Fayet-Iliopoulos $D$ term and its renormalization in the minimal supersymmetric standard model

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We consider the renormalization of the Fayet-Iliopoulos $D$ term in a softly broken supersymmetric gauge theory with a nonsimple gauge group containing an Abelian factor, and present the associated $\beta$ function through three loops. We also include in an appendix the result for several Abelian factors. We specialize to the case of the minimal supersymmetric standard model, and investigate the behavior of the Fayet-Iliopoulos coupling $\xi$ for various boundary conditions at the unification scale. We focus particularly on the case of nonstandard soft supersymmetry breaking couplings, for which $\xi$ evolves significantly between the unification scale and the weak scale.

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I. INTRODUCTION

In Abelian gauge theories with $N=1$ supersymmetry there exists a possible invariant that is not allowed in the non-Abelian case: the Fayet-Iliopoulos $D$ term:

$$L = \xi \int V(x, \theta, \bar{\theta}) \, d^4 \theta = \xi D(x). \quad (1.1)$$

In previous papers [1,2] we have discussed the renormalization of $\xi$ in the presence of the standard soft supersymmetry-breaking terms

$$L_{SB} = - (m^2)_{ij} \phi^i \phi^j - \left( \frac{1}{6} h^{ijk} \phi^i \phi^j \phi^k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.} \right). \quad (1.2)$$

The result for $\beta_\xi$ is as follows:

$$\beta_\xi = \frac{\beta}{g} \xi + \beta_\xi \quad (1.3)$$

where $\beta_\xi$ is determined by $V$-tadpole (or in components $D$-tadpole) graphs, and is independent of $\xi$. Although in Refs. [1,2] we restricted ourselves to the Abelian case, it is evident that a $D$ term can occur with a direct product gauge group $(G_1 \otimes G_2 \cdots)$ if there is an Abelian factor: as is the case for the minimal supersymmetric standard model (MSSM). In the MSSM context one may treat $\xi$ as a free parameter at the weak scale [3], in which case there is no need to know $\beta_\xi$. However, if we know $\xi$ at gauge unification, for example, then we need $\beta_\xi$ to predict $\xi$ at low energies. Our purpose in this paper is first of all to give the result for $\beta_\xi$ through three loops for a general direct product gauge group. For simplicity of exposition, we restrict ourselves in the main body of the paper to the case of one Abelian factor, postponing the more general result (which is complicated by the possibility of “kinetic mixing” [4] between different Abelian factors) to an appendix. We shall then specialize to the case of the MSSM, and perform some running analyses to determine the size of $\xi(M_Z)$ for various choices of boundary conditions at the unification scale $M_X$.

II. GENERAL CASE

First of all, for completeness and to establish the notation, let us recapitulate the standard results for supersymmetric theory. We take an $N=1$ supersymmetric gauge theory with gauge group $\Pi_a G_a$ and with superpotential

$$W(\Phi) = \frac{1}{6} Y^{ijk} \Phi^i \Phi^j \Phi^k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j. \quad (2.1)$$

We will be assuming here that the gauge group has one Abelian factor, which we shall take to be $G_1$. We shall denote the hypercharge matrix for $G_1$ by $\mathcal{Y}$. At one loop we have

$$16\pi^2 \beta_1^{(1)} = g_a^3 Q_a = g_a^3 [T(R_a) - 3 C(G_a)], \quad (2.2a)$$

$$16\pi^2 \gamma_1^{(1)} = P^{ij} = \frac{1}{2} Y^{ijkl} Y_{kl} - \sum_a g_a^2 [C(R_a)]^i_j, \quad (2.2b)$$

where $R_a$ is the group representation for $G_a$ acting on the chiral fields, $C(R_a)$ the corresponding quadratic Casimir, and $T(R_a) = (r_a)^{-1} \text{Tr}[C(R_a)]$, $r_a$ being the dimension of $G_a$. For the adjoint representation, $C(R_a) = C(G_a) I_{1a}$, where $I_{1a}$ is the $r_a \times r_a$ unit matrix. Note that $T(R_1) = \text{Tr}(\mathcal{Y}^2)$, $[C(R_1)]^i_j = (\mathcal{Y}^2)^i_j$. At two loops we have

$$(16\pi^2)^2 \beta_2^{(2)} = 2 g_a^5 C(G_a) Q_a - 2 g_a^3 r_a^{-1} \text{Tr}[P C(R_a)] \quad (2.3a)$$

$$(16\pi^2)^2 \gamma_2^{(2)} = \left[ - Y_{jma} Y^{mpi} - 2 \sum_a g_a^2 C(R_a)^p_j \delta^n_p \right] P^n_p + 2 \sum_a g_a^4 C(R_a)^i_j Q_a. \quad (2.3b)$$

For completeness and later reference, we also quote here the general result for $\beta_3^{(3)}$, which is a straightforward generalization of the result of Ref. [5]:

$$(16\pi^2)^3 \beta_3^{(3)} = \left[ - Y_{jma} Y^{mpi} - 2 \sum_a g_a^2 C(R_a)^p_j \delta^n_p \right] P^n_p + 2 \sum_a g_a^4 C(R_a)^i_j Q_a. \quad (2.3c)$$
\[ \beta_{gg}^{\text{DRED}(3)} = 3 r_a^{-1} s_a X Y_{d}^i + 6 r_a^{-1} s_a^3 \sum_{\beta} \frac{1}{2} \beta(\beta C(R_\beta)^{\prime}) + 3 r_a^{-1} \beta \left[ \beta C(R_\beta)^{\prime} \right] - 6 r_a^{-1} s_a^3 \sum_{\beta} \beta(\beta C(R_\beta)^{\prime}) - 4 r_a^{-1} s_a^5 C(G_a) \beta \left[ \beta C(R_\beta)^{\prime} \right] + g_a \beta(\beta C(R_\beta)^{\prime}) \left[ C(R_\beta)^{\prime} \right]. \] (2.4)

