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Non-standard soft supersymmetry breaking

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We explore a more general class of soft supersymmetry-breaking masses and interactions than that usually considered, both in general and in the MSSM context, where our results for the one-loop β-functions correct some errors in the literature. We identify a new class of one-loop finite supersymmetric theories.

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1. The new soft breakings

The minimal supersymmetric standard model (MSSM) consists of a supersymmetric extension of the standard model, with the addition of a number of dimension 2 and dimension 3 supersymmetry-breaking mass and interaction terms. It became popular when it was demonstrated that such a structure is a natural consequence of supergravity when supersymmetry is broken in a hidden sector. (For a review, see Ref. [1].) The purpose of this paper is a preliminary exploration of the consequences of a more general set of supersymmetry-breaking terms. For a general $\mathcal{N}=1$ theory, let us write

$$L = L_{\rm SUSY} + L_{\rm SOFT}. \tag{1.1}$$

Here $L_{\rm SUSY}$ is the Lagrangian for the supersymmetric gauge theory, containing the gauge multiplet $\{A_\mu, \lambda\}$ (λ being the gaugino) and a matter multiplet $\{\phi_i, \psi_i\}$ transforming as a representation R of the gauge group $\mathcal{G}$. We assume a superpotential of the form

$$W = \tfrac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k. \tag{1.2}$$

A renormalisable superpotential will in general also contain quadratic and linear terms. We suppose that there are no gauge singlet fields so there is no linear term; and as will become clear below, we do not need an explicit quadratic term because such a term will be included as a special case from our new soft breakings.

The soft terms usually considered are those contained in the following Lagrangian:

$$L^{(1)}_{\rm SOFT} = (m^2)^j{}_i\phi^i\phi_j + \left(\tfrac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{2}M\lambda\lambda + \text{h.c.}\right). \tag{1.3}$$

Indeed, in the MSSM context one often sees the (incorrect) assertion that $L^{(1)}_{\rm SOFT}$ contains all possible soft terms [1]. The designation "soft" refers to the fact that the inclusion of $L^{(1)}_{\rm SOFT}$ breaks supersymmetry but does not introduce quadratic divergences[3], and is hence said to preserve naturalness[2]. However in the case of a wide range of theories there are further possible dimension 3 terms which preserve naturalness, as follows:

$$L^{(2)}_{\rm SOFT} = \tfrac{1}{2}r_i^{jk}\phi^i\phi_j\phi_k + \tfrac{1}{2}{m_F}^{ij}\psi_i\psi_j + {m_A}^{ia}\psi_i\lambda_a + \text{h.c.} \tag{1.4}$$

[1] For a recent honourable exception and a nice MSSM review, see Ref. [2]

[2] In a U_1 theory naturalness also requires $\mathrm{tr}Y = 0$, where Y is the U_1 hypercharge[4]

The m_A term (first discussed in Ref. [5]) is only possible given adjoint matter fields; not a feature of the MSSM, but often encountered in GUTs. The reason the terms exhibited in Eq. (1.4) do not appear in the classification of Ref. [3] is that *in general* they engender quadratic divergences. These divergences are in scalar tadpoles, and hence absent if there are no gauge singlet matter fields; as is the case in the MSSM[3]. Thus a truly model-independent approach to the MSSM should include terms of the form shown in Eq. (1.4).

2. The one-loop β-functions

We now present the one-loop β-functions for $L_{\rm SOFT} = L^{(1)}_{\rm SOFT} + L^{(2)}_{\rm SOFT}$. The β-functions for scalar masses and interactions may be calculated using the following equation for the tree scalar potential V_0:

$$\left[\sum_I \beta_I \frac{\partial}{\partial\lambda_I} - (\phi_i {\gamma_L}^i{}_j \frac{\partial}{\partial\phi^j} + \text{h.c.})\right] V_0 = \frac{1}{32\pi^2}\text{STr}\,\mathcal{M}^4. \tag{2.1}$$

The sum over I includes all masses and couplings, and STr stands for the usual spin-weighted trace. (Note that γ_L is the Landau gauge scalar anomalous dimension, which differs from the chiral superfield anomalous dimension, γ.) This equation, in fact, was employed in Ref. [5] to seek and classify one-loop finite theories. In the case of the β-functions for the fermion mass terms the explicit calculation is very simple.

