Fayet-Iliopoulos D term and its renormalization in softly broken supersymmetric theories

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We consider the renormalization of the Fayet-Iliopoulos D term in a softly broken Abelian supersymmetric theory, and calculate the associated β function through three loops. We show that there exists (at least through three loops) a renormalization group invariant trajectory for the coefficient of the D term, corresponding to the conformal anomaly solution for the soft masses and couplings.

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I. INTRODUCTION

In Abelian gauge theories with $N=1$ supersymmetry there exists a possible invariant that is not allowed in the non-Abelian case: the Fayet-Iliopoulos D term,

$$L=\xi\int V(x,\theta,\overline{\theta})d^4\theta=\xi D(x). \tag{1.1}$$

In this paper we discuss the renormalization of ξ in the presence of the standard soft supersymmetry-breaking terms

$$L_{\mathrm{SB}}=-(m^2)^j_i\phi^i\phi_j -\left(\frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k+\frac{1}{2}b^{ij}\phi_i\phi_j+\frac{1}{2}M\lambda\lambda+\mathrm{H.c.}\right). \tag{1.2}$$

Let us begin by reviewing the position when there is no supersymmetry breaking, i.e. for $L_{\mathrm{SB}}=0$. Many years ago, Fischler *et al.* [1] proved an important result concerning the renormalization of the D term (see also Ref. [2]). Since it is a $\int d^4\theta$-type term, one may expect that the D term will undergo renormalization in general. Moreover, by simple power counting it is easy to show that the said renormalization is in general quadratically divergent. Evidently this poses a naturalness problem since (if present) it would introduce the cutoff mass scale into the scalar potential. At the one loop level it is easy to show that the simple condition $\mathrm{Tr}\,\mathcal{Y}=0$ (where $\mathcal{Y}$ is the U_1 hypercharge and the trace is taken over the chiral supermultiplets) removes the divergence. Remarkably, although one may of course easily draw individual diagrams proportional (for example) to $\mathrm{Tr}\,\mathcal{Y}^5,\mathcal{Y}^7$, etc., this condition suffices to all orders.

In the presence of supersymmetry breaking, however, it is clear that ξ will suffer *logarithmic* divergences. If calculations are done in the component formalism with D eliminated by means of its equation of motion, then these divergences are manifested via contributions to the β function for m^2. It is in this manner that the results for the soft β functions were given in, for example, Ref. [3]. Here we prefer to consider the renormalization of ξ separately; an advantage of this is that it means that the exact results for the soft β functions presented in Refs. [4–7] (see also Refs. [8,9]) apply without change to the Abelian case. The result for β_ξ is as follows:

$$\beta_\xi=\frac{\beta_g}{g}\xi+\hat{\beta}_\xi \tag{1.3}$$

where $\hat{\beta}_\xi$ is determined by V-tadpole (or in components D-tadpole) graphs, and is independent of ξ. In the supersymmetric case, we have $\hat{\beta}_\xi=0$, whereupon Eq. (1.3) is equivalent to the statement that the D term, Eq. (1.1), is unrenormalized. In the presence of Eq. (1.2), however, $\hat{\beta}_\xi$ depends on m^2, h and M (it is easy to see that it cannot depend on b). The main result of this paper is a complete calculation of $\hat{\beta}_\xi$ through three loops; it is interesting that the dependence on h and M arises first at this order. (A partial calculation was presented in Ref. [10].)

Although in this paper we restrict ourselves to the Abelian case, it is evident that a D term can occur with a direct product gauge group ($G_1\otimes G_2\cdots$) if there is an Abelian factor: as is the case for the minimal supersymmetric standard model (MSSM). In the MSSM context one may treat ξ as a free parameter at the weak scale [11], in which case there is no need to know $\hat{\beta}_\xi$. However, if we know ξ at gauge unification, then we need $\hat{\beta}_\xi$ to predict ξ at low energies. Now in the D-uneliminated case it is possible to express all the β functions associated with the soft supersymmetry-breaking terms given in Eq. (1.2) in terms of the gauge β function β_g, the chiral supermultiplet anomalous dimension γ and a certain function X which appears only in β_{m^2}; moreover in a special renormalization scheme [the Novikov-Shifman-Vainshtein-Zakharov (NSVZ) scheme], β_g can also be expressed in terms of γ, and X takes a particularly simple form [7,12]. It is clearly of interest to ask whether an analogous exact expression exists for β_ξ. Moreover, there exists an exact solution to the soft renormalization group (RG) equations for m^2, M and h corresponding to the case when all the supersymmetry breaking arises from the conformal anomaly [13] and it is also interesting to ask whether this solution can be extended to the non-zero ξ case.

The key to the derivation of the exact results for the soft β functions is the spurion formalism. The obstacle to deriving an analogous result for $\hat{\beta}_\xi$ is the fact that individual superspace diagrams are (as already mentioned) quadratically divergent. We do, however, present a solution for ξ related to the conformal anomaly solution, but which must be constructed order by order in perturbation theory.

II. RENORMALIZATION AND NON-PROPAGATING FIELDS

A. Non-supersymmetric case

This paper is concerned with the renormalization of the coefficient of an auxiliary field term, and it is perhaps useful to begin with a (we hope) pedagogical discussion of this in a non-supersymmetric context. One often sees the statement that the field theory

$$L=\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}F^2+\frac{1}{2}hF\phi^2, \tag{2.1}$$

where $\phi^2=\Sigma_{a=1}^{a=N}\phi^a\phi^a$, is equivalent to the theory

$$L=\frac{1}{2}(\partial_\mu\phi)^2-\frac{\lambda'}{24}\phi^4 \tag{2.2}$$

[where $\phi^4=(\phi^2)^2$], by virtue of the equation of motion for the non-propagating field F, which is

$$F=-\frac{1}{2}h\phi^2 \tag{2.3}$$

so that

$$\lambda'=3h^2. \tag{2.4}$$

There is a trap for the unwary here, however, in that Eq. (2.1) is not multiplicatively renormalizable, and as a consequence Eq. (2.4) is not renormalization group invariant. Let us replace Eq. (2.1) by

$$L=\frac{1}{2}(\partial_\mu\phi)^2+\frac{1}{2}F^2+\frac{1}{2}hF\phi^2-\frac{\lambda}{24}\phi^4. \tag{2.5}$$

We then obtain (eliminating F)

$$\lambda'=\lambda+3h^2, \tag{2.6}$$

and it follows that

$$\beta_{\lambda'}=\beta_\lambda+6h\beta_h, \tag{2.7}$$

which is easy to verify at one loop by direct calculation:

$$16\pi^2\beta_{\lambda'}=\frac{N+8}{3}\lambda'^2 \tag{2.8a}$$

$$16\pi^2\beta_\lambda=\frac{N+8}{3}\lambda^2+12\lambda h^2+12h^4 \tag{2.8b}$$

$$16\pi^2\beta_h=\frac{N+4}{2}h^3+\frac{N+2}{3}h\lambda \tag{2.8c}$$

and it is easy to see that Eq. (2.7) indeed holds. The minor subtlety here is that β_λ does not vanish when $\lambda=0$, so that the naive relation Eq. (2.4) is *not* RG invariant. Consequently, if we set $\lambda=0$, then Eqs. (2.8a) and (2.8c) and are not compatible with the (naive) result of taking $\mu\, d/d\mu$ of Eq. (2.4).

One may generalize this example as follows, by introducing a mass for ϕ and a linear F term:

$$\begin{aligned}L&=\frac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}m^2\phi^2+\frac{1}{2}F^2+\xi F+\frac{1}{2}hF\phi^2-\frac{\lambda}{24}\phi^4\\&=\frac{1}{2}(\partial_\mu\phi)^2-\frac{1}{2}m'^2\phi^2-\frac{\lambda'}{24}\phi^4\end{aligned} \tag{2.9}$$

where

$$m'^2=m^2+h\xi. \tag{2.10}$$

We now have the additional identity

$$\beta_{m'^2}=\beta_{m^2}+h\beta_\xi+\xi\beta_h \tag{2.11}$$

which can be verified at one loop using the results

$$\begin{aligned}16\pi^2\beta_{m'^2}&=\frac{N+2}{3}\lambda' m'^2\\16\pi^2\beta_\xi&=\frac{N}{2}h^2\xi+Nhm^2\\16\pi^2\beta_{m^2}&=\frac{N+2}{3}\lambda m^2+2h^2m^2\end{aligned} \tag{2.12}$$

together with the result for β_h which is unaffected.

