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Hadronic decay of a vector meson from the lattice

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Abstract

We explore the decay of a vector meson to two pseudoscalar mesons on the lattice with $N_f = 2$ flavours of sea quark. Although we are working with quark masses that do not allow a physical decay, we show how the transition rate can be evaluated from the amplitude for $\rho \to \pi\pi$ and from the annihilation component of $\pi\pi \to \pi\pi$. We explore the decay amplitude for two different pion momenta and find consistent results. The coupling strength we find is in agreement with experiment. We also find evidence for a shift in the ρ mass caused by mixing with two pion states.

1 Introduction

The study of hadronic decays using lattice techniques has long been known to be feasible in principle [1, 2]. Pioneering attempts were made to study glueball decay [3] and ρ meson decay [4] in quenched QCD where no actual decay takes place. This underlines the approach: one can study the mixing of hadronic states on a lattice, provided that the energies of the states are close (see ref [5, 6] for quantitative analysis). Indeed string breaking has been explored this way [7, 8] as has scalar meson mixing [5] and hybrid meson decay [6]. The common feature of these examples is that there is little experimental knowledge of the relevant transition matrix elements. Here we rectify this by studying ρ meson decay. Another motivation is that non-leptonic weak decays ($K \to \pi\pi$ especially) are very important to understand from non-perturbative QCD and a study of simpler purely hadronic decays will be a useful step in this direction.

On a lattice many features are different from in the real world. The most significant for our purposes is that periodic spatial boundary conditions are imposed. Note that the continuum limit in such a finite volume is defined and, if the spatial extent is L, the momentum is discrete in units of $2\pi/L$. This implies that the two particle spectrum is discrete. For decay of a vector meson in the centre of mass, in a P-wave (actually the T_1^{--} representation of the cubic rotation group), the decay momentum must be non-zero and hence the lightest $\pi\pi$ state will be with momentum 1 and -1 in these units. For the lattices we will use, the energy levels neglecting interactions are illustrated in fig. 1. As has been noted before [4], a closer match in energy can be achieved by considering the decay of a moving vector meson, see also fig. 1, since this effectively allows a relative momentum in the centre of mass of π/L. Note that in neither case is the ρ meson unstable.

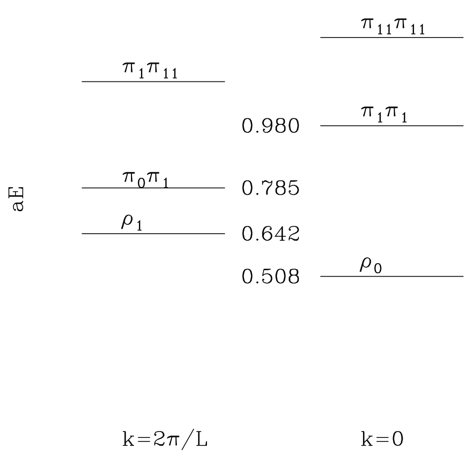

Figure 1: The energy spectrum (in lattice units with $a \approx 0.11$ fm) on a lattice of the ρ meson and two pion states considered here. Here k is the overall momentum and the suffixes are the components (in units of $2\pi/L$) of momentum of the mesons. The energy values come from UKQCD fit [9] central values for zero momentum states.

As an illustration, consider the transfer matrix with two states, a ρ meson with energy $m - \Delta/2$ and a $\pi\pi$ state with energy $m + \Delta/2$ with a transition amplitude $x = \langle \rho | \pi\pi \rangle$:

$$e^{-ma} \begin{pmatrix} e^{-a\Delta/2} & ax \\ ax & e^{a\Delta/2} \end{pmatrix} \tag{1}$$

