times SO(22)$ coset–space of the toroidally compactified manifold. The h_{ij} untwisted moduli fields arise from the Neveu–Schwarz sector,

$$|\chi^I\rangle_L \ \otimes |\bar\Phi^{+J}\bar\Phi^{-J}\rangle_R \ , \tag{3.4}$$

given in terms of the 22 complex right–handed world–sheet fermions $\bar\Phi^{+J}$ and their complex conjugates $\bar\Phi^{-J}$. The corresponding marginal operators are given as

$$J^i_L(z)\bar J^j_R(\bar z) \ \equiv \ \ : y^i(z)\omega^i(z)(z) :: \bar\Phi^{+j}(\bar z)\bar\Phi^{-j}(\bar z) : \ . \tag{3.5}$$

It is noted that the transformation properties of χ^i, which appear in the moduli (3.4), are the same as those of $y^i\omega^i$, which appear in the Abelian Thirring interactions (3.5).

The next stages in the free fermionic model building consist of adding consecutive boundary condition basis vectors. In the first instance we can add basis vectors that preserve the $N=4$ space–time supersymmetry, and with no periodic left–moving fermions . Those that produce massless states may contain either four, or eight, right–moving periodic fermions. At least one basis vector with eight periodic right–moving fermions is needed to produce space–time spinors under the GUT gauge group. The free fermionic models correspond to $Z_2\times Z_2$ orbifolds and can then be classified into models that produce spinorial representations from one, two, or three of the twisted planes of the $Z_2\times Z_2$ orbifold [15]. It turns out that the entire space of models may be classified using a specific set of basis vectors, and the varying cases are produced by the various choices of the GSO projection coefficients. This covering basis contains two basis vectors with eight non–overlapping right–moving periodic fermions, and all left–moving world–sheet fermions are antiperiodic. Hence, with no loss of generality, these are the vector ξ_1 in eq. (2.7) and the vector ξ_2

$$\xi_2 = (0,\cdots,0|0,\cdots,0,\underbrace{1,\cdots,1}_{\bar\phi^{1,\cdots,8}}) \ . \tag{3.6}$$

where the notation introduced in section 2 has been used. We note that all the untwisted moduli and marginal operators in eqs. (3.4) and (3.5) are invariant under the projections induced by (2.7) and (3.6). The four dimensional right–moving gauge group is now $SO(12)\times E_8\times E_8$.

The next step in the construction is the inclusion of the basis vectors b_1 and b_2 that correspond to the action of the $Z_2\times Z_2$ orbifold twists. The assignment of boundary condition in b_1 and b_2 may vary [15], and one specific choice is given by

$$b_1 \ = \ (\underbrace{1,\cdots\cdots\cdots,1}_{\psi^\mu,\chi^{12},y^{3,\ldots,6},\bar y^{3,\ldots,6}},0,\cdots,0|\underbrace{1,\cdots,1}_{\bar\psi^{1,\ldots,5},\bar\eta^1},0,\cdots,0) \tag{3.7}$$

$$b_2 \;=\; (\underbrace{1,\cdots\cdots\cdots\cdots\cdots,1}_{\psi^\mu,\chi^{34},y^{1,2},\omega^{5,6},\bar{y}^{1,2}\bar{\omega}^{5,6}},0,\cdots,0|\underbrace{1,\cdots,1}_{\bar{\psi}^{1,\dots,5},\bar{\eta}^2},0,\cdots,0) \tag{3.8}$$

In the notation of section 2 we can write the untwisted moduli fields, eq. (3.4) in the form

$$|\chi^i\rangle_L \;\otimes |\bar{y}^j\bar{\omega}^j\rangle_R \;, \tag{3.9}$$

$$|\chi^i\rangle_L \;\otimes |\bar{\Phi}^{+J}\bar{\Phi}^{-J}\rangle_R \;, \tag{3.10}$$

$$\tag{3.11}$$

and the (3.5)

$$J_L^i(z)\bar{J}_R^j(\bar{z}) \;\equiv\; :y^i(z)\omega^i(z)::\bar{y}^j(z)\bar{\omega}^j(z): \qquad (j=1,\cdots,6); \tag{3.12}$$

$$J_L^i(z)\bar{J}_R^J(\bar{z}) \;\equiv\; :y^i(z)\omega^i(z)::\bar{\Phi}^{+J}(\bar{z})\bar{\Phi}^{-J}(\bar{z}): \qquad (J=7,\cdots,22)\;. \tag{3.13}$$

The effect of the additional basis vectors b_1 and b_2 is to make some of the chiral currents, J_L^i or $\bar{J}_R^j$, antiperiodic. As a result some of the Abelian Thirring interaction terms in eq. (3.3) are not invariant when the world–sheet fermions are parallel transported around the noncontractible loops of the world–sheet torus. The corresponding moduli fields are projected from the massless spectrum by the GSO projections.

Under the basis vector b_1 as defined above the chiral currents transform as:

$$J_L^{1,2} \to J_L^{1,2}\;, \quad J_L^{3,4,5,6} \to -J_L^{3,4,5,6} \tag{3.14}$$

$$\bar{J}_R^{1,2} \to \bar{J}_R^{1,2}\;, \quad \bar{J}_R^{3,4,5,6} \to -\bar{J}_R^{3,4,5,6} \tag{3.15}$$

and $\bar{J}_R^j(j=7,\cdots,22)$ are always periodic. Similarly, under b_2 we have

$$J_L^{3,4} \to J_L^{3,4}\;, \quad J_L^{1,2,5,6} \to -J_L^{1,2,5,6} \tag{3.16}$$

$$\bar{J}_R^{1,2} \to \bar{J}_R^{3,4}\;, \quad \bar{J}_R^{1,2,5,6} \to -\bar{J}_R^{1,2,5,6} \tag{3.17}$$

As a consequence the only allowed Thirring interaction terms are

$$J_L^{1,2}\bar{J}_R^{1,2},\, J_L^{3,4}\bar{J}^{3,4},\, J_L^{5,6}\bar{J}_R^{5,6} \tag{3.18}$$

Correspondingly there are three sets of untwisted moduli scalars

$$h_{ij} = |\chi^i\rangle_L \otimes |\bar{y}^j\bar{\omega}^j\rangle_R = \begin{cases} (i,j=1,2) \\ (i,j=3,4) \\ (i,j=5,6) \quad , \end{cases} \tag{3.19}$$

which parameterize the moduli space

$$\mathcal{M} = \left(\frac{SO(2,2)}{SO(2)\times SO(2)}\right)^3 . \tag{3.20}$$

We can define six complex moduli from the six real ones. For example, on the first complex plane we can write,

$$H_1^{(1)} = \frac{1}{\sqrt{2}}(h_{11}+ih_{21}) = \frac{1}{\sqrt{2}}|\chi^1+i\chi^2\rangle_L \otimes |\bar{y}^1\bar{\omega}^1\rangle_R \quad (3.21)$$

$$H_2^{(1)} = \frac{1}{\sqrt{2}}(h_{12}+ih_{22}) = \frac{1}{\sqrt{2}}|\chi^1+i\chi^2\rangle_L \otimes |\bar{y}^2\bar{\omega}^2\rangle_R \quad (3.22)$$