We recall that gauge anomaly cancellation requires
\[ \text{Tr}[\mathcal{Y} C(R_\alpha)] = 0 \] (2.5)
and naturalness (or cancellation of \( U_1 \)-gravitational anomalies) requires
\[ \text{Tr}[\mathcal{Y}] = 0. \] (2.6)

The diagrams contributing to \( \tilde{\beta}_\xi \) through three loops for a general non-simple gauge group are essentially the same as those depicted for the pure Abelian case in Ref. [2], but reinterpreting internal gauge and gaugino propagators as
\[ (16\pi^2)^3 \tilde{\beta}_\xi^{(3)\text{DRED'}} = -6(16\pi^2)^2 \text{Tr}[\text{YM} m^2 \mathcal{Y}^2] - 4 \text{Tr}[\text{YP}^2 m^2] - 5 \text{Tr}[\text{YH}^4] + 2 \text{Tr}[\text{YP}^2 m^2] - 24 \zeta(3) \sum_a \frac{1}{2} \text{Tr}[\text{YM} C(R_\alpha)] \]
\[ + 12 \zeta(3) \sum_a s_a^2 \text{Tr}[\text{Y} M a H C(R_\alpha) + \text{c.c.}] - 96 \zeta(3) \sum_{a,\beta} s_a^2 \beta M a M a \text{Tr}[\text{YM} C(R_\alpha) C(R_\beta)] - 24 \zeta(3) \]
\[ \times \left[ \sum_{a,\beta} s_a^2 \beta M a M a \text{Tr}[\text{YM} C(R_\alpha) C(R_\beta)] + \text{c.c.} \right] \] (2.9)

where [6]
\[ W_{ij} = \left( \frac{1}{2} Y^2 m^2 + \frac{1}{2} m^2 Y^2 + h^2 \right)_{ij} + 2 Y_{ip} Y_{jp} (m_q Y_{ij}) \]
\[ - 8 \sum_{\beta} \beta M a M a \text{Tr}[\text{YM} C(R_\alpha)] \] (2.10)
\[ H_{ij} = h^{ikl} Y_{kjl} + 4 \beta M a M a \text{Tr}[\text{YM} C(R_\alpha)] \] (2.11)

with \( (Y^2)_{ij} = Y^{ikl} Y_{kjl} \), \( (h^2)_{ij} = h^{ikl} h_{kjl} \). These results are computed using the DRED' scheme, which is a variant of the dimensional reduction scheme (DRED) defined so as to ensure that \( \beta \) functions for physical couplings have no dependence on the \( e \)-scalar mass [7]. Most of the terms in Eq. (2.9) correspond in a simple way to the analogous terms in Eq. (5.2) of Ref. [2], the only subtle point being the \( M a M a \) terms, where one sees easily that only in the case of Fig. 15(e) of [2] can the two gaugino masses belong to different gauge groups \( (G_a) \). Thus the last term in Eq. (2.9) and the \( M M a g^4 \) terms from the terms involving \( H \) come entirely from this particular figure.

It was proved in Ref. [1] in the pure Abelian case that if the \( m^2 \) dependence in \( \tilde{\beta}_\xi \) takes the form \( \text{Tr}[m^2 A] \), then we have
\[ \text{Tr}[\mathcal{Y} A] = 2 \frac{\beta_{\xi \beta}}{\xi}. \] (2.12)

It is easy to see that the proof extends to the direct product case, and indeed we can check Eq. (2.12) explicitly using Eqs. (2.7)–(2.9) and (2.2a), (2.3a) and (2.4).

### III. MSSM

We now specialize to the case of the MSSM. The relevant part of the MSSM superpotential is
\[ W = H_3 \tau^* Y_{iQ} + H_1 b^* Y_{bQ} + H_1 t^* Y_{tL} \] (3.1)
where \( Y_{iQ}, Y_{bQ}, Y_{tL} \) are \( 3 \times 3 \) Yukawa flavor matrices.

The gauge \( \beta \) functions are given at one loop by...
At two loops [8] the anomalous dimensions are given by
\[
(16\pi^2)^2 \gamma^{(2)}_{\rho} = -2 Y_\rho (P_{\rho} + P_{H_1}) Y_\rho - 2 P_{\rho} C_{\rho} \\
+ 2 \left( \frac{1}{16} b_1 g_1^4 + \frac{1}{3} b_3 g_3^4 \right),
\]
(3.6a)
\[
(16\pi^2)^2 \gamma^{(2)}_{\rho'} = -2 Y_\rho (P_{\rho'} + P_{H_1}) Y_\rho \\
- 2 P_{\rho'} C_{\rho'} + 2 \left( \frac{1}{16} b_1 g_1^4 + \frac{1}{3} b_3 g_3^4 \right),
\]
(3.6b)
\[
(16\pi^2)^2 \gamma^{(2)}_{Q} = -Y_Q (P_{Q} + P_{H_1}) Y_Q - 2 P_{Q} C_{Q} \\
- 2 P_{Q} C_{Q} + 2 \left( \frac{1}{3} b_1 g_1^4 + \frac{3}{8} b_2 g_2^4 \right),
\]
(3.6c)
\[
(16\pi^2)^2 \gamma^{(2)}_{\tau} = -Y_\tau (P_{\tau} + P_{H_1}) Y_\tau \\
- 2 P_{\tau} C_{\tau} + \frac{1}{2} b_1 g_1^4,
\]
(3.6d)
\[
(16\pi^2)^2 \gamma^{(2)}_{L} = -Y_L [P_{\tau} + P_{H_1}] Y_\tau - 2 P_{L} C_{L} + \frac{1}{3} b_1 g_1^4 + \frac{1}{2} b_2 g_2^4 \rightarrow \frac{1}{3} b_1 g_1^4 + \frac{1}{2} b_2 g_2^4,
\]
(3.6e)
\[
(16\pi^2)^2 \gamma^{(2)}_{H_1} = -3 [Y_\rho P_{Q} Y_\rho + Y_{\rho'} P_{Q} Y_{\rho'}] \\
- 3 [Y_\rho P_{Q} Y_\rho + Y_{\rho'} P_{Q} Y_{\rho'}] - 2 C_{L} P_{H_1} \\
+ \frac{3}{16} b_1 g_1^4 + \frac{1}{2} b_2 g_2^4,
\]
(3.6f)
\[
(16\pi^2)^2 \gamma^{(2)}_{H_2} = -3 [Y_\rho P_{Q} Y_\rho + Y_{\rho'} P_{Q} Y_{\rho'}] - 2 C_{L} P_{H_2} \\
+ \frac{3}{16} b_1 g_1^4 + \frac{1}{2} b_2 g_2^4.
\]
(3.6g)

