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One loop $\overline{\text{MS}}$ gluon pole mass from the LCO formalism

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Abstract. We compute the one loop corrections to the pole mass of the gluon in the $\overline{\text{MS}}$ scheme in the Landau gauge in both the Curci-Ferrari model and the local composite operator formalism with N_f flavours of massless quarks. For the latter we determine an estimate for the gluon mass using the effective potential of a local dimension two composite operator and find, for example, $m_{\text{gluon}} = 2{\cdot}10\Lambda_{\overline{\text{MS}}}$ in Yang-Mills theory.

The issue of whether the gluon obtains a dynamically generated mass has been a popular topic of investigation in recent years. Following the work of [1, 2, 3, 4] who observed that the perturbative vacuum of QCD is unstable, one of the main activities has been on the numerical evaluation of the vacuum expectation value of the square of the gauge potential, $\langle \frac{1}{2} A^2_\mu \rangle$. Various methods have been used to achieve this ranging from combinations of lattice computations with the operator product expansion and instanton considerations, [5, 6, 7, 8, 9, 10, 11], to a more theoretical approach of the local composite operator formalism of, for instance, [12, 13, 14, 15]. Moreover, there is evidence from phenomenology that the existence of a gluon mass in the range of 500-800 MeV may provide a more accurate explanation of various experimental data. Indeed, a valuable summary table of current gluon mass estimates has been given in the article by Field, [16]. Whilst the operator $\frac{1}{2} A^2_\mu$ suffers from the immediate objection of not being a gauge invariant entity, it has been shown how to relate it to a dimension two gauge invariant physical operator, which is the minimization of A^2_μ over all gauge configurations [12, 17, 18].This operator, albeit non-local, reduces to a local operator in the Landau gauge and it is solely in this gauge that, for example the lattice results of [5, 6, 10, 11] have been determined. Indeed the local composite operator (LCO) formalism of [12, 13] was originally developed in the Landau gauge but recently an estimate of $\langle \frac{1}{2} A^2_\mu \rangle$ has been determined in arbitrary linear covariant gauge, [17].

Whilst there is much activity in trying to ascertain the existence of a dynamical gluon mass, there appears to be less effort into standardizing mass estimates. For instance, in the quark sector of QCD the estimates of the various quark masses by methods such as sum rules, lattice regularization and the operator product expansion are all expressed as the $\overline{\text{MS}}$ running mass at the scale of 2GeV. Although clearly measurements are not always made at this scale. To connect the mass estimates one requires an as accurate as possible evaluation in perturbation theory of the quark mass anomalous dimension in the $\overline{\text{MS}}$ scheme. This is currently available at four loops, [19, 20, 21, 22, 23]. Moreover, the relation between the quark pole masses and the running mass is known at three loops, [24]. For the same problem for a gluon mass the analogous quantities are not available to as high an order. For instance, the running of the naive gluon mass operator, $\frac{1}{2} A^2_\mu - \alpha \bar{c} c$, in the non-linear Curci-Ferrari gauge, [25], is known at three loops, [26]. In the Landau gauge, it transpires that it is not an independent renormalization being the sum of the gluon and ghost anomalous dimensions, which is a result that derives from a Slavnov-Taylor identity, [27]. This has recently been exploited to obtain the four loop running in the Landau gauge for the $SU(3)$ colour group, [28]. However, the relation between the pole mass of the gluon and the running gluon mass is not yet available for QCD at *one* loop. Therefore, it is the aim of this article to provide such a relation for QCD which will build on the Yang-Mills expression recently given in [29] for the Curci-Ferrari gauge. Moreover, since the LCO formalism has provided estimates for a dynamically generated gluon mass which are comparable with other methods we will also determine the relation for that approach as well. This will provide a clean estimate for a gluon mass, since in [12] the effective potential for the operator $\frac{1}{2} A^2_\mu$ was developed at two loops in the Landau gauge. However, there the estimate for a dynamical gluon mass was based on determining the value of an *effective* gluon mass which was by definition a *classical* mass. It seems to us that a more appropriate quantity to estimate through the effective potential approach would be a one loop quantity derived from the gluon two-point function such as the pole mass. This is the second aim of the article.

