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R-symmetry, Yukawa textures and anomaly mediated supersymmetry breaking

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We explore, in the MSSM context, an extension of the Anomaly Mediated Supersymmetry Breaking solution for the soft scalar masses that is possible if the underlying theory has a gauged R-symmetry. The slepton mass problem characteristic of the scenario is resolved, and a context for the explanation of the fermion mass hierarchy provided.

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Recently there has been interest in a specific and predictive framework for the origin of soft supersymmetry breaking within the MSSM, known as Anomaly Mediated Supersymmetry Breaking (AMSB). The supersymmetry-breaking terms originate in a vacuum expectation value for an F-term in the supergravity multiplet, and the gaugino mass M, the ϕ^3 coupling h^{ijk} and the $\phi\phi^*$-mass $(m^2)^i{}_j$ are all given in terms of the gravitino mass, m_0, and the β-functions of the unbroken theory by simple relations that are renormalisation group (RG) invariant [1] –[20]. Direct application of this idea to the MSSM leads, unfortunately, to negative (mass)^2 sleptons: in other words, to a theory without a vacuum preserving the U_1 of electromagnetism. Various resolutions of this dilemma have been investigated; here we explore a particularly minimalist one, which requires the introduction of no new fields into the low energy theory. The key lies in a compelling generalisation of the RG invariant solution described above[5]. The basic AMSB solution is given by:

$$M = m_0 \frac{\beta_g}{g}, \tag{1a}$$

$$h^{ijk} = -m_0 \beta_Y^{ijk}, \tag{1b}$$

$$(m^2)^i{}_j = \tfrac{1}{2}|m_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu}. \tag{1c}$$

Now β_{m^2} is given by[3] (see also [21]–[25])

$$(\beta_{m^2})^i{}_j(m^2, \cdots) = \left[2\mathcal{O}\mathcal{O}^* + 2MM^* g^2 \frac{\partial}{\partial g^2} + \tilde{Y} \frac{\partial}{\partial Y} + \tilde{Y}^* \frac{\partial}{\partial Y^*} + X \frac{\partial}{\partial g} \right] \gamma^i{}_j, \tag{2}$$

where

$$\mathcal{O} = \left(M g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y^{lmn}} \right), \tag{3}$$

$$\tilde{Y}^{ijk}(m^2, Y) = (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} + (m^2)^k{}_l Y^{ijl} \tag{4}$$

and (in the NSVZ scheme)[4][26] [1]

$$X(m^2, M) = -2 \frac{g^3}{16\pi^2} \frac{r^{-1}\mathrm{Tr}[m^2 C(R)] - MM^* C(G)}{[1 - 2g^2 C(G)(16\pi^2)^{-1}]}. \tag{5}$$

(Here r is the number of generators of the gauge group and $C(R)$ and $C(G)$ are the quadratic matter and adjoint Casimirs respectively.)

[1] In Ref. [25] the existence of X (absent in Ref. [24]) is confirmed

It is immediately clear that, given a solution to Eq. (2), $m^2 = m_1^2$, then $m^2 = m_1^2 + m_2^2$ is also a solution, where m_2^2 satisfies the equation (linear and homogeneous in m_2^2):

$$\mu \frac{d}{d\mu} m_2^2 = \left[\tilde{Y}^*(m_2^2, Y^*) \frac{\partial}{\partial Y^*} + \tilde{Y}(m_2^2, Y) \frac{\partial}{\partial Y} + X(m_2^2, 0) \frac{\partial}{\partial g} \right] \gamma. \tag{6}$$