These can be combined into three Kähler (T_i) structure and three complex structure (U_i) moduli. For example, on the first complex plane we define

$$T_1 = \frac{1}{\sqrt{2}}(H_1^{(1)} - iH_2^{(1)}) = \frac{1}{\sqrt{2}}(\chi^1+i\chi_2)|0\rangle_L \otimes (\bar{y}^1\bar{\omega}^1 - i\bar{y}^2\bar{\omega}^2)|0\rangle_R \quad (3.23)$$

$$U_1 = \frac{1}{\sqrt{2}}(H_1^{(1)} + iH_2^{(1)}) = \frac{1}{\sqrt{2}}(\chi^1+i\chi_2)|0\rangle_L \otimes (\bar{y}^1\bar{\omega}^1 + i\bar{y}^2\bar{\omega}^2)|0\rangle_R \quad (3.24)$$

and similarly for $T_{2,3}$ and $U_{2,3}$. These span the coset moduli space

$$\mathcal{M} = \left[\frac{SU(1,1)}{U(1)} \otimes \frac{SU(1,1)}{U(1)}\right]^3 , \quad (3.25)$$

which is the untwisted moduli space of the symmetric $Z_2 \times Z_2$ orbifold. We can write the allowed Thirring interaction terms, for the untwisted moduli of (3.19), in terms of the complex coordinates $Z_k^{\pm}$. For example, for the first set we have,

$$\sum_{i,j=1,2} h_{ij} J_L^i \bar{J}_R^j = \frac{1}{2}\left(T_1 \partial Z_1^- \bar{\partial} Z_1^+ + \bar{T}_1 \partial Z_1^+ \bar{\partial} Z_1^- + U_1 \partial Z_1^- \bar{\partial} Z_1^- + \bar{U}_1 \partial Z_1^+ \bar{\partial} Z^+\right) \quad (3.26)$$

where T and U are the complex fields defined in eqs. (3.23) and (3.24). The Thirring interactions for the other two complex planes can be written similarly. The boundary condition basis vectors b_1 and b_2 translate into twists of the complex planes [14]. Thus, b_1 leaves the first complex plane invariant, and twists the second and third plane, *i.e.* $Z_1 \to Z_1$ and $Z_{2,3} \to -Z_{2,3}$, whereas b_2 leaves the second plane invariant and twists the first and third plane. It is apparent that the Thirring terms in eq. (3.26) are invariant under the action of the $Z_2 \times Z_2$ twists, whereas all the mixed terms are projected out by the GSO projections. From eq. (3.26) it is seen that the T field is associated the Kähler structure moduli, whereas the U field is associated with a complex structure moduli. This agrees with the fact that the untwisted sector of the $Z_2 \times Z_2$ orbifold produces three complex and three Kähler structure moduli.

4 Moduli in the three generation free fermionic models

Next I turn to examine the untwisted moduli in the quasi–realistic three generation free fermionic models. As discussed in section 2 the reduction to three generation is achieved by the inclusion of three or four additional boundary condition basis vectors, beyond the NAHE–set. This are typically denoted in the literature as b_i $(i = 4, 5, \cdots)$ for basis vectors that do not break the $SO(10)$ GUT symmetry, and by $\alpha, \beta, \gamma \cdots$, for basis vectors that do. From the point of view of the untwisted moduli beyond the NAHE–set models, the relevant boundary conditions are those of the world–sheet fermions $\{y, \omega|\bar{y}, \bar{\omega}\}^{1,\cdots,6}$.

Left–right symmetric boundary conditions cannot eliminate any additional moduli beyond those that exist in the NAHE–set model. This is a general consequence of the requirement that the supercurrent (2.1) is well defined under the parallel transport of the world–sheet fermions, as well as the requirement of $N = 1$ space–time supersymmetry. It is instructive to recall that the moduli of the NAHE–based models correspond to the non–vanishing world–sheet Thirring interactions, eq. (3.18), and are those of the $Z_2 \times Z_2$ orbifold, eq. (3.26). An example of a left–right three generation free fermionic model is given in table [2.12]. Rewriting the boundary conditions for the internal world–sheet fermions in the notation of table [4.1]

	$y^1\omega^1$	$\bar{y}^1\bar{\omega}^1$	$y^2\omega^2$	$\bar{y}^2\bar{\omega}^2$	$y^3\omega^3$	$\bar{y}^3\bar{\omega}^3$	$y^4\omega^4$	$\bar{y}^4\bar{\omega}^4$	$y^5\omega^5$	$\bar{y}^5\bar{\omega}^5$	$y^6\omega^6$	$\bar{y}^6\bar{\omega}^6$
α	00	00	00	00	10	10	01	01	01	01	10	10
β	01	01	10	10	00	00	00	00	01	01	10	10
γ	10	10	01	01	10	10	10	10	00	00	00	00

(4.1)

it is evident that all the Thirring interaction terms in eq. (3.18) are invariant under the transformations in table [4.1]. As discussed in [20] and section 3 there is a one–to–one correspondence between the Thirring terms, eq. (3.18) and the moduli fields, eq. (3.19). Therefore, it is sufficient to examine either the moduli or the Thirring terms, and this relation guarantees the existence, or exclusion, of the corresponding Thirring terms/moduli fields.

Left–right symmetric boundary conditions preserve the untwisted moduli space of the $Z_2 \times Z_2$ orbifold. To see that this is indeed a general result, and a consequence of world–sheet, and space–time, supersymmetry, let us recall that the world–sheet supercurrent eq. (2.1) restricts that each triplet $\chi_i y_i \omega_i$ must transform as $\psi^\mu \partial X_\mu$. The boundary conditions of each triplets are therefore restricted to be

$$\begin{aligned} b(\psi^\mu) &= 1 \rightarrow b(\chi_i, y_i, \omega_i) = (1,0,0);\ (0,1,0);\ (0,0,1);\ (1,1,1) \\ b(\psi^\mu) &= 0 \rightarrow b(\chi_i, y_i, \omega_i) = (0,1,1);\ (1,0,1);\ (1,1,0);\ (0,0,0) \end{aligned} \tag{4.2}$$

The requirement of $N = 1$ supersymmetry further restricts that

$$b(\chi_{2k-1}) = b(\chi_{2k}) \quad k = 1, 2, 3 \tag{4.3}$$

Symmetric boundary conditions means that the boundary condition of the left–moving $b\{y_i, \omega_i\}$ are identical to their corresponding right–moving fields $b\{\bar{y}_i, \bar{\omega}_i\}$.