B. Supersymmetric case: *D* terms

After this warm-up exercise, let us turn to a softly broken Abelian supersymmetric gauge theory. The relevant part of the Lagrangian is as follows:

$$L=\frac{1}{2}D^2+\xi D+gD\phi^*\mathcal{Y}\phi-\phi^*m^2\phi+\cdots \tag{2.13}$$

where $\mathcal{Y}^i{}_j$ is the charge matrix of the chiral supermultiplet, and m^2 is a supersymmetry-breaking term. After eliminating D this becomes

$$L=-\phi^*\overline{m^2}\phi-\frac{1}{2}g^2(\phi^*\mathcal{Y}\phi)^2, \tag{2.14}$$

where

$$\overline{m^2}=m^2+g\xi\mathcal{Y}. \tag{2.15}$$

RG invariance of this result gives

$$\begin{aligned}\beta_{\overline{m^2}}(\overline{m^2},\ldots)&=\beta_{m^2}(m^2,\ldots)+\beta_g\xi\mathcal{Y}+g\beta_\xi\mathcal{Y}\\&=\beta_{m^2}(m^2,\ldots)+2\beta_g\xi\mathcal{Y}+g\mathcal{Y}\hat{\beta}_\xi(m^2,\ldots),\end{aligned} \tag{2.16}$$

where

$$\beta_\xi=\frac{\beta_g}{g}\xi+\hat{\beta}_\xi \tag{2.17}$$

with $\hat{\beta}_\xi$ independent of ξ. For a derivation of Eq. (2.17), see Ref. [10]. What about the pitfall in the toy model which led us to introduce λ? We are saved by supersymmetry: if we add a ϕ^4 term to Eq. (2.13), then supersymmetry would be broken, at the dimension 4 level; contrariwise, if we omit it, then it will not be generated. Therefore, Eq. (2.16) is valid.

There is an important distinction between $\beta_{\overline{m}^2}(\overline{m}^2,\ldots)$ and $\beta_{m^2}(m^2,\ldots)$, which both appear in Eq. (2.16), and determine the mass renormalization with D eliminated and uneliminated respectively. Because β functions are determined by one particle irreducible (1PI) diagrams, β_{m^2} does not contain any D-tadpole contributions; the renormalization of these is dealt with separately by β_ξ. However, in the D-eliminated formalism, there is no β_ξ, and there is a distinct set of contributions to $\beta_{\overline{m}^2}$ involving the four-point vertex created by eliminating D. It follows that

$$\beta_{\overline{m}^2}(\overline{m}^2,\ldots)=\beta_{m^2}(\overline{m}^2,\ldots)+g\mathcal{Y}\hat{\beta}_\xi(\overline{m}^2,\ldots), \tag{2.18}$$

since diagrams corresponding to one or more insertions of a D-tadpole-type contribution on the *internal* line of a diagram do not contribute to the β function because the corresponding Feynman integral is factorized [14].

Let us now define our notation for the calculation. We take an Abelian $N=1$ supersymmetric gauge theory with superpotential

$$W(\Phi)=\frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k+\frac{1}{2}\mu^{ij}\Phi_i\Phi_j, \tag{2.19}$$

and at one loop we have

$$16\pi^2\beta_g^{(1)}=g^3Q=g^3\mathrm{Tr}[\mathcal{Y}^2], \tag{2.20a}$$

$$16\pi^2\gamma^{(1)i}{}_j=P^i{}_j=\frac{1}{2}Y^{ikl}Y_{jkl}-2g^2(\mathcal{Y}^2)^i{}_j. \tag{2.20b}$$

In the spurion formalism the soft-breaking Lagrangian is given by

$$\begin{aligned}L_{\mathrm{soft}}=-\bigg[\int d^2\theta\theta^2\bigg(&\frac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k\\&+\frac{1}{2}b^{ij}\Phi_i\Phi_j+\frac{1}{2}MW^\alpha W_\alpha\bigg)+\mathrm{H.c.}\bigg]\\&-\int d^4\theta(m^2)^i{}_j\theta^2\overline{\theta}^2\overline{\Phi}_i e^{-g\mathcal{Y}V}\Phi^j,\end{aligned} \tag{2.21}$$

where V is the vector superfield and W^α the corresponding field strength. The equivalent expression in terms of components is given in Eq. (1.2). With the explicit all orders result for β_{m^2}, we prove a remarkably simple result for $\hat{\beta}_\xi$. The aforementioned exact result for β_{m^2} is [6]

$$\begin{aligned}(\beta_{m^2})^i{}_j(m^2,\ldots)=\bigg[&2\mathcal{O}\mathcal{O}^*+2MM^*g^2\frac{\partial}{\partial g^2}+\tilde{Y}\frac{\partial}{\partial Y}\\&+\tilde{Y}^*\frac{\partial}{\partial Y^*}+X\frac{\partial}{\partial g}\bigg]\gamma^i{}_j,\end{aligned} \tag{2.22}$$

where

$$\mathcal{O}=\bigg(Mg^2\frac{\partial}{\partial g^2}-h^{lmn}\frac{\partial}{\partial Y^{lmn}}\bigg), \tag{2.23}$$

$$\tilde{Y}^{ijk}=(m^2)^i{}_lY^{ljk}+(m^2)^j{}_lY^{ilk}+(m^2)^k{}_lY^{ijl} \tag{2.24}$$

and (in the NSVZ scheme)

$$16\pi^2X^{\mathrm{NSVZ}}=-2g^3\mathrm{Tr}[m^2\mathcal{Y}^2]. \tag{2.25}$$

Once again we should emphasize that, whereas in a non-Abelian theory Eq. (2.22) holds in both the D-eliminated and D-uneliminated formalisms, in a theory with Abelian factors it is only true for D uneliminated.

It is now easy to show that

$$\beta_{m^2}(\overline{m}^2,\ldots)=\beta_{m^2}(m^2,\ldots). \tag{2.26}$$

This follows simply by substituting for $\overline{m}^2$ from Eq. (2.15) and then using the facts that

$$(\mathcal{Y})^i{}_lY^{ljk}+(\mathcal{Y})^j{}_lY^{ilk}+(\mathcal{Y})^k{}_lY^{ijl}=0 \tag{2.27}$$

by gauge invariance and

$$\mathrm{Tr}(\mathcal{Y}^3)=0 \tag{2.28}$$

for anomaly cancellation.

The result for X, Eq. (2.25), applies in the NSVZ scheme, which is one of a class of schemes related by redefinitions of g and M, the ramifications of which are described in Ref. [4]. Now X transforms non-trivially under these redefinitions [7], but it can be shown using Eqs. (2.27), (2.28) that X is unchanged by the replacement $m^2\to\overline{m}^2$ in any member of this class of schemes; consequently Eq. (2.26) always applies. We then find immediately from Eqs. (2.16), (2.18) that

$$\hat{\beta}_\xi(\overline{m}^2,\ldots)=2\frac{\beta_g}{g}\xi+\hat{\beta}_\xi(m^2,\ldots). \tag{2.29}$$

Now on dimensional grounds we may write

$$\begin{aligned}\hat{\beta}_\xi=&m^2A_1(g,Y,Y^*)+hh^*A_2(g,Y,Y^*)+MM^*A_3(g,Y,Y^*)\\&+(Mh^*+M^*h)A_4(g,Y,Y^*),\end{aligned} \tag{2.30}$$

where we have suppressed $(i,j,\ldots)$ indices for simplicity. [In the conventional dimensional reduction (DRED) scheme, $\hat{\beta}_\xi$ will also depend on the ϵ scalar (mass)^2, $\tilde{m}^2$, and this dependence, as we shall see, arises first at three loops. Our three-loop result, therefore, will be in the DRED′ scheme [15].] Hence we have at once that

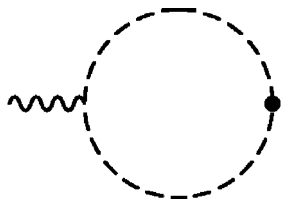

FIG. 1. Feynman diagram for the one-loop calculation in components. Dashed lines are scalar propagators and the external field is a D. Blobs denote m^2 insertions.

$$\mathrm{Tr}(\mathcal{Y}A_1)=2\frac{\beta_g}{g^2}. \tag{2.31}$$

So if we take the D-tadpole contributions to β_ξ, then the terms proportional to m^2 will reduce to $2\beta_g/g$ if we replace m^2 by $g\mathcal{Y}$. This result is, in fact, clear from a diagrammatic point of view, since the aforesaid replacement converts the diagrams into D self-energy graphs, and hence indeed gives rise to β_g.

III. ONE LOOP CALCULATION

Here we describe the one-loop calculation of $\hat{\beta}_\xi$; this is straightforward, of course. In a softly broken theory, the $\hat{\beta}_\xi$ calculation may be carried out in components or using the superfield spurion formalism. Usually, superfield techniques (once mastered) offer a substantial reduction in labor compared to component calculations; we will begin to suspect from the one loop calculation, however, and confirm in the next section, that this is not the case here.