Remarkably, Eq. (6) has a solution of the form[5][8]

$$(m_2^2)^i{}_j = \overline{m}_0^2 (\gamma^i{}_j + \overline{q}^i \delta^i{}_j) \tag{7}$$

where $\overline{m}_0^2$ and $\overline{q}^i$ are constants, as long as a set $\overline{q}^i$ exists that satisfy the following constraints:

$$(\overline{q}^i + \overline{q}^j + \overline{q}^k) Y_{ijk} = 0 \tag{8a}$$

$$2\mathrm{Tr}\,[\overline{q} C(R)] + Q = 0, \tag{8b}$$

where Q is the one loop β_g coefficient. It is easy to show[5] that Eq. (8) corresponds precisely to requiring that the theory have a non-anomalous R-symmetry (which we will denote $\mathcal{R}$, to avoid confusion with our notation R for group representations). Setting

$$\overline{q}^i = 1 - \tfrac{3}{2} r^i, \tag{9}$$

we see that Eq. ($8a$) corresponds to $(r^i + r^j + r^k) Y_{ijk} = 2Y_{ijk}$, which is the conventional $\mathcal{R}$-charge normalisation. Moreover, it is then easy to show (recall that the gaugino has $\mathcal{R}$-charge of 1) that Eq. ($8b$) is simply the anomaly cancellation condition for the $\mathcal{R}$-charges.

Our strategy in this paper will be to take the AMSB solution Eq. (1), but with Eq. ($1c$) generalised to

$$(m^2)^i{}_j = \tfrac{1}{2} |m_0|^2 \mu \frac{d\gamma^i{}_j}{d\mu} + \overline{m}_0^2 (\gamma^i{}_j + \overline{q}^i \delta^i{}_j). \tag{10}$$

For a discussion of a possible origin of $\overline{m}_0^2$ as the vacuum expectation value of a U_1 D-term, see Ref. [8].

In a theory with direct product structure there is a relation of the form Eq. ($8b$) for each gauged subgroup; so in the MSSM case there are three conditions, corresponding to cancellation of the $\mathcal{R}(SU_3)^2$, $\mathcal{R}(SU_2)^2$ and $\mathcal{R}(U_1)^2$ anomalies. As discussed in [27], these anomalies could be cancelled by the Green-Schwarz mechanism, but this would not be appropriate for us here since in that case Eq. ($8b$) would no longer be satisfied. We also impose cancellation of the $(\mathcal{R})^2 U_1$ anomaly, although this is not required to render Eq. (10)

RG-invariant. Cancellation of the remaining $(\mathcal{R})^3$ and $\mathcal{R}$-gravitational anomalies can also be achieved if we assume the existence of a MSSM-singlet sector (at high energies)[2].

Now for the MSSM superpotential

$$W_{MSSM} = \mu_s H_1 H_2 + (\lambda_u)_{ab} H_2 Q_a (u^c)_b + (\lambda_d)_{ab} H_1 Q_a (d^c)_b + (\lambda_e)_{ab} H_1 L_a (e^c)_b, \qquad (11)$$

there is no possible $\mathcal{R}$-symmetry, satisfying the constraints described above, such that all the Yukawa couplings are non-zero[3]. One way out of this dilemma would be to add extra particles[28]; here we instead persist with the minimal field content, and are hence forced to distinguish between the generations. Apart from simplicity this also provides a context for explaining the fermion mass hierarchy. We therefore presume an $\mathcal{R}$-charge assignment such that only the third generation Yukawa couplings are permitted (we will return later to the origin of the first two generation masses). We will, however, enact the constraint that the first two generations have identical $\mathcal{R}$-charges. As we shall see, this will alleviate potential Flavour Changing Neutral Current (FCNC) problems.