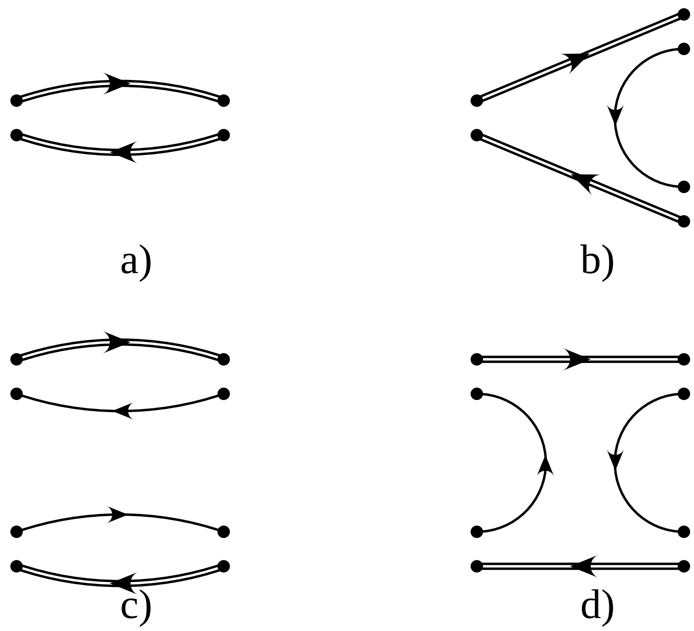

Figure 2: The quark diagrams studied here, where (a) is ρ propagation, (b) is $\rho \to \pi\pi$, (c) and (d) are the direct and 'box' components of $\pi\pi \to \pi\pi$. The valence quarks are shown by a double line and the quark-antiquark pair created in the decay by a single line, although in this work both quarks are taken as having the mass of the sea quark (approximately of strange quark mass). As described in the text, we take the P-wave component by antisymmetrizing on exchange of the final state pions. Note that the crossed quark diagram ($\overline{\underline{\mathrm{X}}}$)does not contribute.

This transfer matrix has eigenvalues $\lambda = e^{-Ea}$ with

$$E \approx m \pm (\Delta^2/4 + x^2)^{1/2} \tag{2}$$

which is larger than the unmixed splitting by a shift of $\epsilon = (\Delta^2/4+x^2)^{1/2} - \Delta/2$ up (assuming $\Delta > 0$) of the two pion state and down for the ρ state.

We expect the lightest state with the relevant quantum numbers (isospin 1, T_1^{--} representation) to be dominantly created by a local (or fuzzed) quark antiquark operator, as conventionally used to study the ρ meson. Likewise, the lightest two pion state is expected to be dominantly created by a lattice operator made of two pion operators. These statements are qualitative, and Lüscher has emphasised [2] that a quantitative description can be based on a careful measurement of the two-particle energy levels (using in principle any lattice operators whatsoever) for different sizes L. To determine this energy shift accurately is a challenging task. As well as attempting to determine it directly, we measure the mixing $x = \langle\rho|\pi\pi\rangle$ which allows us to estimate the energy shift from eq. 2, as well as to estimate the transition amplitude directly.

The essence of the determination of this transition amplitude is that when the energies of the two hadronic states (ρ meson and $\pi\pi$ system with the same quantum numbers) are close (specifically when $t\Delta \ll 5$) then the exponentials in t corresponding to the two energy states conspire to give a linear dependence (see ref.[5] for a discussion). The transition measured on a lattice then gives access [5, 6] to the required matrix element since for large t (specifically $(E' - E)t \gg 1$ with $E' - E$ the energy gap to the first excited state),

$$\frac{\langle\rho(0)|\pi(t)\pi(t)\rangle}{\langle\rho(0)|\rho(t)\rangle^{1/2}\langle\pi(0)\pi(0)|\pi(t)\pi(t)\rangle^{1/2}} = xt + \text{const} \tag{3}$$

provided that the transition rate is not too large, namely $xt \ll 1$.

