The η and η' mesons in QCD.

UKQCD Collaboration

C. McNeile and C. Michael
Theoretical Physics Division, Dept. of Mathematical Sciences,
University of Liverpool, Liverpool L69 3BX, UK

Abstract

We study the flavour singlet pseudoscalar mesons from first principles using lattice QCD. We explore the quark content of the η and η' mesons and we discuss their decay constants.

1   Introduction

There is considerable interest in understanding hadronic decays involving η and η' in the final state. The phenomenological study of hadronic processes involving flavour singlet pseudoscalar mesons makes assumptions about their composition. Here we address the issue of the nature of the η and η' from QCD directly, making use of lattice techniques.

Lattice QCD directly provides a bridge between the underlying quark description and the non-perturbative hadrons observed in experiment. The amplitudes to create a given meson from the vacuum with a particular operator made from quark fields are measurable, an example being the determination of f_π. It also allows a quantitative study of the disconnected quark contributions that arise in the flavour singlet sector. The lattice approach provides other information such as that obtained by varying the number of quark flavours and their masses.

In the case of pseudoscalar mesons, the chiral perturbation theory approach also provides links between a quark description and the hadronic states. For the pion, this has the well known consequence that the decay constant f_π describes quantitatively both the $\mu\nu$ and $\gamma\gamma$ decays. For the flavour singlet states (η and η'), the situation is more complicated [1]. The axial anomaly now involves a gluonic component and the definition of decay constants is not straightforward. From the chiral perturbation theory description, one expects the mixing of η and η' to be most simply described in a quark model basis. In the flavour singlet sector, for pseudoscalar mesons, we then have contributions to the mass squared matrix with quark model content $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ (which we label as nn and ss respectively):

$$\begin{pmatrix} m_{nn}^2 + 2x_{nn} & \sqrt{2}x_{ns} \\ \sqrt{2}x_{ns} & m_{ss}^2 + x_{ss} \end{pmatrix} \qquad (1)$$

Here m corresponds to the mass of the flavour non-singlet eigenstate and is the contribution to the mass coming from connected fermion diagrams while x corresponds to the contribution from disconnected fermion diagrams. In the limit of no mixing (all $x = 0$, the OZI suppressed case), then we have the quenched QCD result that the η is degenerate with the π meson and the η' would correspond to the $s\bar{s}$ pseudoscalar meson. This is not the case, of course, and the mixing contributions x are important.

Using as input m_{nn}, m_{ss}, m_η and $m_{\eta'}$, the three mixing parameters x cannot be fully determined. It is usual to express the resulting one parameter freedom in terms of a mixing angle, here defined by

$$\eta = \eta_{nn} \cos\phi - \eta_{ss} \sin\phi \quad \eta' = \eta_{nn} \sin\phi + \eta_{ss} \cos\phi \qquad (2)$$

We show the resulting values of the mixing parameters x in the figure (the input value for m_{ss} will be discussed later).

The η and η' mesons are often described in an SU(3) motivated quark basis, namely $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. The mixing angle θ in this basis would be given by $\phi - 54.7^0$ in a lowest order chiral perturbation theory. In order to have $f_K \neq f_\pi$, one needs higher order terms in the chiral perturbation theory treatment and then the mixing scheme becomes more complicated [1] in this basis with more than one angle needed.

In the SU(3) symmetric limit, $m_{nn} = m_{ss} = m$ and $x_{nn} = x_{ns} = x_{ss} = x$, so that only one mixing parameter is relevant and the mixing matrix simplifies considerably to a diagonal form with elements m^2 (octet) and $m^2 + 3x$ (singlet). Previous lattice studies [2] have used degenerate quarks, so have explored this case and have found that the mixing parameter x is of a magnitude which can explain qualitatively the observed splitting between the η and η' mesons.