In terms of the internal conformal field theory this means that symmetric boundary conditions correspond to the case in which all the internal left–moving world–sheet fermions from the set $\{y,\omega\}^{1,\cdots,6}$ are paired with 12 right–moving world–sheet real fermions $\{\bar{y},\bar{\omega}\}^{1,\cdots,6}$, to form 12 Ising model operators [21, 22]. Enumerating all the possible boundary conditions for the internal fermions, subject to these restrictions we have

$$b(\psi^\mu)=1\ \&\ b(\chi_{2k-1}=\chi_{2k})=1\rightarrow b(\{y,\omega\}_{2k-1},\{y,\omega\}_{2k})=\begin{cases}0\,0\,0\,0\\0\,0\,1\,1\\1\,1\,0\,0\\1\,1\,1\,1\end{cases}$$

$$b(\psi^\mu)=1\ \&\ b(\chi_{2k-1}=\chi_{2k})=0\rightarrow b(\{y,\omega\}_{2k-1},\{y,\omega\}_{2k})=\begin{cases}1\,0\,1\,0\\1\,0\,0\,1\\0\,1\,1\,0\\0\,1\,0\,1\end{cases}$$

$$b(\psi^\mu)=0\ \&\ b(\chi_{2k-1}=\chi_{2k})=1\rightarrow b(\{y,\omega\}_{2k-1},\{y,\omega\}_{2k})=\begin{cases}1\,0\,1\,0\\1\,0\,0\,1\\0\,1\,1\,0\\0\,1\,0\,1\end{cases}$$

$$b(\psi^\mu)=0\ \&\ b(\chi_{2k-1}=\chi_{2k})=0\rightarrow b(\{y,\omega\}_{2k-1},\{y,\omega\}_{2k})=\begin{cases}0\,0\,0\,0\\0\,0\,1\,1\\1\,1\,0\,0\\1\,1\,1\,1\end{cases}\qquad(4.4)$$

i.e. there are in total four different possibilities for the transformation of the pairs $\{y_i,\omega_i\}$

$$b(\{y_i,\omega_i\})\ =\ (0,0);\ (1,0);\ (0,1);\ (1,1)\qquad(4.5)$$

Using the relations (2.14) these translate to transformations of the corresponding bosonic coordinates, *i.e.*

y,ω	X_L	X_R	$X=X_L+X_R$
0 0	$X_L+\pi$	$X_R+\pi$	$X+2\pi$
1 0	$-X_L$	$-X_R$	$-X$
0 1	$-X_L+\pi$	$-X_R+\pi$	$-X+2\pi$
1 1	X_L	X_R	X

(4.6)

In terms of the complexified coordinates Z_k, eq. (2.15) these translate into

$$Z_k\rightarrow\pm Z_k+\delta_1 2\pi+\delta_2 i2\pi,\qquad(4.7)$$

where $\delta_{1,2}=0;1$ signify possible shifts in the real and complex directions. Examining the complexified Thirring terms, eq. (3.26), it is noted that these always involve pairs of complex coordinates that transform with the same sign, and hence are always invariant under symmetric boundary conditions.

Relaxing the condition of $N = 1$ space–time supersymmetry means that we may have

$$b(\chi_{2k-1}) \;\neq\; b(\chi_{2k}) \tag{4.8}$$

for some k. For example, for $k = 1$ we may have

$$\begin{array}{cccc} y_1 & \omega_1 & y_2 & \omega_2 \\ 1 & 1 & 1 & 0 \end{array} \tag{4.9}$$

In this case the moduli fields h_{11} and h_{22} are retained, whereas h_{12} and h_{21} are projected out. Hence, if we break $N = 1$ space–time supersymmetry by the assignment of boundary conditions we may project six additional untwisted moduli, and six will remain.

Next, I turn to examine the presence of untwisted moduli in models with asymmetric boundary conditions. The assignment of asymmetric boundary conditions entails that a left–moving real fermion from the set $\{y, \omega\}$ is paired with another left–moving real fermion from this set, and with which it has identical boundary conditions in all basis vectors. For every such pair of left–moving fermions, there is a corresponding pair of right–moving fermions. These combinations therefore give rise to a global $U(1)_L$ symmetry, and a corresponding gauged $U(1)_R$ symmetry. These complex pairings therefore allow the assignment of asymmetric boundary conditions, with important phenomenological consequences [11, 26, 27, 13]. The simplest case is that of table [4.10] which involves a single such complexified fermion.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,...,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,...,8}$
α	1	1	0	0	1 1 1 0 0	1	0	1	1 1 1 1 0 0 0 0
β	1	0	1	0	1 1 1 0 0	0	1	1	1 1 1 1 0 0 0 0
γ	1	0	0	1	$\frac12$ $\frac12$ $\frac12$ $\frac12$ $\frac12$	$\frac12$	$\frac12$	$\frac12$	$\frac12$ 0 1 1 $\frac12$ $\frac12$ $\frac12$ 0

	$y^3\bar y^3$	$y^4\bar y^4$	$y^5\bar y^5$	$y^6\bar y^6$	$y^1\bar y^1$	$y^2\bar y^2$	$\omega^5\bar\omega^5$	$\omega^6\bar\omega^6$	$\omega^2\omega^3$	$\omega^1\bar\omega^2$	$\omega^4\bar\omega^4$	$\bar\omega^2\bar\omega^3$
α	1	0	0	1	0	0	1	0	0	0	1	1
β	0	0	0	1	0	1	1	0	0	1	0	1
γ	1	1	0	0	1	1	0	0	0	0	0	1

(4.10)

With some choice of GSO projection coefficients. The model of table [4.10] was published in [11], and the entire spectrum is given there. In this model it is easily seen that the moduli fields,

$$h_{11},\; h_{21},\; h_{34},\; h_{44},\; h_{55},\; h_{56},\; h_{65},\; h_{66}, \tag{4.11}$$

and their corresponding Thirring terms, are retained in the spectrum, whereas the moduli fields,

$$h_{12},\; h_{22},\; h_{33},\; h_{43}, \tag{4.12}$$

are projected out, and the corresponding Thirring terms are not invariant under the transformations. Using the bosonic coordinates eq. (2.15) we can translate the transformations of the internal fermions to the corresponding action on bosonic variables. In terms of

$$Z_i = X_{2i-1} + iX_{2i} = X^L_{2i-1} + X^R_{2i-1} + i(X^L_{2i} + X^R_{2i}) \quad (i = 1,2,3), \tag{4.13}$$

we have

	Z_1	Z_2	Z_3
α	$X_1 + i(X^L_2 - X^R_2) + 2\pi + i2\pi$	$-X^L_3 + X^R_3 - iX_4 + i2\pi$	$-Z_3 + 2\pi$
β	$-X_1 + i(-X^L_2 + X^R_2) + 2\pi$	$X^L_3 - X^R_3 - iX_4 + 2\pi + i2\pi$	$-Z_3 + 2\pi$
γ	$-X_1 + i(-X^L_2 + X^R_2)$	$-X^L_3 + X^R_3 - iX_4$	$Z_3 + 2\pi + i2\pi$