In components there is a single diagram, shown in Fig. 1, and we have

$$\begin{aligned}\text{Fig. }1&=-g(\mathcal{Y})^i{}_j\int d^dk\left(\frac{1}{k^2+m^2}\right)^j{}_i\\&=-g(\mathcal{Y})^i{}_j\int d^dk\left(\frac{1}{k^2}\delta^j{}_i-\frac{1}{k^4}(m^2)^j{}_i+\cdots\right)\\&=-g\,\mathrm{Tr}(\mathcal{Y})\int\frac{d^dk}{k^2}+g\,\mathrm{Tr}(\mathcal{Y}m^2)\frac{2}{16\pi^2(4-d)}\\&\quad+\text{finite terms},\end{aligned} \tag{3.1}$$

where our integration measure d^dk includes the usual $(2\pi)^{-d}$ factor. Note that we present this Feynman integral calculation (and subsequent ones) in Euclidean space. In order to extract the ultraviolet divergence from the logarithmically divergent term in Eq. (3.1) we have made the replacement

$$\int\frac{d^dk}{k^4}\to\int\frac{d^dk}{(k^2+m_0^2)^2} \tag{3.2}$$

where m_0 is an infrared (IR) regulator mass. Naturally we could have directly evaluated the diagram without first expanding in powers of m^2, but this procedure would be cumbersome at higher loops; it is simpler to treat m^2 as an insertion and introduce regulator masses only for those propagators which are IR dangerous. This technique was described in Ref. [16], and is generally more convenient than the alternative of "threading" a single momentum through the diagram [17]. The pole result for a graph of any number of loops, *when all sub-divergences are subtracted*, is independent of the precise details of how the IR divergences are regulated. All this is, of course, well known to higher-loop calculators but may, perhaps, be of some pedagogical interest.

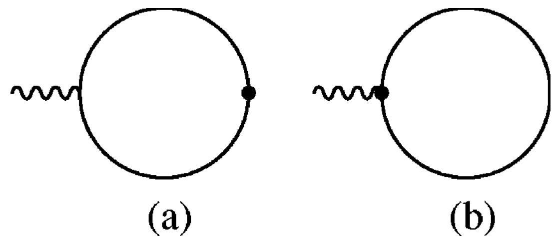

FIG. 2. Feynman diagrams for the one-loop calculation in superspace. Solid lines are chiral propagators and the external lines are vector superfields. Blobs denote m^2 insertions.

We see that to remove the quadratic divergence we must have $\mathrm{Tr}\,\mathcal{Y}=0$, and that at one loop

$$\hat{\beta}_\xi=\frac{1}{16\pi^2}g\,\mathrm{Tr}(\mathcal{Y}m^2). \tag{3.3}$$

In the superfield spurion calculation we have two graphs, shown in Fig. 2.

The results are as follows:

$$\begin{aligned}\text{Fig. }2\text{a}=-g\int d^dk\int d^4\theta\, V(\theta,\overline{\theta})D^2\Bigg[\mathrm{Tr}(\mathcal{Y})\frac{\delta^{(4)}(\theta-\theta')}{k^2}\\+\mathrm{Tr}(\mathcal{Y}m^2)\frac{1}{k^4}\overline{D}^2\theta^2\overline{\theta}^2D^2\delta^{(4)}(\theta-\theta')\Bigg]\overline{D}^2\Big|_{\theta=\theta'}\end{aligned} \tag{3.4}$$

while

$$\begin{aligned}\text{Fig. }2\text{b}&=-g\,\mathrm{Tr}(\mathcal{Y}m^2)\int d^dk\int d^4\theta\,\theta^2\overline{\theta}^2V(\theta,\overline{\theta})D^2\\&\quad\times\frac{\delta^{(4)}(\theta-\theta')}{k^2}\overline{D}^2\Big|_{\theta=\theta'}\\&=-g\,\mathrm{Tr}(\mathcal{Y}m^2)\int d^4\theta\,\theta^2\overline{\theta}^2V(\theta,\overline{\theta})\int\frac{d^dk}{k^2}.\end{aligned} \tag{3.5}$$

The first term from Eq. (3.4) vanishes via $\mathrm{Tr}\,\mathcal{Y}=0$, and the second may be reduced using the identity Eq. (A5) to give

$$\text{Fig. }2\text{a}=g\,\mathrm{Tr}(\mathcal{Y}m^2)\int\frac{d^dk}{k^4}\int d^4\theta V(\theta,\overline{\theta})e^{2\theta\not{k}\overline{\theta}}. \tag{3.6}$$

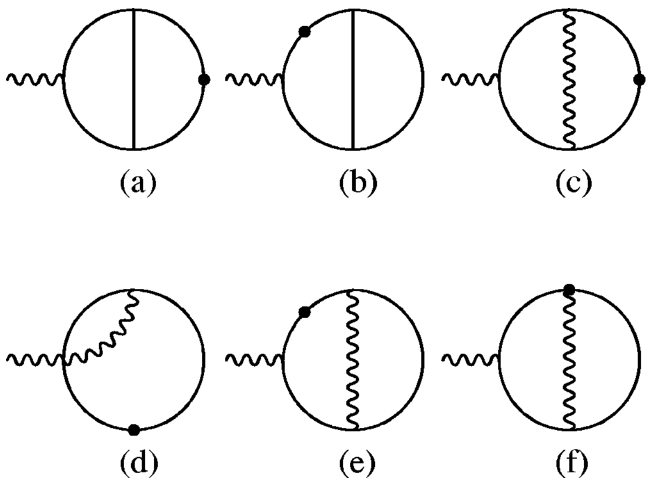

FIG. 3. Feynman diagrams for the two-loop calculation in superspace. Solid lines are chiral propagators and internal wavy lines are vector propagators. Blobs denote m^2 insertions.

If we expand the exponential in Eq. (3.6), the quadratically divergent $\theta^2\overline{\theta^2}$ term cancels Fig. 2b, while the remaining term reproduces the component calculation, Eq. (3.1).

IV. TWO LOOP CALCULATION

In this section we discuss the two-loop calculation of $\hat{\beta}_\xi$ in some detail. Calculations of β functions for soft-breaking parameters may be carried out in components or using the spurion formalism. Indeed, as mentioned earlier, in the case of β_h, β_M and β_{m^2} the fact that the spurion diagrams are only logarithmically divergent means that these quantities have simple all-orders expressions in terms of γ and β_g. However, as we have emphasized, individual diagrams contributing to β_ξ are quadratically divergent. This means that if, for example, we represent a h^{ijk} vertex in superspace by $h^{ijk}\theta^2$, then we cannot simply factor the θ^2 out, because it can be ''hit'' by a superspace D derivative; indeed, as is clear from the one-loop calculation, the contribution when the θ^2 is not ''hit'' will not give a logarithmic divergence, and must cancel. The simple relationship between a graph with a h^{ijk} and the corresponding one with a supersymmetric Yukawa vertex which holds for the soft breaking β functions is thereby lost. Nevertheless, the spurion formalism may still be used. In this section we shall describe both the spurion approach and the component calculation. Normally a superspace calculation would be expected to be more efficient than the component version. In this case, however, we shall see that the advantages of the spurion calculation are by no means so obvious. The fact that in components the D insertion can only be on a scalar line considerably reduces the number of diagrams in this case.

The two-loop diagrams in the spurion formalism are depicted in Fig. 3. Standard superspace manipulations are used to reduce the graphs to basic momentum integrals, together with a single remaining $\int d^4\theta$; by power counting, the logarithmically divergent contributions come from terms with no θ's and $\overline{\theta}$'s remaining in the integrand. Some useful identities are collected in Appendix A. Note that we have omitted graphs with a mass insertion on the leftmost vertex, where the external V is attached; these graphs, like Fig. 2b, do not contribute to the logarithmic divergence, and are canceled by the quadratic divergences (terms with an integrand involving $\theta^2\overline{\theta^2}$) from the graphs shown. We have also omitted a graph like Fig. 3d, but with the mass insertion on the rightmost vertex, because it also gives rise to a quadratic divergence only. The divergent contributions to ξ^B from each graph are listed in Table I.

TABLE I. Results for two-loop Feynman diagrams.

	a	b	c	d	e	f
Fig. 3	$-JS_1$	0	$8JS_2$	$-4JS_2$	$4JS_2$	$-4JS_2$
Fig. 5	$-JS_1$	$2JS_2$	$4JS_2$	$-2JS_2$		

Here J denotes the standard two loop momentum integral shown in Fig. 4, and also

$$S_1=\mathrm{Tr}[\mathcal{Y}m^2Y^2],\quad S_2=g^2\mathrm{Tr}[\mathcal{Y}^3m^2]. \tag{4.1}$$

The calculation of J proceeds as follows (note that here and in all subsequent integrals we subtract all subdivergences):

$$\begin{aligned}J&=\int\frac{d^dkd^dq}{q^2(k-q)^2(k^2+m_0^2)^2}-\frac{2}{16\pi^2\epsilon}\int\frac{d^dk}{(k^2+m_0^2)^2}\\&=\frac{2}{(4\pi)^d\epsilon^2}\left(1+\frac{\epsilon}{2}-\gamma\epsilon\right)-\frac{4}{(4\pi)^{(d/2)+2}\epsilon^2}\left(1-\frac{\gamma\epsilon}{2}\right)\\&=\frac{1}{(16\pi^2)^2}\left(-\frac{2}{\epsilon^2}+\frac{1}{\epsilon}\right).\end{aligned} \tag{4.2}$$

We have ignored contributions of the form I^2, where

$$I=\int\frac{d^dk}{(k^2+m_0^2)^2}, \tag{4.3}$$

such as that from Fig. 3b, because I^2 has no simple pole after sub-divergence subtraction; I^2 is the simplest possible example of a *factorized* Feynman integral, which quite generally give no simple pole [14]. Subsequently we will ignore any graph which reduces to factorized form.