The one-loop results for the gauge coupling β-function β_g and for γ are:

$$16\pi^2\beta_g = g^3 Q \quad \text{and} \quad 16\pi^2\gamma^i{}_j = P^i{}_j, \tag{2.2}$$

where

$$Q = T(R) - 3C(G), \quad \text{and} \quad P^i{}_j = \tfrac{1}{2}Y^{ikl}Y_{jkl} - 2g^2 C(R)^i{}_j. \tag{2.3}$$

Here

$$T(R)\delta_{ab} = \text{Tr}(R_a R_b), \quad C(G)\delta_{ab} = f_{acd}f_{bcd} \quad \text{and} \quad C(R)^i{}_j = (R_a R_a)^i{}_j, \tag{2.4}$$

and as usual $Y^*_{ijk} = Y^{ijk}$ etc. For the new soft terms from Eq. (1.4) we find:

$$16\pi^2(\beta_{m_F})_{ij} = P^k{}_i m_{Fkj} + P^k{}_j m_{Fik}, \tag{2.5a}$$

$$16\pi^2(\beta_{m_A})_{ia} = P^j{}_i m_{Aja} + g^2 Q m_{Aia}, \tag{2.5b}$$

[3] Although singlets are a popular addition in not-so-minimal models; recently in the context of (Dirac) neutrino masses

and

$$16\pi^2(\beta_r)_i^{jk} = \tfrac{1}{2}P^l{}_i r_l^{jk} + P^k{}_l r_i^{jl} + \tfrac{1}{2}r_i^{mn}Y_{lmn}Y^{ljk} + 2r_l^{mj}Y_{imn}Y^{kln} + 2g^2 r_l^{jk}C(R)^l{}_i$$
$$+ 2g^2 r_l^{mj}(R_a)^k{}_i(R_a)^l{}_m - 2m_{Flm}Y^{mnj}Y^{plk}Y_{npi} - 4g^2 m_{Fil}C(R)^l{}_m Y^{mjk}$$
$$- 4g\sqrt{2}\left[g^2 C(G)m_A^{ja}(R_a)^k{}_i + (R_a)^j{}_l Y^{lmk}Y_{mni}m_A^{na}\right] \quad + (k \leftrightarrow j). \tag{2.6}$$

For the original soft terms in Eq. (1.3) we find

$$16\pi^2\beta_h^{ijk} = U^{ijk} + U^{kij} + U^{jki}, \tag{2.7a}$$
$$16\pi^2\beta_b^{ij} = V^{ij} + V^{ji}, \tag{2.7b}$$
$$16\pi^2[\beta_{m^2}]^i{}_j = W^i{}_j, \tag{2.7c}$$
$$16\pi^2\beta_M = 2g^2 QM, \tag{2.7d}$$

where

$$U^{ijk} = h^{ijl}P^k{}_l + Y^{ijl}X^k{}_l, \tag{2.8a}$$
$$V^{ij} = b^{il}P^j{}_l + r_{lm}^i h^{jlm} + r_l^{im}r_m^{jl} - m_{Fkl}Y^{ilm}m_{Fmn}Y^{jnk}$$
$$+ 4g^2 M m_F^{ik}C(R)^j{}_k - 4g^2 C(G)m_A^{ia}m_A^{ja}, \tag{2.8b}$$
$$W^i{}_j = \tfrac{1}{2}Y_{jpq}Y^{pqn}(m^2)^i{}_n + \tfrac{1}{2}Y^{ipq}Y_{pqn}(m^2)^n{}_j + 2Y^{ipq}Y_{jpr}(m^2)^r{}_q + h_{jpq}h^{ipq}$$
$$+ r_j^{kl}r_{kl}^i + 2r_{jl}^k r_k^{il} - 4(m_F^{kl}m_{Flm} + m_{Ama}m_A^{ka})Y^{imn}Y_{jkn}$$
$$- 8g^2(MM^*C(R)^i{}_j + m_F^{kl}m_{Fjk}C(R)^i{}_l + C(G)m_A^{ia}m_{Aja} + (R_aR_b)^i{}_j m_{Aka}m_A^{kb})$$
$$- 4\sqrt{2}g(Y^{iml}m_{Fmn}(R_a)^n{}_j m_{Ala} + Y_{jml}m_F^{mn}(R_a)^i{}_n m_A^{la}) \tag{2.8c}$$

