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Gauge Singlet Renormalisation in Softly-Broken Supersymmetric Theories

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We consider the renormalisation of a softly-broken supersymmetric theory with singlet fields and a superpotential with a linear term. We show that there exist exact β-functions for both the linear term in the superpotential and the associated linear term in the Lagrangian. We also construct exact renormalisation group invariant trajectories for these terms, corresponding to the conformal anomaly solution for the soft masses and couplings.

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In supersymmetric gauge theories which contain gauge singlet fields (such as the MSSM with the addition of right-handed neutrinos) there is the possibility of a linear term in the superpotential W, so that we have (for a renormalisable theory)

$$W(\phi) = a^i\phi_i + \tfrac{1}{2}\mu^{ij}\phi_i\phi_j + \tfrac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k. \qquad (1)$$

In the component formalism the a^i term leads to a term in the scalar potential which is linear in the auxiliary field F. In some of our previous work on the renormalisation group functions of softly-broken supersymmetric theories[1]–[4], we have excluded singlets and hence such terms; so our purpose here is to extend the formalism to incorporate them.

The renormalisation issues raised by a linear F-term are similar to those associated with a Fayet-Iliopoulos (FI) linear D-term, which is possible when the gauge group contains an abelian factor. In previous papers[5]–[7] we computed the β-function for the coefficient ξ of this term, and showed that upon eliminating D using its equation of motion, β_ξ is associated with additional terms in the β-function for the soft masses. We also showed the existence of a solution to the renormalisation group (RG) equations for ξ, related to the exact anomaly mediated supersymmetry breaking (AMSB) solutions for the soft breaking parameters[8][9], but which in this case could only be constructed order by order in perturbation theory. In the present paper we shall perform the analogous analysis for the linear F-term. In this case the β-function for a^i is associated, after elimination (or, as we shall see, redefinition) of F, with additional terms in the β-functions for the quadratic and linear soft scalar couplings. Therefore the treatment of a linear F-term involves both generalising our previous analysis of the quadratic soft term and a discussion of the linear soft term.

The analysis will be simpler than the D-term case, since (by superspace power counting in the spurion formalism) a^i can only receive logarithmically divergent corrections, whereas the individual diagrams contributing to β_ξ are quadratically divergent, and so although this quadratic divergence cancels when the graphs are summed, the evaluation of an individual contribution to β_ξ in the spurion formalism is non-trivial. By contrast, in the linear F case the full power of the spurion formalism may be brought to bear, leading to exact results for the relevant β functions[1] and corresponding exact AMSB solutions. For pedagogical reasons, however, we will begin in the component formalism.

[1] This analysis was to an extent anticipated in Ref. [10].

The scalar potential of our component Lagrangian is

$$V = V_{\rm susy} + V_{\rm soft}, \tag{2}$$

where

$$V_{\rm susy} = -F^i F_i - F^i \frac{\partial W^*}{\partial \phi^i} - F_i \frac{\partial W}{\partial \phi_i} - \tfrac{1}{2}(D^a)^2 - g D^a \phi^* R^a \phi, \tag{3}$$

and

$$V_{\rm soft} = \left(c^i \phi_i + \tfrac{1}{2} b^{ij} \phi_i \phi_j + \tfrac{1}{6} h^{ijk} \phi_i \phi_j \phi_k - \kappa^i{}_j F^j \phi_i + \text{c.c.}\right) + (m^2)^i{}_j \phi_i \phi^j, \tag{4}$$

where as usual $\phi^i = (\phi_i)^*$, and the supermultiplet (ϕ, F, ψ) transforms according to the representation R^a. We have included the standard soft-breaking terms together with the additional terms involving c and κ required for multiplicative renormalisability. Notice that although we have a $F^*\phi$ term in Eq. (4), we do not add a $F\phi$ one because the latter would lead (in general) to quadratically divergent tadpoles; for the same reason there is no $\phi^*\phi^2$ term.