Thus using the simple pole given by

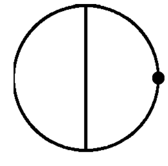

FIG. 4. Momentum integral for the two-loop calculation. The dot denotes a double propagator.

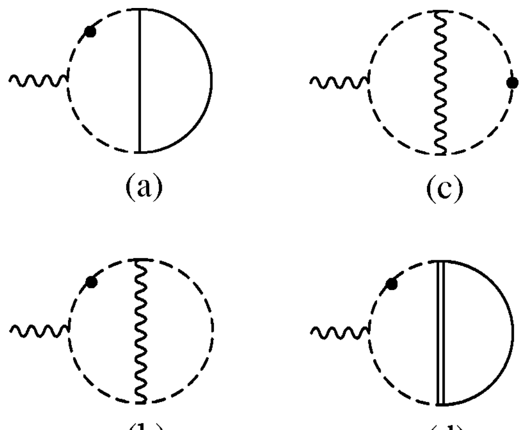

FIG. 5. Feynman diagrams for the two-loop calculation in components. Dashed lines are scalar propagators, solid lines are chiral fermion propagators, double solid lines are gaugino propagators and internal wavy lines are vector propagators. Blobs denote m^2 insertions.

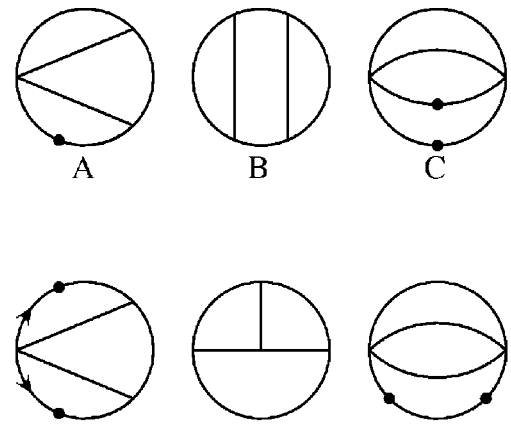

FIG. 6. Momentum integrals for the three-loop calculation. One (two) dots denote a double (triple) propagator. The two arrows in D denote contracted momenta.

$$J_{\rm simple} = \frac{1}{(16\pi^2)^2\epsilon}, \tag{4.4}$$

and recalling that to get the L-loop contribution to the β function we need to multiply the Feynman diagram simple pole result by L, we find that at two loops we have

$$16\pi^2\hat{\beta}_\xi = 2g\,{\rm Tr}[\mathcal{Y}m^2] - 4g\,{\rm Tr}[\mathcal{Y}m^2\gamma^{(1)}] + \cdots, \tag{4.5}$$

so we see that in fact only A_1 is non-zero through this order.

The calculation may equally well be performed in the component formalism. The relevant diagrams are shown in Fig. 5. As we mentioned earlier, there are relatively few, indeed fewer than in the spurion case. Their divergent contributions are again listed in Table I, and upon adding we find again the result of Eq. (4.5). It is apparent from Fig. 5 that there is no DRED or DRED′ distinction at this order, because the vector boson couples only via the $\phi^*\phi W_\mu$ vertex, which projects out the ϵ scalar. A further consistency check is provided by Eq. (2.31); since

$$16\pi^2\beta_g = g^3{\rm Tr}[\mathcal{Y}^2] - 2g^3{\rm Tr}[\mathcal{Y}^2\gamma^{(1)}] + \cdots, \tag{4.6}$$

we see that Eq. (4.5) is indeed consistent with Eq. (2.31). Finally, it is easy to verify that our result reproduces the relevant terms from the calculation of β_{m^2} (with D eliminated) presented in Refs. [3,18]. (The other two-loop calculation of the soft β functions [8] was performed with D uneliminated.)

V. THREE LOOP RESULTS

We have calculated $\hat{\beta}_\xi^{(3){\rm DRED}'}$ in full. As we found in the previous section, the calculation in terms of component fields is generally more straightforward than that using the spurion formalism. In the case of terms proportional to $m^2\mathcal{Y}Y^4$ we have performed both spurion and component calculations, while for $m^2\mathcal{Y}^3Y^2$-type terms we have used the spurion formalism, which could be streamlined by systematic use of the identities in Appendix A. Both these calculations were sensitive to the check provided by Eq. (2.31). The rest of the calculation was done using components. Although the number of diagrams is large, the amount of algebra involved in each diagram is not great.

In both component and superfield formalisms, every graph can be reduced to a sum of terms consisting of a product of a group theory factor and one of a set of logarithmically divergent three loop graphs, which are shown in Fig. 6.

These graphs may be evaluated by the introduction of infrared regulator masses as described for J in the previous section. The results for the simple pole contributions (after subtraction of subdivergences) are as follows:

$$A_{\rm simple} = \frac{4}{3}\frac{1}{(16\pi^2)^3\epsilon}, \quad B_{\rm simple} = -\frac{2}{3}\frac{1}{(16\pi^2)^3\epsilon},$$
$$C_{\rm simple} = \frac{2}{3}\frac{1}{(16\pi^2)^3\epsilon}, \quad D_{\rm simple} = -\frac{2}{3}\frac{1}{(16\pi^2)^3\epsilon},$$
$$E_{\rm simple} = 4\zeta(3)\frac{1}{(16\pi^2)^3\epsilon}, \quad F_{\rm simple} = -\frac{3}{4}\frac{1}{(16\pi^2)^3\epsilon}. \tag{5.1}$$

We relegate details of the calculation to Appendix B. The final result may be written as follows:

$$\begin{aligned}(16\pi^2)^3\frac{\hat{\beta}_\xi^{(3){\rm DRED}'}}{g} &= -6(16\pi^2)^2{\rm Tr}[\mathcal{Y}m^2\gamma^{(2)}] - 4\,{\rm Tr}[WP\mathcal{Y}]\\ &\quad -\frac{5}{2}{\rm Tr}[HH^*\mathcal{Y}] + 2\,{\rm Tr}[P^2m^2\mathcal{Y}] - 24g^2\zeta(3){\rm Tr}[W\mathcal{Y}^3]\\ &\quad +12\zeta(3)g^2{\rm Tr}[M^*H\mathcal{Y}^3 + {\rm c.c.}]\\ &\quad -144\zeta(3)g^4MM^*{\rm Tr}[\mathcal{Y}^5]\end{aligned} \tag{5.2}$$

where [3]

$$W^i{}_j=\left(\frac{1}{2}Y^2m^2+\frac{1}{2}m^2Y^2+h^2\right)^i{}_j+2Y^{ipq}Y_{jpr}(m^2)^r{}_q$$
$$-8g^2MM^*(\mathcal{Y}^2)^i{}_j, \tag{5.3}$$

$$H^i{}_j=h^{ikl}Y_{jkl}+4g^2M(\mathcal{Y}^2)^i{}_j \tag{5.4}$$

and

$$(16\pi^2)^2\gamma^{(2)i}{}_j=[-Y_{jmn}Y^{mpi}-2g^2(\mathcal{Y}^2)^p{}_j\delta^i{}_n]P^n{}_p$$
$$+2g^4\mathrm{Tr}[\mathcal{Y}^2](\mathcal{Y}^2)^i{}_j, \tag{5.5}$$

with $(Y^2)^i{}_j=Y^{ikl}Y_{jkl}, (h^2)^i{}_j=h^{ikl}h_{jkl}$. We can now check the m^2 terms in this result, using Eq. (2.31). Replacing m^2 by $g\mathcal{Y}$, we obtain

$$g(16\pi^2)^3\mathrm{Tr}(\mathcal{Y}A_1^{(3)})=6X_1+12X_3+2X_4$$
$$-12g^6\mathrm{Tr}[\mathcal{Y}^2]\mathrm{Tr}[\mathcal{Y}^4], \tag{5.6}$$

where

$$X_1=g^2Y^{klm}P^n{}_l(\mathcal{Y}^2)^p{}_mY_{knp},$$
$$X_3=g^4\mathrm{Tr}[P\mathcal{Y}^4],$$
$$X_4=g^2\mathrm{Tr}[P^2\mathcal{Y}^2], \tag{5.7}$$

in precise agreement with the result for $\beta_g^{(3)}$, given in [19], which for an Abelian theory is

$$(16\pi^2)^3\beta_g^{(3)\mathrm{DRED}}=g\{3X_1+6X_3+X_4-6g^6\mathrm{Tr}[\mathcal{Y}^2]\mathrm{Tr}[\mathcal{Y}^4]\}. \tag{5.8}$$

(Of course for β_g there is no distinction between DRED and DRED′.) Note that $\hat{\beta}_\xi^{(3)\mathrm{DRED}}$ would only differ from $\hat{\beta}_\xi^{(3)\mathrm{DRED}'}$ by the inclusion of terms of the form $g^5\tilde{m}^2\mathrm{Tr}[\mathcal{Y}^5]$ and $g^3\tilde{m}^2\mathrm{Tr}[P\mathcal{Y}^3]$, arising from ϵ-scalar mass insertions. We have not calculated these explicitly because it is clear they can be removed by a redefinition of m^2, as follows:

$$\delta m^2=-2\frac{g^2}{16\pi^2}\tilde{m}^2\mathcal{Y}^2+\alpha_1\left(\frac{g^2}{16\pi^2}\right)^2\tilde{m}^2\mathcal{Y}^4$$
$$+\alpha_2\frac{g^2}{(16\pi^2)^2}\tilde{m}^2P\mathcal{Y}^2, \tag{5.9}$$

where the first term was derived in [15]. It would be interesting to verify that the appropriate redefinition also renders the three-loop contribution to β_{m^2} independent of $\tilde{m}^2$.