We begin by defining our notation. We recall that the QCD Lagrangian in a linear covariant gauge is

$$L^{\text{QCD}} = - \frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2 - \bar{c}^a \partial^\mu D_\mu c^a + i \bar{\psi}^{iI} \not{D} \psi^{iI} \tag{1}$$

where α is the gauge fixing parameter, $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu - g f^{abc} A^b_\mu A^c_\nu$ and ψ^{iI} is the quark field. The indices take the following ranges $1 \leq a \leq N_A$, $1 \leq I \leq N_F$ and $1 \leq i \leq N_f$ where N_F and N_A are the dimensions of the fundamental and adjoint representations respectively, N_f is the number of quark flavours and f^{abc} are the colour group structure constants. The covariant derivatives which determine $G^a_{\mu\nu}$ are

$$D_\mu c^a ~=~ \partial_\mu c^a ~-~ g f^{abc} A^b_\mu c^c ~~,~~ D_\mu \psi^{iI} ~=~ \partial_\mu \psi^{iI} ~+~ i g T^a A^a_\mu \psi^{iI} ~. \tag{2}$$

In [12], the LCO formalism was derived which involves an additional scalar field σ which is related to the dimension two composite operator $\frac{1}{2} A^2_\mu$. The relevant Lagrangian is

$$L^{\mathrm{LCO}} ~=~ L^{\mathrm{QCD}} ~-~ \frac{\sigma^2}{2g^2\zeta(g)} ~+~ \frac{1}{2g\zeta(g)} \sigma A^a_\mu A^{a\,\mu} ~-~ \frac{1}{8\zeta(g)} \left(A^a_\mu A^{a\,\mu} \right)^2 \tag{3}$$

where there is an extra contribution to the quartic gluon interaction and L^{LCO} contains the usual covariant gauge fixing terms though we will only consider the Landau gauge case, $\alpha = 0$. The quantity $\zeta(g)$ is a function of the coupling constant which has been computed to $O(g^2)$ in the Landau gauge in [12, 13, 14] and is such that it ensures the generating functional underlying the formalism satisfies a homogeneous renormalization group equation, [12]. For this article we note that the relevant terms are

$$\begin{aligned} \frac{1}{g^2\zeta(g)} ~=~ & \left[\frac{(13C_A - 8T_F N_f)}{9N_A} \right. \\ & + \Big(2685464 C_A^3 T_F N_f - 1391845 C_A^4 - 213408 C_A^2 C_F T_F N_f - 1901760 C_A^2 T_F^2 N_f^2 \\ & \quad + ~ 221184 C_A C_F T_F^2 N_f^2 + 584192 C_A N_f^3 T_F^3 - 55296 C_F T_F^3 N_f^3 \\ & \quad \left. - ~ 65536 T_F^4 N_f^4 \Big) \frac{g^2}{5184\pi^2 N_A (35C_A - 16T_F N_f)(19C_A - 8T_F N_f)} \right] ~. \end{aligned} \tag{4}$$

In [12, 13, 14] the σ field develops a non-zero vacuum expectation value when one computes the one loop effective potential of σ which is

$$\begin{aligned} V(\sigma) ~=~ & \frac{9N_A}{2} \lambda_1 \sigma'^2 \\ & + \left[\frac{3}{64} \ln \left(\frac{g\sigma'}{\mu^2} \right) + C_A \left(- \frac{351}{8} C_F \lambda_1 \lambda_2 + \frac{351}{16} C_F \lambda_1 \lambda_3 - \frac{249}{128} \lambda_2 + \frac{27}{64} \lambda_3 \right) \right. \\ & \quad + ~ C_A^2 \left(- \frac{81}{16} \lambda_1 \lambda_2 + \frac{81}{32} \lambda_1 \lambda_3 \right) \\ & \quad \left. + \left(- \frac{13}{128} - \frac{207}{32} C_F \lambda_2 + \frac{117}{32} C_F \lambda_3 \right) \right] \frac{g^2 N_A \sigma'^2}{\pi^2} ~+~ O(g^4) \end{aligned} \tag{5}$$