It is a simple matter to show that if we define

$$\overline{F}_i = F_i + \kappa^j{}_i \phi_j + a_i \tag{5}$$

then we obtain

$$\begin{aligned} V = &-\overline{F}^i \overline{F}_i - \overline{F}^i \frac{\partial \overline{W}^*}{\partial \phi^i} - \overline{F}_i \frac{\partial \overline{W}}{\partial \phi_i} - \tfrac{1}{2}(D^a)^2 - g D^a \phi^* R^a \phi \\ &+ \left(\overline{c}^i \phi_i + \tfrac{1}{2} \overline{b}^{ij} \phi_i \phi_j + \tfrac{1}{6} \overline{h}^{ijk} \phi_i \phi_j \phi_k + \text{c.c.}\right) + (\overline{m}^2)^i{}_j \phi_i \phi^j, \end{aligned} \tag{6}$$

where

$$\overline{W}(\phi) = \tfrac{1}{2} \mu^{ij} \phi_i \phi_j + \tfrac{1}{6} Y^{ijk} \phi_i \phi_j \phi_k, \tag{7}$$

and

$$\left(\overline{m}^2\right)^i{}_j = \left(m^2\right)^i{}_j + \left(\kappa \kappa^\dagger\right)^i{}_j, \tag{8a}$$

$$\overline{h}^{ijk} = h^{ijk} + Y^{l(jk} \kappa^{i)}{}_l, \tag{8b}$$

$$\overline{b}^{ij} = b^{ij} + Y^{ijl} a_l + \mu^{l(i} \kappa^{j)}{}_l, \tag{8c}$$

$$\overline{c}^i = c^i + \mu^{il} a_l + \kappa^i{}_l a^l, \tag{8d}$$

with

$$Y^{l(jk} \kappa^{i)}{}_l = Y^{ljk} \kappa^i{}_l + Y^{ilk} \kappa^j{}_l + Y^{ijl} \kappa^k{}_l$$

and

$$\mu^{l(i}\kappa^{j)}{}_l = \mu^{li}\kappa^j{}_l + \mu^{lj}\kappa^i{}_l.$$

The relations Eqs. $(8a-d)$ are renormalisation group invariant, because Eq. (2) has all the interactions necessary for multiplicative renormalisability. Although κ and a are necessary for this RG invariance, it is clear that they are not independent couplings, since they do not appear in the reduced potential, Eq. (6). We have only examined the scalar sector above, but note that we can simply replace W by $\overline{W}$ in the fermion sector, since this depends on the second derivatives of W with respect to ϕ. The equivalence of a theory with a superpotential like W to one with a superpotential like $\overline{W}$ might seem puzzling, since in a theory with no soft terms, linear terms are a *sine qua non* for F-type spontaneous supersymmetry breaking[11]. The resolution lies, of course, in the $Y^{ijl}a_l$ and $\mu^{il}a_l$ terms in Eq. $(8c, d)$ respectively, which are generated by the redefinition of F.

We now essentially present the above analysis again, but using the spurion formalism, which, allied with the non-renormalisation theorem, will enable us to derive a series of exact relations among the β-functions.

In the spurion context, the Lagrangian corresponding to Eqs. (3), (4) is given by

$$L = L_{\rm susy} + L_{\rm soft} + L_{\rm GF} + L_{\rm FP}, \tag{9}$$

where

$$L_{\rm susy} = \int d^4\theta \overline{\Phi}^j \left(e^{2gV}\right)^i{}_j \Phi_i + \left[\int d^2\theta \left(W(\Phi) + \tfrac{1}{4}W^\alpha W_\alpha\right) + \text{c.c.}\right], \tag{10}$$

and

$$\begin{aligned} L_{\rm soft} = &- \left[\int d^2\theta\theta^2 \left(c^i\Phi_i + \tfrac{1}{2}b^{ij}\Phi_i\Phi_j + \tfrac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k + \tfrac{1}{2}MW^\alpha W_\alpha\right) + \text{c.c.}\right] \\ &+ \int d^4\theta \left[-(m^2)^k{}_j\theta^2\overline{\theta}^2\overline{\Phi}^j \left(e^{2gV}\right)^i{}_k\Phi_i + \overline{\Phi}^j \left(\theta^2\kappa^k{}_j + \overline{\theta}^2\kappa^{\dagger k}{}_j\right)\left(e^{2gV}\right)^i{}_k\Phi_i\right], \end{aligned} \tag{11}$$