Thus for the superpotential to have $\mathcal{R}$-charge 2, we require (henceforth we work with the fermionic charges, related to the $\mathcal{R}$-charges by $q_f = r - 1$):

$$q_3 + u_3 + h_2 = q_3 + d_3 + h_1 = l_3 + e_3 + h_1 = -1 \qquad (12a)$$

$$h_1 + h_2 = 0, \qquad (12b)$$

while for cancellation of the mixed anomalies we require

$$q_3 + \tfrac{1}{2}(u_3 + d_3) + 2\left(q_1 + \tfrac{1}{2}(u_1 + d_1)\right) + 3 = 0 \quad (13a)$$

$$\tfrac{1}{2}l_3 + \tfrac{3}{2}q_3 + 2(\tfrac{1}{2}l_1 + \tfrac{3}{2}q_1) + \tfrac{1}{2}(h_1 + h_2) + 2 = 0 \quad (13b)$$

$$\tfrac{1}{6}q_3 + \tfrac{1}{3}d_3 + \tfrac{4}{3}u_3 + \tfrac{1}{2}l_3 + e_3 + 2(\tfrac{1}{6}q_1 + \tfrac{1}{3}d_1 + \tfrac{4}{3}u_1 + \tfrac{1}{2}l_1 + e_1) + \tfrac{1}{2}(h_1 + h_2) = 0 \quad (13c)$$

$$-l_3^2 + e_3^2 + q_3^2 - 2u_3^2 + d_3^2 + 2(-l_1^2 + e_1^2 + q_1^2 - 2u_1^2 + d_1^2) - h_1^2 + h_2^2 = 0 \quad (13d)$$

Eqs. $(13a - d)$ correspond to cancellation of the $\mathcal{R}(SU_3)^2$, $\mathcal{R}(SU_2)^2$, $\mathcal{R}(U_1)^2$ and $\mathcal{R}^2 U_1$ anomalies respectively. It is easy to show that even without imposing the quadratic constraint Eq. $(13d)$, the system of equations Eqs. (12), (13) has no solution if we set $q_1 = q_3, u_1 = u_3$ etc. Thus, as asserted above, there is no possible generation independent

[2] Note that the gravitino also contributes to these anomalies [28] [29].

[3] Application of the scenario to the MSSM was dismissed in Ref. [5], presumably for this reason.

$\mathcal{R}$-charge assignment. The above constraints may be solved (for arbitrary values of the *leptonic* charges) as follows:

$$q_3 = \tfrac{4}{9} - \tfrac{1}{3}l_3 - \tfrac{1}{9}\tfrac{\overline{\kappa}}{\kappa} \tag{14a}$$
$$u_3 = -\tfrac{22}{9} - \tfrac{2}{3}l_3 - e_3 + \tfrac{1}{9}\tfrac{\overline{\kappa}}{\kappa} \tag{14b}$$
$$d_3 = -\tfrac{4}{9} + \tfrac{4}{3}l_3 + e_3 + \tfrac{1}{9}\tfrac{\overline{\kappa}}{\kappa} \tag{14c}$$
$$q_1 = -\tfrac{101}{90} - \tfrac{1}{3}\kappa + \tfrac{1}{15}l_3 + \tfrac{1}{5}e_3 + \tfrac{1}{30}\overline{\kappa} + \tfrac{1}{18}\tfrac{\overline{\kappa}}{\kappa} \tag{14d}$$
$$u_1 = -\tfrac{79}{90} - \tfrac{2}{3}\kappa - \tfrac{16}{15}l_3 - \tfrac{6}{5}e_3 - \tfrac{1}{30}\overline{\kappa} - \tfrac{1}{18}\tfrac{\overline{\kappa}}{\kappa} \tag{14e}$$
$$d_1 = \tfrac{101}{90} + \tfrac{4}{3}\kappa + \tfrac{14}{15}l_3 + \tfrac{4}{5}e_3 - \tfrac{1}{30}\overline{\kappa} - \tfrac{1}{18}\tfrac{\overline{\kappa}}{\kappa} \tag{14f}$$
$$h_2 = -h_1 = l_3 + e_3 + 1, \tag{14g}$$

where $\kappa = l_1 - l_3 + e_1 - e_3 - 3$, and $\overline{\kappa} = -12l_3 - 16e_3 + 10e_1 - 23$. Thus for any set of rational values for the leptonic charges there exist rational values for all the charges.