Finally, let us compare our result with the form of $\hat{\beta}_\xi^{(3)\mathrm{DRED}'}$ that we obtained in Ref. [10] (note that we did not there distinguish DRED from DRED′). We see that our result Eq. (5.2) indeed confirms the conjectured form given in Eq. (4.10) of Ref. [10], and that the two then undetermined constants are given by $\nu_1=24\zeta(3)$ and $\nu_2=0$.

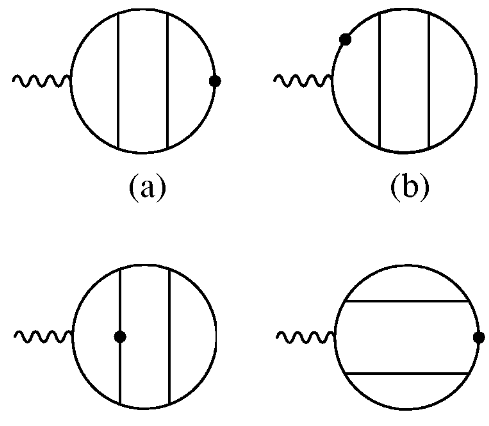

FIG. 7. Feynman diagrams in superspace for the three-loop contribution of the form $m^2Y^4\mathcal{Y}$, i.e. $T_{1\ldots3}$. Blobs denote m^2 insertions.

VI. CONFORMAL ANOMALY TRAJECTORY

The following set of equations provide an exact solution to the renormalization group equations for M, h and m^2:

$$M=M_0\frac{\beta_g}{g}, \tag{6.1a}$$

$$h^{ijk}=-M_0\beta_Y^{ijk}, \tag{6.1b}$$

$$(m^2)^i{}_j=\frac{1}{2}|M_0|^2\mu\frac{d\gamma^i{}_j}{d\mu}. \tag{6.1c}$$

Moreover, these solutions indeed hold if the only source of supersymmetry breaking is the conformal anomaly, when M_0 is in fact the gravitino mass.

This set of soft breakings has generated considerable interest, but there are clear difficulties for the MSSM, since it is easy to see that sleptons are predicted to have negative $(\mathrm{mass})^2$. Most studies of this scenario have resolved this dilemma by adding a constant m_0^2, presuming another source of supersymmetry breaking. A non-zero ξ alone is not an alternative, unfortunately, as is easily seen from Eq. (2.15); the two selectrons, for example, have oppositely signed hypercharge, so one of them at least remains with negative $(\mathrm{mass})^2$. This stumbling block may be overcome by introducing an extra U_1 [20,21]; for alternative treatments see Refs. [13,22].

It is immediately obvious that, given Eq. (6.1), there is a RG invariant solution for ξ through two loops (for $\hat{\beta}_\xi$) given by

$$16\pi^2\xi=g|M_0|^2\mathrm{Tr}[\mathcal{Y}(\gamma-\gamma^2)], \tag{6.2}$$

since differentiating with respect to μ and using Eq. (6.1c) leads at once to Eqs. (2.17), (4.5). Interestingly, however, this result for ξ *vanishes* at leading and next-to-leading order, since one easily demonstrates that

$$\mathrm{Tr}[\mathcal{Y}\gamma^{(1)}]=0 \tag{6.3}$$

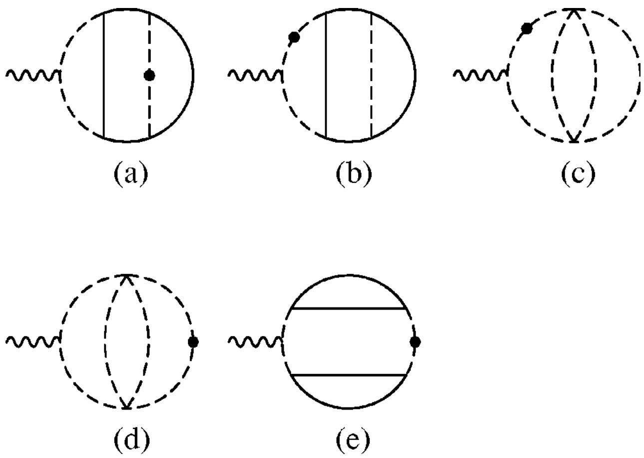

FIG. 8. Feynman diagrams in components for the three-loop contribution of the form $m^2Y^4\mathcal{Y}$, i.e. $T_{1\cdots 3}$.

and

$$\mathrm{Tr}[\mathcal{Y}\gamma^{(2)}]=\mathrm{Tr}[\mathcal{Y}(\gamma^{(1)})^2]. \tag{6.4}$$

It is interesting to ask whether the trajectory can be extended beyond two loops, and whether it in fact continues to vanish order by order. We have shown that there is indeed a generalization of Eq. (6.2) to at least three loops (for $\hat{\beta}_\xi$), and that at this order the result for ξ is non-zero.

Our result is as follows:

$$\frac{\xi^{\mathrm{DRED}'}}{g|M_0|^2}=(16\pi^2)^{-4}\{-3I_1-12\zeta(3)\times(I_2-2g^6\mathrm{Tr}[\mathcal{Y}^2]\mathrm{Tr}[\mathcal{Y}^5])\}, \tag{6.5}$$

where

$$I_1=\mathrm{Tr}[\mathcal{Y}P^3]-\frac{1}{2}(\mathcal{Y})^i{}_jY^{jkl}Y_{imn}P^m{}_kP^n{}_l+2g^2\mathrm{Tr}[\mathcal{Y}^3P^2]-2g^4\mathrm{Tr}[\mathcal{Y}^2]\mathrm{Tr}[\mathcal{Y}^3P]$$

$$I_2=g^2(\mathcal{Y}^3)^i{}_jY^{jkl}Y_{ikm}P^m{}_l+g^2\mathrm{Tr}[\mathcal{Y}^3P^2]+2g^4\mathrm{Tr}[\mathcal{Y}^5P]. \tag{6.6}$$

It is easy to verify that the result of taking $\mu\,\partial/\partial\mu$ of Eq. (6.5) is identical to that obtained by substituting Eqs. (6.1) in Eqs. (4.5), (5.2). This is a non-trivial result in that the number of candidate terms for inclusion in Eq. (6.5) is considerably less than the number of distinct terms which arise when Eqs. (4.5), (5.2) are placed on the RG trajectory. We therefore conjecture that the trajectory extends to all orders.

It is natural to ask what the result for $\hat{\beta}_\xi^{(3)}$ is in the NSVZ scheme, which is obtained (at the relevant order) by the redefinitions [4]

