

Do we understand the unquenched value of f_B ?

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Abstract. I review our qualitative understanding of the increase in the value of the B meson decay constant (f_B), when dynamical fermions are included in lattice QCD calculations.

1. Introduction

The over determination of the CKM matrix, using the new data from the B factories at SLAC and KEK, is a sensitive test of the standard model [1]. The determination of the CKM matrix elements from experiment depends critically on hadronic parameters, such as f_B , B_B , B_{B_s} and B_K , most of which are inaccessible to experiment, but can be calculated from lattice QCD.

In a recent survey of the latest lattice QCD results for the f_B (the decay constant of the B meson) Bernard [2] quotes: $f_B^{quenched} = 175(20)$ MeV from quenched QCD and $f_B = 200(30)$ MeV for the value of the B decay constant in full QCD. Although all lattice QCD calculations [2] have seen an increase in f_B between quenched and unquenched QCD, the effect in the world “average” is only at the one σ level. The increase in f_B between quenched and full QCD is more significant for an individual collaboration’s results, for example CP-PACS, obtain [3] $\frac{f_B^{n_f=2}}{f_B^{n_f=0}} = 1.11(6)$. As unquenched simulations are so computationally demanding, it seems useful to review the additional arguments that support the increase in decay constants due to the inclusion of dynamical fermions.

2. What are unquenching effects?

Lattice QCD is a “clever” finite difference approximation to continuum QCD [4]. Lattice QCD calculations involve computing the partition function

$$\mathcal{Z} = \int \prod_x dU(x) \exp(-S_G) (\det(M))^{n_f} \quad (1)$$

where U describe the gauge fields, S_G is the lattice representation of the gauge action ($\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$) and M is a lattice representation of the Dirac operator for quarks. The quark fields have been integrated out. The dynamics of the gluon fields depends on

the determinant of the Dirac operator. The determinant describes the dynamics of the sea quarks and is very expensive to compute. Until recently, most phenomenological lattice calculations did not include the determinant in the dynamics (quenched QCD). Surprisingly, quenched QCD calculations describe experiment reasonably well. The biggest recent study of quenched QCD by the CP-PACS collaboration [5] found that the spectrum of light hadrons disagreed with experiment at the 10% level.

The results from an individual calculation depend on the lattice spacing a and physical volume. These errors may be removed by repeating the calculation at different lattice spacings and volumes and then extrapolating the results to the continuum and infinite volume limit. For example, in the recent CP-PACS calculation [3] they obtain $f_B = 287(7)$ MeV, $234(8)$ MeV, and $208(10)$ MeV, at lattice spacings: 0.22 fm, 0.16 fm, and 0.11 fm respectively, from a calculation of $n_f = 2$ QCD. CP-PACS prefer to quote $f_B = 208(10)(11)$ MeV as their continuum result, however Bernard [2] prefers to extrapolate the CP-PACS data to the continuum assuming a quadratic dependence on the lattice spacing (there are good, but not totally rigorous arguments for this type of extrapolation) and obtains $f_B = 190(12)(26)$ MeV. This kind of ambiguity in the final analysis of lattice results is the cause of the large systematic errors in the final results of lattice calculations. The cost of lattice QCD calculations that include dynamical fermions goes (something) like [6] $\frac{1}{a^{6.5}}$ where a is the lattice spacing, so halving the lattice spacing is very computationally expensive.

3. How to understand unquenching

There is a simple model of the effect of unquenching that is based on the quark model [7, 8, 9]. Consider the Richardson heavy quark potential [10].

$$V(q) \sim \frac{4\pi}{(11 - \frac{2n_f}{3})} \frac{1}{q^2 \ln(1 + q^2/\Lambda^2)} \quad (2)$$

where n_f is the number of flavours. Equation 2 or the potential extracted from a lattice calculation is used in Schrödinger's equation to calculate the wave function of mesons. The decay constant is computed using the Van Royen-Weisskopf formulae

$$f \propto |\psi(0)| \quad (3)$$

In position space the potential in equation 2 at small radial separations is deeper in the full theory ($n_f=3$) relative to the quenched theory ($n_f=0$). So the decay constant (computed from equation 3) is higher in the full theory than in quenched QCD. There is evidence from many lattice QCD calculations of the heavy quark potential, that dynamical fermions produce a similar effect to the n_f dependence of the Richardson potential [11].