(4.14)

From table [4.14] we see that the complex structure of two of the complex planes is broken, whereas the third is retained. Furthermore, the transformations with respect to two of the internal bosonic coordinates are asymmetric between the left– and the right–moving part, and are therefore not geometrical. This entails that the corresponding moduli must be projected out, and the coordinates are frozen at fixed radii. Indeed, using the identity (3.26), and the definitions of the complex moduli fields in eq. (3.21–3.24) we relate the projection of the moduli fields h_{ij} to constraints on the complex and Kähler moduli of the $Z_2 \times Z_2$ orbifold, U_i and T_i $(i = 1, 2, 3)$. Thus, in the case of the model of table [4.10], the projection of the $h_{11},\ h_{22},\ h_{33},\ h_{43}$ moduli fields translate to the conditions

$$T_1 = U_1 \quad , \quad T_2 = -U_2 \quad \text{whereas} \quad T_3; \ = U_3 \quad \text{are unconstrained} \tag{4.15}$$

The next example is a model with two complexified left– and right–moving internal fermions from the set $\{y, \omega|\bar{y}, \bar{\omega}\}$. Table [4.16] provides an example of such a model.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	0	0	0	0	1 1 1 0 0	0	0	1	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	1	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ 0 0 0 $\frac{1}{2}$ $\frac{1}{2}$ 1

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\bar{\omega}^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^1\bar{\omega}^1$	$\omega^2\bar{\omega}^2$	$\omega^3\bar{\omega}^3$	$\omega^4\bar{\omega}^4$
α	1	0	0	0	0	0	1	1	0	0	1	0
β	0	0	1	1	1	0	0	0	1	0	0	0
γ	0	1	0	1	0	0	0	1	0	0	0	1

(4.16)

With some choice of generalized GSO coefficients. The model of table [4.16] produces three generations from the twisted sector b_1, b_2 and b_3, and a standard–like observable gauge group. It produces Electroweak super–Higgs doublets from the first two untwisted planes, and a color triplet from the third. In this model it is easily seen that the moduli fields,

$$h_{12},\ h_{22},\ h_{34},\ h_{44}, \tag{4.17}$$

and their corresponding Thirring terms, are retained in the spectrum, whereas the moduli fields,

$$h_{11},\ h_{22},\ h_{33},\ h_{43},\ h_{55},\ h_{56},\ h_{65},\ h_{66}, \tag{4.18}$$

are projected out, and the corresponding Thirring terms are not invariant under the transformations. In terms of the complex bosonic coordinates (4.13) we have on the first two complex planes

	Z_1	Z_2
α	$(X_1^L - X_1^R) + iX_2 + (1+2i)\pi$	$(X_3^L - X_3^R) + iX_4 + (1+2i)\pi$
β	$(X_1^L - X_1^R) + iX_2 + (1+2i)\pi$	$(X_3^L - X_3^R) + iX_4 + (1+2i)\pi$
γ	$(X_1^L - X_1^R) + iX_2 + (1+2i)\pi$	$(X_3^L - X_3^R) + iX_4 + \pi$

(4.19)

and on the third

	Z_3
α	$(X_5^L - X_5^R) + i(X_6^L - X_6^R) + (2+i)\pi$
β	$(X_5^L - X_5^R) + i(X_6^L - X_6^R) + i\pi$
γ	$(X_5^L - X_5^R) + i(X_6^L - X_6^R) + (2+i)\pi$

(4.20)

the projection of the moduli fields in eq. (4.18) translate in this case to the conditions

$$T_1 = -U_1 \quad , \quad T_2 = -U_2 \quad \text{whereas} \quad T_3; \ = U_3 \ = 0 \tag{4.21}$$

It is seen here that the third complex plane is completely fixed, whereas on the first and second planes X_2 and X_4 retain their geometrical character, and X_1 and X_3 do not.

From the two examples above, and eq. (3.26) we can draw the general constraints on the complex and Kähler moduli, which can be written for $k = 1, 2, 3$ as,

$$U_k = h_{2k-1\ 2k-1} + h_{2k\ 2k} + i(h_{2k\ 2k-1} - h_{2k-1\ 2k}) \tag{4.22}$$

$$T_k = h_{2k-1\ 2k-1} - h_{2k\ 2k} + i(h_{2k\ 2k-1} + h_{2k-1\ 2k}). \tag{4.23}$$

From the identity eq. (3.26), the vanishing of certain real Thirring terms on the left–hand side, translate to the conditions

$$T_k + U_k = 0 \ \Leftrightarrow \ h_{2k-1\ 2k-1} \ \&\ h_{2k\ 2k-1} \ \text{are projected out.} \tag{4.24}$$

$$T_k - U_k = 0 \ \Leftrightarrow \ h_{2k\ 2k} \ \&\ h_{2k-1\ 2k} \ \text{are projected out.} \tag{4.25}$$

Hence, if both $T_k + U_k = 0$ and $T_k - U_k = 0$ hold, then both the Kähler and complex structure moduli of the k^{th} plane are projected out, and the radii and angles of the corresponding internal torus are frozen. In this case there is no extended geometry in the effective low energy field theory. This situation is similar to the manner in which gauge symmetries are broken in string theory by the GSO projections. Namely, the gauge symmetry is not realized in the effective low energy field theory, but exists as an organizing principle at the string theory level. That is parts of the string spectrum obeys the symmetry, but the entire string spectrum does not.

The above two examples demonstrates the interesting possibility of correlating between the number of complexified fermions and the surviving untwisted moduli, which would suggest that for every complex fermion, four additional untwisted moduli are projected out. However, in the following I show that this is not necessarily the case, and the situation is more intricate.

The retention or projection of untwisted moduli in the case with three complex fermions depends on the choice of pairings of the left–moving real fermions from the set $\{y, \omega\}^{1,\cdots,6}$. The distinction between the different choices of pairings, and some phenomenological consequences, was briefly discussed in ref. [13]. To demonstrate the effect on the untwisted moduli I consider the two models in tables [2.13] [9] and [4.26] [6].