Here we undertake a non-perturbative study in QCD from first principles which will be able to establish the values of the mixing parameters x, including the pattern of SU(3) breaking. This more comprehensive study would take into account the different masses of the light (u and d) quarks and the heavier s quark. Within the lattice approach, it is not at present feasible to evaluate using quarks as light as the nearly massless u and d quarks and also it is more tractable to use an even number of degenerate quarks in the vacuum. As we shall show, despite these restrictions, a thorough study of the mixing between η and η' is possible.

Our lattice study uses dynamical configurations with $N_f = 2$ flavours of sea quarks of type 1 and we consider the properties of pseudoscalar mesons made of either quark 1 or quark 2, where quark 2 corresponds to a heavier quark. Thus quark 2 is treated as partially quenched. Here we have in mind exploring a situation which will be relevant to treating strange quark propagation (quark

2) in a vacuum containing only lighter quarks (quark 1). We focus here on the results of lattice evaluations, for background to the methods used see ref.[3]. We address three topics where lattice input permits us to construct a firm foundation for the η, η' mixing:

- From comparing pseudoscalar meson masses with valence quarks of two different masses (namely meson masses m_{11}, m_{12} and m_{22}), we can estimate the mass m_{ss} of the unmixed $\bar{s}s$ meson, given the observed m_{ns} and m_{nn} masses (ie K and π respectively).

- From measuring the mixing parameters x_{11}, x_{12} and x_{22} between initial and final flavour singlet states consisting of either quark 1 or 2 with different masses as above, we can establish the pattern of SU(3) breaking in the mixing.

- For $N_f = 2$ degenerate flavours of quark, we determine the pseudoscalar decay constants for the flavour singlet (P_0) and non-singlet (P_1) meson. This input allows us to discuss the relation between the observed $\gamma\gamma$ decay modes of π^0, η and η' and the underlying quark content.

2   Lattice results

2.1   The $s\bar{s}$ pseudoscalar mass

Chiral symmetry considerations lead to the expectation that the pseudoscalar meson composed of quarks of mass M_q has mass squared m^2 which behaves linearly with M_q at small quark mass. However, at large quark mass (c and b quarks for instance), one expects the meson mass to vary approximately linearly with the quark mass. Here we are not concerned with the region of very small quark mass where chiral logs are important [1], so we summarise this behaviour by

$$m^2 = bM_q + cM_q^2 + O(M_q^3) \tag{3}$$

For a pseudoscalar meson made of two different quarks of mass M_n and M_s, we shall assume its mass only depends on $(M_n + M_s)/2$ and not on $(M_s - M_n)/2$ as found in lattice studies [4] and in lowest order chiral perturbation theory. If eq. 3 were valid with just the linear term in the quark mass(ie $c = 0$), then one directly obtains the required mass of the pseudoscalar meson composed of s quarks, $m_{ss}^2 = 2m_{sn}^2 - m_{nn}^2$, that is $2K^2 - \pi^2$, leading to $m_{ss} = 0.687$ GeV.

This can be explored on a lattice by measuring the pseudoscalar meson mass for valence quarks in combinations 11, 22 and 12. Then, for small c/b, we have

$$\frac{c}{4b^2} = \frac{\frac{1}{2}(m_{11}^2 + m_{22}^2) - m_{12}^2}{(m_{22}^2 - m_{11}^2)^2} \tag{4}$$

This has been studied in the quenched approximation giving evidence [5] for a positive coefficient c in eq. 3. In the quenched approximation, however, the chiral behaviour at small quark mass is anomalous since the theory is not unitary. A better way to study this issue on the lattice is then to use dynamical configurations with sea quarks of type 1 and to consider the propagation of mesons made of either quark 1 or quark 2, where quark 2 corresponds to a heavier quark.

We present results from UKQCD configurations [6] with $N_f = 2$ flavours of sea quark with SW-clover coefficient $C_{SW} = 1.76$, lattice size $12^3 24$, and with sea quarks having $\kappa = 0.1398$, corresponding to sea quarks of mass around the strange quark mass ($m_P/m_V = 0.67$). Then we take the heavier valence quark (with $\kappa = 0.1380$) as corresponding to approximately twice the strange mass ($m_P/m_V = 0.81$).