We will presently exhibit a set of sum-rules for the sparticle masses that are completely independent of the set of values $l_3, e_3, \kappa, \overline{\kappa}$. Let us first see whether we can gain any insight on the $\mathcal{R}$-charge assignments by relating them to a possible origin of the light quark and lepton masses. Suppose [30] there are higher-dimension terms in the effective field theory of the form (for the up-type quarks) $H_2 Q_i u^c_j (\frac{\theta}{M_U})^{a_{ij}}$ or $H_2 Q_i u^c_j (\frac{\overline{\theta}}{M_U})^{\overline{a}_{ij}}$, where $\theta, \overline{\theta}$ is a pair of MSSM singlet fields with $\mathcal{R}$-charges $\pm r_\theta$ that get equal vacuum expectation values, and M_U represents some high energy new physics scale (with similar terms for the light down quarks and leptons). Evidently the $\mathcal{R}$-charge assignments will then dictate the texture of the Yukawa couplings, via the relation $h_2 + q_1 + u_1 + a_{11} r_\theta = -1$ and similar identities.

We thus obtain Yukawa textures of the general form:

$$\Delta_u = \begin{pmatrix} \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa+\delta_q|} & \epsilon^{\sigma|\kappa+\delta_q|} & 1 \end{pmatrix}, \quad \Delta_d = \begin{pmatrix} \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa-\delta_q|} & \epsilon^{\sigma|\kappa-\delta_q|} & 1 \end{pmatrix} \tag{15}$$

for the up and down quarks, and

$$\Delta_L = \begin{pmatrix} \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\delta_L|} \\ \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\delta_L|} \\ \epsilon^{\sigma|\kappa+3-\delta_L|} & \epsilon^{\sigma|\kappa+3-\delta_L|} & 1 \end{pmatrix} \tag{16}$$

for the leptons, where

$$\begin{aligned} \delta_q &= \frac{1}{30\kappa}(-47\kappa + 12l_3\kappa + 6e_3\kappa + \kappa\overline{\kappa} - 10\kappa^2 + 5\overline{\kappa}) \\ \delta_L &= \frac{7}{10} - \frac{6}{5}l_3 - \frac{3}{5}e_3 - \frac{1}{10}\overline{\kappa} + \kappa, \end{aligned} \tag{17}$$

$\epsilon = \left|\frac{<\theta>}{M_U}\right|$ and $\sigma = (|r_\theta|)^{-1}$ (provided r_θ is such that all the exponents in Eqs. (15), (16) are integers). More complex scenarios may be contemplated in which there are more than one pairs of θ, $\overline{\theta}$ fields, but we do not consider this further.

In work on Yukawa textures it is common to assume that they are symmetric: this assumption is not dictated by the theoretical structure of our model. Moreover, it is easy to show that to obtain symmetric textures for both up and down quarks requires $\kappa = \overline{\kappa} = 0$. This then implies that the up and down quark Yukawa couplings amongst the 1st and 2nd generations are also allowed (and presumably of $O(1)$, leaving the fermion mass hierarchy unexplained). We therefore abandon the symmetric paradigm; as an alternative simplifying assumption, motivated by the similarity of the hierarchies of the down quark and lepton masses, we impose $\Delta_d = \Delta_L$. This requires

$$\kappa = -\tfrac{3}{2}, \qquad \overline{\kappa} = -\tfrac{21}{2} - \tfrac{9}{4}\lambda, \tag{18}$$

where $\lambda = 2l_3 + e_3$. We then find $\delta_q = \frac{3}{8}\lambda - \frac{1}{4}$. The only value of λ we have found which leads to nice textures with only one pair of θ, $\overline{\theta}$ fields is $\lambda = -\frac{1}{3}$; with $r_\theta = \frac{3}{8}$, we then obtain texture matrices of the form

$$\Delta_u = \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^5 & \epsilon^5 & 1 \end{pmatrix}, \quad \Delta_d = \Delta_L = \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^3 & \epsilon^3 & 1 \end{pmatrix}. \tag{19}$$