A further cross check [5, 6] is also possible from the box diagram (see fig. 2d) under similar conditions, since

$$\frac{\langle\pi(0)\pi(0)|\pi(t)\pi(t)\rangle_{\text{box}}}{\langle\pi(0)\pi(0)|\pi(t)\pi(t)\rangle_{\text{direct}}} = \frac{1}{2}x^2t^2 + \mathcal{O}(t) \tag{4}$$

Here the notation $\pi(t)$, $\rho(t)$, etc refers to creating a state with those quantum numbers from the vacuum at that time on the lattice. The denominators are to normalise the states to unity. To relate the matrix element x to the usual continuum large volume formalism, one has to relate this unit normalisation condition to the usual relativistic one - see ref [2, 10, 11] for a full discussion. One simple way to do this is by considering the formula for the decay width, though we must emphasise that no actual decay takes place in our case because of the unrealistic quark masses. Then first order perturbation theory (Fermi's Golden Rule) implies a transition rate $\Gamma = 2\pi\langle x^2\rangle\rho(E)$ where the angle brackets indicate that an average over spatial directions will be needed. For a decay from the centre of mass with relative momentum k, the density of states $\rho(E) = L^3kE/(8\pi^2)$.

We use the mixing we establish on the lattice to evaluate the mass shift ϵ of the two-particle state. This mass shift turns out to be too small to measure accurately by a direct determination for the two-pion state, but we are able determine this shift from a study of the ρ mass. From our estimate of the energy shift, we are able to use Lüscher's formalism to determine the ρ decay parameters, obtaining results in agreement with the method described above.

Here we present our preliminary study of this problem. We establish methods that give a good signal on a lattice and discuss the cross-checks that can be made. Our analysis is restricted to mesons made of quarks of mass close to the strange quark mass. We discuss the phenomenological implications. Overall we find that lattice study of hadronic transitions is feasible and we find good agreement between our determinations and the expectations from experiment. The vector meson transition to two pseudoscalar mesons is actually rather strong and our methods will apply even better to weaker transitions, such as hybrid meson decay [6].

2 Lattice results

We use the UKQCD data set with $N_f = 2$ flavours of sea quarks with a NP clover fermionic action and Wilson glue at $\beta = 5.2$ and $C_{SW} = 2.0171$, $\kappa_{\rm sea} = \kappa_{\rm valence} = 0.1355$, volume $16^3 \times 32$ and configurations separated by 40 trajectories [9]. This corresponds [9] to a quark mass for which $m_\pi/m_\rho = 0.578^{+13}_{-19}$ which is approximately the strange quark mass, with a lattice spacing $a = 0.110(4)$ fm.

We define ρ and $\pi\pi$ states as being in a given T_1^{--} representation of the cubic group in the centre of mass. We consider the ρ meson state as having polarisation in a given direction (eg using $\bar{q}\gamma_z q$ to create it). We shall consider $\rho^+ \to \pi^+\pi^0$ so that the pions are distinguishable and this has contributions from two triangle quark diagrams (see fig. 2(b)) with $u\bar{u}$ and $d\bar{d}$ creation respectively: each with factors of $1/\sqrt{2}$ from the π^0. We normalise the P-wave $\pi\pi$ state with relative momentum k in the centre of mass as $[\pi^+(k)\pi^0(-k) - \pi^+(\bar{k})\pi^0(-\bar{k})]/\sqrt{2}$ where $\bar{k}$ has the z-component of k reversed. In the cases we will consider, k only has a z-component, so $\bar{k} = -k$. In order to have a smaller relative momentum, we also consider transitions from a ρ with nonzero momentum (where we take the polarisation along the momentum direction). In that case we define x as $\langle \rho_1 | \pi_1\pi_0 - \pi_0\pi_1 \rangle / \sqrt{2}$, also the z-axis is now privileged and we study the A_2^- representation of the symmetry group D_{4h}.

For the three and four-point correlators, we use a stochastic method to evaluate them from every space-time point on each lattice. Because of this volume averaging, we are able to get results from only 10 lattice configurations. The stochastic sources are Gaussian on one timeslice only [12] and extended propagator techniques are used to evaluate 3 and 4-point correlators. This study of the three and four-point correlators involved 2880 inversions which is comparable to the number of inversions needed to evaluate propagators from one space-time point on 208 configurations (as used for the two-point correlators [9]).