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
b_4	1	1	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
α	1	0	0	1	1 1 1 0 0	1	0	1	1 1 1 1 0 0 0 0
β	1	0	1	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^6$	$y^2\bar{y}^2$	$\omega^5\bar{\omega}^5$	$\bar{y}^1\bar{\omega}^6$	$\omega^1\omega^3$	$\omega^2\bar{\omega}^2$	$\omega^4\bar{\omega}^4$	$\bar{\omega}^1\bar{\omega}^3$
α	1	0	0	1	0	0	1	0	0	0	1	0
β	0	0	0	1	0	1	0	1	1	0	1	0
γ	0	0	1	1	1	0	0	1	0	1	0	0

(4.26)

The choice of generalized GSO projection coefficients of the model of table [4.26], as well as the complete mass spectrum and its charges under the four dimensional gauge group are given in ref. [6].

One of the important constraints in the construction of the free fermionic models is the requirement that the supercurrent, eq. (2.1), is well defined. In the models that utilize only periodic and anti–periodic boundary conditions for the left–moving sector, the eighteen left–moving fermions are divided into six triplets in the adjoint representation of the automorphism group $SU(2)^6$. These triplets are denoted by $\{\chi_i, y_i, \omega_i\}$ $i = 1, \cdots, 6$, and their boundary conditions are constrained as given in eq. (4.2). In the type of models that are considered here a pair of real fermions which are combined to form a complex fermion or an Ising model operator must have the identical boundary conditions in all sectors. In practice it is sufficient to require that a pair of such real fermions have the same boundary conditions in all the boundary basis vectors which span a given model. In the model of table [2.13] the pairings are:

$$\{(y^3y^6, y^4\bar{y}^4, y^5\bar{y}^5, \bar{y}^3\bar{y}^6), \\ (y^1\omega^6, y^2\bar{y}^2, \omega^5\bar{\omega}^5, \bar{y}^1\bar{\omega}^6), \\ (\omega^2\omega^4, \omega^1\bar{\omega}^1, \omega^3\bar{\omega}^3, \bar{\omega}^2\bar{\omega}^4)\}, \tag{4.27}$$

where the notation emphasizes the original division of the world–sheet fermions by the NAHE–set [8, 13]. Note that with this pairing the complexified left-moving pairs mix between the six $SU(2)$ triplets of the left–moving automorphism group. That is the boundary condition of y^3y^6 fixes the boundary condition of $y^1\omega^5$. In this model we find that under the α projection

$$J_L^{1,\cdots,6} \rightarrow -J_L^{1,\cdots,6}, \bar{J}_R^{1,\cdots,6} \rightarrow \bar{J}_R^{1,\cdots,6} \ . \tag{4.28}$$

As a result all of the Thirring terms, and hence all the untwisted moduli are projected out in this model. In terms of the complex bosonic coordinates (4.13) the fermionic boundary conditions on the first two complex planes translate to

	Z_1	Z_2
α	$(X_1^L - X_1^R) + i(X_2^L - X_2^R) + (1+2i)\pi$	$(X_3^L - X_3^R) + i(X_4^L - X_4^R) + (1+2i)\pi$
β	$(X_1^L - X_1^R) + i(X_2^L - X_2^R) + (1+2i)\pi$	$(X_3^L - X_3^R) + i(X_4^L - X_4^R) + (1+2i)\pi$
γ	$(X_1^L - X_1^R) + i(X_2^L - X_2^R) + \pi$	$(X_3^L - X_3^R) + i(X_4^L - X_4^R) + \pi$

(4.29)

and on the third

	Z_3
α	$(X_5^L - X_5^R) + i(X_6^L - X_6^R) + (2+i)\pi$
β	$(X_5^L - X_5^R) + i(X_6^L - X_6^R) + i\pi$
γ	$(X_5^L - X_5^R) + i(X_6^L - X_6^R) + (2+i)\pi$

(4.30)

Hence, the boundary conditions in this case correspond to asymmetric action on all six real internal coordinates. In terms of the Kähler and complex structure fields we have that the projection of the 12 real h_{ij} translate to

$$T_k + U_k = 0 \ \text{ and } \ T_k - U_k = 0 \ \text{ for } \ k = 1, 2, 3 \ . \tag{4.31}$$

Therefore,

$$T_k = U_k = 0.$$

and, and all the untwisted geometrical moduli are projected out in this model.

On the other hand the pairings in the model of table [4.26] are:

$$\{(y^3y^6, y^4\bar{y}^4, y^5\bar{y}^5, \bar{y}^3\bar{y}^6), \\ (y^1\omega^6, y^2\bar{y}^2, \omega^5\bar{\omega}^5, \bar{y}^1\bar{\omega}^6), \\ (\omega^1\omega^3, \omega^2\bar{\omega}^2, \omega^4\bar{\omega}^4, \bar{\omega}^1\bar{\omega}^3)\} \tag{4.32}$$

Note that with this pairing the complexified left-moving pairs in Eqs. (4.32) mix between the first, third and sixth $SU(2)$ triplets of the left–moving automorphism group. In this model it is easily seen that the moduli fields,

$$h_{11},\ h_{12},\ h_{21},\ h_{22},\ h_{34},\ h_{44},\ h_{55},\ h_{65} \tag{4.33}$$

and their corresponding Thirring terms, are retained in the spectrum, whereas the moduli fields,

$$h_{33},\ h_{43},\ h_{56},\ h_{66} \tag{4.34}$$

are projected out, and the corresponding Thirring terms are not invariant under the transformations. In terms of the bosonized variables the boundary conditions translate to

	Z_1	Z_2
α	$Z_1+(2+i2)\pi$	$-Z_2+i2\pi$
β	$-Z_1+\pi$	$-Z_2+(2+i)\pi$
γ	$-Z_1+i2\pi$	$(X_3^L-X_3^R)+iX_4+(1+i2)\pi$

(4.35)

and on the third

	Z_3
α	$-Z_3+2\pi$
β	$Z_3+(2+i)\pi$
γ	$-X_5+i(-X_6^L+X_6^R)+i\pi$

(4.36)

Hence, in this case the asymmetric fermionic boundary conditions translate to asymmetric action only on two of the six real bosonic coordinates, whereas the other four retain their geometrical interpretation. In terms of the Kähler and complex structure fields the projection of the 4 real h_{ij} fields in eq. (4.34) translate to

$$T_1;\ U_1 \quad \text{are unconstrained}, \quad T_2=-U_2 \quad , \quad T_3=U_3 \tag{4.37}$$

Furthermore, with the choice of pairings in eqs. (4.32) we can construct a model in which all the untwisted moduli are retained in the spectrum. An example of such a model is given in table [4.38].