We now turn to the soft couplings. The quantities \( W \) and \( H \) defined in Eqs. (2.10), (2.11) are given by
\[
W_\rho = (2 m_{\rho}^2 + 4 m_{H_2}^2) Y_\rho + 4 Y_\rho m_{Q} Y_\rho \\
+ 2 Y_\rho m_{Q} Y_\rho - 8 C_{\rho} M^M,
\]
(3.7a)
\[
W_{\rho'} = (2 m_{\rho'}^2 + 4 m_{H_2}^2) Y_{\rho'} + 4 Y_{\rho'} m_{Q} Y_{\rho'} \\
+ 2 Y_{\rho'} m_{Q} Y_{\rho'} - 8 C_{\rho'} M^M,
\]
(3.7b)
\[
W_{Q} = (2 m_{Q}^2 + 4 m_{H_2}^2) Y_{Q} + (2 m_{Q}^2 + 2 m_{H_2}^2) Y_{Q} Y_{Q} \\
+ Y_{Q} Y_{Q} m_{Q} Y_{Q} + 2 Y_{Q} m_{Q} Y_{Q} + 2 Y_{Q} m_{Q} Y_{Q} + 2 Y_{Q} m_{Q} Y_{Q} + 2 h_{I} Y_{Q} + 2 h_{I} Y_{Q} - 8 C_{Q} M^M,
\]
(3.7c)
\[
W_{\tau} = (2 m_{\tau}^2 + 4 m_{H_2}^2) Y_{\tau} + 2 Y_{\tau} m_{Q} Y_{\tau} \\
+ 4 h_{I} h_{I} + 8 C_{\tau} M^M,
\]
(3.7d)
\[
W_{L} = (2 m_{L}^2 + 4 m_{H_2}^2) Y_{L} + 2 Y_{L} m_{Q} Y_{L} \\
+ 2 h_{I} h_{I} - 8 C_{L} M^M,
\]
(3.7e)
\[
W_{H_2} = 6 m_{H_2}^2 Y_{H_2} + 2 m_{Q} Y_{Q} + 2 m_{Q} Y_{Q} + 2 m_{Q} Y_{Q} + 2 h_{I} h_{I} - 8 C_{L} M^M,
\]
(3.7f)

where
\[
C_{\rho} M^M = \frac{1}{2} |M_{1}|^2 g_1^2 + \frac{1}{3} |M_{1}|^2 g_1^2,
\]
(3.8a)
\[
C_{\rho'} M^M = \frac{1}{2} |M_{1}|^2 g_1^2 + \frac{1}{3} |M_{1}|^2 g_1^2,
\]
(3.8b)
\[
C_{Q} M^M = \frac{1}{2} |M_{1}|^2 g_1^2 + \frac{1}{3} |M_{1}|^2 g_1^2,
\]
(3.8c)
\[
C_{\tau} M^M = \frac{1}{2} |M_{1}|^2 g_1^2,
\]
(3.8d)
\[
C_{L} M^M = \frac{1}{2} |M_{1}|^2 g_1^2 + \frac{1}{3} |M_{1}|^2 g_1^2.
\]
and

\[ H' = 4h_iY' + 4C'M', \]
\[ H_{bc} = 4h_iY_{L} + 4C'M', \]
\[ H_Q = 2(Y_ih_i + Y_ih_i) + 4C'M', \]
\[ H_r = 4h_iY'_r + 4C'M', \]
\[ H_L = 2Y'_i h_i + 4C'M', \]
\[ H_{H_1} = \text{Tr}[6Y_i h_i + Y_i h_i] + 4C'M', \]
\[ H_{H_2} = 6\text{Tr}[Y_i h_i] + 4C'M', \]

(3.9)

where

\[ C'_r = \frac{1}{3} M_3 g^2 + \frac{1}{\pi} M_4 g^2, \]
\[ C'_b = \frac{1}{3} M_3 g^2 + \frac{1}{\pi} M_4 g^2, \]
\[ C'_Q = \frac{1}{\pi} M_1 g^2, \]
\[ C'_L = \frac{1}{3} M_2 g^2 + \frac{1}{\pi} M_1 g^2. \]

(3.10)

With all these subsidiary definitions we can now give the results for \( \beta^\xi \) up to three loops. We have

\[ (16\pi^2)^2 \beta_\xi^{(2)} = 2 \sqrt{3} g_1 \text{Tr}[m_Q^2 - m_L^2 - 2m_r^2] \]
\[ + m_{bc}^2 + m_{L_r}^2 - 2m_{H_1}^2 + m_{H_2}^2 \]