$$(16\pi^2)^2\delta g=-\frac{1}{2}g^3\mathrm{Tr}[P\mathcal{Y}^2]$$

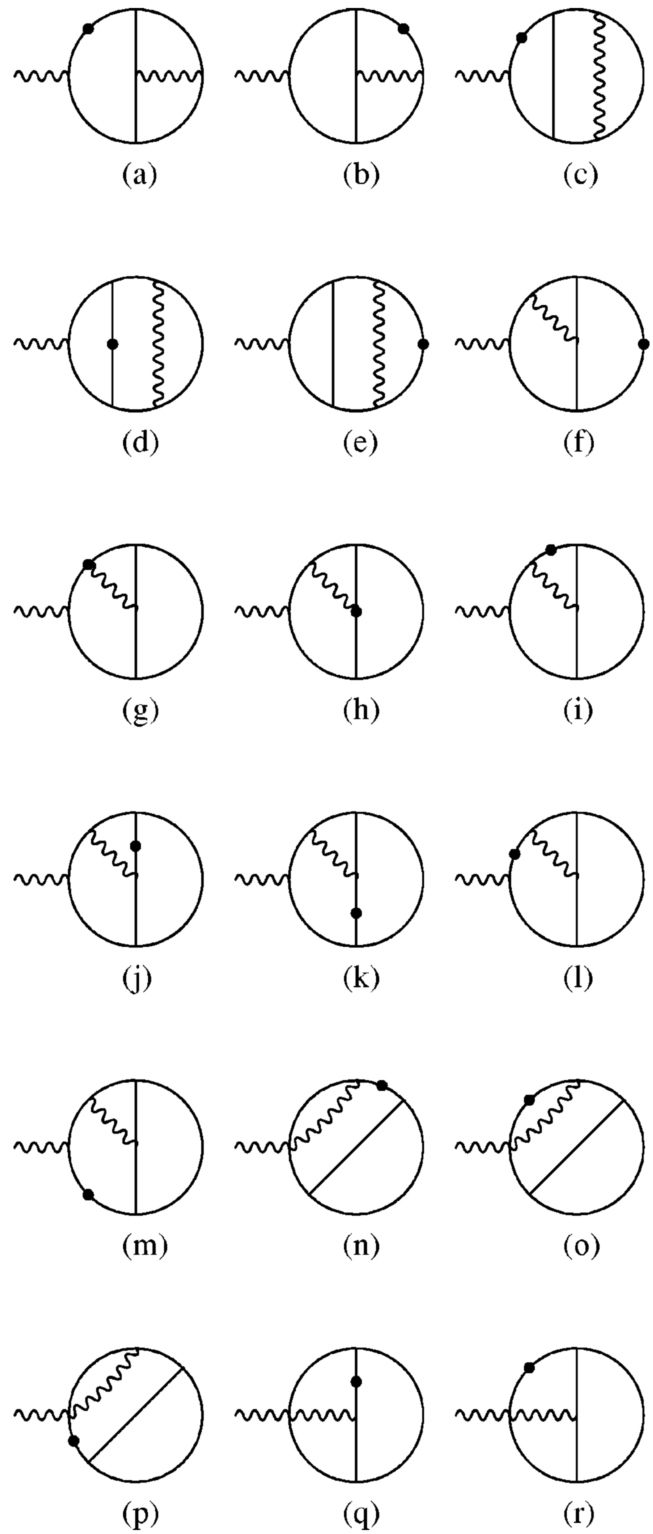

FIG. 9. Feynman diagrams in superspace for the three-loop contribution of the form $g^2m^2Y^2\mathcal{Y}^3$, i.e. $T_{4\cdots 6}$.

$$(16\pi^2)^2\delta M=-Mg^2\{\mathrm{Tr}[P\mathcal{Y}^2]-2g^2\mathrm{Tr}[(\mathcal{Y}^2)^2]\}+\frac{1}{2}g^2h^{ikl}Y_{jkl}(\mathcal{Y}^2)^j{}_i. \tag{6.7}$$

It is straightforward to show that in order to obtain the re-

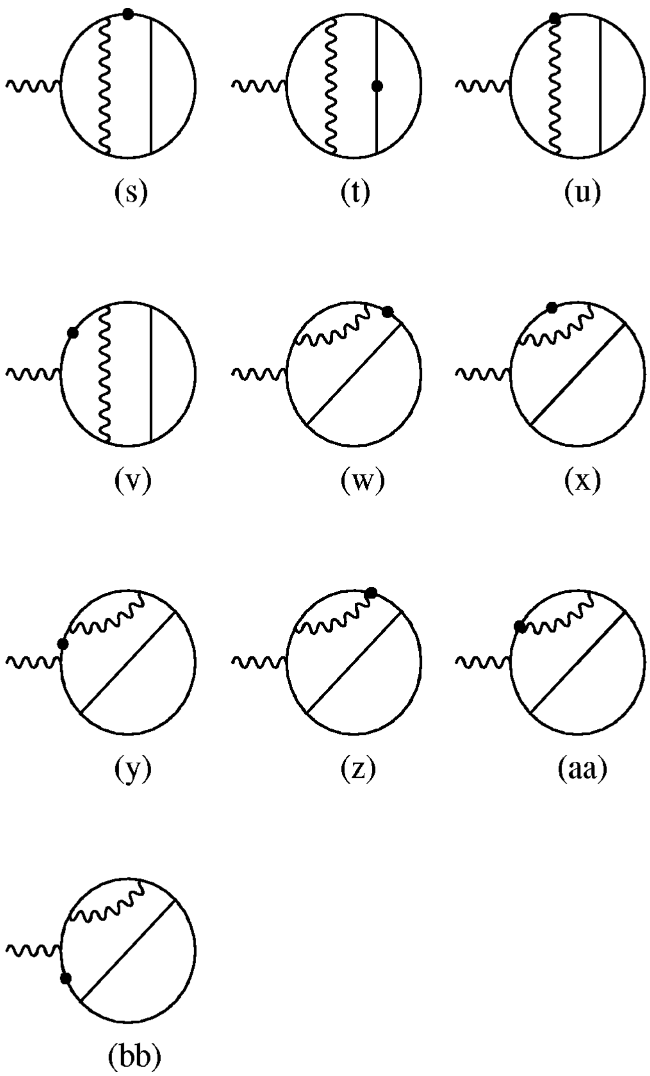

FIG. 9 (*Continued*).

sults, Eqs. (2.17) and (2.31), in the NSVZ scheme, we must also redefine ξ as follows:

$$(16\pi^2)^2\delta\xi = -\frac{1}{2}g^2\mathrm{Tr}[P\mathcal{Y}^2]\xi - g\mathrm{Tr}[m^2P\mathcal{Y}]. \quad (6.8)$$

The effect of this is to replace Eq. (6.5) by

$$\frac{\xi^{\mathrm{NSVZ}}}{g|M_0|^2} = (16\pi^2)^{-4}\{-4I_1 - 12\zeta(3)(I_2 - 2g^6\mathrm{Tr}[\mathcal{Y}^2]\mathrm{Tr}[\mathcal{Y}^5])\} \quad (6.9)$$

and Eq. (5.2) by

$$(16\pi^2)^3\frac{\hat{\beta}_\xi^{(3)\mathrm{NSVZ}}}{g} = -4(16\pi^2)^2\mathrm{Tr}[\mathcal{Y}m^2\gamma^{(2)}] - \frac{5}{2}(2\,\mathrm{Tr}[WP\mathcal{Y}] + \mathrm{Tr}[HH^*\mathcal{Y}]) - 24g^2\zeta(3)\mathrm{Tr}[W\mathcal{Y}^3] + 12\zeta(3)g^2\mathrm{Tr}[M^*H\mathcal{Y}^3 + \mathrm{c.c.}] - 144\zeta(3)g^4MM^*\mathrm{Tr}[\mathcal{Y}^5]. \quad (6.10)$$

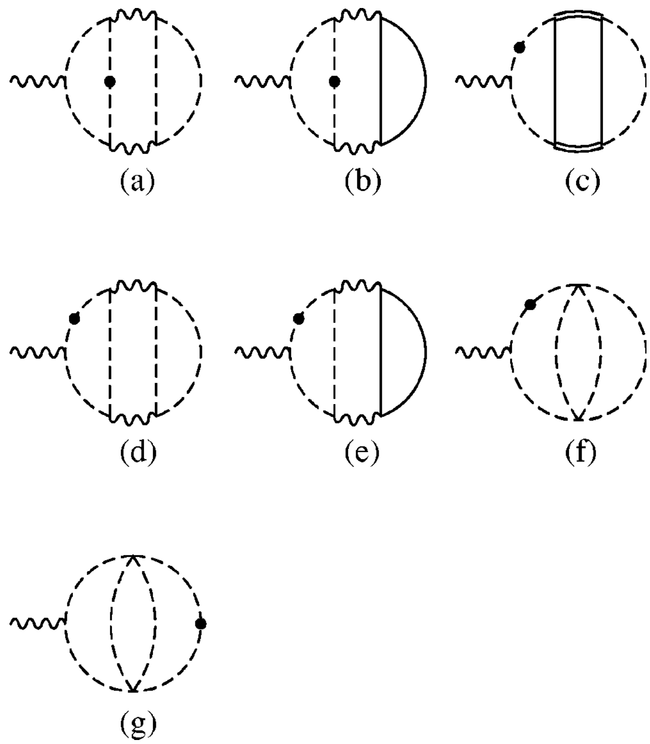

FIG. 10. Feynman diagrams in components for the three-loop contribution to T_8.

It is disappointing that this expression does not immediately suggest an all orders result. At this point it is worth recalling that, while to connect the DRED′ and NSVZ schemes via Eq. (6.7) we redefined g and M, there exists also a redefinition of Y [involving $\zeta(3)$] which has the pleasant property of extending to three loops the existence of finite $N=1$ theories [23]. Unfortunately this redefinition disturbs Eq. (2.22), which leads one to imagine that there might be a combined

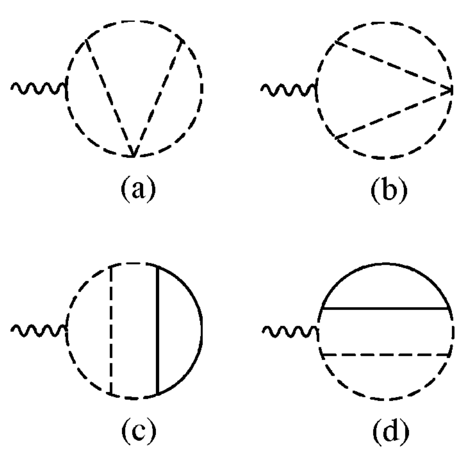

FIG. 11. Feynman diagrams in components for the three-loop contribution to T_9, T_{10}.