where V is the vector superfield, W^α the corresponding field strength and M is the gaugino mass. $L_{\rm GF}$ and $L_{\rm FP}$ are the gauge-fixing and ghost Lagrangians whose exact form will not be important to us. Now by making the redefinition

$$\Phi_i = \Phi'_i - \theta^2(\kappa^j{}_i\Phi'_j + a_i), \tag{12}$$

(which corresponds precisely to Eq. (5)) we find that the Lagrangian adopts the form

$$L' = L'_{\rm susy} + L'_{\rm soft} + L_{\rm GF} + L_{\rm FP}, \tag{13}$$

where

$$L'_{\rm susy} = \int d^4\theta \overline{\Phi}'^j \left(e^{2gV}\right)^i{}_j \Phi'_i + \left[\int d^2\theta \left(\overline{W}(\Phi') + \tfrac{1}{4}W^\alpha W_\alpha\right) + {\rm c.c.}\right], \tag{14}$$

and

$$\begin{aligned} L'_{\rm soft} = &- \left[\int d^2\theta\theta^2 \left(\overline{c}^i\Phi'_i + \tfrac{1}{2}\overline{b}^{ij}\Phi'_i\Phi'_j + \tfrac{1}{6}\overline{h}^{ijk}\Phi'_i\Phi'_j\Phi'_k + \tfrac{1}{2}MW^\alpha W_\alpha\right) + {\rm c.c.}\right] \\ &- \int d^4\theta (\overline{m}^2)^k{}_j \theta^2\overline{\theta}^2 \overline{\Phi}'^j \left(e^{2gV}\right)^i{}_k \Phi'_i, \end{aligned} \tag{15}$$

where $\overline{h}$, $\overline{b}$, $\overline{c}$ and $\overline{m}^2$ are exactly as defined in Eq. (8), and $\overline{W}$ as defined in Eq. (7). Once again, note that a and κ no longer appear explicitly. We shall refer to L in Eq. (9) as the unreduced Lagrangian, and L' in Eq. (13) as the reduced Lagrangian; it is the latter that one would use in practical applications.

We may now obtain a set of consistency conditions by requiring L and L' in Eqs. (9), (13) to be equivalent as functions of the renormalised couplings (i.e. equal for all renormalisation scales μ). We use $\overline{\beta}$, β to represent a β-function calculated in the reduced, unreduced formalisms respectively. Moreover, for each β function we separate out the part $\hat{\beta}$ corresponding to 1PI graphs. For example, we write

$$\beta_i^a(a, b, \cdots) = \gamma^m{}_i a_m + \hat{\beta}_i^a, \tag{16}$$

where $\hat{\beta}^a = \hat{\beta}^a(Y, Y^*, g, b, \mu, M, h^*)$ is determined by 1PI tadpole graphs. Writing

$$\mu\frac{d}{d\mu}\Phi'_i = -\gamma'^j{}_i\Phi'_j = -\gamma^j{}_i\Phi'_j - 2\theta^2\overline{\gamma}_{1i}^j\Phi'_j + \theta^2\overline{\sigma}_i, \tag{17}$$

it follows from RG invariance of Eq. (12) that

$$\overline{\gamma}_{1j}^i = -\tfrac{1}{2}\hat{\beta}_{\kappa j}^i - \kappa^i{}_k\gamma^k{}_j, \tag{18}$$

and

$$\overline{\sigma}_i = \hat{\beta}_i^a + 2\gamma^m{}_i a_m. \tag{19}$$

Eqs. (18) and (19) give $\overline{\gamma}_{1j}^i$ and $\overline{\sigma}_i$ in terms of the unreduced parameters; we shall shortly give prescriptions for calculating them directly in terms of reduced parameters.