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,\dots,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,\dots,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 0 0 0 0 0 0 1
γ	0	0	0	0	$\frac12$ $\frac12$ $\frac12$ $\frac12$ $\frac12$	$\frac12$	$\frac12$	$\frac12$	$\frac12$ 0 1 1 $\frac12$ $\frac12$ $\frac12$ 1

	y^3y^6	$y^4\bar y^4$	$y^5\bar y^5$	$\bar y^3\bar y^6$	$y^1\omega^6$	$y^2\bar y^2$	$\omega^5\bar\omega^5$	$\bar y^1\bar\omega^6$	$\omega^1\omega^3$	$\omega^2\bar\omega^2$	$\omega^4\bar\omega^4$	$\bar\omega^1\bar\omega^3$
α	1	0	0	0	1	0	0	0	1	0	0	0
β	0	1	0	1	0	0	0	1	0	0	1	1
γ	0	0	1	1	0	0	1	1	0	0	0	1

(4.38)

with a choice of generalized GSO projection coefficients. The model in table [4.38] is a variant of the model of ref. [6]. This model utilizes asymmetric boundary conditions. There are three complexified internal fermions, which are the same as those of the model of table [4.26], and the pairing of the left–moving world–sheet fermions is identical. In terms of its phenomenological characteristics the model is a three generation model, each arising from the sectors b_1, b_2 and b_3. The four dimensional gauge group is the same as that of ref. [6]. The model utilizes the doublet–triplet splitting mechanism of ref. [26], which arises from the asymmetric boundary condition assignments in the basis vectors that break the observable $SO(10) \to SO(6) \times SO(4)$. Similarly, the model yields tri–level Yukawa couplings to the three $+2/3$ charged quarks, but not to the $-1/3$ charged quarks, which is a result of the asymmetric boundary condition assignment in the basis vector that breaks $SO(10) \to SU(5) \times U(1)$. In the model of table [4.38] all the untwisted moduli are left in the massless spectrum. It is instructive to rewrite the boundary conditions of the internal fermions in this model in the notation of table [4.1]

	$y^1\omega^1$	$\bar{y}^1\bar{\omega}^1$	$y^2\omega^2$	$\bar{y}^2\bar{\omega}^2$	$y^3\omega^3$	$\bar{y}^3\bar{\omega}^3$	$y^4\omega^4$	$\bar{y}^4\bar{\omega}^4$	$y^5\omega^5$	$\bar{y}^5\bar{\omega}^5$	$y^6\omega^6$	$\bar{y}^6\bar{\omega}^6$
α	11	00	00	00	11	00	00	00	00	00	11	00
β	00	11	00	00	00	11	11	11	00	00	00	11
γ	00	11	00	00	00	11	00	00	11	11	00	11

(4.39)

In this notation it is apparent that despite the utilization of asymmetric boundary conditions, the specific pairing of world–sheet fermions allows the retention of all the untwisted moduli. In terms of the bosonized variables the boundary conditions translate to

	Z_1	Z_2	Z_3
α	$Z_1 + (1+i2)\pi$	$Z_2 + (1+i2)\pi$	$Z_3 + (2+i)\pi$
β	$Z_1 + (1+i2)\pi$	$Z_2 + 2\pi$	$Z_3 + (2+i)\pi$
γ	$Z_1 + (1+i2)\pi$	$Z_2 + (2+i2)\pi$	$Z_3 + i\pi$

(4.40)

From eq. (4.40) it is evident that all three complex planes retain the complex geometry, and hence the three Kähler and three complex moduli remain in the spectrum. In this model the reduction to three generations is attained solely by the shift identifications in the real and complex directions, which are asymmetric in terms of the fermionic boundary conditions, but symmetric in terms of the bosonic variables.

The investigation above of the moduli in the three generation free fermionic models is, of course, not exhaustive, but rather illustrative. The moduli of other models in this class may be similarly investigated. I give here a cursory view of several additional models. As illustrated above the determinantal factor in regard to the untwisted moduli is the pairing of the left– and right–moving fermions from the set

$\{y,\omega|\bar{y},\bar{\omega}\}^{1,\cdots,6}$. In the case of symmetric boundary conditions, with 12 left–moving real fermions combined with 12 real right–moving fermions to form 12 Ising model operators, the moduli fields always remain in the spectrum and the six compactified coordinates maintain their geometrical character. When the real fermions are combined to form complex fermions the situation is more varied, as illustrated above.

The determination of the moduli, however, does not depend on the choice of the observable four dimensional gauge group. To exemplify that I consider the model of ref. [7], which utilizes the same complexification of the real fermions as that of the model of table [4.10]. It is then found that the retained and projected moduli in this model are those in eqs. (4.11) and (4.12), respectively. Of course, there may be a correlation between the assignment of boundary conditions to the real fermions $\{y,\omega\}_{L,R}$ and those that determine the four dimensional gauge group. Such dependence may arise because of the modular invariance constraints [17]. But typically we may relegate this correlation to the hidden sector, and therefore the observable sector is not affected.

In the case of three complex right–moving fermions, and their corresponding left–moving complexified fermions, each pair being associated with a distinct complex planes, we noted in eqs. (4.27) and (4.32), two cases of pairings, with differing consequences for the untwisted moduli fields. The case of (4.32) was further investigated in the model of table [4.38]. The moduli with the pairing of eq. (4.27) may be similarly investigated in models that utilize this pairing. An example of such a model is the model in table [4.41],

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$
α	1	1	1	0	1	1	1	0	1	1	1	0
β	0	1	0	1	0	1	0	1	1	0	0	0
γ	0	0	1	1	1	0	0	0	0	1	0	1

(4.41)

with a suitable choice of GSO projection coefficients. This model is published in [10], and has similar features to the model of table [2.13], with the difference being that [4.41] yields bottom–quark and tau–lepton Yukawa couplings at the quartic level of the superpotential [10], whereas [2.13] yields such couplings only at the quintic order [9]. In the model of table. [4.41] one finds again that all the untwisted moduli fields are projected out. Another example of a model in this class is the model on page 14 of [13], which utilizes the pairings of eq. (4.27). This model is a variation of the model of table [4.41], with the boundary conditions in the vector γ chosen such that the sector b_2 produces trilevel bottom–type Yukawa coupling rather than top–type. Again in this model all the untwisted moduli are projected out. In fact, we may conjecture that in general in free fermionic models that utilize the NAHE–set of boundary condition basis vectors, and the pairing of eq. (4.27), the moduli are always projected out. The reason is that in order to reduce the number of generations to three, one from each of the twisted sectors, b_1, b_2 and b_3, we have to break the degeneracy between the complexified left–moving and right–moving fermions in each sector. This entails the assignment of asymmetric boundary conditions with respect to these complexified fermions, in at least one of the basis vectors, α, β or γ. In the case of the pairing of eq. (4.27) this leads to the projection of all the untwisted moduli, as noted above, because of the fact that this pairing mixes all the six left–moving triplets of the $SU(2)^6$ automorphism algebra.