with

$$X^i{}_j = h^{ikl}Y_{jkl} + 4g^2 MC(R)^i{}_j. \tag{2.9}$$

In the expression corresponding to Eq. (2.7c) in Ref [6], there is an additional contribution of the form $g^2(R_a)^i{}_j\mathrm{Tr}[R_a m^2]$. This term arises only for $U(1)$ and amounts to a renormalisation of the linear D-term that is allowed in that case.

In the special case when $m_A^{ia} = h^{ijk} = M = b^{ij} = 0$, $m_F = \mu$, $r_i^{jk} = Y^{jkl}\mu_{il}$ and $(m^2)^i{}_j = \mu^{il}\mu_{jl}$ then the theory becomes supersymmetric, with

$$16\pi^2(\beta_\mu)_{ij} = P^k{}_i\mu_{kj} + P^k{}_j\mu_{ik}. \tag{2.10}$$

It is easy to check that Eqs. (2.5a), (2.6), (2.8c) are consistent with this result. In the case $m_A^{ia} = 0$, $m_F = \mu$, $r_i^{jk} = Y^{jkl}\mu_{il}$ and $(m^2)^i{}_j \to (m^2)^i{}_j + \mu^{il}\mu_{jl}$ our results reduce to the usual soft β-functions, as given in Ref. [6] (see also Ref. [7]). It is easy to see that this corresponds to the inclusion of a term $\frac{1}{2}\mu^{ij}\phi_i\phi_j$ in the superpotential. This is why we do not need to include such a term in Eq.(1.2). Indeed, a plausible common origin for the new and usual soft terms would form the basis for a solution to the so-called "μ problem".

An interesting special case is provided by one-loop finite theories such that $P = Q = 0$. Theories with $r_i^{jk} = m_F = m_A = 0$ were considered in Ref. [5]; but there are other possibilities. Note that we have immediately that $\beta_{m_F} = \beta_{m_A} = 0$ and if we set[4]

$$r_i^{jk} = \sqrt{2}g\left[(R_a)^j{}_i m_A^{ka} + (R_a)^k{}_i m_A^{ja}\right] \tag{2.11}$$

and $m_F = 0$, we find that $\beta_r = \beta_b = 0$. If we additionally set

$$m^{Aia}m_{Aja} = \rho\delta^i{}_j, h = -MY, (m^2)^i{}_j = (2\rho + \tfrac{1}{3}MM^*)\delta^i{}_j \quad \text{and} \quad C(R)^i{}_j = C(G)\delta^i{}_j, \tag{2.12}$$

then we have $W^i{}_j = X^i{}_j = 0$ and one-loop finiteness. A theory that can satisfy these constraints is one with $\mathcal{G} = SU(N)$, three adjoint matter superfields and the superpotential[9]

$$W = gN\sqrt{\frac{2}{N^2-4}}d^{abc}\phi_1^a\phi_2^b\phi_3^c, \tag{2.13}$$

where the unbroken theory has the field content of $\mathcal{N} = 4$, but no higher supersymmetry.

3. The MSSM

We now turn to the case of the MSSM, in the approximation where we retain only the third generation Yukawa couplings. In this context, in fact, the existence of both r_i^{jk} and m_F-type terms was entertained in a pioneering paper on the MSSM[10] so we adopt some of their notation for convenience of comparison. Thus we write

$$W = \lambda_t H_2 Q\bar{t} + \lambda_b H_1 Q\bar{b} + \lambda_\tau H_1 L\bar{\tau}, \tag{3.1}$$

$$\begin{aligned} L_{\text{SOFT}}^{(1)} = \sum_\phi m_\phi^2\phi^*\phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^3 \tfrac{1}{2}M_i\lambda_i\lambda_i + \text{h.c.}\right] \\ + \left[m_{10}\lambda_t H_2 Q\bar{t} + m_8\lambda_b H_1 Q\bar{b} + m_6\lambda_\tau H_1 L\bar{\tau} + \text{h.c.}\right] \end{aligned} \tag{3.2}$$