The MILC collaboration [9] have systematically studied this model in an unquenched simulation using staggered fermions. From the graphs in the paper [9], at $m_{PS}/m_V \sim 0.58$, the effect of unquenching is 3% for f_π and 7% for f_B . The MILC collaboration found that unquenching was smaller on the ratio of decay constants that

on individual decay constants in this model. A key qualitative prediction of this model is that the decay constant in unquenched QCD is greater than the quenched value. Also the unquenching effect should be larger for f_B than f_π .

Another way to understand the effect of unquenching is to use a quenched Lagrangian [12, 13]. The idea is similar to Chiral Lagrangian's, where a Lagrangian is written in terms of hadron fields with the same symmetries as QCD. A number of unknown parameters enter the Chiral Lagrangian, that must be fixed from experiment (or from lattice calculations). Once the parameters are fixed, the Lagrangian can be used to make predictions. For the case of quenched QCD, ghost fields are introduced to cancel the quark determinants. The resulting theory has a different symmetry to QCD, but the idea is basically the same as for continuum chiral perturbation theory.

The formalism for heavy-light mesons was developed by Booth [14] and Sharpe and Zhang [15]. The quenching errors are estimated by comparing the chiral logs (some of the loop effects) in quenched and full QCD. This is thought to be an upper bound on the magnitude of the quenching effects [15]. Unfortunately, the estimate of the quenching errors involves 7 parameters that are hard to determine accurately. The message from this analysis was that f_B in full QCD could be either greater than or less than the quenched value. Also, it was possible that the ratio f_{B_s}/f_B had large quenching errors (this has not been found in simulations [2]).

4. Lattice results for known decay constants

The results of the first systematic studies of decay constants in quenched QCD have been reviewed by Sharpe [6]. Sharpe concluded that $\frac{f_\pi}{m_\rho}$ was lower than experiment, after the continuum limit had been taken. The recent large CP-PACS collaboration [5] study of quenched QCD found that in quenched QCD $f_\pi^{quenched} (J_K^{quenched}) = 120.0 \pm 5.7$ (138.8 ± 4.4) MeV, that are smaller than experiment by 2σ (5σ).

After the continuum extrapolation, the light decay constants from quenched QCD are lower than experiment, consistent with the quark model in section 3. The SESAM collaboration [16] found at one lattice spacing that the pion decay constant was approximately one sigma larger with dynamical fermions, than from an equivalent quenched simulation. The opposite trend was seen in the QCDSF/UKQCD [17] data. It will be difficult to determine the effect of dynamical fermions on light decay constants, until a continuum extrapolation is done.

Including heavy quarks (Charm and Bottom) in lattice QCD calculations requires the introduction of new techniques, such as effective field theories for the heavy quarks [2]. As a test of the new heavy quark methods, the f_{D_s} decay constant is computed and compared against experiment. The value for f_{D_s} quoted in the particle data table [18] is $280 \pm 19 \pm 28 \pm 34$ MeV. In table 1, I have collected some results for f_{D_s} from lattice gauge theory calculations (all the various errors have been added in quadrature). All the lattice results for quenched QCD are lower than the experimental value. This is consistent with the picture that unquenching raises the value of a decay

Group	Comments	f_{D_s} MeV	σ
Average (lattice) [19]	Review in 1998	220(25)	2.2
JLQCD [20]	Continuum limit	224(20)	2.8
MILC [21]	Continuum limit	210(27)	2.6
Collins et al. [22]	$a = 0.18 \text{ fm}$	223(54)	1.1
UKQCD [23]	$a = 0.068 \text{ fm}$	241(30)	1.3

Table 1. Summary of recent quenched lattice results for f_{D_s} . The σ column is the number of lattice errorbars below the central experimental value of 280 MeV.

constant. The latest results [2] from lattice calculations show a 3-8% increase in f_{D_s} , when dynamical fermions are included.

5. Conclusions

A consistent picture does seem to emerge from lattice QCD calculations, that unquenching does raise the value of decay constants. However, the errors on the calculations need to be reduced. Ideally, a calculation of f_B similar to the recent CP-PACS calculation is required [3], but at lighter sea masses and smaller lattice spacings. Improvements in lattice techniques and faster computers will reduce the errors on f_B .

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