(3.11)

\[ (16\pi^2)^2 \beta_\xi^{(3)} = -4 \sqrt{3} g_1 \text{Tr}[m_Q^2 P_Q - m_L^2 P_L] \]
\[ - 2m_r^2 P_{r'} + m_{bc}^2 P_{bc} + m_{L_r}^2 P_{L_r} \]
\[ - 2m_{H_1} P_{H_1} + 2m_{H_2} P_{H_2} \]

(3.12)

\[ (16\pi^2)^2 \beta_\xi^{(3)} = \sqrt{3} g_1 \{ -6(16\pi^2)^2 \beta_\xi^{(3)} \}
\[ - 4\beta_\xi^{(3)} - \frac{1}{2} \beta_\xi^{(3)} \]
\[ + 2\beta_\xi^{(3)} + \xi(3)(-24\beta_\xi^{(3)}) \]
\[ + 12\beta_\xi^{(3)} - 96\beta_\xi^{(3)} - 48\beta_\xi^{(3)} \}, \]

(3.13)

where

\[ \beta_\xi^{(3)} = \text{Tr}(m_Q^2 Y_Q^2 - m_L^2 Y_L^2 - 2m_r^2 Y_{r'}^2 + m_{bc}^2 Y_{bc}^2) \]
\[ + m_{L}^2 Y_{L}^2 - 2m_{H_1} Y_{H_1}^2 + m_{H_2} Y_{H_2}^2), \]
\[ \beta_\xi^{(3)} = \text{Tr}(W_Q P_Q - W_L P_L - 2W_{r'} P_{r'} + W_{bc} P_{bc}) \]
\[ + W_{r'} P_{r'} - W_{H_1} P_{H_1} + W_{H_2} P_{H_2} \], \]

(3.14)

We shall now present our MSSM results specialized to the commonly considered case where only the 3rd generation Yukawa masses are significant. We also take the gaugino masses to be real. Writing \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) for the 3rd generation couplings, Eq. (3.4) becomes

\[ P_{r'} = 2\lambda_2 - 2C_{r'} \]
\[ P_{bc} = 2\lambda_2 - 2C_{bc} \]
\[ P_Q = \lambda_3 + \lambda_3 - 2C_Q \]
\[ P_{r'} = 2\lambda_2 - 2C_{r'} \]
\[ P_{L} = \lambda_2 - 2C_{L} \]
\[ P_{u'} = -2C_{r'} \]
\[ P_{d'} = -2C_{bc} \]
\[ P_{\tau} = -2C_{\tau} \]
\[ P_{\tau} = -2C_{\tau} \]
\[ P_{H_1} = \lambda_2^2 + 3\lambda_2^2 - 2C_{L} \]
\[ P_{H_2} = 3\lambda_2^2 - 2C_{L}. \]

(3.15)
where \{i,b,Q,\tau,L\} now refers to the 3rd generation, and \{u,d,R,e,N\} refers to either of the 1st or 2nd generation. Equation (3.6a) now takes the form

\[
(16\pi^2)^2 \gamma_{P_i}^{(2)} = -2\lambda_2^2 (P_Q + P_{H_1}) - 2P_i C_{iP} + 2\left(\frac{1}{\pi^2} b_1 g_1^4 + \frac{1}{\pi^2} b_3 g_3^4\right),
\]

(3.16a)

\[
(16\pi^2)^2 \gamma_{P_{ib}}^{(2)} = -2\lambda_2^2 (P_Q + P_{H_1}) - 2P_{ib} C_{ib} + 2\left(\frac{1}{\pi^2} b_1 g_1^4 + \frac{1}{\pi^2} b_3 g_3^4\right),
\]

(3.16b)

\[
(16\pi^2)^2 \gamma_{P_Q}^{(2)} = -\lambda_2^2 (P_{P_{ib}} + P_{H_1}) - \lambda_2^2 (P_{P_{ib}} + P_{H_1}) + 2P_Q C_{P_Q} + 2\left(\frac{1}{\pi^2} b_1 g_1^4 + \frac{1}{\pi^2} b_2 g_2^4 + \frac{1}{\pi^2} b_3 g_3^4\right),
\]

(3.16c)

\[
(16\pi^2)^2 \gamma_{P_{L_i}}^{(2)} = -2\lambda_2^2 (P_{P_{ib}} + P_{H_1}) - 2P_{L_i} C_{P_{L_i}} + \frac{1}{4} b_1 g_1^4,
\]

(3.16d)

\[
(16\pi^2)^2 \gamma_{P_{L_{ib}}}^{(2)} = -2\lambda_2^2 (P_{P_{ib}} + P_{H_1}) - 2P_{L_{ib}} C_{P_{L_{ib}}} + \frac{1}{4} b_1 g_1^4 + \frac{1}{4} b_3 g_3^4,
\]

(3.16e)

\[
(16\pi^2)^2 \gamma_{\psi_{ib}}^{(2)} = -2P_{\psi_{ib}} C_{\psi_{ib}} + 2\left(\frac{1}{\pi^2} b_1 g_1^4 + \frac{1}{\pi^2} b_3 g_3^4\right),
\]

(3.16f)

\[
(16\pi^2)^2 \gamma_{\psi_{ib}}^{(2)} = -2P_{\psi_{ib}} C_{\psi_{ib}} + 2\left(\frac{1}{\pi^2} b_1 g_1^4 + \frac{1}{\pi^2} b_3 g_3^4\right),
\]