From the non-renormalisation theorem we have

$$\beta_Y^{ijk} = Y^{l(jk}\gamma^{i)}{}_l, \tag{20}$$

with a similar expression for β_μ. However, as foreshadowed in Eq. (16), in the presence of soft terms the non-renormalisation theorem does not protect β_a from 1PI contributions, and $\hat{\beta}_a$ is non-zero. We do, however, have

$$\beta_h^{ijk} = h^{l(jk}\gamma^{i)}{}_l, \tag{21}$$

with a similar result for β_b; but again $\hat{\beta}_c$ is non-zero. To derive the soft β-functions in the reduced formalism, we impose

$$\mu\frac{d}{d\mu}\left(L_{\rm susy} + L_{\rm soft}\right) = \mu\frac{d}{d\mu}\left(L'_{\rm susy} + L'_{\rm soft}\right). \tag{22}$$

Then using the results for the unreduced β-functions such as Eqs. (20), (21), inserting Eq. (17) and using Eqs. (18) and (19), we obtain

$$\overline{\beta}_{\overline{h}}^{ijk} = \overline{h}^{l(jk}\gamma^{i)}{}_l - 2Y^{l(jk}\overline{\gamma}_1{}^{i)}{}_l, \tag{23a}$$

$$\overline{\beta}_{\overline{b}}^{ij} = \overline{b}^{l(i}\gamma^{j)}{}_l - 2\mu^{l(i}\overline{\gamma}_1{}^{j)}{}_l + Y^{ijl}\overline{\sigma}_l, \tag{23b}$$

together with consistency conditions relating reduced and unreduced quantities (analogous to Eqs. (18) and (19))

$$\hat{\overline{\beta}}_{\overline{c}}^i = \hat{\beta}_c^i - 2a^l\left(\overline{\gamma}_1\right)^i{}_l + \hat{\beta}_a^l\kappa^i{}_l + \mu^{il}\overline{\sigma}_l, \tag{24}$$

and

$$\left(\hat{\overline{\beta}}_{\overline{m}^2}\right)^i{}_j = \left(\hat{\beta}_{m^2}\right)^i{}_j - 2\left(\kappa\overline{\gamma}_1^\dagger\right)^i{}_j - 2\left(\overline{\gamma}_1\kappa^\dagger\right)^i{}_j - 2\left(\kappa\gamma\kappa^\dagger\right)^i{}_j. \tag{25}$$

Eqs. (23)–(25) may also be obtained (perhaps more simply) by operating with $\mu\frac{d}{d\mu}$ on the RG-invariant relations Eqs. (8). Eqs. (23) were given in Refs. [1][12], but excluding singlet fields. Results in the presence of singlet fields were given up to the two-loop level in [10][13][14].

We now consider the explicit forms of $\overline{\gamma}_1$, $\overline{\beta}_{\overline{m}^2}$, $\overline{\sigma}$ and $\hat{\overline{\beta}}_{\overline{c}}$. From Eq. (17), we see that $\overline{\gamma}_1$ and $\overline{\sigma}$ are obtained from θ^2-dependent contributions to the two-point function and the one-point function respectively, calculated from L'. We also see from Eq. (15), on rewriting

$$\int d^2\theta\theta^2 c^i\Phi_i = \int d^4\theta\theta^2\overline{\theta}^2 c^i\Phi_i, \tag{26}$$

that $\overline{\beta}_{\overline{m}^2}$ and $\overline{\beta}_{\overline{c}}$ are obtained from the $\theta^2\overline{\theta}^2$-dependent contributions to the two-point function and the one-point function respectively, again calculated from L'. We are led to the following prescription: consider superspace diagrams contributing to the two-point function in the supersymmetric theory, with superpotential $\overline{W}$. These diagrams are only logarithmically divergent, and so for the purposes of the D-algebra the $\theta^2, \overline{\theta}^2$ associated with the spurion may be taken as constants. In each such diagram we can simply replace Y^{ijk} by $Y^{ijk} - \overline{h}^{ijk}\theta^2$, μ^{ij} by $\mu^{ij} - \overline{b}^{ij}\theta^2$, and gauge couplings g^2 by by $g^2(1 + M\theta^2 + M^*\overline{\theta}^2 + MM^*\theta^2\overline{\theta}^2)$[10]. We also replace each factor $\delta^k{}_l$ in an internal chiral propagator by $\delta^k{}_l + (\overline{m}^2)^k{}_l\theta^2\overline{\theta}^2$. This procedure may be implemented using differential operators; for instance, we obtain for $\overline{\gamma}_1$ and $\overline{\beta}_{\overline{m}^2}$