We find that the $\pi\pi \to \pi\pi$ P-wave amplitude is dominated by the product of two two-point pion correlators (see fig. 2c) which is accurately known, whereas the correlations between the pions and the box contributions are more noisy. Thus we choose to normalise by the two-point pion correlators. Moreover, in order to suppress contributions from excited states we normalise by the ground state contributions to the (ρ and π) two-point correlators as determined by 2-state fits to the UKQCD correlator data [9] (with local and fuzzed sources and sinks) for a t-range of 4-12.

We present our results for three-point correlators from eq. 3 in fig. 3. Note that the relevant ratio is quite large, becoming comparable to unity at larger

t. The requirements of our method should be satisfied for $3 \ll t \ll 12$ for $\rho_1 \to \pi_1\pi_0$, since we require $xt \ll 1$, $\Delta t \ll 5$ where $a\Delta = 0.14$ and $(E'-E)t \gg 1$ where $E' - E \approx 0.3$. The results in this case have quite small errors and do show well the required linear dependence on t in this region. The curve is from the two-state transfer matrix model with $ax = 0.06$. The departure at small t is presumably due to excited state contributions to the three-point correlator. These cannot produce a linear dependence on t, however, although they can contribute a constant even at large t. Since such constant contributions from excited states are allowed in principle, we show in fig. 4 the slope of the correlator ratio of eq. 3. This is consistent with constant at $ax = 0.06$ in this region, for both local and non-local ρ sources.

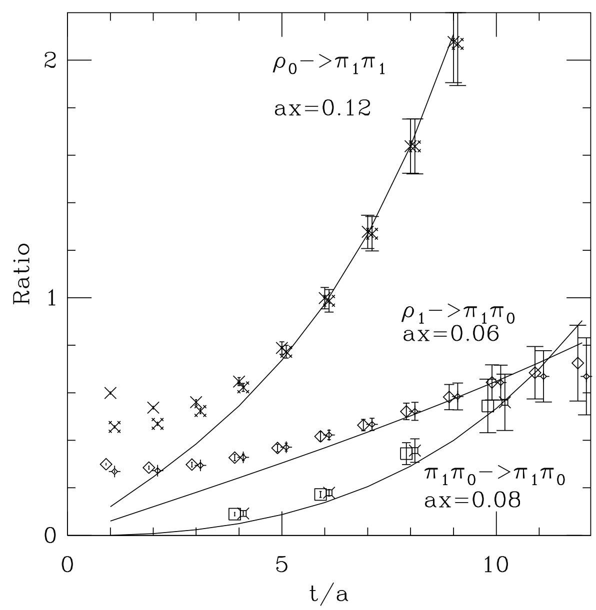

Figure 3: Normalised three and four particle correlators versus t/a. For the three particle ($\rho \to \pi\pi$) case, the crosses(fancy crosses) are for relative momentum $2\pi/L$ with local(fuzzed) operators while the diamonds(small diamonds) are for π/L with local(fuzzed) ρ operators. The curves are with transition amplitudes $ax = 0.12$ (upper) and $ax = 0.06$ (lower) as described in the text. For the P-wave annihilation component of the four particle pion correlator (the 'box diagrams') with pion relative momentum π/L, the results are shown by squares(fancy squares) for local pion operators (one pion fuzzed). The model curve is shown here for $ax = 0.08$.

As emphasised before [5], a cross check is available from the box diagram (eq. 4). Note that we need to evaluate this for both the momentum direct

(10 → 10) and crossed (10 → 01). These contributions are computationally difficult to measure and we have chosen t values of 4, 6, 8 and 10 here, as illustrated in fig. 3, again normalising by the ground state two-point correlators, where a comparison is made with the two-state transfer matrix model with $ax = 0.08$. Since excited state contributions can produce both constant and linear terms for this quantity, we form the linear combination of three adjacent t-values that eliminates them and so determine x by comparing with the two-state transfer matrix model. The resulting values are shown in fig. 4 where they are seen to be compatible with those from the three particle analysis.