5 Twisted moduli

I now turn to show that is the class of three generation string models under consideration here moduli which arise from the twisted sectors are also projected from the massless spectrum. To see how this comes about we start with the set of basis vectors $\{1, S, \xi_1, \xi_2, b_1, b_2\}$ [14]. This set of basis vectors generates a model with $E_6 \times U(1)^2 \times E_8 \times SO(4)^3$, with 24 matter states in the 27 representation of the observable E_6 gauge group, arising from the twisted sectors. These states are decomposed in the following way under $E_6 \to SO(10)$. The spinorial 16 representations of $SO(10)$ arise from the sectors b_1, b_2 and b_3, whereas the vectorial 10 representations arise from the sectors $b_j + \xi_1$. Here, the basis vector ξ_1 produces the space-time vector gauge bosons that enhance the $N = 4$ observable $SO(16)$ gauge group to E_8. In addition to the vectorial 10 representation the sectors $b_j + \xi_1$, also produce a pair of $SO(10)$ singlets, one of which is embedded in the 27 of E_6, whereas the second is indentified with a twisted moduli. Therefore, the models with an E_6 observable gauge group contain additional 24 twisted moduli, which matches the number of chiral matter states in the model, as it should.

In the realistic free fermionic models the observable gauge group is broken from E_6 to $SO(10) \times U(1)$. This can be achieved in two equivalent ways. One possibility is to replace the vector ξ_1 with the vector 2γ. In this case the gauge group of the $N = 4$ vacuum, generated by the subset of basis vectors $\{1, S, 2\gamma, \xi_2\}$, is $SO(12) \times SO(16) \times SO(16)$. The space-time vector bosons of the four dimensional gauge group in this case are obtained from the NS sector, the sector ξ_2, and the sector $\xi_2 + 2\gamma$. An equivalent way to produce the same $N = 4$ string vacuum is to break the $E_8 \times E_8$ gauge group by the choice of GSO phase eq. (2.11) in the one–loop string partition function. One choice of this phase preserves the states from the sectors ξ_1 and ξ_2, and therefore enhances the NS $SO(16) \times SO(16)$ gauge symmetry to $E_8 \times E_8$. The second choice projects out those states. In the second case the sectors $b_j + 2\gamma$, or $b_j + \xi_1$ produce states in the 16 vectorial representation of the hidden $SO(16)$ gauge group, whereas the vectorial states in the $(5+\bar{5})$ and $(1+\bar{1})$, of the observable $SO(10)$ group, generated by the world–sheet fermions $\bar{\psi}^{1,\cdots,5}$ are projected out from the spectrum. Hence, in this case all the moduli that arise from twisted sectors are projected out. The only states that arise from the twisted sectors in three generation models are observable or hidden matter states.

6 Discussion and conclusions

In this paper I investigated the untwisted moduli fields in the three generation free fermionic heterotic–string models. This class of string vacua produced some of the most realistic string models constructed to date. Not only does it produce three generations of chiral fermions under the Standard–Model gauge group with potentially phenomenologically viable couplings to the electroweak Higgs doublet fields, but it also affords the attractive embedding of the Standard Model spectrum in $SO(10)$ representations. This property ensures the canonical GUT normalization of the weak hypercharge, which in turn facilitates the agreement of the heterotic–string coupling unification with the experimental data. Thus, these models produce a gross structure, which is compelling from a phenomenological point of view. It ought to be emphasized that one should not regard any of the existing models as providing a completely realistic phenomenology, but merely as providing a probe into what may be some of the ingredients of the eventually true vacuum. From this perspective, a vital property of the free fermionic models is their connection to $Z_2 \times Z_2$ orbifold compactifications.

The free fermionic formalism facilitated the construction of the three generation models and the analysis of some of their phenomenological characteristics. However, this method is formulated, a priori, at a fixed point in the moduli space and the immediate notion of the underlying geometry of the six dimensional compactified manifold is lost. In particular, the identification of the untwisted moduli fields, and their role in the low energy effective theory, is encumbered. These are reincorporated into the models by identifying the moduli fields as the coefficients of the Abelian Thirring interactions [20]. The moduli fields in the free fermionic models are therefore in correspondence with the Abelian Thirring terms that are invariant under the GSO projections, induced by the basis vectors that define the models.

In this paper the issue of untwisted moduli in the realistic three generation free fermionic models was investigated. The existence of models in which all the untwisted Kähler and complex structure moduli are projected out by the generalized GSO projections was demonstrated. The conditions for the projection of all the moduli were identified, and compared to other similar models in which the untwisted moduli are retained. The basic condition that enable the projection of all the untwisted moduli is when the fermionic boundary conditions are such that they correspond to left–right asymmetric boundary conditions with respect to all the six real coordinates of the six dimensional internal manifold. Additionally, it was shown that in this class of three generation models the E_6 symmetry is broken to $SO(10) \times U(1)$ by a GSO projection. As a consequence all the moduli that arise from the twisted sectors are also projected out in these models.

The existence of quasi–realistic models in which all the untwisted Kähler and complex structure moduli are projected out is fascinating. It offers a novel perspective on the existence of extra dimensions in string theory, and on the problem of moduli stabilization. The untwisted moduli are those that govern the underlying geometry, and hence the physics of the extra dimensions. Reparameterization invariance in string theory introduces the need for additional degrees of freedom, beyond the four space–time dimensions, to maintain the classical symmetry in the quantized theory. These additional degrees of freedom may be interpreted as extra dimensions, which are compactified and hidden from contemporary experimental observations. Thus, the consistency of string theory gives rise to the notion of extra dimensions, which in every other respect is problematic. In particular, it raises the issue of what is the mechanism that selects and fixes the parameters of the compactified space. However, if there exist string models in which all the untwisted moduli are projected out by the GSO projections, it means that in these vacua the parameters of the extra dimensions are frozen. In fact, in these string vacua the extra degrees of freedom needed for consistency cannot be interpreted as extra dimensions, as it is not possible to deform from their fixed values, and there is no notion of a continuous classical geometry.

The problem of moduli stabilization in string theory attracted considerable attention in the literature [1]. Most of the studies have been directed toward stabilization of the extra dimensions in the effective low energy field theory that emerges from the underlying string theory. The primary obstacle to this is the fact that there are no potential terms for the moduli fields to all orders in perturbation theory. One then, in general, has to rely on the appearance of nonperturbative potential terms, or the utilization of internal fluxes [1]. However, the mechanism that fixes the moduli in the free fermionic models is an intrinsically string theoretic mechanism. The reason is that this mechanism utilizes asymmetric boundary conditions. The possibility to separate the internal dimensions into left– and right–movers, and to assign different transformation properties to them, is intrinsically string theoretic and is nonsensical in the effective low energy point quantum field theories, considered to date.