$$(\overline{\gamma}_1)^i{}_j = \mathcal{O}\gamma^i{}_j, \tag{27a}$$

$$\left(\overline{\beta}_{\overline{m}^2}\right)^i{}_j = \Delta\gamma^i{}_j, \tag{27b}$$

where

$$\mathcal{O} = Mg^2\frac{\partial}{\partial g^2} - \overline{h}^{lmn}\frac{\partial}{\partial Y^{lmn}} - \overline{b}^{lm}\frac{\partial}{\partial \mu^{lm}}, \tag{28a}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2MM^*g^2\frac{\partial}{\partial g^2} + \left[\tilde{\overline{Y}}^{lmn}\frac{\partial}{\partial Y^{lmn}} + \tilde{\overline{\mu}}^{lm}\frac{\partial}{\partial \mu^{lm}} + \text{c.c.}\right] + X\frac{\partial}{\partial g}, \tag{28b}$$

with

$$\tilde{\overline{Y}}^{ijk} = (\overline{m}^2)^{(i}{}_l Y^{jk)l} \quad \text{and} \quad \tilde{\overline{\mu}}^{ij} = (\overline{m}^2)^{(i}{}_l \mu^{j)l}. \tag{29}$$

Eq. (27a) was given in Refs. [1],[12]; note however the inclusion of the derivatives with respect to μ, which give zero acting on γ but will be important presently. The full understanding of Eq. (27b), in particular the necessity for, and form of, the term involving X in Eq. (28b), was developed in Refs. [1], [3] and also Refs. [12], [15] (see also Ref. [16]). In particular, it was shown in Ref. [3] that a form for X derived for a particular RG trajectory in Ref. [17] was in fact valid in general. (Note that the X term, hitherto written separately, has now been included in the definition of Δ.)

For $\overline{\sigma}$ and $\hat{\overline{\beta}}_{\overline{c}}$ we should consider superspace tadpole diagrams. By chirality, such diagrams can be obtained by taking a graph contributing to the two-point function with an external leg attached to a Y or Y^*, and replacing this Y or Y^* by a μ or μ^* respectively. After making the substitutions described above, $\overline{\sigma}$ and $\hat{\overline{\beta}}_{\overline{c}}$ are derived from the θ^2 terms and $\theta^2\overline{\theta}^2$ terms respectively in these tadpole diagrams. This process may be accomplished using the operators $\mathcal{O}$ and Δ defined above. We obtain

$$\overline{\sigma}_i = -2\mathcal{O}\left(Z_i\right), \tag{30}$$

where

$$Z_i = Y_{imn} K^{mn}{}_{pq} \mu^{pq}, \tag{31}$$

with $K^{mn}{}_{pq}$ defined by the condition

$$Y_{imn} K^{mn}{}_{pq} Y^{pqj} a_j = \gamma^j{}_i a_j. \tag{32}$$

We also have

$$\hat{\beta}^i_{\overline{c}} = \Delta Z^i + \mu^{il}\overline{\sigma}_l - (\overline{m}^2)^i{}_k Z^k. \tag{33}$$

These results hold in the reduced case; however, the calculations of $\hat{\beta}^a$ and $\hat{\beta}_c$ in the unreduced case are closely related, corresponding to θ^2 terms and $\theta^2\overline{\theta}^2$ terms respectively in tadpole diagrams derived from L rather than L'. The difference is that there are now contributions from insertions of κ on internal propagators, which correspond to substituting for $\overline{h}$, $\overline{b}$, $\overline{c}$, $\overline{m}^2$ in terms of h, b, c, m^2 in accordance with Eq. (8). However, making these substitutions in Eqs. (30), (33) overcounts by including contributions (from a, and from insertions of κ on external legs) which would correspond to one-particle reducible diagrams. This leads precisely to the consistency conditions Eqs. (19), (24). Similar reasoning applies when considering the relation between $\overline{\gamma}_1$ and $\hat{\beta}_\kappa$. In this case the substitution of $\overline{h}, \ldots$ in terms of $h, \ldots$ does not yield all the κ contributions to $\hat{\beta}^i_{\kappa j}$, which also contains a contribution $\kappa^i{}_k[Y^{kmn}K_{mn}{}^{pq}Y_{pqj} - \gamma^k{}_j]$, with K as in Eq. (32). However, the substitution overcounts by including $-\kappa^i{}_k Y^{kmn} K_{mn}{}^{pq} Y_{pqj}$ which would correspond to one particle reducible diagrams and must be removed. Combining these contributions leads to Eq. (18).