(3.16g)

\[
(16\pi^2)^2 \gamma_{\psi_{ib}}^{(2)} = -2P_{\psi_{ib}} C_{\psi_{ib}} + 2\left(\frac{1}{\pi^2} b_1 g_1^4 + \frac{1}{\pi^2} b_3 g_3^4\right),
\]

(3.16h)

\[
(16\pi^2)^2 \gamma_{\psi_{ib}}^{(2)} = -2P_{\psi_{ib}} C_{\psi_{ib}} + \frac{1}{4} b_1 g_1^4,
\]

(3.16i)

\[
(16\pi^2)^2 \gamma_{\psi_{ib}}^{(2)} = -2P_{\psi_{ib}} C_{\psi_{ib}} + \frac{1}{4} b_1 g_1^4 + \frac{1}{4} b_2 g_2^4,
\]

(3.16j)

\[
(16\pi^2)^2 \gamma_{H_{ib}}^{(2)} = -3\lambda_2^2 [P_{P_{ib}} + P_{H_1}] - \lambda_2^2 [P_{P_{ib}} + P_{H_1}] - 2C_{P_{H_1}} + \frac{3}{\pi^2} b_4 g_4^4 + \frac{3}{\pi^2} b_2 g_2^4,
\]

(3.16k)

\[
(16\pi^2)^2 \gamma_{H_{ib}}^{(2)} = -3\lambda_2^2 [P_{P_{ib}} + P_{H_1}] - \lambda_2^2 [P_{P_{ib}} + P_{H_1}] - 2C_{H_{ib}} + \frac{3}{\pi^2} b_4 g_4^4 + \frac{3}{\pi^2} b_2 g_2^4 + \frac{1}{4} b_2 g_2^4.
\]

(3.16l)

Correspondingly, we retain only the three 3rd generation tri-linear soft couplings \(h_1 = A, \lambda_2, \), \(h_1 = A_b \lambda_2, \) and \(h_2 = A_2 \lambda_2, \) Equation (3.7) now becomes

\[
W_{\psi} = 4\lambda_2^2 (m_{\psi}^2 + m_{Q}^2 + m_{H_2}^2 + A_2^2) - 8C_{\psi}^{MM},
\]

(3.17)

\[
W_{\psi} = 4\lambda_2^2 (m_{\psi}^2 + m_{Q}^2 + m_{H_2}^2 + A_2^2) - 8C_{\psi}^{MM},
\]

(3.17)

Equation (3.9) now becomes

\[
H_{\psi} = 4A, \lambda_2^2 + 4C_{\psi}^{L},
\]

(3.17)

\[
H_{\psi} = 4A, \lambda_2^2 + 4C_{\psi}^{L},
\]

(3.17)

\[
H_Q = 2(A_2 \lambda_2^2 + A_1 \lambda_2^2) + 4C_Q^L,
\]

(3.17)

\[
H_{\psi} = 4A, \lambda_2^2 + 4C_{\psi}^{L},
\]

(3.17)

\[
H_L = 2A_2 \lambda_2^2 + 4C_{\psi}^{L},
\]

(3.17)

\[
H_{\psi} = 4C_{\psi}^{L},
\]

(3.17)

\[
H_{H_{ib}} = 4C_{\psi}^{L},
\]

(3.17)

\[
H_{H_1} = 6A_2 \lambda_2^2 + 2A_2 \lambda_2^2 + 4C_{\psi}^{L}.
\]

(3.17)

\[
H_{H_2} = 6A_2 \lambda_2^2 + 4C_{\psi}^{L}.
\]

(3.17)
16\pi^2\beta^{(1)}_\xi = 2\sqrt{\frac{\hat{g}}{8}} (m^2_{\tilde{g}} + 2m^2_{\tilde{R}} - m^2_{\tilde{L}} - 2m^2_{\tilde{N}} - 2m^2_{\tilde{\nu}} - 4m^2_{\tilde{e}} \\
+ m^2_{\tilde{\nu}'} + 2m^2_{\tilde{e}'} + m^2_{\tilde{\nu}} + 2m^2_{\tilde{e}} - m^2_{\tilde{H}_1} + m^2_{\tilde{H}_2}).

(3.19)

(16\pi^2)^2\beta^{(2)}_\xi = -4\sqrt{\frac{\hat{g}}{8}} (m^2_{\tilde{Q}}P_{\tilde{Q}} + 2m^2_{\tilde{R}}P_{\tilde{R}} - m^2_{\tilde{L}}P_{\tilde{L}} - 2m^2_{\tilde{N}}P_{\tilde{N}} \\
- 2m^2_{\tilde{\nu}}P_{\tilde{\nu}} - 4m^2_{\tilde{e}}P_{\tilde{e}} + 2m^2_{\tilde{\nu}'}P_{\tilde{\nu}'} + 2m^2_{\tilde{e}'}P_{\tilde{e}'} \\
+ m^2_{\tilde{\nu}}P_{\tilde{\nu}} - m^2_{\tilde{H}_1}P_{\tilde{H}_1} + m^2_{\tilde{H}_2}P_{\tilde{H}_2}).

(3.20)

Finally, Eq. (3.14) is replaced by

\beta^{(3)} = m^2_{\tilde{Q}}\gamma_{\tilde{Q}} + 2m^2_{\tilde{R}}\gamma_{\tilde{R}} - m^2_{\tilde{L}}\gamma_{\tilde{L}} - 2m^2_{\tilde{N}}\gamma_{\tilde{N}} - 2m^2_{\tilde{\nu}}\gamma_{\tilde{\nu}} \\
- 4m^2_{\tilde{\nu}'}\gamma_{\tilde{\nu}'} + 2m^2_{\tilde{e}}\gamma_{\tilde{e}} + 2m^2_{\tilde{e}'}\gamma_{\tilde{e}'} \\
+ 2m^2_{\tilde{\nu}}\gamma_{\tilde{\nu}} - m^2_{\tilde{H}_1}\gamma_{\tilde{H}_1} + m^2_{\tilde{H}_2}\gamma_{\tilde{H}_2}.