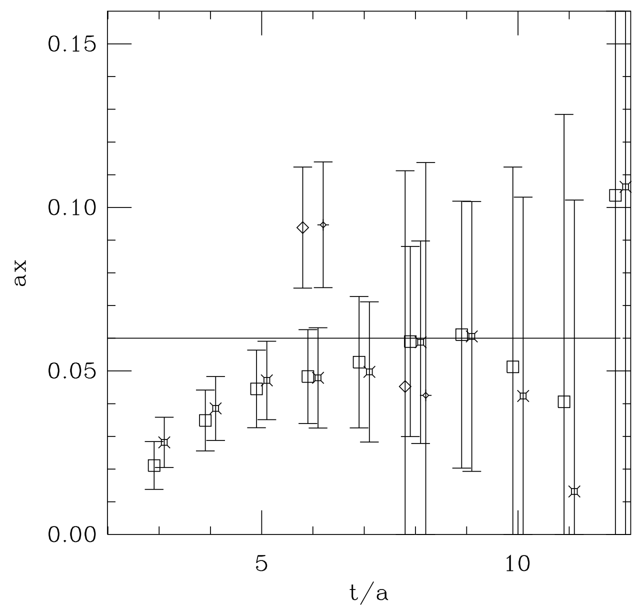

Figure 4: The matrix element ax versus t/a for the transition $\rho_1 \rightarrow \pi_1\pi_0$. These results come from finite differences to remove excited state contaminations in the data of fig. 3. The squares and fancy-squares are from the three-point analysis (with local and fuzzy ρ operator respectively) with a finite difference taken over two t intervals. The diamond (small diamond) is from the Box diagram with all local π operators (one fuzzed) evaluated by subtracting linear and constant terms using $t/a = 4 - 8$, and $6 - 10$.

Thus we conclude that we do have a consistent lattice determination of $ax = 0.06^{+2}_{-1}$ for this momentum combination. From this determination of the transition amplitude, we can determine an energy shift, as given by eq. 2, assuming that only the two nearest levels mix. This yields a shift of $a\epsilon = 0.022^{+17}_{-7}$ - up for the $\pi_1\pi_0$ state and down for the ρ_1 state.

For the ρ meson, a general investigation of the energy shift due to two pion

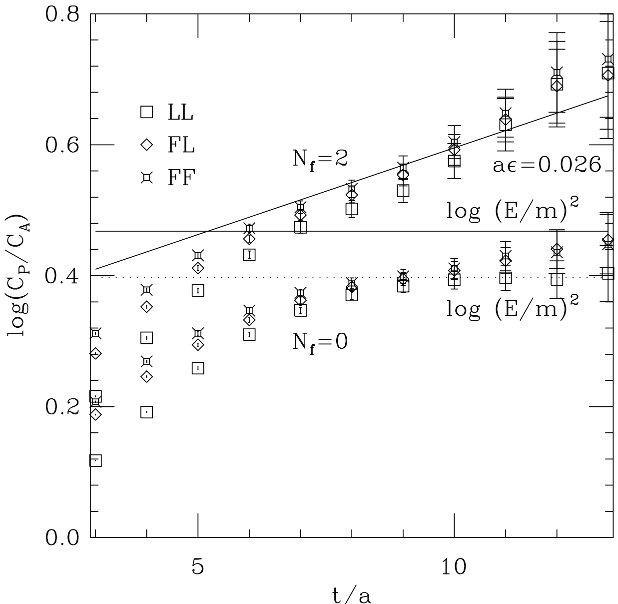

Figure 5: The ratio of two-point ρ correlators with momentum $2\pi/L$ and polarisation parallel (P) or perpendicular (A) to the momentum. These are for sources/sinks as shown from UKQCD [9] configurations with $N_f = 2$ (upper) and quenched (lower with similar quark masses at $\beta = 5.93$ and $\kappa = 0.1339$). If there were no mixing between these ρ states and the nearby $\pi_0\pi_1$ state, the ratio should be $(E/m)^2$ (shown as a line) for the ground state and nearer to 1.0 for heavier excited states. Thus we would expect the ratio to reach a plateau at $t > 6$ where the ground state dominates. Instead for the dynamical quark data, we see a significantly higher ratio and the line shows the slope obtained from the mass splitting ϵ we find in fitting. For quenched lattices the ratio just rises to $(E/m)^2$ (shown dotted) at larger t but does not cross it.