The results Eqs. (30) and (33) mean that our knowledge of $\overline{\sigma}$ and $\hat{\beta}_{\overline{c}}$ is limited only by our knowledge of γ. Thus all the β-functions that depend on soft-breaking parameters are determined by the underlying supersymmetric theory, except for the one associated with a FI-term. We have verified Eq. (30) by an explicit calculation of F-tadpole diagrams through three loops, using the Feynman gauge component formalism and supersymmetric dimensional regularisation.

We may now obtain exact solutions of the RG equation for a_i and c^i (in the unreduced case), or equivalently exact solutions to the RG equations for $\overline{b}^{ij}$ and $\overline{c}^i$ (in the reduced case). It is already well-known[8][9] that the following set of equations provide an exact solution (the AMSB solution) to the renormalisation group equations for $M, \overline{h}, \overline{b}$ and $\overline{m}^2$ in the case where there are no singlet fields and the gauge group contains no abelian factors:

$$M = M_0 \frac{\beta_g}{g}, \tag{34a}$$
$$\overline{h}^{ijk} = -M_0 \beta_Y^{ijk}, \tag{34b}$$
$$\overline{b}^{ij} = -M_0 \beta_\mu^{ij}, \tag{34c}$$
$$(\overline{m}^2)^i{}_j = \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu}. \tag{34d}$$

In fact these solutions are realised if the only source of supersymmetry breaking is the conformal anomaly, when M_0 is the gravitino mass[8]. However, Eq. (34d) acquires extra terms[5] if the gauge group contains abelian factors via non-zero FI terms, and (as we shall show) Eq. (34c) acquires extra terms if there are singlet fields in the theory.

In the unreduced case, the solutions corresponding to Eq. (34) are

$$M = M_0 \frac{\beta_g}{g}, \tag{35a}$$
$$h^{ijk} = 0, \tag{35b}$$
$$b^{ij} = 0, \tag{35c}$$
$$(m^2)^i{}_j = |M_0|^2 \left[\tfrac{1}{2}\mu \frac{d\gamma^i{}_j}{d\mu} - (\gamma^2)^i{}_j \right], \tag{35d}$$
$$\kappa^i{}_j = -M_0 \gamma^i{}_j. \tag{35e}$$

RG invariance of Eqs. (35b, c) follows trivially from Eq. (21) and the corresponding equation for β_b; RG invariance of Eq. (35e) follows from Eq. (18), using the fact that that on the AMSB trajectory,

$$\overline{\gamma}^i_{1j} = \tfrac{1}{2} M_0 \mu \frac{d}{d\mu} \gamma^i{}_j, \tag{36}$$

a relation established in Ref. [9]. Finally, the RG-invariance of Eq. (35d) then follows from that of Eq. (34d) using Eqs. (8a), (35e).

We now claim that solutions to the RG equations for a_i, c^i corresponding to Eqs. (35) are

$$a_i = -M_0 Z_i, \tag{37a}$$
$$c^i = \tfrac{1}{2}|M_0|^2 \left[\mu \frac{d}{d\mu} Z^i - (\gamma Z)^i \right], \tag{37b}$$

with Z as defined in Eq. (31). It is straightforward to show that this works; let us begin with Eq. (37a). Eq. (30) now becomes simply

$$\overline{\sigma}_i = \frac{2}{M_0}\mathcal{O}a_i, \tag{38}$$

where, on applying Eqs. (8) and (35) in Eq. (28a), we find

$$\mathcal{O} = \tfrac{1}{2}M_0\left(\beta_g\frac{\partial}{\partial g} + 2\mathcal{Q}\right) - Y^{klm}a_m\frac{\partial}{\partial\mu^{kl}}, \tag{39}$$

with

$$\mathcal{Q} = \sum_{klm}\beta_Y^{klm}\frac{\partial}{\partial Y^{klm}} + \sum_{kl}\beta_\mu^{kl}\frac{\partial}{\partial\mu^{kl}}. \tag{40}$$