The fits with two states to a 4×4 matrix of mesonic correlators for t range 3 to 10 give results for the spectrum shown in Table 1.

Taking account of the correlation among the errors, we obtain the dimensionless ratio

$$m_{11}^2 c/(4b^2) = 0.011(3) \tag{5}$$

which indicates a statistically significant curvature from the c term. Setting the scale [6] using $a^{-1} = 1.47$ GeV then the value of c/b^2 in physical units can be obtained from $m_{11} = 698$ MeV.

Applying this value of c to the determination of the m_{ss} mass from the π and K masses, gives a relative shift upwards due to the curvature term (c) of $1.1(3)\%$, corresponding to a value of $m_{ss} = 0.687 + 0.008$ GeV.

Note that this value also helps us to identify the meson mass ratio corresponding to strange quarks, namely $m_P/m_V = m_{ss}/m_\phi = 0.682$.

This lattice study thus answers the question of the likely deviation in the pseudoscalar mass formula from the result given by the lowest order chiral expression.

2.2 Flavour-singlet mixing

The mass splitting between flavour non-singlet and singlet mesons can be measured using lattice evaluation of disconnected quark propagators. This is not an easy task: the contamination from excited states is difficult to remove and the statistical errors turn out to be relatively large. Initial studies have been in the quenched approximation [2, 7, 3]. Here, although there is no flavour splitting of the masses, the mass splitting matrix element x can be evaluated. It is, however, preferable to be able to study the mass splitting directly and hence here we focus on results from full QCD simulations [8, 9].

The study of the mass spectrum of flavour singlet (P_0) and non-singlet pseudoscalar meson (P_1) using $N_f = 2$ flavours of sea quark 1 leads to singlet mass

κ_s	κ_{v1}	κ_{v2}	$m_1 a$	af_1/Z	$m_0 a$	af_0/Z
0.1398	0.1398	0.1398	0.477(5)	0.171(6)	0.56(4)	0.196(12)
					0.56(5)	0.176(18)
0.1398	0.1380	0.1398	0.563(5)	0.182(4)		
0.1398	0.1380	0.1380	0.640(4)	0.190(4)		

Table 1: Pseudoscalar meson masses with $N_f = 2$, labelled 1 for non-singlet (isospin 1) and 0 for singlet (isospin 0), and decay constants for the sea and valence quarks of hopping parameter shown (here 0.1398 corresponds to strange quarks and 0.138 to quarks twice as heavy as strange) from lattice studies with scale $a^{-1} = 1.47$ GeV. The singlet results are shown for fits with 1 state and $3 \le t \le 7$ (upper) and with 2 states and $2 \le t \le 7$ (lower).

$m_0 = (m_{11}^2 + 2x_{11})^{1/2}$ and non-singlet mass $m_1 = m_{11}$ respectively which allows x_{11} to be extracted. We shall also be interested in the dependence of x on quark masses and on non-diagonal mixings. These can be studied with a little less rigour as we discuss later.

We use the UKQCD lattices introduced in the previous section. The disconnected diagrams were evaluated using a variance reduction method [8] which uses all the data available with no dilution from the stochastic method used. We use as many different operators for the pseudoscalar meson as possible to have the largest basis in which to extract the ground state - local and non-local (fuzzed) in space with both γ_5 and $\gamma_5\gamma_4$ spin structure. Unfortunately, even with this basis of four operators, we are unable to determine the singlet mass precisely. For example for both valence and sea quarks having $\kappa = 0.1398$, as shown in Table 1, we obtain $am_0 = 0.56(4)$ from a one state fit to $t = 3$ to 7 with a 4×4 matrix of meson correlators. A two state fit to a wider t range (2-7) gives a similar mass value. The corresponding non-singlet pseudoscalar mass is also given in Table 1, so the determination of x from $m_1^2 + 2x_{11} = m_0^2$ has relatively large errors ($x_{11} = 0.10(4)$ GeV^2 using $a^{-1} = 1.47$ GeV). As an alternative, we also fit the ratio of the singlet to non-singlet correlators directly to a ground state mass difference. Using the t range 2-7 and local and fuzzed pseudoscalar operators, we obtain $am_0 - am_1 = 0.12(7)$. This method gives a slightly larger mass value (indicating $x_{11} = 0.13(8)$ GeV^2) but has even larger errors. These results indicate that much larger ensembles of gauge configurations will be needed to make more precise this approach of determining x from masses.