[4] One loop finite theories with $\mathcal{N} = 2$ supersymmetry and nonzero r_i^{jk} were constructed in Ref. [8]

and

$$L^{(2)}_{\rm SOFT} = m_4\psi_{H_1}\psi_{H_2} + m_9\lambda_t H_1^* Q\overline{t} + m_7\lambda_b H_2^* Q\overline{b} + m_5\lambda_\tau H_2^* L\overline{\tau} + \text{h.c.} \tag{3.3}$$

Nowadays $m_{6,8,10}$ are usually written $A_{\tau,b,t}$ respectively. We note en passant that if R-parity violation is allowed then, as is well known, there are various additional terms allowed in W; the extra allowed terms of the $\phi^2\phi^*$ and $\psi\psi$-type are as follows (for one generation):

$$L^{(2)RPV}_{\rm SOFT} = \rho_1 L^* Q\overline{t} + \rho_2 H_2^* H_1\overline{\tau} + m_\rho\psi_L\psi_{H_2} + \text{h.c.}, \tag{3.4}$$

but we do not pursue this possibility here.

It is straightforward to show from our results that

$$\begin{aligned}
16\pi^2\beta_{m_1^2} &= 2\lambda_\tau^2(m_1^2 + m_6^2 + m_L^2 + m_{\overline{\tau}}^2) + 6\lambda_b^2(m_1^2 + m_8^2 + m_Q^2 + m_{\overline{b}}^2)\\
&\quad + 6\lambda_t^2 m_9^2 - 8C_H m_4^2 - 6g_2^2 M_2^2 - 2g'^2 M_1^2, && (3.5a)\\
16\pi^2\beta_{m_2^2} &= 6\lambda_t^2(m_2^2 + m_{10}^2 + m_Q^2 + m_{\overline{t}}^2) + 2\lambda_\tau^2 m_5^2 + 6\lambda_b^2 m_7^2\\
&\quad - 8C_H m_4^2 - 6g_2^2 M_2^2 - 2g'^2 M_1^2, && (3.5b)\\
16\pi^2\beta_{m_3^2} &= (\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_t^2)m_3^2 + 2\lambda_\tau^2 m_5 m_6 + 6\lambda_b^2 m_7 m_8 + 6\lambda_t^2 m_9 m_{10}\\
&\quad - 4C_H m_3^2 + 6g_2^2 m_4 M_2 + 2g'^2 M_1 m_4, && (3.5c)\\
16\pi^2\beta_{m_4} &= (\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_t^2 - 4C_H)m_4, && (3.5d)\\
16\pi^2\beta_{m_5} &= (\lambda_\tau^2 - 3\lambda_b^2 + 3\lambda_t^2)m_5 + 6m_7\lambda_b^2 + (4m_5 - 8m_4)C_H, && (3.5e)\\
16\pi^2\beta_{m_6} &= 8\lambda_\tau^2 m_6 + 6\lambda_b^2 m_8 + 6g_2^2 M_2 + 6g'^2 M_1, && (3.5f)\\
16\pi^2\beta_{m_7} &= (-\lambda_\tau^2 + 3\lambda_b^2 + 5\lambda_t^2)m_7 + 2m_5\lambda_\tau^2 + 2\lambda_t^2(m_9 - 2m_4)\\
&\quad + (4m_7 - 8m_4)C_H, && (3.5g)\\
16\pi^2\beta_{m_8} &= 2\lambda_\tau^2 m_6 + 12\lambda_b^2 m_8 + 2\lambda_t^2 m_{10}\\
&\quad + \tfrac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \tfrac{14}{9}g'^2 M_1, && (3.5h)\\
16\pi^2\beta_{m_9} &= (\lambda_\tau^2 + 5\lambda_b^2 + 3\lambda_t^2)m_9 + 2m_7\lambda_b^2 - 4m_4\lambda_b^2 + (4m_9 - 8m_4)C_H, && (3.5i)\\
16\pi^2\beta_{m_{10}} &= 2\lambda_b^2 m_8 + 12\lambda_t^2 m_{10} + \tfrac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \tfrac{26}{9}g'^2 M_1, && (3.5j)\\
16\pi^2\beta_{m_Q^2} &= 2X_b + 2X_t - \tfrac{32}{3}g_3^2 M_3^2 - 6g_2^2 M_2^2 - \tfrac{2}{9}g'^2 M_1^2, && (3.5k)\\
16\pi^2\beta_{m_{\overline{t}}^2} &= 4X_t - \tfrac{32}{3}g_3^2 M_3^2 - \tfrac{32}{9}g'^2 M_1^2, && (3.5l)\\
16\pi^2\beta_{m_{\overline{b}}^2} &= 4X_b - \tfrac{32}{3}g_3^2 M_3^2 - \tfrac{8}{9}g'^2 M_1^2, && (3.5m)\\
16\pi^2\beta_{m_L^2} &= 2X_\tau - 6g_2^2 M_2^2 - 2g'^2 M_1^2, && (3.5n)\\
16\pi^2\beta_{m_{\overline{\tau}}^2} &= 4X_\tau - 8g'^2 M_1^2, && (3.5o)\\
16\pi^2\beta_{M_i} &= 2b_i M_i g_i^2, && (3.5p)
\end{aligned}$$