On the other hand,

$$\mu\frac{d}{d\mu} = \beta_g\frac{\partial}{\partial g} + \mathcal{R}, \tag{41}$$

where

$$\mathcal{R} = \mathcal{Q} + \mathcal{Q}^*. \tag{42}$$

Now in Ref. [9] it was shown that for a tensor ${X^i}_j$ we have

$$(\mathcal{Q}X)^i{}_j - (\mathcal{Q}^*X)^i{}_j = \gamma^i{}_k X^k{}_j - X^i{}_k\gamma^k{}_j, \tag{43}$$

and in particular that

$$\mathcal{Q}\gamma = \mathcal{Q}^*\gamma. \tag{44}$$

Eq. (44), in fact, is the result one needs to establish Eq. (36). The generalisation of Eq. (43) to a tensor with an arbitrary number of indices is obvious; but for our purposes all we need is the result

$$(\mathcal{Q}Z)_i - (\mathcal{Q}^*Z)_i = -\gamma^j{}_i Z_j. \tag{45}$$

Armed with this equation and

$$Y^{klm}a_m\frac{\partial}{\partial\mu^{kl}}a_i = -M_0\gamma^k{}_i a_k, \tag{46}$$

(which follows easily from Eqs. (37a), (31), (32)), we can show (using Eq. (19), (38)–(46)) that

$$\hat{\beta}_i^a = \mu\frac{d}{d\mu}a_i - \gamma^m{}_i a_m, \tag{47}$$

reproducing Eq. (16), and thereby proving the RG invariance of Eq. (37a). We now turn to Eq. (37b). To prove RG invariance of this solution, we require two identities, generalising similar results proved in Ref. [9]. The first (which follows by repeated application of Eq. (45)) is

$$\begin{aligned}\mathcal{R}^2 Z^i =&\Big(4\beta_Y^{klm}\beta^Y_{pqr}\frac{\partial^2}{\partial Y^{klm}\partial Y_{pqr}}+4\beta_Y^{klm}\beta^\mu_{pq}\frac{\partial^2}{\partial Y^{klm}\partial\mu_{pq}}\\ &+4\beta_\mu^{kl}\beta^Y_{pqr}\frac{\partial^2}{\partial\mu^{kl}\partial Y_{pqr}}+4\beta_\mu^{kl}\beta^\mu_{pq}\frac{\partial^2}{\partial\mu^{kl}\partial\mu_{pq}}\\ &+(\mathcal{R}\gamma)^{(k}{}_nY^{lm)n}\frac{\partial}{\partial Y^{klm}}+(\mathcal{R}\gamma)^n{}_{(k}Y_{lm)n}\frac{\partial}{\partial Y_{klm}}\\ &+(\mathcal{R}\gamma)^{(k}{}_n\mu^{l)n}\frac{\partial}{\partial\mu^{kl}}+(\mathcal{R}\gamma)^n{}_{(k}\mu_{l)n}\frac{\partial}{\partial\mu_{kl}}\Big)Z^i+(\gamma^2Z)^i.\end{aligned} \tag{48}$$

The second identity is that if Eqs. (34) are imposed, then

$$|M_0|^2\mu\frac{d\beta_g}{d\mu}=3\frac{\beta_g^2|M_0|^2}{g}+2X. \tag{49}$$

(Note that this identity is true for a range of regularisation schemes which includes standard dimensional reduction[3].) Using these identities in conjunction with Eqs. (41), (28b), (39), (37a) and (46), it follows that when Eqs. (34) are imposed, we have

$$\Delta Z^i=\tfrac{1}{2}|M_0|^2\left[\left(\mu\frac{d}{d\mu}\right)^2Z^i-(\gamma^2Z)^i+2\mu\frac{d\gamma^i{}_k}{d\mu}Z^k\right]. \tag{50}$$

Using Eqs. (24), (47), (37a), (50), (34d), (35d), (33), we find

$$\beta_c^i=\tfrac{1}{2}|M_0|^2\left[\left(\mu\frac{d}{d\mu}\right)^2Z^i-\mu\frac{d}{d\mu}(\gamma Z)^i\right], \tag{51}$$

which shows that Eq. (37b) is RG-invariant.