If one studies correlations of meson operators made from valence quarks of type 2 in a sea of quarks of type 1, one will find the ground state pseudoscalar meson to be that composed of quarks of type 1 (assuming type 2 quarks are heavier than type 1). Because of this, we need to explore in more detail to study the SU(3) breaking of the mixing parameters.

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t	x_{11}	x_{12}	x_{22}
2	.089(9)	.073(6)	.058(6)
3	.089(12)	.072(10)	.063(10)

Table 2: Mass mixing matrix elements x in GeV^2 from quenched lattices for strange valence quarks (1) and quarks of twice the strange mass (2).

To get a first look at this issue, we consider a quenched lattice and measure the ratio of the disconnected to connected diagrams for pseudoscalar meson propagation. We present results for $\beta = 5.7$, $C_{SW} = 1.57$, $12^3 \times 24$ with 100 configurations with $\kappa = 0.14077$ and 0.13843. The non-singlet spectrum at these parameters was studied previously [10] giving m_V/m_P values of 0.65 and 0.78 which correspond approximately to strange quarks and quarks twice as heavy as strange. The scale was set as $a^{-1} = 1.2$ GeV. The disconnected meson correlator was determined using a stochastic method with variance reduction [8].

Assuming dominance by ground state meson contributions, the ratio of disconnected to connected diagrams at time separation t is

$$\frac{D_{ij}}{C_{ij}} = \frac{N_f x_{ij}(t+1)}{2(m_{ii}m_{jj})^{1/2}} \qquad (6)$$

with flavour non-singlet pseudoscalar mass m_{ii} for quarks of type i. The factor of $t+1$ comes from the number of lattice sites at which the disconnected diagram can be split. To clarify the pattern of SU(3) breaking, we also study the non-diagonal case where we also measure the disconnected to connected ratio (here C_{ij} is taken as $(C_{ii}C_{jj})^{1/2}$). In extracting x_{12} we can make an additional correction for the contribution from the propagation of mesons with different masses m_{11} and m_{22} although, in practice, this correction is very small.

Using local meson operators, we obtain for x the values in Table 2. The largest t value has least contributions from excited state contamination and the consistency of the results versus t suggests that such contamination is small. As was found previously [2], x increases as the quark mass is decreased. Moreover, we can check to see if there is a factorisation of x as expected in some chiral perturbation theory descriptions [1], namely $x_{12}^2 = x_{11}x_{22}$, and we find that x_{12} lies somewhat below the value given by this assumption.

We now revert to discussing the more realistic (partially quenched) case: with heavier quarks of type 2 in a sea of two flavours of quarks of type 1. The method described above for the quenched case can be applied here too. In principle this method is now only valid for small $N_f xt$ for the propagation of quark 1. From this analysis of the measured D/C values, we get the x values shown in Table 3. The values of x_{11} are similar to those obtained above (with larger errors) directly from the rigorous method of using the mass differences. This suggests that the strong assumptions made in determining x directly from

t	x_{11}	x_{12}	x_{22}
2	.100(9)	.072(6)	.054(5)
3	.112(12)	.083(11)	.063(9)
4	.106(16)	.077(15)	.059(9)
4F	.093(13)	.073(12)	.052(11)

Table 3: Mass mixing matrix elements x in GeV^2 from lattices with $N_F = 2$ flavours of sea quark (of type 1) with strange valence quarks (1) and valence quarks of twice the strange mass (2). The meson operators used are local except for the case labelled 4F which has non-local (fuzzed) creation and annihilation operators

D/C are actually reasonable in practice. This, and the consistency of values from different t and different mesonic operators, gives us confidence to use the x values from quarks of type 2 (which are partially quenched anyway) as a guide to the quark mass dependence of x. The x values again show an increase with decreasing quark mass and also approximate factorisation.