(3.21)

IV. Running Analysis

As we mentioned in the Introduction, if we have no prejudice as to the value of \xi at the gauge unification scale \MX, then we may as well treat \xi as a free parameter at the weak scale [3], and the running of \xi is irrelevant. However, it is conceivable that the underlying theory at scales beyond \MX may favor certain values of \xi(\MX), and then the running of \xi would need to be considered. We shall see that for currently popular choices of boundary conditions at \MX—namely, the minimal supergravity scenario and the anomaly mediated supersymmetry breakdown (AMSB) scenario—the running of \xi is determined predominantly by the first term on the right-hand side of Eq. (1.3) between \MX and \MZ, and hence to a good approximation we have

\xi(\MZ) \simeq \frac{g_1(\MZ)}{g_1(\MX)} \xi(\MX).

(4.1)

For instance, we find from Eqs. (2.7), (2.8) that universal soft masses at \MX imply \beta^{(1)}_\xi(\MX) = \beta^{(2)}_\xi(\MX) = 0, using Eq. (2.6), and the fact that it follows immediately from Eqs. (2.2b) using gauge invariance and anomaly cancellation [Eq. (2.5)] that

Tr[\hat{\gamma}^{(1)}] = 0.

(4.2)

Moreover, it is easy to show, using the result for \beta^{(1)} from Ref. [6], that if we work consistently at one loop, then Tr[\hat{\gamma} m^2] is scale invariant. So if initially \xi = Tr[\hat{\gamma} m^2] = 0, then \xi remains zero under (one loop) renormalization group (RG) evolution. With typical universal conditions at \MX with soft masses \m_0 and \m - \m_0, A - \m_0, we find (using three loops for \beta^{(2)}_\xi and two loops for the other \beta functions) that \xi \approx 0.001\m_0^2 at \MZ.

Another favored set of boundary conditions is those derived from AMSB [9]. Here the soft masses are given by

\langle m^2 \rangle_j = \frac{1}{2} |m_{32}|^2 \mu \frac{d^j}{d\mu^j} [\hat{\gamma} m^2].

(4.3)

where \m_{32} is the gravitino mass. In fact, since the AMSB result is RG invariant, it applies at all scales between \MX and \MZ. We then find from Eqs. (2.7), (2.8) that up to two loops, we may write

16\pi^2\beta_\xi = g_1 |m_{32}|^2 \mu \frac{d}{d\mu} [\hat{\gamma} m^2].

(4.4)

Gauge invariance and anomaly cancellation combined with Eqs. (2.2b) and (2.3b) yield [1]

Tr[\hat{\gamma}^{(1)}] = Tr[\hat{\gamma} (\gamma^{(2)} - (\gamma^{(1)})^2)] = 0.

(4.5)

and so \beta_\xi vanishes through two loops. Therefore to a good approximation \xi(\MZ) will be given by Eq. (4.1), and once again will be negligible at \MZ if it is zero at \MX.

However, if non-universal scalar masses at \MX are contemplated, then the effects of \beta_\xi might be significant—as
was noted in Ref. [10], for instance. Another context where $\beta_\xi$ might play a role is that of non-standard soft supersymmetry breaking [11]. This is because with the non-standard terms (for example $\phi^2 \phi^*$ terms) the result that $\text{Tr}[\lambda m^2]$ is one-loop scale invariant is not preserved. It follows that even with universal boundary conditions for $m^2$ and $\xi = 0$ at $M_X$, $\xi$ becomes non-zero at $M_Z$ even with one-loop running. In the current context of the MSSM with the 3rd generation dominating, the additional soft terms are given by

$$L_{\text{soft}}^{\text{new}} = m_{\phi} \psi_{H_1}^\dagger \psi_{H_2} + A_{\lambda} \lambda H_1^8 Q t^c + A_{\phi} \lambda H_2^8 Q b^c + A_{\lambda} \lambda H_1^8 L t^c + \text{H.c.}$$

Now in Ref. [11] we assumed, in fact, that $\xi$ was zero at $M_Z$; here we explore the more natural assumption that $\xi = 0$ at the unification scale. We follow Ref. [11] in dropping the explicit $\mu$ term from the superpotential, since it can be subsumed into $L_{\text{soft}}^{\text{new}}$. With given values at $M_X$ for $m_\phi$ and for the universal parameters $A$, $M$ and $m_0$, and for a given $\tan \beta$, we adjust $A_t = A_b = A_{\lambda} = A$ (at $M_X$) to obtain an acceptable electroweak vacuum. As in Ref. [11], we have made allowance for radiative corrections by using the tree Higgs minimization conditions, but evaluated at the scale $M_{\text{SUSY}} \approx m_0$. In Fig. 1 we show (for illustrative values of $M$, $m_\phi$ and $A$) the region of the $m_0 \tan \beta$ plane where this can be achieved.

For comparison, we show in Fig. 2 the corresponding region for $\xi(M_{\text{SUSY}}) = 0$. We notice that it is qualitatively similar, though slightly larger.

Note that this figure differs slightly from Fig. 1 of Ref. [11]. This is because we have incorporated one-loop corrections to the Higgs boson mass and because we have taken

<table>
<thead>
<tr>
<th>$\xi(M_X) = 0$</th>
<th>$\xi(M_{\text{SUSY}}) = 0$</th>
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TABLE I. Spectra (in GeV) for $\xi(M_X) = 0$ and for $\xi(M_{\text{SUSY}}) = 0$, with $M=200$ GeV, $m_\phi=640$ GeV, $A=0$, $m_0=150$ GeV at $M_X$, and with $\tan \beta = 8$. 