intermediate states is complicated by the need to regulate the sum over pion momenta. Here we are interested primarily in shifts arising from significant mixing with nearly degenerate energy levels in a finite volume. One signature [14] of such a shift is that the cubic invariance will be broken: the energy of a ρ meson with momentum $2\pi/L$ will be different when it is polarised in the momentum direction or perpendicular to it. We have explored this energy shift directly from the study of 2-point ρ correlators with momentum $2\pi/L$. When the polarisation is along the momentum, there will be mixing with the nearby $\pi_1\pi_0$ state for dynamical simulations (but not for the quenched case), while when the polarisation is perpendicular there will be no such mixing. From fits to the correlations (using 207 configurations with local and fuzzed sources at

4 time-points and requiring the excited state mass to be the same for the two fits to the t-range 4-12), we see a significant mass shift: namely the parallel ρ is $a\epsilon = 0.026(7)$ lighter than the perpendicular. We illustrate this in fig. 5. This value is in excellent agreement with the mass shift we deduced above by mixing arguments. We measured this mass shift for other sea quark masses, but have a significant signal only for the case ($\kappa = 0.1355$) studied here. Note that this directly observed mass-shift for the ρ state should not be present in a quenched study and it is not, see fig. 5, so we have here one of the few observables that directly come from the sea-quark contributions.

An even more useful result would be a direct determination of the energy shift for the $\pi\pi$ state. The optimum way to determine the energies of the two close levels would be by a joint fit (or variational analysis) of the matrix of ρ and $\pi\pi$ correlations. For this we need the 4-point $\pi\pi$ correlator, for which the ratio of contributions to the product of 2-point pion correlators at $t = 8$ is 1.07(4) for the direct term, -0.04(4) for the momentum-swapped direct term and 0.35(6) for the box terms. The relative error on the total 4-point correlator at $t = 8$ is 6%. From fitting the matrix of correlators for t from 8 to 12 with two states, we form the combination orthogonal to the ground state, so selecting the first excited (ie mainly $\pi\pi$) state. From this we evaluate the 'un-binding energy', the $\pi\pi$ energy difference from the sum of pion energies, finding $a\epsilon = 0.02(2)$. This value is in excellent agreement with our values obtained from the mixing analysis and from the ρ mass shift. The error, however, is still extremely large and a major increase in computational resource would be needed to determine this energy shift directly with sufficient precision.

For the case when $\rho_0 \to \pi_1\pi_1$, the energies are further apart, the next excited state is closer (see fig. 1) and the expected value of x is twice as big since it increases like the relative momentum k for a P-wave decay. We estimate that $4 \ll t \ll 8$ is required in this case. As shown in fig. 3, the values of the three-point correlator obtained for this case are approximately twice as large, but there is no longer a region showing linear behaviour. We show the result from the two-state transfer matrix model with the expected larger energy gap ($a\Delta = 0.47$ in this case) and normalised by the unmixed π two-point correlators (as we have used in the figures) and this gives the curve shown in the figure for $ax = 0.12$. This value is close to twice the value obtained with half of the relative momentum above, as expected. In this latter case, however, we do not have substantial cross-checks and this is qualitative rather than quantitative.

The method we have used to determine x depends on eliminating excited state contributions that appear at subleading powers of t. It is impossible to exclude that a combination of such terms do modify our results significantly. However, the cross checks that we have available are strong: we see consistent values for x from local or fuzzed sources from 3-point correlators, from the box contribution to the 4-point correlator and from the 3-point correlator with larger momentum release. We also see consistent evidence for the energy shift of the ρ meson with different polarisation directions.