where we have set

$$\lambda_1 ~=~ [13C_A - 8T_F N_f]^{-1} ~~,~~ \lambda_2 ~=~ [35C_A - 16T_F N_f]^{-1} ~~,~~ \lambda_3 ~=~ [19C_A - 8T_F N_f]^{-1} ~~, \tag{6}$$

$$\sigma ~=~ \frac{9N_A}{(13C_A - 8T_F N_f)} \sigma' \tag{7}$$

and μ is the usual $\overline{\mathrm{MS}}$ renormalization scale, which is introduced to retain a dimensionless coupling constant in dimensional regularization.

Now we consider the relation between the pole mass and the running gluon mass in the Curci-Ferrari model, [25], which includes the BRST invariant mass operator

$$L^{\mathrm{mass}} ~=~ \frac{1}{2} m^2 A^a_\mu A^{a\,\mu} ~-~ \alpha m^2 \bar{c}^a c^a \tag{8}$$

where m is the bare mass. With this term the gluon and ghost propagators in the Landau gauge are

$$-\frac{\delta^{ab}}{(k^2-m^2)}\left[\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right] \quad , \quad \frac{\delta^{ab}}{k^2} \tag{9}$$

respectively. With these it is a straightforward exercise to compute the one loop correction to the gluon two-point function. In this respect the one loop snail diagram derived from the quartic gluon interaction cannot be neglected in the massive case. The result of our computation for the pole mass in the Curci-Ferrari model is

$$\begin{aligned} m^2_{\mathrm{CF}} &= \left[1 + \left(\left(\frac{313}{576} - \frac{35}{192}\ln\left(\frac{m^2(\mu)}{\mu^2}\right) - \frac{11\pi\sqrt{3}}{128}\right)C_A \right.\right. \\ &\quad \left.\left. + \left(\frac{1}{12}\ln\left(\frac{m^2(\mu)}{\mu^2}\right) - \frac{5}{36}\right)T_FN_f\right)\frac{g^2}{\pi^2} + O(g^4)\right] m^2(\mu) \end{aligned} \tag{10}$$

where $m_{\mathrm{o}} = m(\mu)Z_m$ is the bare mass, $m(\mu)$ is the running mass and μ is the renormalization mass scale. We have renormalized with the usual one loop $\overline{\mathrm{MS}}$ renormalization constants. As a check on the expression, we note that it reduces to the same relation given in [29] when $N_f = 0$. Moreover, we have verified the expression of [29] for arbitrary α prior to specifying the Landau gauge which provided a non-trivial check on the symbolic manipulation programmes we used for this article.

We have repeated the above computation for the LCO Lagrangian where the bare mass is now defined to be $\sigma/[g\zeta(g)]$, [12, 13, 14], which at leading order is

$$m^2_{\mathrm{o}} = \frac{(13C_A - 8T_FN_f)}{9N_A}g\sigma \ . \tag{11}$$

With the additional interactions the expression for the LCO pole mass is of a similar form

$$\begin{aligned} m^2_{\mathrm{LCO}} &= \left[1 + \left(\left(\frac{287}{576} - \frac{3}{64}\ln\left(\frac{m^2(\mu)}{\mu^2}\right) - \frac{11\pi\sqrt{3}}{128}\right)C_A \right.\right. \\ &\quad \left.\left. - \frac{1}{9}T_FN_f\right)\frac{g^2}{\pi^2} + O(g^4)\right] m^2(\mu) \end{aligned} \tag{12}$$