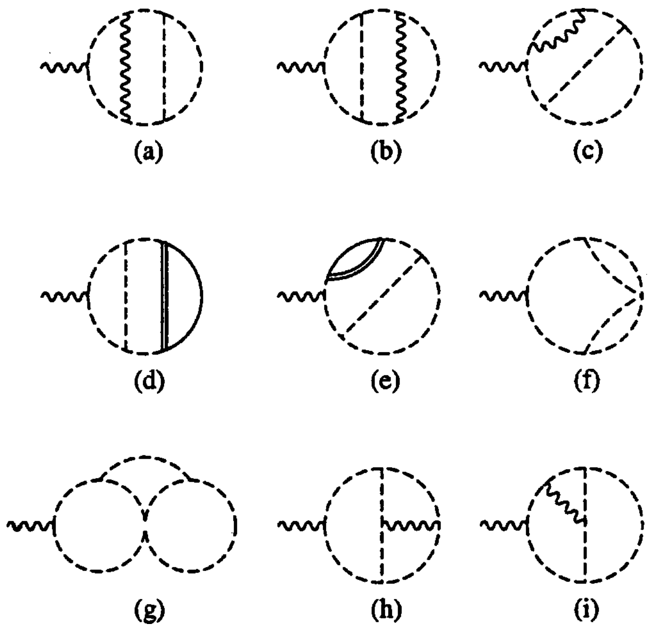

FIG. 12. Feynman diagrams in components for the three-loop contribution to T_{11}.

redefinition of m^2, Y that both preserves Eq. (2.22) and simplifies $\hat{\beta}_\xi^{(3)}$. We have not yet succeeded in constructing such a transformation.

VII. FINAL REMARKS

We have presented a detailed, and we hope a reasonably self-contained, description of the calculation of $\hat{\beta}_\xi$ through three loops. It is intriguing that in the Abelian case we are unable to express the renormalization of the theory completely in terms of β_g and γ, which, in the non-Abelian case, suffice to describe the renormalization of both the unbroken theory and also the theory with the standard soft terms. Although there exists perturbatively a solution related to the AMSB solution for the soft parameters, once again we are unable at the moment to extend this solution to all orders.

The next step is obviously an extension of our calculation to the case of a product gauge group including both Abelian and non-Abelian factors, such as the MSSM; this is not a trivial deduction from the results we have presented. Although it is clear that if ξ is assumed to be small at gauge

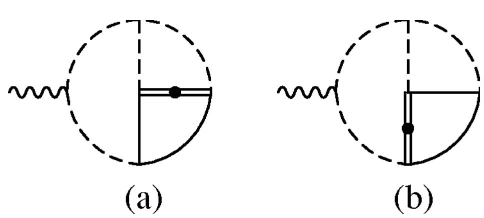

FIG. 13. Feynman diagrams in components for the three-loop contribution to T_{12}. Blobs denote gaugino mass insertions..

TABLE II. Results for Fig. 9.

a	$-2E$	$\frac{1}{2}T_4-T_5$
b	$-4A$	$\frac{1}{4}T_4+\frac{1}{2}T_5+T_6$
c	$-4A$	T_5
d	$-2C$	$-T_5-T_6$
e	$-2(2A+B+2D)$	$-\frac{1}{2}T_4$
f	$-4A$	$-\frac{1}{4}T_4+\frac{1}{2}T_5$
g	$4A$	$-\frac{1}{2}T_4$
h	$4A$	$\frac{1}{4}T_4-\frac{1}{2}T_5-T_6$
i	$-4(4A-C-E)$	$-\frac{1}{2}T_4$
j	$-4(2A-2D-E)$	$\frac{1}{4}T_4-\frac{1}{2}T_5-T_6$
k	$-8(A+D)$	$\frac{1}{4}T_4-\frac{1}{2}T_5-T_6$
l	$-4A$	$-\frac{1}{2}T_4$
m	$-4A$	$-\frac{1}{2}T_4$
n	$2B$	T_4
o	$2A$	T_4
p	$2B$	T_4
q	$4A$	$\frac{1}{4}T_4-\frac{1}{2}T_5-T_6$
r	$4A$	$-\frac{1}{2}T_4$
s	$-4(A+B)$	T_4
t	$-2(2A+B+2D)$	T_6
u	$2A$	T_4
v	$-2A$	T_4
w	$-2B$	T_4
x	$-4(A+B)$	T_4
y	$-2B$	T_4
z	$2B$	T_4
aa	$2B$	T_4
bb	$-2B$	T_4

unification, then it does not have much effect at low energies, it should be remembered that this is an assumption, and that the MSSM has one more parameter than is commonly supposed.

ACKNOWLEDGMENTS

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APPENDIX A: *D*-ALGEBRA IDENTITIES

In this appendix we list some identities that we found useful in superspace calculations of contributions to $\hat{\beta}_\xi$. (An early reference for superspace calculations incorporating soft breaking is Ref. [24].) The soft terms given in Eq. (2.21) are treated as insertions in the superfield diagrams and standard

TABLE III. Results for Fig. 10 (all multiplied by T_8).

	a+b	c	d+e	f	g
Fig. 10	$(C-4A-2B)$	$4(A-F)$	$(2B+2F-4A)$	$2F$	C

TABLE IV. Results for Fig. 12 (all multiplied by T_{11}).

	a	b	c	d	e	f	g	h	i
Fig. 12	$-A-B$	A	$-2B$	$-A$	$2B$	$\frac{1}{2}B$	$-A$	$\frac{1}{2}(4A-B-2E)$	$A-B-E$

superspace manipulations may then be used to reduce supergraphs to ordinary momentum space integrals. It is convenient to denote the momentum space version of the superspace covariant derivatives by

$$(D_{p1})_\alpha = \frac{1}{2}\left[\frac{\partial}{\partial\theta_1^\alpha} - \not p_{\alpha\dot\alpha}\overline{\theta}_1^{\dot\alpha}\right],$$

$$(\overline{D}_{p1})_{\dot\alpha} = -\frac{1}{2}\left[\frac{\partial}{\partial\overline{\theta}_1^{\dot\alpha}} - \not p_{\alpha\dot\alpha}\theta_1^\alpha\right], \qquad \text{(A1)}$$

where

$$\not p_{\alpha\dot\alpha} \equiv p_\mu \sigma^\mu_{\alpha\dot\alpha} \equiv i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu . \qquad \text{(A2)}$$

We then have the fundamental supersymmetry algebra

$$\{(D_{p1})_\alpha, (\overline{D}_{p1})_{\dot\alpha}\} = \frac{1}{2}\not p_{\alpha\dot\alpha}. \qquad \text{(A3)}$$

We find

$$D_{p1}^2\theta_1^2 = -e^{\theta_1\not p\overline{\theta}_1}, \quad \overline{D}_{p1}^2\overline{\theta}_1^2 = -e^{-\theta_1\not p\overline{\theta}_1}, \qquad \text{(A4)}$$

and moreover

$$D_{p1}^2\overline{D}_{p1}^2\theta_1^2\overline{\theta}_1^2 D_{p1}^2\overline{D}_{p1}^2\delta_{12} = e^{\theta_1\not p\overline{\theta}_1 + \theta_2\not p\overline{\theta}_2},$$

$$\overline{D}_{p1}^2 D_{p1}^2\theta_1^2\overline{\theta}_1^2\overline{D}_{p1}^2 D_{p1}^2\delta_{12} = e^{-(\theta_1\not p\overline{\theta}_1 + \theta_2\not p\overline{\theta}_2)}, \qquad \text{(A5)}$$

where

$$\delta_{12} = \delta^{(4)}(\theta_1 - \theta_2). \qquad \text{(A6)}$$

We also have

$$\delta_{12}\overline{D}_{q1}^2 D_{q1}^2 e^{2\theta_1\not p\overline{\theta}_1}\overline{D}_{q1}^2 D_{q1}^2\delta_{12} = \delta_{12}[(p-q)^2 - 2p^2\theta_1\not q\overline{\theta}_1 + 2q^2\theta_1\not p\overline{\theta}_1 + p^2q^2\theta_1^2\overline{\theta}_1^2], \qquad \text{(A7a)}$$

$$\delta_{12}D_{q1}^2\overline{D}_{q1}^2 e^{2\theta_1\not p\overline{\theta}_1}D_{q1}^2\overline{D}_{q1}^2\delta_{12} = \delta_{12}[(p+q)^2 + 2p^2\theta_1\not q\overline{\theta}_1 + 2q^2\theta_1\not p\overline{\theta}_1 + p^2q^2\theta_1^2\overline{\theta}_1^2]. \qquad \text{(A7b)}$$

Finally,

$$\delta_{12}D_{r1}^2\overline{D}_{r1}^2[(p-q)^2 - 2p^2\theta_1\not q\overline{\theta}_1 + 2q^2\theta_1\not p\overline{\theta}_1 + p^2q^2\theta_1^2\overline{\theta}_1^2]D_{r1}^2\overline{D}_{r1}^2\delta_{12}$$
$$= \delta_{12}[(p-q)^2r^2 + 2q^2p\cdot r - 2p^2q\cdot r + p^2q^2] + \text{terms in } \theta_1, \overline{\theta}_1, \qquad \text{(A8a)}$$

$$\delta_{12}\overline{D}_{r1}^2 D_{r1}^2[(p-q)^2 - 2p^2\theta_1\not q\overline{\theta}_1 + 2q^2\theta_1\not p\overline{\theta}_1 + p^2q^2\theta_1^2\overline{\theta}_1^2]\overline{D}_{r1}^2 D_{r1}^2\delta_{12}$$
$$= \delta_{12}[(p-q)^2r^2 - 2q^2p\cdot r + 2p^2q\cdot r + p^2q^2] + \text{terms in } \theta_1, \overline{\theta}_1. \qquad \text{(A8b)}$$

TABLE V. Results for Fig. 14 (all multiplied by T_{13}).

	a	b	c	d
Fig. 14	$-C$	$-(A+B+2D)$	$2C$	$2(A-B)$

Note that the right-hand sides of Eqs. (A7a), (A7b) are related by $q\leftrightarrow -q$, and similarly those of Eqs. (A8a), (A8b) are related by $r\leftrightarrow -r$.