where $b_{1,2,3} = (33/5, -1, -3)$, $g'^2 = 3g_1^2/5$, $C_H = \frac{3}{4}g_2^2 + \frac{3}{20}g_1^2$ and

$$\begin{aligned} X_t &= \lambda_t^2(m_Q^2 + m_{\bar{t}}^2 + m_2^2 + m_9^2 + m_{10}^2 - 2m_4^2), \\ X_b &= \lambda_b^2(m_Q^2 + m_{\bar{b}}^2 + m_1^2 + m_7^2 + m_8^2 - 2m_4^2), \\ X_\tau &= \lambda_\tau^2(m_L^2 + m_{\bar{\tau}}^2 + m_1^2 + m_5^2 + m_6^2 - 2m_4^2). \end{aligned} \tag{3.6}$$

The terms linear in the gaugino masses M_i differ by a sign from Ref. [10]; this is a matter of convention. The results for β_{m_7} and β_{m_9}, however, disagree. This appears to arise from the omission in Ref. [10] of some contributions which cancel in the supersymmetric limit.

4. IR fixed points

In this section we discuss the RG evolution of $m_{4,5,7,9}$, with emphasis on possible fixed point (or quasi-fixed point) structure. In a recent paper[11], we showed that in a wide range of theories the existence of stable infra-red fixed points for the Yukawa couplings implies stable infra-red fixed points for the A-parameters and soft scalar masses.[5] We shall see that there is no such simple correspondence for the new soft interactions.

It follows from Eq. (3.5) that there is a fixed point of the RG evolution such that

$$\frac{m_5}{m_4} = \frac{m_7}{m_4} = \frac{m_9}{m_4} = 1. \tag{4.1}$$

This fixed point corresponds to the supersymmetric limit for these parameters (supersymmetry is not fully restored since we do not have, for example, that $m_1/m_4 = 1$ is a fixed point). An obvious question is whether Eq. (4.1) represents an *infra-red* fixed point of our theory, and if so whether fixed point (or, more likely, quasi-fixed-point) behaviour is exhibited in the standard evolution down to M_Z. The stability matrix for the evolution of $\frac{m_5}{m_4}$, $\frac{m_7}{m_4}$ and $\frac{m_9}{m_4}$ is given by:

$$S = \begin{pmatrix} 8C_H - 6\lambda_b^2 & 6\lambda_b^2 & 0 \\ 2\lambda_\tau^2 & 8C_H - 2\lambda_\tau^2 + 2\lambda_t^2 & 2\lambda_t^2 \\ 0 & 2\lambda_b^2 & 8C_H + 2\lambda_b^2 \end{pmatrix} \tag{4.2}$$

which has eigenvalues $8C_H, 8C_H + \Lambda_{1,2}$ where $\Lambda_{1,2}$ are the roots of the quadratic

$$\Lambda^2 - 2(\lambda_t^2 - \lambda_\tau^2 - 2\lambda_b^2)\Lambda - 4(3\lambda_b^2 + 3\lambda_t^2 + \lambda_\tau^2)\lambda_b^2 = 0. \tag{4.3}$$