for massless quarks in the Landau gauge. Equipped with this result we can now extend the method of [12, 13, 14] for estimating a gluon mass. In [12, 14] the minimum of the effective potential (5) was determined by solving $\frac{dV(\sigma)}{d\sigma} = 0$. Since the factors multiplying the classical effective mass are coupling constant dependent, this is equivalent to extremizing $V^{\mathrm{eff}}(m^2_{\mathrm{o}})$. However, it seems to us that an alternative approach is to compute instead the extremum of $V^{\mathrm{eff}}(m^2_{\mathrm{LCO}})$ where one inverts (12) to obtain $m(\mu)$ as a function of m^2_{LCO} and then substitutes this into (5). Thus we find

$$\begin{aligned} V^{\mathrm{eff}}\left(m^2_{\mathrm{LCO}}\right) &= \left[\frac{9}{2}\lambda_1 + \left(-\frac{29}{128} - \frac{207}{32}C_F\lambda_2 + \frac{117}{32}C_F\lambda_3 \right.\right. \\ &\quad + C_A\left(-\frac{351}{8}C_F\lambda_1\lambda_2 + \frac{351}{16}C_F\lambda_1\lambda_3 - \frac{183}{64}\lambda_1 \right. \\ &\quad \left. - \frac{249}{128}\lambda_2 + \frac{27}{64}\lambda_3 + \frac{99}{128}\pi\sqrt{3}\lambda_1\right) \\ &\quad + C_A^2\left(-\frac{81}{16}\lambda_1\lambda_2 + \frac{81}{32}\lambda_1\lambda_3\right) + \frac{3}{64}\ln\left(\frac{m^2_{\mathrm{LCO}}}{\mu^2}\right) \\ &\quad \left.\left. + \frac{27}{64}C_A\lambda_1\ln\left(\frac{m^2_{\mathrm{LCO}}}{\mu^2}\right)\right)\frac{g^2}{\pi^2}\right]\frac{(13C_A - 8T_FN_f)^2}{81N_A}g^2\zeta^2(g)m^4_{\mathrm{LCO}} \end{aligned} \tag{13}$$

Repeating the process to find a minimum necessitates solving

$$
\begin{aligned}
0 \;=\; & \left[\frac{9}{2}\lambda_1 + \left(-\frac{13}{64} - \frac{207}{32} C_F \lambda_2 + \frac{117}{32} C_F \lambda_3 \right.\right. \\
& + C_A \left(-\frac{351}{8} C_F \lambda_1 \lambda_2 + \frac{351}{16} C_F \lambda_1 \lambda_3 - \frac{339}{128}\lambda_1 \right. \\
& \left. \quad - \frac{249}{128}\lambda_2 + \frac{27}{64}\lambda_3 + \frac{99}{128}\pi\sqrt{3}\lambda_1 \right) \\
& + C_A^2 \left(-\frac{81}{16}\lambda_1\lambda_2 + \frac{81}{32}\lambda_1\lambda_3 \right) + \frac{3}{64}\ln\left(\frac{m_{\mathrm{LCO}}^2}{\mu^2}\right) \\
& \left.\left. + \frac{27}{64} C_A \lambda_1 \ln\left(\frac{m_{\mathrm{LCO}}^2}{\mu^2}\right)\right) \frac{g^2}{\pi^2} \right] \frac{(13C_A - 8T_F N_f)^2}{81 N_A} g^2 \zeta^2(g) m_{\mathrm{LCO}}^4 \qquad (14)
\end{aligned}
$$

which corresponds to the condition

$$
\frac{dV\left(m_{\mathrm{LCO}}^2\right)}{dm_{\mathrm{LCO}}^2} \;=\; 0 \; . \qquad (15)
$$