3 Phenomenology

We have evaluated the transition $\rho \to \pi\pi$ at an unrealistic quark mass (approximately the strange quark mass), in a finite volume (of size $16a \approx 1.76$fm) and with $N_f = 2$ flavours of sea quark. Note that the lattice technique enables the transition to be measured off the energy-shell.

For the $\rho_1 \to \pi_1\pi_0$ case, the relative momentum in the centre of mass (with boost given by $\gamma = 1.156$ to the lattice frame) corresponds to $ka = \pi/L\gamma$, hence 305 MeV/c which is close to the experimental momentum of 358 MeV/c for $\rho \to \pi\pi$.

We have normalised the P-wave $\pi\pi$ state so that relative momentum k and $\bar{k}$ are identified which reduces the density of states by 0.5. Since we have evaluated x for momentum aligned with the polarisation, there will be an angular average contributing a factor of 1/3. Hence we expect

$$\Gamma = x^2 L^3 kE/(24\pi) \tag{5}$$

Our lattice determination is most precise for the transition $\rho_1 \to \pi_1\pi_0$ which is not in the centre of mass. We proceed first by evaluating the decay width in the lattice frame. We use the expression of eq. 5 with $k = \pi/L$, the relative momentum between the pions in the lattice frame. There are additional factors of γ from the phase space and from transforming the width to the centre of mass frame. Then, defining a dimensionless coupling constant directly from the reduced width as $\bar{g}^2 = \Gamma ME/k^3$ (corresponding to $g^2/(6\pi)$ where g is the conventional definition of the coupling constant), we find a value of $\bar{g} = 1.40^{+47}_{-23}$ from $ax = 0.06^{+2}_{-1}$, where we have used our best estimates of the centre of mass values E and k as discussed below.

One can relate the lattice frame to centre of mass frame by considering the boost of the two pions to the centre of mass (with $K = 2\pi/L$ and $\gamma = (1-K^2/E^2)^{-1/2} = 1.156$). Then in the centre of mass, the cubic spatial volume is modified by $L \to \gamma L$ in the direction of K (taken as the z-axis here). Moreover in this extended cube, pion momenta $k = 2\pi n/\gamma L$ are allowed with n_z half-integral, coming from anti-periodic spatial boundary conditions in the relative pion z-coordinate. Although this boost (by γ) reduces the two pion system to the centre of mass, the ρ will not have zero momentum. This arises since energy is not conserved in the lattice frame, as we measure t-directed correlations, which implies that momentum is not conserved in the centre of mass. So our estimate above is uncertain by factors of $\gamma(\pi\pi)/\gamma(\rho) = 1.156/1.264$. As the energy gap between the ρ and its decay products gets smaller, this problem will decrease. The transition $\rho_0 \to \pi_1\pi_1$ is in the centre of mass, which avoids the above problem, and $ax = 0.12$ corresponds to $\bar{g} = 1.57$ which is consistent, although systematic errors in this case are large.

A more rigorous approach is to focus on the two pion energy as in the Lüscher formalism [2]. This has been generalised [13] to non-zero overall momentum

$\bar{g}$	q(sea)	q(val)	method
1.40^{+47}_{-23}	s	s	eqs.3,5
1.56^{+21}_{-13}	s	s	$\epsilon(\rho)$, eq.6
1.39	n	n	$\rho \to \pi\pi$
1.44	n	n/s	$K^* \to K\pi$
1.52	n	s	$\phi \to \bar{K}K$
$1.46 - 1.74$	s	n	$\rho\bar{K}K$ Regge