The charges now have the form shown in Table 1.

q_3	l_3	u_3	d_3	e_3
$\frac{e}{6} - \frac{2}{9}$	$-\frac{e}{2} - \frac{1}{6}$	$-\frac{2e}{3} - \frac{29}{18}$	$\frac{e}{3} + \frac{1}{18}$	e

q_1	l_1	u_1	d_1	e_1	H_1	H_2
$\frac{e}{6} - \frac{43}{72}$	$-\frac{e}{2} + \frac{5}{24}$	$-\frac{2e}{3} + \frac{19}{72}$	$\frac{e}{3} - \frac{77}{72}$	$e + \frac{9}{8}$	$-\frac{e}{2} - \frac{5}{6}$	$\frac{e}{2} + \frac{5}{6}$

Table 1: The fermionic $\mathcal{R}$-charges for the case $\Delta_d = \Delta_L$

It is easy to show that as long as $-\frac{1}{3} < e < \frac{1}{3}$ and $\overline{m}_0^2 < 0$, the contribution to each slepton mass term due to the $\overline{q}$ term in Eq. (10) will be positive, and we may expect to achieve a viable spectrum; however, it turns out that it is still non-trivial to obtain an acceptable minimum because, for example, if $e = 0$ and $\overline{m}_0^2 < 0$, the $\overline{m}_0^2\overline{q}$ contributions to

Eq. (10) from u_3, q_1 and d_1 are negative. Reverting to the Yukawa texture issue, we see that $\Delta_{u,d,L}$ are not in the class of forms for the texture matrix most frequently considered in the literature, where more attention has focussed on the possibility of texture zeroes. They are of interest, however, in that Δ_u has one zero eigenvalue, and $\Delta_{d,L}$ have two zero eigenvalues. It follows from these properties that mass hierarchies may be produced with matrices of this generic structure. For example, given the following up and down-quark Yukawa matrices,

$$\lambda_u \propto \begin{pmatrix} -0.28\epsilon^4 & 1.3\epsilon^4 & 0.4\epsilon \\ -0.32\epsilon^4 & 1.45\epsilon^4 & 1.36\epsilon \\ -0.36\epsilon^5 & 1.67\epsilon^5 & 1 \end{pmatrix} \quad \lambda_d \propto \begin{pmatrix} -1.75\epsilon^4 & 1.99\epsilon^4 & 0.25\epsilon \\ -3.01\epsilon^4 & 2.53\epsilon^4 & 1.18\epsilon \\ 0.26\epsilon^3 & -0.48\epsilon^3 & 0.95 \end{pmatrix}, \tag{20}$$

with $\epsilon = 0.25$, we obtain ratios for the quark masses and a CKM matrix within experimental limits[4]. Let us consider the issue of FCNC contributions (for a review, see for example Ref. [31]). The matrices λ_u and λ_d are both diagonalised by matrices which are approximately of the general form

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

from which it follows, because we chose identical $\mathcal{R}$-charge assignments for the first two generations, that if we rotate the squark masses to the basis that diagonalises both the quark masses and the quark-squark-gluino coupling, then all the off-diagonal terms are small, so FCNC contributions mediated by the gluino will be suppressed. Of course even in the absence of squark-flavour mixing there are susy FCNC contributions; consider for example the wino-squark box diagram contribution to $K - \overline{K}$ mixing. Here the up/charm squark contributions will be GIM suppressed and the top squark contribution suppressed by CKM angles, just as the analogous Standard Model top quark diagram is. For the charged leptons, we are less constrained given the lack of a (or, if we generalised to the massive neutrino case, our ignorance of the) leptonic CKM matrix.

Naturally because the off-diagonal squark and slepton masses are (though relatively small) not zero, it follows that the whole issue of FCNCs deserves a more detailed analysis.