Let us consider two special cases:

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5 We first showed IR-focussing of soft parameters for some GUTs in Ref. [12]; see also Ref. [13]. For recent analyses in the MSSM context, see Ref. [14] (small $\tan\beta$) and Ref. [15] (large $\tan\beta$).

4.1. The Quasi Fixed Point

Suppose that we are near the quasi-infra-red fixed point (QIRFP) for λ_t, $\lambda_t(M_Z) \approx 1.1$. This corresponds to $\tan\beta \approx 1.7$ and means we can neglect λ_b and λ_τ, and it is easy to see that our fixed point is stable. With the Yukawa couplings and other soft parameters, one finds (given a stable fixed point) QIRFP behaviour rather than convergence to the fixed point. In this case, m_7/m_4 shows good fixed point convergence, while m_9/m_4 and m_5/m_4 approach much more slowly, with no marked QIRFP behaviour. If, for example, we have $m_5 = m_7 = m_9 = 0$ and $m_4 \neq 0$ at the gauge unification scale, M_U, then at M_Z we find

$$\frac{m_5}{m_4} \approx \frac{m_9}{m_4} \approx 0.5, \quad \text{and} \quad \frac{m_7}{m_4} \approx 0.9, \tag{4.4}$$

whereas if we take $m_5 = m_7 = m_9 = 2m_4$ at M_U then at M_Z we find:

$$\frac{m_5}{m_4} \approx \frac{m_9}{m_4} \approx 1.5, \quad \text{and} \quad \frac{m_7}{m_4} \approx 1.1. \tag{4.5}$$

The fact that m_5 and m_9 remain approximately equal is easy to understand from Eqs. $(3.5e, i)$ using $\lambda_b \approx \lambda_\tau \approx 0$.

4.2. Trinification

There is a region of parameter space giving acceptable electro-weak breaking that corresponds to Yukawa trinification: $\lambda_t(M_U) \approx \lambda_b(M_U) \approx \lambda_\tau(M_U) \approx 0.6$. The corresponding value of $\tan\beta$ is $\tan\beta \approx 50$. The two eigenvalues $8C_H + \Lambda_{1,2}$ are both positive at M_U but one of them is negative at M_Z. Consequently we cannot anticipate that the fixed point (Eq. (4.1)) will be relevant. Indeed, taking $m_5 = m_7 = m_9 = 0$ and $m_4 \neq 0$ at M_U, we find (at M_Z):

$$\frac{m_9}{m_4} \approx 0.7, \quad \frac{m_7}{m_4} \approx 0.7, \quad \text{and} \quad \frac{m_5}{m_4} \approx 0.4, \tag{4.6}$$

whereas if we take $m_5 = m_7 = m_9 = 2m_4$ at M_U then at M_Z we find:

$$\frac{m_7}{m_4} \approx \frac{m_9}{m_4} \approx 1.3, \quad \text{and} \quad \frac{m_5}{m_4} \approx 1.6, \tag{4.7}$$

so in this case none of the parameters show fixed point behaviour, as expected. This time m_7 and m_9 remain approximately equal, and again this is easy to understand from Eqs $(3.5g, i)$, using $\lambda_b \approx \lambda_\tau \approx \lambda_t$.

We turn now to a full running analysis of the theory, with the assumption that there is no explicit Higgs μ-term.