FIG. 1. The region of the $m_0 \tan \beta$ plane corresponding to an acceptable electroweak vacuum, for $M = 200$ GeV, $m_\phi = 150$ GeV, $A=0$ and $\xi(M_X) = 0$. The shaded region corresponds to one or more sparticle or Higgs boson masses in violation of current experimental bounds.

FIG. 2. The region of the $m_0 \tan \beta$ plane corresponding to an acceptable electroweak vacuum, for $M = 200$ GeV, $m_\phi = 150$ GeV, $A=0$ and $\xi(M_{\text{SUSY}}) = 0$. The shaded region corresponds to one or more sparticle or Higgs boson masses in violation of current experimental bounds.
account of the increasingly stringent experimental bounds (in particular increasing \( m_\phi \) at \( M_X \) to get acceptable chargino masses). For \( m_0 = 640 \) GeV and \( \tan \beta = 8 \), we find \( \tilde{A} = 1.07(1.01) \) TeV, \( \tilde{A}_t(M_{\text{SUSY}}) \approx 661(627) \) GeV, \( \tilde{A}_e(M_{\text{SUSY}}) \approx 664(630) \) GeV, \( \tilde{A}_\mu(M_{\text{SUSY}}) \approx 491(469) \) GeV. [The pairs of numbers correspond to \( \xi(M_X) = 0 \), \( \xi(M_{\text{SUSY}}) = 0 \) respectively.] The spectra obtained for \( \xi(M_X) = 0 \) and for \( \xi(M_{\text{SUSY}}) = 0 \) are given in Table I. We see that there are significant differences, especially in the masses of \( H, A \) and \( H^\pm \). On the other hand, the chargino and neutralino masses are unaffected, with a lightest supersymmetric particle (LSP) neutralino.

Finally, in Table II we give the values of \( \xi(M_{\text{SUSY}}) \) for some typical points in the allowed region of Fig. 1. We see indeed that \( \xi(M_{\text{SUSY}}) \) is quite sizable.

We have verified that the same results are obtained if we either (1) perform the RG evolution in the \( \xi \)-uneliminated theory and then eliminate \( \xi \) (via its equation of motion) at low energies or (2) eliminate \( \xi \) at \( M_X \), and evolve to low energies with the (modified) \( \xi \)-eliminated \( \beta \) functions. For a general discussion of the equivalence of these procedures, see Refs. [1,2].

### APPENDIX: GENERAL RESULT FOR SEVERAL ABELIAN FACTORS

In this appendix we give the general results for the case of a direct product group with several Abelian factors. As we mentioned earlier, the situation is complicated by the possibility of "kinetic mixing" [4] between the different Abelian factors. We can accommodate this possibility by introducing a matrix of couplings for the Abelian factors. Suppose that the gauge group is \((U_1)^a \Pi_n G_a \), where the \( G_a \), \( \alpha = a \) + 1, ..., \( n \) are non-Abelian. The gauge couplings are then \( g_{\alpha \beta} \), where \( g_{\alpha \beta} = g_a \delta_{\alpha \beta} \), \( \alpha = a + 1 \ldots n \), with a similar form for the gauge \( \beta \) functions. The gaugino masses also form a matrix \( M_{\alpha \beta} \) with an analogous structure, as do their \( \beta \) functions. Suppose the hypercharges of the Abelian factors for a given representation are \( Y_{\alpha \beta} \), \( \alpha = 1 , \ldots , a \). Then we define

\[
\hat{Y}_{\alpha \beta} = \sum_{\beta = 1}^{a} Y_{\alpha \beta} g_{\beta \alpha}, \quad \alpha = 1 , \ldots , a,
\]

and a generalized quadratic Casimir matrix

\[
C(R) = \sum_{\alpha = 1}^{a} \hat{Y}_{\alpha \beta} \hat{Y}_{\beta \alpha} + \sum_{\alpha = a + 1}^{n} g_{\alpha \beta}^2 C(R_\alpha).
\]

The Fayet-Iliopoulos couplings now form a vector \( \hat{\xi}_{\alpha \beta} \), \( \alpha = 1 , \ldots , a \), and we have the matrix equation

\[
\beta \hat{\xi} = g^{-1} \beta \hat{\xi} + \hat{\beta} \hat{\xi}.
\]

We can now give the explicit general results, starting with the gauge \( \beta \) functions and anomalous dimension. At one loop,

\[
16\pi^2 \beta^{(1)} = g \hat{Q}
\]

where

\[
\hat{Q}_{\alpha \beta} = \text{Tr} \left[ \hat{Y}_{\alpha \beta} \hat{Y}_{\beta \alpha} \right], \quad \alpha , \beta = 1 , \ldots , a,
\]

\[
\hat{Q}_{\alpha \beta} = g_{\alpha \beta}^2 Q_a C(R_\alpha), \quad \alpha = a + 1 , \ldots , n,
\]

and

\[
16\pi^2 (\gamma^{(1)})^j = P^j = \frac{1}{2} Y_{ikl} Y_{jkli} - 2 \hat{C}(R)^j_i.
\]