Setting the quark mass to strange (since $m_P/m_V = 0.682$ in nature for s quarks) in both quenched and $N_f = 2$ evaluations leads to a consistent lattice estimate of x_{ss} in the range 0.09 to 0.13 GeV^2. This value is also consistent with that reported from a study of $N_f = 2$ by the CP-PACS collaboration [11] with $m_P/m_V = 0.69$ and $a^{-1} = 1.29$ GeV giving values of $x_{ss} = 0.10$ GeV^2 and 0.14 GeV^2 (depending on using $t_{\min} = 2,\ 3$ in fits, respectively). These lattice values are obtained at quite coarse lattice spacings and there may be some additional systematic error arising from the extrapolation to the continuum limit. We have, however, chosen to use a clover improved fermion action [6] to minimise this extrapolation error.

We are unable to determine the mixing strengths x for lighter quarks than strange. So we assume that the value of x continues to increase as the quark mass is decreased below strange in a similar way to the decrease we see from twice strange (type 2) to strange (type 1).

Consider now the consequence of this determination of the mixing. We use input masses $m_{nn} = 0.137$ GeV, $m_{ss} = 0.695$ GeV (as discussed above) and aim to have x values in line with our results above, namely $x_{ss} \approx 0.12$ GeV^2, $x_{ns}^2 \approx x_{nn}x_{ss}$ and we also expect, though with big errors from the extrapolation, $x_{nn}/x_{ss} \approx 2$. The figure shows the x values needed to reproduce the known η and η' masses for each mixing angle ϕ. The lattice determination of x_{ss} is shown by the dotted horizontal band. Keeping close to this band while satisfying the other lattice constraints is possible for the mixing illustrated by the vertical line. This has $x_{nn} = 0.292$, $x_{ns} = 0.218$, $x_{ss} = 0.13$ GeV^2 which gives a description of the observed η and η' masses while being consistent with our QCD inspired evidence about the mixing strengths. This assignment corresponds to a mixing angle ϕ in the η_{nn}, η_{ss} basis of 44.5^0. Note that this is almost maximal which

implies that the quark content (apart from the relative sign) of the η and η' meson is the same. The corresponding mixing angle in the η_8, η_1 basis (modulo comments above) is a value of θ of -10.2^0.

2.3   Flavour-singlet decay constants

The decays of π^0, η and η' to $\gamma\gamma$ are expected to proceed via the quark triangle diagram. The quark model gives a decay proportional to Q_i^2 for the contribution from a quark of charge Q_i. Thus for the π^0 meson and the flavour-singlet nn and ss mesons, the quark charge contributions to the decay amplitudes would be in the ratio $1 : 5/3 : \sqrt{2}/3$. The experimental [12] reduced decay amplitudes for π^0, η, and η' are in the ratio $1.0 : 1.00(10) : 1.27(7)$. This information can be used to analyse the quark content of the pseudoscalar mesons subject to a quantitative understanding of the decay mechanisms.

The conventional approach assumes that the decay constants for the decays of the three mesons are the same and then the relative decay amplitudes give information on the quark content. This suggests a mixing angle of $\theta \approx -20^0$ is preferred [1, 12].

We now address the issue of determining these decay constants directly from QCD using lattice methods. Our study uses 2 flavours of degenerate quark and we define the decay constants by

$$\langle 0|A^\mu|P_1(q)\rangle = f_1 q^\mu \qquad \langle 0|A^\mu|P_0(q)\rangle = f_0 q^\mu \qquad (7)$$

For the isospin 1 state P_1 (π - like), this is on a firm footing because of the anomaly hence f_1 will be scale invariant. For the flavour singlet pseudoscalar meson P_0, the decay constant defined as above will not be scale invariant because of gluonic contributions to the anomaly [1]. In this exploratory study we determine the decay constants with lattice regularisation and we shall compare the singlet and non-singlet values.