5. RG evolution

In general, if we admit these new soft breakings the effect is to enlarge the (already gargantuan) parameter space of the MSSM. This parameter space is customarily controlled in the MSSM by assumptions of unification for the soft scalar masses (to m_0), gaugino masses (to M) and A-parameters (to A). A distinctive possibility within our scenario is as follows: suppose we adopt this unification, the non-standard soft terms are present, $m_{5,7,9}$ unify to m_r, *and there is no μ-term in the superpotential.* In the special case that the soft terms satisfy $m_{4,5,7,9} = 0$, this corresponds to the MSSM without a μ-term. Now in the standard running analysis, the Higgs potential minimisation is used to determine m_3^2 and μ^2 (at M_Z). We are, however, constrained by the absence of a μ term and the fact that we are still requiring m_1^2 and m_2^2 to unify at M_U.

As discussed recently by Falk[16], the MSSM with a μ term such that $|\mu| < 0.4M$, say, is restricted to a very small region of parameter space at $m_0 >> M$. As a consequence, it is difficult to arrange for a Higgsino-like lightest neutralino. In our scenario, however, it turns out that the fact that m_4 and m_r are "divorced" from μ means we are able to achieve acceptable vacua with $m_4 \leq M$ while retaining unification for both scalar and gaugino masses. Values for m_0 are lower than in the MSSM ($\mu = 0$) case but for an acceptable vacuum we find that $m_0 \geq 595$ GeV.

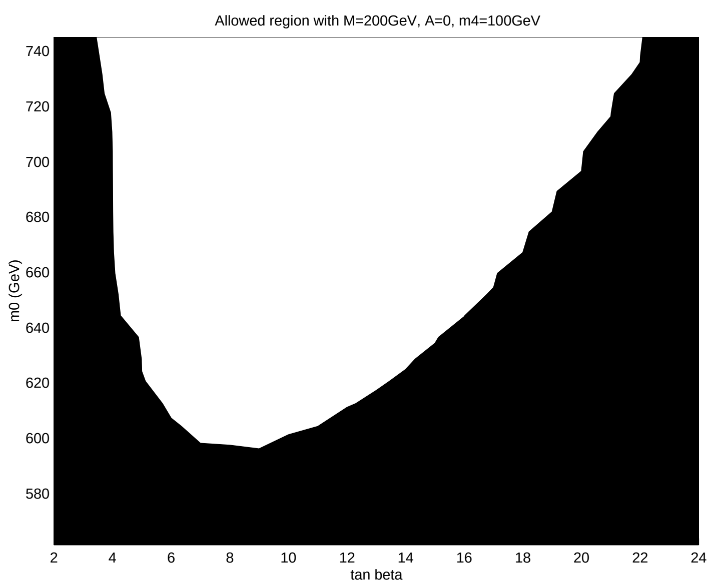

Fig.1: The region of the $m_0, \tan\beta$ plane corresponding to an acceptable electroweak vacuum, for $M = 200\text{GeV}$, $m_4(M_U) = 100\text{GeV}$ and $A = 0$. The shaded region corresponds to one or more sparticle or Higgs masses in violation of current experimental bounds.

In Fig. 1, we show the region of the $m_0, \tan\beta$ plane where we are able to obtain (by varying m_r) an acceptable electroweak vacuum for illustrative values of M, m_4, and A. We have made allowance for radiative corrections by using the tree Higgs minimisation conditions, but evaluated at the scale m_0. While a crude approximation, this suffices to demonstrate our main point that even with $\mu = 0$ there are substantial regions of parameter space available, including ones with $m_4 < M$ and hence a Higgsino-like light neutralino. The lowest value of m_0 ($m_0 \approx 590$GeV) corresponds to a value of $\tan\beta \approx 8$; at this value of $\tan\beta$ we find that $m_{5,7,9}$ behave as in section (4.1), i.e. $m_5 \approx m_9$ at M_Z. For $m_0 = 600$GeV and $\tan\beta = 8$, for example, we find $m_r = 1.06$TeV, $m_5 \approx m_9 \approx 590$GeV, $m_7 \approx 410$GeV, a Higgs with mass 84GeV and a LSP neutralino with mass 50GeV. For convenience we collect the sparticle mass matrices which are affected by the new soft breakings in an appendix.