At two loops,

\[
(16\pi^2)^2 (\gamma^{(2)})^j = - [ Y_{jmn} Y_{mpi}^\ast + 2 \hat{C}(R)^j_i \delta_{pi}^m ] P^m_p + 2 \hat{Q}_{\alpha \beta} \hat{Y}_{\alpha \beta} + g_{\alpha \beta}^2 Q_a C(R_\alpha)^j_i
\]

and

\[
(16\pi^2)^2 (\beta^{(2)})_{\alpha \beta} = - 2 g_{\alpha \gamma} \text{Tr} \left[ \hat{P} \hat{Y}_{\alpha \gamma} \hat{Y}_{\beta \gamma} \right], \quad \alpha , \beta = 1 , \ldots , a,
\]

\[
(16\pi^2)^2 (\beta^{(2)})_a = 2 g_{\alpha \gamma}^5 C(G_\alpha) Q_a - 2 g_{\alpha \gamma}^3 \text{Tr} \left[ P C(R_\alpha) \right],
\]

\[
\alpha = a + 1 , \ldots , n.
\]

At three loops we have

### V. CONCLUSIONS

In this paper we have extended the results of Ref. [2] for the renormalization of the Fayet-Iliopoulos D term to the case of a direct product gauge group, and applied the result to the MSSM. With standard soft supersymmetry breaking and universal boundary conditions at \( M_X \), then \( \xi \) is negligible at low energies if \( \xi(M_X) = 0 \). However, with non-standard soft breakings (and/or non-universal boundary conditions for the standard ones) we find significant effects even for \( \xi(M_X) = 0 \). In the non-standard breaking case, the effect is especially marked for the masses of \( H, A \) and \( H^\pm \), which decrease significantly when \( \xi \) is taken into account.

### ACKNOWLEDGMENTS

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\[ (16\pi^2)^3(\beta^{DRED(3)})_{a\beta} = g_{a\gamma} \left\{ 3Y^i m Y_{jkn} p_a \nabla^a (\nabla^\gamma \nabla^\beta)^i_j + 6 \text{Tr}[P \nabla^\gamma \nabla^\beta C(R)] + \text{Tr}[P^2 \nabla^\gamma \nabla^\beta] \right\} \]

\[ -6 \sum_{k, \lambda = 1}^a \bar{Q}_{k\lambda} \text{Tr}[\nabla^\gamma \nabla^\beta \nabla^\kappa \nabla^\lambda] - 6 \sum_{k = a + 1}^n g_{a\lambda}^4 Q_{k\lambda} \text{Tr}[\nabla^\gamma \nabla^\beta C(R_\lambda)] \right\}, \quad \alpha, \beta = 1, \ldots, a, \]

\[ (\beta^{DRED(3)})_a = 3z^a \left\{ 3Y^i m Y_{jkn} p_a C(R^a) + 6z^a \text{Tr}[P C(R_a)] + z^a \text{Tr}[P^2 C(R_a)] \right\} \]

\[ -6z^a \sum_{k, \lambda = 1}^a \bar{Q}_{k\lambda} \text{Tr}[C(R_a) \nabla^\gamma \nabla^\beta \nabla^\kappa \nabla^\lambda] - 6z^a \sum_{k = a + 1}^n g_{a\lambda}^4 Q_{k\lambda} \text{Tr}[C(R_a) C(R_\lambda)] \right\} \]

\[ -4z^a \sum_{k = a + 1}^n g_{a\lambda} C(G_a) \text{Tr}[P C(R_a)] + \sum_{k = a + 1}^n g_{a\lambda}^7 Q_{k\lambda} C(G_a) [4C(G_a) - Q_a], \quad \alpha = a + 1, \ldots, n. \] (A9)

For the Fayet-Iliopoulos couplings we have, at one loop,

\[ 16\pi^2[\beta^{(1)}_\xi]_a = \text{Tr}[\bar{\nabla}_a m^2], \quad \alpha = 1, \ldots, a, \] (A10)

and, at two loops,

\[ 16\pi^2[\beta^{(2)}_\xi]_a = -4 \text{Tr}[\bar{\nabla}_a m^2 \psi(1)]. \] (A11)

Finally,

\[ (16\pi^2)^3(\beta^{(3)DRED}_\xi)_{a\beta} = -6(16\pi^2)^2 \text{Tr}[\bar{\nabla}_a m^2 \psi^2] - 4 \text{Tr}[\bar{\nabla}_a WP] - \frac{5}{2} \text{Tr}[\bar{\nabla}_a HH'] + 2 \text{Tr}[\bar{\nabla}_a P^2 m^2] \]

\[ -24z(3) \text{Tr}[\bar{\nabla}_a WC(R)] + 12z(3) \text{Tr}[\bar{\nabla}_a H\bar{C}(R) + \text{c.c.}] - 96z(3) \text{Tr}[\bar{\nabla}_a \tilde{C}^{MM}(R) \bar{C}(R)] \]

\[ -24z(3) \{ \text{Tr}[\bar{\nabla}_a \tilde{C}(R) \bar{C}^M(R)] + \text{c.c.} \}, \] (A12)

where

\[ W^i_j = \left[ \frac{1}{2} Y^2 m^2 + \frac{1}{2} m^2 Y^2 + h^2 \right] i_j + 2Y^i p j k p^j (m^2) i_j - 8 \tilde{C}^{MM}(R)^i_j, \]

\[ H^i_j = h^{ik} Y^j k + 4 \tilde{C}(R)^i_j, \]

\[ \tilde{C}(R) = \sum_{a, \beta = 1}^a M_{a\beta} \bar{\nabla}_a \nabla_\beta + \sum_{a = a + 1}^n g_{a\lambda}^2 M_{a\lambda} C(R_\lambda), \]

\[ \tilde{C}^{MM}(R) = \sum_{a, \beta = 1}^a (MM)^{a\beta} \bar{\nabla}_a \nabla_\beta + \sum_{a = a + 1}^n M_{a}^\beta M_{a}^\beta M_{a}^2 C(R_\lambda). \] (A13)

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