These decay constants can be thought of as giving the quark wave function at the origin of the pseudoscalar meson. Since the mass splitting between singlet and non-singlet is not reproduced directly in quenched QCD, it is essential to use lattice studies that do include sea quark effects in this study of decay matrix elements.

Results were obtained using fits to full (connected and disconnected) meson propagation with 4 different types of meson creation and destruction operator. These are local and fuzzed operators with either γ_5 or $\gamma_4\gamma_5$ couplings, so giving 4×4 matrix of pseudoscalar correlators. We used the $N_F = 2$ UKQCD configurations [6] referred to above. For the disconnected correlators, the variance reduction technique [8] is essential to get a reasonable signal to noise ratio, particularly for the operators involving the $\gamma_4\gamma_5$ factor.

The lattice result for f with various valence quark masses with fixed sea quark mass (quark 1) as above is shown in Table 1. For the non-singlet results

using $a^{-1} = 1.47$ GeV and the tadpole-improved perturbative value of Z of 0.81 (and of c_A which is involved in mixing of the lattice pseudoscalar and axial currents but has a very small effect in practice) we get $f_{11} = 198(8)$ MeV. Since this corresponds to strange quarks, it is in reasonable agreement with experiment [12] assuming a steady increase from $f_{nn} = 131$ MeV and $f_{ns} = 160$ MeV to f_{ss}. We do see evidence for this increase in f with quark mass directly on the lattice going from quarks of type 1 (strange) to type 2 (twice strange) as shown in Table 1.

The flavour singlet results are shown in Table 1. They are determined by fits to the appropriate (connected plus disconnected) meson correlators which are a 4×4 matrix at each t value. Despite this extensive data set, the determinations of f have relatively large statistical errors and the systematic error from changing the type of fit is also comparable. For our case with $N_f = 2$ degenerate quarks, the comparison of the flavour singlet and non-singlet shows that the singlet decay constants appear to be somewhat larger, though the errors are too big to substantiate this.

Combining the mass dependence we find in the flavour non-singlet sector with the near equality of singlet and non-singlet decay constants, we can deduce properties of the physical case with three light quarks. Thus, in terms of the conventional treatment [12], we would expect $f_\eta/f_\pi > 1$ and $f_{\eta'}/f_\pi > 1$. One way to minimise the effects of mixing is to consider $X = (a_\eta^2 + a_{\eta'}^2)/a_\pi^2$ where a refers to the reduced decay amplitude. Using the conventional formulae for the decay amplitudes would then give a value of $X = 3r^2$ (where r is a suitably weighted average of f_η/f_π and $f_{\eta'}/f_\pi$ which are both greater than 1). Thus the conventional treatment gives $X > 3$ which is significantly larger than the experimental value [12] of 2.64(24). Thus it appears unlikely that the conventional treatment (with the decay to $\gamma\gamma$ being given by the analogue of the formula for pions) is correct for any mixing angle.

We conclude that there is no support for the conventional assumption that the singlet decays are given by a similar expression to the non-singlet. As has been pointed out by many authors [1], this is plausible for at least two reasons: (i) the η and η' mesons are heavier and therefore less likely to dominate the axial current or, equivalently, higher order corrections to chiral perturbation theory will be more important (ii) the flavour-singlet axial anomaly has a gluonic component which will give additional contributions to any hadronic process.

3   Conclusion

From our careful non-perturbative study of mass formulae for flavour non-singlet pseudoscalar mesons made of different quarks, we deduce that the ss state lies at 695 MeV. We then determine the pattern of mixing for the flavour singlet sector, obtaining $x_{ss} \approx 0.12$ GeV^2, $x_{nn}/x_{ss} \approx 2$ and $x_{ns}^2 \approx x_{nn}x_{ss}$. These conditions are indeed consistent and point to a mixing close to maximal ($\phi = 45 \pm 2^0$)

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in the nn, ss basis (this corresponds to a conventional (η_8, η_1) mixing θ of -10 ± 2^0). We are able to explore the decay constants for singlet pseudoscalar mesons for the first time. Our results show similar decay constants for singlet and non-singlet states of the same mass but with quite large errors.