APPENDIX B: THREE LOOP DETAILS

In this Appendix we give a complete graph-by-graph description of the three-loop calculation. We start by giving a list of the distinct tensor structures involved:

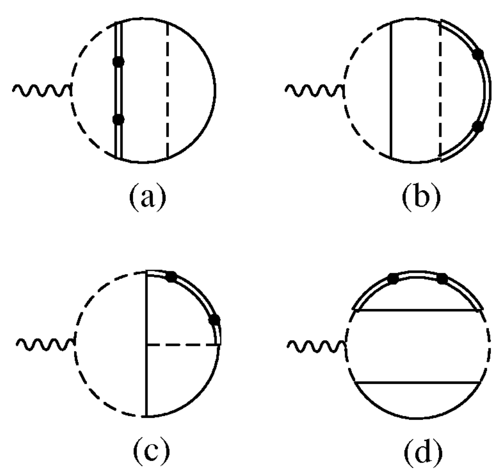

FIG. 14. Feynman diagrams in components for the three-loop contribution to T_{13}.

TABLE VI. Results for Fig. 15 (all multiplied by T_{14}).

	a	b	c	d	e	f
Fig. 15	$8(E+\frac{1}{2}C-A+\frac{1}{2}B)$	$4(B-C)$	$-2C$	$8(B-A)$	$8(\frac{1}{2}B+\frac{1}{2}E-A)$	$-2C$
	g	h				
Fig. 15	$4(A+B+2D)$	$8(A-B)$				

$$T_1=(Y^2)^i{}_jY^{jkl}Y_{ikm}(m^2\mathcal{Y})^m{}_l,$$

$$T_2=(Y^2)^i{}_jY^{jkl}Y_{imn}(m^2)^m{}_k\mathcal{Y}^n{}_l,$$

$$T_3=\mathrm{Tr}[Y^2Y^2m^2\mathcal{Y}],\quad T_4=g^2\mathrm{Tr}[Y^2m^2\mathcal{Y}^3],$$

$$T_5=g^2Y^{ikl}Y_{imn}(m^2\mathcal{Y})^m{}_k(\mathcal{Y}^2)^n{}_l,$$

$$T_6=g^2Y^{ikl}Y_{imn}(m^2)^m{}_k(\mathcal{Y}^3)^n{}_l,\quad T_7=g^4\mathrm{Tr}[m^2\mathcal{Y}^5],$$

$$T_8=g^4\mathrm{Tr}[\mathcal{Y}^2]\mathrm{Tr}[m^2\mathcal{Y}^3],\quad T_9=Y^{ikl}Y_{imn}h_{jkl}h^{pmn}\mathcal{Y}^p{}_j,$$

$$T_{10}=\mathrm{Tr}[Y^2h^2\mathcal{Y}],\quad T_{11}=g^2\mathrm{Tr}[h^2\mathcal{Y}^3],$$

$$T_{12}=g^2Mh_{ikl}Y^{jkl}(\mathcal{Y}^3)^i{}_j,\quad T_{13}=g^2MM^*\mathrm{Tr}[Y^2\mathcal{Y}^3],$$

$$T_{14}=g^4MM^*\mathrm{Tr}[\mathcal{Y}^5]. \tag{B1}$$

We now give a list of diagrams contributing to these tensor structures, with the exception of T_7, for which there are a very large number of separate diagrams; note that when m^2 is replaced by $g\mathcal{Y}$, T_7 and only T_7 produces $\mathrm{Tr}[\mathcal{Y}^6]$, so that we can in fact infer the coefficient of T_7 in our final result via Eq. (2.31). We did, however, perform the explicit T_7 calculation, and indeed obtained the expected result.

We begin with a comparison between superspace and component formalisms. The results from Fig. 7 (the superspace calculation) are

$$\text{Fig. 7a}=-\frac{1}{2}(B+2D)(T_1+T_2),\quad \text{Fig. 7b}=AT_1,$$

$$\text{Fig. 7c}=\frac{1}{2}CT_2,\quad \text{Fig. 7d}=\frac{3}{4}BT_3, \tag{B2}$$

while from Fig. 8 (the component calculation) we find

$$\text{Fig. 8a}=-\frac{1}{2}(B+C+2D)(T_1+T_2),$$

$$\text{Fig. 8b}=(A-F)T_1,$$

$$\text{Fig. 8c}=FT_1,\quad \text{Fig. 8d}=\frac{1}{2}C(T_1+2T_2),$$

$$\text{Fig. 8e}=\frac{3}{4}BT_3. \tag{B3}$$

[Here and elsewhere, we combine diagrams which clearly give identical results, up to symmetry factors. Consider for example, Fig. 8d. Because the scalar fields are complex, this diagram represents two distinct (by the usual rules) Feynman diagrams.] The totals of Figs. 7, 8 are manifestly identical, which is a good check on our spurion rules.

The results from Figs. 9,10,12,14,15 are given in Tables II–VI respectively.

The results from Fig. 11 are given by

$$\text{Fig. 11a}=-\frac{1}{2}AT_9,\quad \text{Fig. 11b}=\frac{1}{4}BT_9,$$

$$\text{Fig. 11c}=-\frac{1}{4}AT_{10},\quad \text{Fig. 11d}=\frac{1}{2}BT_{10}. \tag{B4}$$

The results from Fig. 13 are

$$\text{Fig. 13a}=(E-2A)[T_{12}+(T_{12})^*],$$

$$\text{Fig. 13b}=B[T_{12}+(T_{12})^*]. \tag{B5}$$

The final total is obtained by combining the tables, substituting the simple pole results for $A,B,\ldots,F$ from Eq. (5.1), and multiplying by 3 (for three loops):

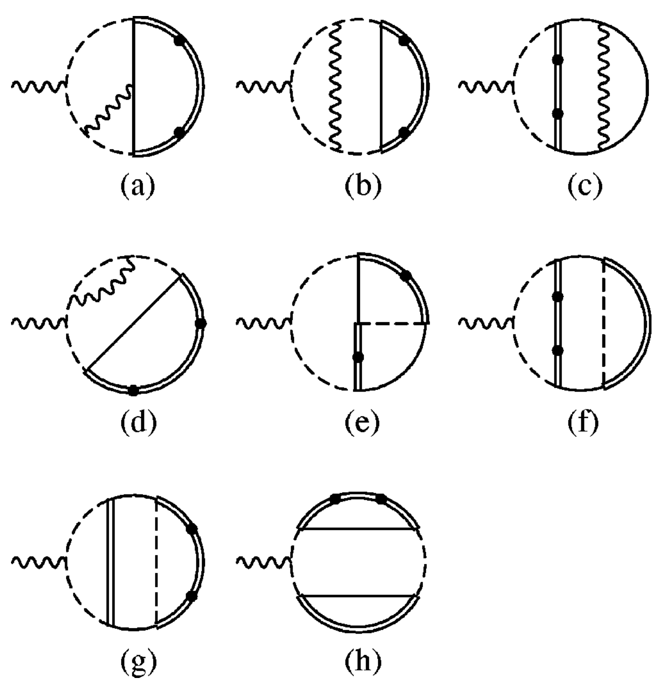

FIG. 15. Feynman diagrams in components for the three-loop contribution to T_{14}.

$$(16\pi^2)^3 \frac{\hat{\beta}_\xi^{(3)\mathrm{DRED}'}}{g} = 7T_1 + 4T_2 - \frac{3}{2}T_3 + [10 - 24\zeta(3)]T_4 - 12T_5 + 16[1 - 3\zeta(3)]T_6 - 16T_7 - 12T_8 - \frac{5}{2}T_9 - 2T_{10} + 8[1 - 3\zeta(3)]T_{11} - 2[5 - 6\zeta(3)](T_{12} + T_{12}^*) + 16T_{13} - 8[13 - 18\zeta(3)]T_{14}, \tag{B6}$$

which can easily be recast into the form given in Eq. (5.2). (As indicated earlier, we have suppressed details of the T_7 computation.)

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