We cannot entirely claim avoidance of fine-tuning, inasmuch as the lightest quark masses ($m_{u,d}$) are somewhat sensitive to small changes in the coefficients shown in Eq. (20);

[4] We have neglected CP-violation, but this can easily be incorporated.

for example if we change 1.3 to 1.4 in λ_u then m_u increases by a factor of 4. However, the CKM matrix, m_s and m_c are remarkably stable under such variations.

The mechanism proposed for generating the light fermion masses raises the following issue. As a symmetry of the low energy effective field theory, our $\mathcal{R}$-symmetry forbids from the superpotential, Eq. (11), not only the light fermion Yukawa couplings but also the well-known set of baryon and lepton-number violating terms of the form QLd^c, $d^c d^c u^c$, LLe^c and $H_2 L$. It is clear that *a priori* the same mechanism we invoke above to generate the light masses might lead to similar contributions to these operators, for example via the operator $d_1^c d_2^c u_3^c (\frac{\theta}{M_U})^p$. However it is easy to check that, with the charge assignment we make above for the $\theta, \overline{\theta}$ fields, the value of p required to render this operator $\mathcal{R}$-invariant is not an integer; and similarly for the other baryon and lepton-number violating operators above. There will in general be higher-dimensional B-violating and L-violating operators but the effects of these will be strongly suppressed.

The phenomenology of AMSB-models has been discussed at length in the literature. If we compare our model here with the constrained MSSM (where the assumption of soft universality at the unification scale means that the theory is characterised by the usual input parameters, $\tan\beta$, m_0, $m_{\frac{1}{2}}$ and A), we see that we have the same number of parameters, $\tan\beta$, m_0, $\overline{m}_0^2$ and the $\mathcal{R}$-charge e. We can try and further constrain the model by demanding that the soft $H_1 H_2$ mass term lies on the same RG trajectory as the other soft terms (see Ref. [3]), but we find it impossible to find a satisfactory vacuum in that case.

A characteristic feature of AMSB models is the near-degenerate light charged and neutral winos; this prediction, depending as it does on Eq. ($1a$), is preserved in the scenario presented here. A variety of mass spectra for $m_0 = 40$TeV (corresponding to a gluino mass of around 1TeV), but with different values of $\tan\beta$, e and $\overline{m}_0^2$, is presented in Table 2; we were unable to find any values of e and $\overline{m}_0^2$ corresponding to an acceptable spectrum for $\tan\beta$ significantly larger than 10. The heaviest sparticle masses scale with m_0 and are given roughly by $M_{\text{SUSY}} = \frac{1}{40} m_0$. Consequently we take account of leading-log corrections by evaluating the mass spectrum at this scale. In other words, before applying Eq. (10), we evolve the dimensionless couplings (together with v_1, v_2) from the weak scale up to the scale M_{SUSY}. A dramatic feature of the spectra is the splitting in the slepton masses for different generations. Moreover, unusual[32] is the possibility (exemplified in the first three columns of Table 2) that the $\tilde{\nu}_\tau$ is the LSP. As is well known, radiative corrections give a sizeable upward contribution to the mass of the light CP-even Higgs, and so we have included the one-loop calculation (in the approximation given by Haber[33]).