We have not addressed here the issue of the origin of these mixing parameters x. Lattice studies [3] have the capability to relate them to topological charge density fluctuations or to other vacuum properties.

Our lattice studies have been hampered by two constraints. One is that the disconnected quark diagrams needed for a study of singlet mesons are intrinsically noisy. Much larger data sets (tens of thousands of gauge configurations) will be needed to increase precision. Another constraint is that we are unable to work with sea quarks substantially lighter than strange. We have also not attempted a continuum limit extrapolation of our lattice results. Although we are using a lattice formalism that should improve this extrapolation, it would be safer to test it directly. The lattice non-perturbative results do, however, show clearly the structure of the mixing in the singlet pseudoscalar mesons.

References

[1] H. Leutwyler, Nucl. Phys. Proc. Suppl. 64 (1998) 223; R. Kaiser and H. Leutwyler, hep-ph/9806336; T. Feldmann et al., Phys. Rev. D58 (1998) and Phys. Lett B449 (1999) 339. For a review see T. Feldmann, Int.J.Mod.Phys. A15 (2000) 159, hep-ph/9907491

[2] Y. Kuramashi et al., Phys. Rev. Lett. 72, 3348 (1994).

[3] S. Güsken, hep-lat/9906034

[4] UKQCD Collaboration, C. R. Allton et al., Phys. Rev. D49, 474 (1994).

[5] T. Bhattacharya, R. Gupta, G. Kilcup and S. Sharpe, Phys. Rev. D53 (1996) 6486.

[6] UKQCD Collaboration, C. R. Allton et al., Phys. Rev. D60, 034507 (1999).

[7] W. Bardeen et al., Nucl. Phys. B (Proc. Suppl.) 73 (1999) 243; ibid 83-84 (2000) 215.

[8] C. Michael, M. S. Foster and C. McNeile (UKQCD Collaboration), Nucl. Phys. B (Proc. Suppl.) 83-84 (2000) 185, hep-lat/9909036

[9] SESAM-TχL Collaboration, T. Struckmann et al., hep-lat/0006012.

[10] UKQCD Collaboration, H. Shanahan et al., Phys. Rev. D 55, 1548 (1997).

[11] CP-PACS Collaboration, A. Ali-Khan et al., Nucl. Phys. B (Proc. Suppl.) 83, (2000) 162, hep-lat/9909045

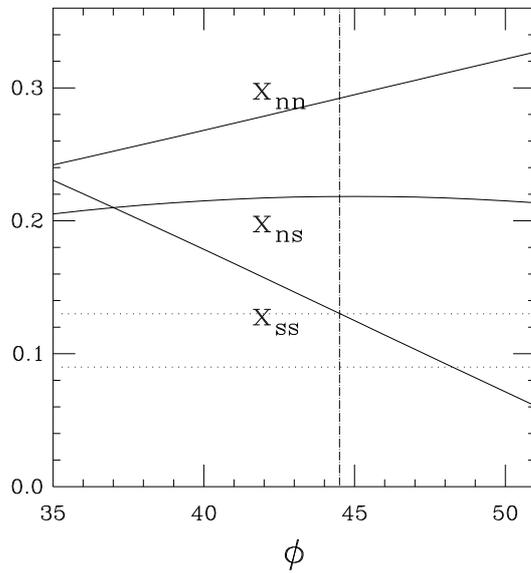[12] Partice Data Group, C. Caso et al., Euro. Phys. Jour. C3 (1998) 1.

Figure 1: The mass mixing parameters x in GeV^2 versus η, η' mixing angle ϕ in the η_{nn}, η_{ss} basis. The horizontal dotted lines give the allowed range from the lattice determination of x_{ss}. The vertical line illustrates our preferred solution.