With the aid of Eqs. (35) and (37) it is now straightforward to write down the AMSB results in the reduced case, by substituting in Eq. (8). One can also check that the resulting expressions are indeed RG invariant. For convenience we first assemble the complete results for the soft β-functions in the reduced formalism:

$$\begin{aligned}\overline{\beta}_M&=2\mathcal{O}\left[\frac{\beta_g}{g}\right],\\ \overline{\beta}_{\overline{h}}^{ijk}&=\overline{h}^{l(jk}\gamma^{i)}{}_l-2Y^{l(jk}\overline{\gamma}_1{}^{i)}{}_l,\\ \overline{\beta}_{\overline{b}}^{ij}&=\overline{b}^{l(i}\gamma^{j)}{}_l-2\mu^{l(i}\overline{\gamma}_1{}^{j)}{}_l+Y^{ijl}\overline{\sigma}_l,\\ \overline{\beta}_{\overline{c}}^i&=\overline{c}^j\gamma^i{}_j+\Delta Z^i+\mu^{il}\overline{\sigma}_l-(\overline{m}^2)^i{}_kZ^k,\\ \left(\overline{\beta}_{\overline{m}^2}\right)^i{}_j&=\Delta\gamma^i{}_j,\end{aligned} \tag{52}$$

where $\overline{\sigma}$, Z^i are defined in Eqs. (30), (31), and then we give the full AMSB solutions:

$$
\begin{aligned}
M &= M_0 \frac{\beta_g}{g}, \\
\overline{h}^{ijk} &= -M_0 \beta_Y^{ijk}, \\
\overline{b}^{ij} &= -M_0 \beta_\mu^{ij} - M_0 Y^{ijk} Z_k, \\
\overline{c}^i &= \tfrac{1}{2}|M_0|^2 \left[\mu \frac{d}{d\mu} Z^i + (\gamma Z)^i\right] - M_0 \mu^{il} Z_l, \\
(\overline{m}^2)^i{}_j &= \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu}.
\end{aligned}
\tag{53}
$$

For a U_1 theory with a FI term, the AMSB solution for $\overline{m}^2$ in the D-eliminated case becomes[5]

$$
(\overline{m}^2)^i{}_j = \tfrac{1}{2}|M_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu} + g\xi^{\mathrm{RG}}(\mathcal{Y})^i{}_j, \tag{54}
$$

where ξ^{RG} is the RG solution for ξ, and $\mathcal{Y}$ is the hypercharge matrix for the U_1 factor, with gauge coupling g. The proof relies on the consistency condition Eq. (2.25) of Ref. [5], which plays a similar rôle to that of Eq. (19).

In conclusion: we have extended our previous exact results for the soft β-functions and the AMSB solution to allow for the presence of gauge singlet matter fields. In the usual formulation of the NMSSM (see for example [18]) it is easy to see that we would have, in fact, $\overline{\sigma} = Z = 0$; for a non-zero $\overline{c}^i$ we obviously need a chiral superfield which is a "universal" singlet (i.e. invariant under both gauge and global transformations). In the standard gravity-mediated supersymmetry-breaking scenario, one may expect on rather general grounds that $\overline{c}^i$ will suffer gravity-induced quadratic divergences[19] leading to contributions $\overline{c}^i \sim O(M_P M_{\mathrm{sparticle}}^2)$, and consequent destabilisation of the hierarchy. However there are frameworks where the gravitational tadpole has a magnitude that is phenomenologically acceptable (or even desirable) [20]. We hope, therefore, that our results may prove of use in the analysis of non-minimal versions of the MSSM.

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