$\tan\beta$	2	2	5	5	10
sign μ_s	$+$	$-$	$+$	$+$	$+$
e	$-1/9$	$-1/9$	$-1/9$	$-2/9$	$-2/9$
$\overline{m}_0^2(\mathrm{TeV}^2)$	-0.1	-0.1	-0.1	-0.25	-0.2
$\tilde{t}_1$	652	615	567	302	404
$\tilde{t}_2$	882	908	876	879	875
$\tilde{b}_1$	865	865	843	853	843
$\tilde{b}_2$	977	977	974	1009	987
$\tilde{\tau}_1$	94	87	75	136	86
$\tilde{\tau}_2$	110	116	127	289	251
$\tilde{u}_L$	918	918	917	880	892
$\tilde{u}_R$	997	997	997	1084	1057
$\tilde{d}_L$	920	920	921	884	896
$\tilde{d}_R$	887	887	887	776	814
$\tilde{e}_L$	260	260	261	473	418
$\tilde{e}_R$	423	423	423	664	590
$\tilde{\nu}_\tau$	83	83	73	277	234
$\tilde{\nu}_e$	251	251	249	467	410
h	96	105	119	114	124
H	598	598	585	121	308
A	593	593	584	110	307
$H^\pm$	599	599	590	137	318
$\tilde{\chi}_1^\pm$	98	116	104	101	106
$\tilde{\chi}_2^\pm$	628	625	663	449	530
$\tilde{\chi}_1$	98	115	103	99	103
$\tilde{\chi}_2$	364	372	367	357	365
$\tilde{\chi}_3$	619	620	662	446	532
$\tilde{\chi}_4$	637	628	672	470	544
$\tilde{g}$	1008	1008	1008	1008	1008

Table 2: The sparticle masses (given in GeV)

A salient feature of the model is the existence of sum rules in which the dependence on the $\mathcal{R}$-charge assignment cancels. These sum rules follow from Eq. (14); and thus for the particular solution exhibited in Table 1, they are independent of e. We find the following relations for the physical masses (in each case independent of e and sign μ_s; in general the numerical results depend on $\tan\beta$, here taken throughout to equal 5, and also on m_0, here taken throughout to be 40TeV, due to the running to $M_{\rm SUSY}$ (which depends on m_0)):

$$m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} + m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2} - 2(m^2_t + m^2_b) - 2.75 m^2_{\tilde{g}} = 0.92 \overline{m}_0^2 \mathrm{TeV}^2, \qquad (21a)$$

$$m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} - 2(m^2_t + m^2_\tau) - 1.14 m^2_{\tilde{g}} = 0.96 \overline{m}_0^2 \mathrm{TeV}^2, \qquad (21b)$$

$$m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} + m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} - 1.70 m^2_{\tilde{g}} = -3.56 \overline{m}_0^2 \mathrm{TeV}^2, \qquad (21c)$$

$$m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} + m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L} - 3.51 m^2_{\tilde{g}} = 0.90 \overline{m}_0^2 \mathrm{TeV}^2, \qquad (21d)$$

$$m^2_A - 2\sec 2\beta \left(m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} - 2m^2_\tau\right) - 0.49 m^2_{\tilde{g}} = 1.05 \overline{m}_0^2 \mathrm{TeV}^2. \qquad (21e)$$

Eqs. $(21c, d)$ above involve only the first (or second) generations, and so the numerical results here are also independent of $\tan\beta$. Thus these two sum rules hold for every column in Table 2, as is easily verified.

It is interesting to compare these sum rules with the corresponding ones in the Fayet-Iliopoulos scenario described in our previous paper[16]; essentially the distinction lies in the non-zero RHS in Eqs. $(21a - e)$.

In conclusion, we have shown that within the MSSM it is possible to construct a solution to the running equations for m^2, M and h that is completely RG invariant, and leads to a phenomenologically acceptable theory, resulting in a distinctive spectrum with sum rules for the sparticle masses. Two sources of supersymmetry-breaking are required, one corresponding to the gravitino mass (at around $m_0 = 40$TeV) and another, related to a $\mathcal{R}$-symmetry, at around $|\overline{m}_0| = 300 - 500$GeV. The magnitude of the latter suggests the idea of a common origin for it, the μ_s term and the associated $H_1 H_2$ soft term. A convincing demonstration of this would considerably enhance the attractiveness of this model. It would also be interesting to consider variations on the same theme; forbidding the Higgs μ_s term, or incorporating massive neutrinos, for example.

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