String theory provides a consistent framework for perturbative quantum gravity. In this context it provides the quantum, albeit perturbative, probe of the underlying geometry. The effective low energy point quantum field theory, on the other hand, treats the underlying geometry as a classical geometry. The existence of quasi–realistic string vacua in which all the untwisted moduli are fixed at the string level may tell us that, although string theory requires the additional degrees of freedom for its consistency, these degrees of freedom are not necessarily realized as extra continuous classical dimensions, in the phenomenologically viable cases. These vacua live intrinsically in four space–time dimensions, and there is no notion of extra geometrical dimensions in the low energy effective point quantum field theory. Thus, while the geometrical notion of the extra degrees of freedom provides a useful mean to classify the string vacua, these do not have a physical realization. This situation is similar to the way in which the GUT symmetries are broken in the quasi–realistic free fermionic heterotic–string models by the generalized GSO projections. Namely, also in this case, while the Standard Model states fall into representations of the underlying $SO(10)$ symmetry, the GUT group is broken directly at the string level, and is not realized as a gauge symmetry in the effective low energy point quantum field theory.

The existence of quasi–realistic string vacua in which all the untwisted moduli are projected out by the GSO projections is fascinating and intriguing. As discussed in this paper, such string vacua exist among the so–called realistic free fermionic models. We may then ask what are the properties of these models that enabled this outcome, and whether it is unique to this class of models. It should be emphasized that although the free fermionic formalism is formulated at a fixed point in the moduli space, this does not yet entail the absence of an underlying geometry, and the geometrical degrees of freedom are reincorporated in the form of the Abelian world–sheet Thirring interactions. The string vacua in which all the untwisted moduli are fixed represent a special subclass of the three generation models, and the projection of the moduli is highly correlated with the reduction of the number of generations. That is, it is only in the special case of the three generation models that one may expect to find vacua in which all the untwisted moduli are projected out.

The defining property of the three generation free fermionic heterotic string models is their relation to $Z_2 \times Z_2$ orbifold compactifications [14]. We may therefore ask whether the Z_2 projections, and the $Z_2 \times Z_2$ orbifold possess some special properties that enables the projection of all the untwisted geometrical moduli, and distinguishes it from other compactifications. Naturally, an answer to this question requires further investigation. However, we may note that the special property of the $Z_2 \times Z_2$ orbifold is that it may act on the internal dimensions as real, rather than complex, dimensions. As discussed in section 4 it is this property that enables the projection of all the untwisted Kähler and complex structure moduli in the free fermionic models that utilize the pairings of eq. (4.27). Whether or not similar results may be obtained in other classes of string compactifications is an interesting question that requires further research.

It should be emphasized that the results of this paper do not imply a complete solution to the moduli problem in string theory. A moduli field, in general, is a field that does not have a potential to all orders in perturbation theory, and therefore its vacuum expectation value is unconstrained, and it does not get a mass term. The string models contain other sources of moduli fields, aside from the perturbative untwisted geometrical moduli, and the twisted sectors moduli. These include: the dilaton field; and the possibility of charged moduli. Furthermore, the class of vacua under consideration here are supersymmetric and there may exist moduli associated with the supersymmetric flat directions. Moreover, in string theory, each continuous coupling naively implies the existence of a moduli field, and therefore prior to fixing all the couplings in a given model, it seems premature to claim that the entire issue of moduli has been addressed. The untwisted moduli fields are, however, those that parameterize the shape and size of the underlying six dimensional compactified space.

In regard to the dilaton field, the models investigated here are perturbative heterotic strings. Following the string duality advances, we now know that the heterotic string is a perturbative limit of the more fundamental quantum M–theory. At present we have no knowledge about this quantum theory, aside from the existence of its effective limits. The heterotic limit is an expansion in vanishing string coupling, and therefore one would not expect to be able to fix the dilaton VEV in this limit (although one may entertain some nonperturbative possibilities [32]). In this paper it was found that fixation of the other untwisted moduli is achieved by an intrinsic string mechanism, and hence at the perturbative quantum gravity level. However, at present the quantum M–theory is not available. The lesson from the current paper is that the quantum M–theory may allow further possibilities to the problem of dilaton stabilization, which are not readily gleaned in the effective low energy point quantum field theory description.

Additional flat direction moduli and charged moduli may, in general, exist in the string models. Their existence, or absence, in the string model is more model dependent and requires a model by model analysis. However, the structure of the three generation free fermionic models suggests that flat, or charged, moduli are not interchanged with untwisted moduli, and hence an underlying continuous geometrical manifold is not restored.

Finally, it should be emphasized that whether or not extra dimensions play a physical role in nature would, of course, require further study and investigation. From the discussion in sections 2 and 3 it is found that the untwisted sector of the NAHE–based free fermionic models produces three complex and three Kähler structure moduli. This outcome remains valid in any free fermionic model which is left–right symmetric. As discussed in section 2 there do exist three generation free fermionic models that are left–right symmetric. Therefore in these models the entire set of moduli fields exist in the effective low energy field theory and the moduli remain unfixed. However, the important point is that the free fermionic models also allow for boundary conditions which are left–right asymmetric. Naturally, the space of models is vast and we can construct models in which asymmetric boundary conditions are assigned on one, two or three of the complex planes. The remarkable fact is the existence of three generation models in which asymmetric boundary conditions are assigned to all three complex planes. In this set of models the entire set of untwisted moduli are projected out by the GSO projections and the untwisted moduli are fixed. The results of this paper indicate the existence of quasi–realistic string vacua in which the extra dimensions do not possess a classical physical realization. On the other hand there are also three generation models in which the moduli are retained and the geometrical description is maintained, and models in which some of the extra dimensions are frozen whereas others are undetermined. Which, if any, of these possibilities is relevant to nature, would, of course, remain, for the time being, a hotly contested issue. Moreover, the models may of course still contain additional moduli and the role of these moduli requires more detailed investigation. We should also remember that we are discussing here perturbative heterotic string models. In this limit the dilaton remains unfixed, and it may be that nonperturbative effects may give rise to additional moduli. Nevertheless, it is extremely intriguing that in the class of three generation free fermionic models, that are constructed in the vicinity of the self–dual point under the T–duality transformations, one finds that the models possess the intrinsic mechanism to fix all the untwisted geometrical moduli. One would anticipate that the self–dual point, being the symmetry point under the T–duality transformations, plays a vital role in the vacuum selection [33]. The three generation free fermionic models then highlight the class of $Z_2 \times Z_2$ orbifolds as the naturally relevant one. The availability, in this class, to act asymmetrically on all the six real compactified dimensions, then affords the possibility of fixing all the untwisted Kähler and complex structure moduli in these models.

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