We have not in fact substituted for the explicit expression for $\zeta(g)$ since this function factorizes off the expression for the location of the minimum. If we were to include that part of the series which was already known it would introduce an unnecessary truncation error into our final estimates for the pole mass. At this point to solve for the mass a scale needs to be chosen for μ. In [12, 13, 14], the choice of scale was such that it removed the logarithm terms. For this potential we will take a more general approach and instead set $m_{\mathrm{LCO}}^2 = s\mu^2$ where s is an arbitrary parameter. This means we have determined an equation for the value of the coupling constant as a function of s. In other words

$$
\begin{aligned}
y \;=\; & 36 C_A (16 T_F N_f - 35 C_A) \left[\left(3465\pi\sqrt{3} + 4620\ln(s) - 25690\right) C_A^2 - 864 C_F T_F N_f \right. \\
& + \left(19240 - 1584\pi\sqrt{3} - 3792\ln(s)\right) C_A T_F N_f \\
& \left. + (768\ln(s) - 3328) T_F^2 N_f^2 \right]^{-1} \qquad (16)
\end{aligned}
$$

where $y = C_A g^2/(16\pi^2)$. Through the definition of the running coupling constant we have the one loop relation

$$
\frac{g^2(\mu)}{16\pi^2} \;=\; \left[\beta_0 \ln\left[\frac{\mu^2}{\Lambda_{\overline{\mathrm{MS}}}^2}\right]\right]^{-1} \qquad (17)
$$

where

$$
\beta_0 \;=\; \frac{11}{3} C_A \;-\; \frac{4}{3} T_F N_f \; . \qquad (18)
$$

Hence, we can relate the coupling constant to the scale μ and $\Lambda_{\overline{\mathrm{MS}}}$ and deduce a value for m_{LCO}^2. We find

$$
\begin{aligned}
m_{\mathrm{LCO}} \;=\; & \Lambda_{\overline{\mathrm{MS}}}^{(N_f)} \exp\Big[- \Big(\left(3465\pi\sqrt{3} - 25690\right) C_A^2 - 864 C_F T_F N_f \\
& + \left(19240 - 1584\pi\sqrt{3}\right) C_A T_F N_f \;-\; 3328 T_F^2 N_f^2 \Big) \\
& \Big(24 (11 C_A - 4 T_F N_f)(35 C_A - 16 T_F N_f) \Big)^{-1} \Big] \qquad (19)
\end{aligned}
$$

which is the main result of this article. It turns out that this is *independent* of the intermediate parameter s. In other words, no matter what scale μ one chooses, one will always obtain the same value for the solution to (14) at one loop.

N_f	$m_{SU(2)}/\Lambda^{(N_f)}_{\overline{\rm MS}}$	$m_{SU(3)}/\Lambda^{(N_f)}_{\overline{\rm MS}}$
0	2.10	2.10
2	1.54	1.74
3	1.24	1.55

Table 1. One loop estimates of the gluon effective mass for $SU(2)$ and $SU(3)$.

We have given the explicit values of the pole mass estimates from (19) for $SU(2)$ and $SU(3)$ in Table 1. Compared with the classical effective gluon mass estimates of [12, 13, 14] the Yang-Mills estimates have increased by about 5% for $SU(3)$. However, for $N_f \neq 0$ there is a significant decrease. Although this is disappointing it is important to recognise that since they have been derived in a scale independent and therefore renormalization group invariant manner, they may be closer to the true result, though the inclusion of quark mass may alter these estimates.

We conclude with several remarks. First, we have constructed a one loop renormalization group invariant pole mass for the gluon using the LCO effective potential of [12, 13, 14]. However, it would be interesting to see whether this feature persists at the next order. This only requires an extension of the present one loop result since the two loop LCO effective potential is available. Although we have ignored quark mass effects it seems that if one could include quark condensates in the LCO formalism in addition to that for $\frac{1}{2}A^{a\,\mu}A^a_\mu$ then it might be possible to ascertain the extent to which condensates could be responsible for the quark and gluon masses. If the renormalization scale invariance persists even at one loop for this scenario then one would not have to worry about solving a multi-scale type renormalization group equation. Our final comment concerns the situation where a gluon mass is dynamically generated through, say, the LCO formalism. If this is the case then one would have to include additional contributions due to a gluon mass to the existing quark pole mass multi-loop estimates.

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