Table 1: The vector-pseudoscalar-pseudoscalar coupling $\bar{g}$

by considering the boost to the centre of mass. Although we are unable to determine the energy shift ϵ directly for the two-pion state, if we assume that two levels only dominate the mixing, then we can use our results from the ρ meson energy shift and from the shift deduced through mixing from our value of x. Taking $a\epsilon = 0.26(7)$ yields an energy shift in the centre of mass frame of $\epsilon\gamma$ (since $E_L^2 = E_{\rm cm}^2 + K^2$). Then the phase shift for $\pi\pi$ scattering in the centre of mass is $\tan\delta = -L^2 E\gamma\epsilon/48$ to leading order in L^{-1} which yields a central value $\tan\delta = -0.109$ (including the known higher order corrections [13] in L^{-1} gives $\tan\delta = -0.131$). Now, for a nearby particle pole at $E = m_\rho$, one can describe the phase shift δ by using the expression for elastic $\pi\pi$ scattering dominated by this pole:

$$\tan\delta = \frac{\Gamma(k)}{2(m_\rho - E)} \tag{6}$$

where the phase shift is negative because the pole is below the two body energy. Here $\Gamma(k)$ is the decay width parametrised as $\bar{g}^2 k^3/(Em_\rho)$ and evaluated with decay momentum k and energy E. Using $am_\rho = 0.508$, $Ea = 0.709$ and $ka = 0.198$, this gives a coupling of $\bar{g} = 1.56^{+21}_{-13}$ which is similar to the value from our simple analysis above. Indeed in the limit of a weak transition and nearly degenerate energy levels, the two approaches would give exactly the same result.

Experimental data exist for the decays $\rho \to \pi\pi$, $K^* \to K\pi$ and $\phi \to K\bar{K}$ which all involve the creation of a light quark pair. This gives us some information on the dependence of the coupling strength on quark mass as shown in table 1 (here the coupling $\bar{g}$ is normalised to the quark diagrams present in ρ decay). In the limit of a heavy spectator quark (as for $B^* \to B\pi$ for example), $\bar{g}^2$ is expected to increase like m_Q which is the heavy quark mass. This implies that the coupling does indeed increase with spectator quark mass. We also quote in table 1 the evidence for the $\rho \to K\bar{K}$ coupling coming from Regge analyses [16] which bears on the spectator quark dependence. It will be interesting to use lattice studies to explore this further.

4 Conclusions

The lattice measurement passes all cross-checks: three different estimates of the transition amplitude x are presented, of which the three-point analysis of $\rho_1 \to \pi_1\pi_0$ is the most comprehensive, yielding a coupling $\bar{g} = 1.40^{+47}_{-23}$. The energy shift of the ρ state with momentum $2\pi/L$ for different polarisation directions is seen directly which is a powerful cross-check and yields a coupling $\bar{g} = 1.60^{+21}_{-13}$. These values are close to the experimental value (for light quark pair creation) of 1.5.

Our result is in a finite volume and it would be appropriate to test it by using a larger volume, consistent with having the same relative momentum: this implies doubling the lattice spatial dimension in at least one direction. Alternatively it is possible to explore $\rho_0 \to \pi_{1/2}\pi_{1/2}$ using antiperiodic spatial boundary conditions [15] which could give a cross check. These avenues require new dynamical quark simulations so are a major computational endeavour. A further step needed would be to extract the continuum limit of this lattice result. This needs finer lattice spacing and the same physical volume: so again substantial resources. We note that we have used a NP clover formulation which does remove order a corrections.

Even though our method can be affected by excited state contamination, the many cross checks we have made are convincing evidence that they are under control. The systematic errors are still very difficult to estimate since we have not made a continuum extrapolation or explored increasing the lattice volume. Moreover we have not extrapolated in sea quark mass, which is possibly the biggest source of systematic error. Our result is for quark pair production with quarks of mass approximately that of strange quarks.

Since we find a strong transition for $\rho \to \pi\pi$, in the sense that xt is big compared to unity, we have to rely on a mixing model to estimate most accurately the coupling strength. This would not be the case for a weaker decay. Even though the mixing is strong, our estimate of the $\pi\pi$ energy shift is that it is 0.02 in lattice units, which will be very difficult to measure accurately.

We have shown that hadronic transitions can be explored in lattice QCD and the result obtained is consistent with phenomenological values. This supports previous studies of unknown phenomena (eg. hybrid meson decay) and will allow studies of scalar meson decays which will help to untangle the confusing experimental situation.

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