In conclusion: if we wish to make *no assumptions* concerning the nature of the underlying theory, supersymmetric μ_{ij}-terms should be replaced by the set $(m^2)^i{}_j, r_i^{jk}, m_F^{ij}, m_A^{ia}$ in general. With minimal unification assumptions this replaces the MSSM μ-parameter with two parameters m_4, m_r.

Note added: when we submitted this paper we were unaware of Ref. [17], in which $\phi^2\phi^*$-type soft-breakings are used to generate flavour mass hierarchies via radiative corrections; and the need to consider such terms in a model–independent analysis was also stressed in Ref. [18]. We thank Nir Polonski for bringing these papers to our attention.

Appendix A. The sparticle mass matrices

In this appendix we collect the sparticle mass matrices which are affected by our generalised soft breaking.

The stop matrix is:

$$\begin{pmatrix} m_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta & m_t(m_{10} - m_9\cot\beta) \\ m_t(m_{10} - m_9\cot\beta) & m_{\overline{t}}^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta \end{pmatrix}. \tag{A.1}$$

Similarly for the bottom squarks we have:

$$\begin{pmatrix} m_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta & m_b(m_8 - m_7\tan\beta) \\ m_b(m_8 - m_7\tan\beta) & m_{\overline{b}}^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta \end{pmatrix} \tag{A.2}$$

and for the tau sleptons:

$$\begin{pmatrix} m_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta & m_\tau(m_6 - m_5\tan\beta) \\ m_\tau(m_6 - m_5\tan\beta) & m_{\overline{\tau}}^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta \end{pmatrix}. \tag{A.3}$$

The neutralino mass matrix is:

$$\begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & M_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & 0 & -m_4 \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & -m_4 & 0 \end{pmatrix} \tag{A.4}$$

while the chargino mass matrix is:

$$\begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & m_4 \end{pmatrix} \tag{A.5}$$

The Higgs $(\text{mass})^2$ matrices and the sneutrino masses are unaffected, except inasmuch as our preferred scenario involves $\mu = 0$.

References

[1] H.-P. Nilles, Phys. Reports C110 (1984) 1

[2] S. Martin, hep-ph/9709356

[3] D.M. Capper, J. Phys. **G**3 (1977) 731;
L. Girardello and M.T. Grisaru, Nucl. Phys. **B**194 (1982) 65

[4] W. Fischler et al, Phys. Rev. Lett. 47 (1981) 757

[5] D.R.T. Jones, L. Mezincescu and Y.-P. Yao, Phys. Lett. **B**148 (1984) 317

[6] I. Jack and D.R.T. Jones, Phys. Lett. **B**333 (1994) 372

[7] S.P. Martin and M.T. Vaughn, Phys. Rev. **D**50 (1994) 2282;
Y. Yamada, Phys. Rev. **D**50 (1994) 3537;
I. Jack et al, Phys. Rev. **D**50 (1994) R5481

[8] J.M. Frère, L. Mezincescu and Y-P. Yao, Phys. Rev. **D**29 (1984) 1196; Phys. Rev. **D**30 (1984) 2238

[9] D.R.T. Jones and L. Mezincescu, Phys. Lett. **B**138 (1984) 293

[10] K. Inoue et al, Prog. Th. Phys. 67 (1982) 1889; erratum *ibid* 70 (1983) 330

[11] I. Jack and D.R.T. Jones, Phys. Lett. **B**443 (1998) 177

[12] P. Ferreira, I. Jack and D.R.T. Jones, Phys. Lett. **B**357 (1995) 359

[13] M. Lanzagorta and G.G. Ross, Phys. Lett. **B**364 (1995) 63

[14] S. Abel and B.C. Allanach, Phys. Lett. **B**415 (1997) 371;
G.H. Yeghiyan, M. Jurcisin and D.I. Kazakov, hep-ph/9807411

[15] M. Jurcisin and D.I. Kazakov, Mod. Phys. Lett. A14 (1999) 671

[16] T. Falk, hep-ph/9902352

[17] F. Borzumati, G.R. Farrar, N. Polonsky and S. Thomas, hep-ph/9902443

[18] L.J. Hall and L. Randall, Phys. Rev. Lett. 65 (1990) 2939